Graph Colouring via Evolutionary Optimisation

Optimisation Midterm Examination

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Problem

Graph Colouring (NP-hard)

Given an undirected graph $G = (V, E)$, assign a colour to each vertex so that adjacent vertices have different colours. The goal is to minimise the number of colours used (the chromatic number – $\chi(G)$).

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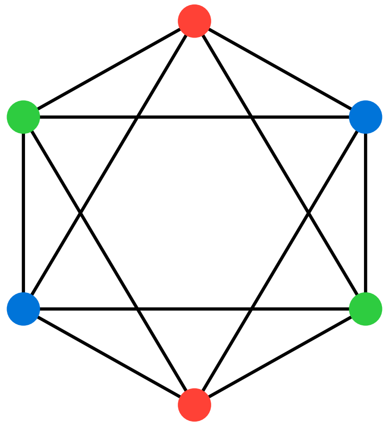

$\chi(G) = 3$

Why it matters

- Frequency assignment in wireless networks
- Exam/meeting timetabling
- Register allocation in compilers
- Map colouring and visualisation

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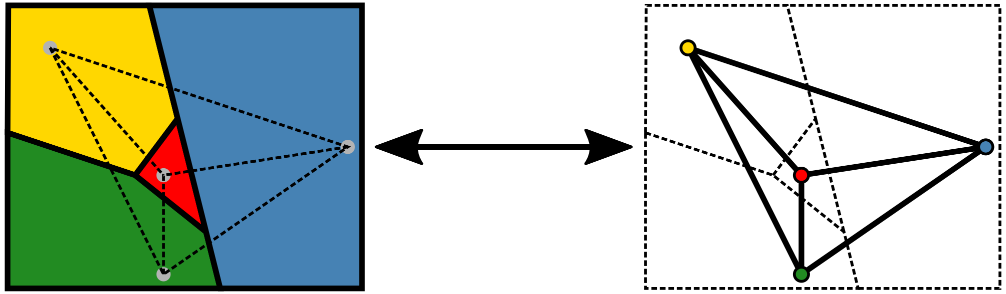

Image: Wikimedia Commons

Related problems

- k-colouring (feasibility)
- Maximum clique / maximum independent set (bounds)
- Graph colouring as a special case of constraint satisfaction

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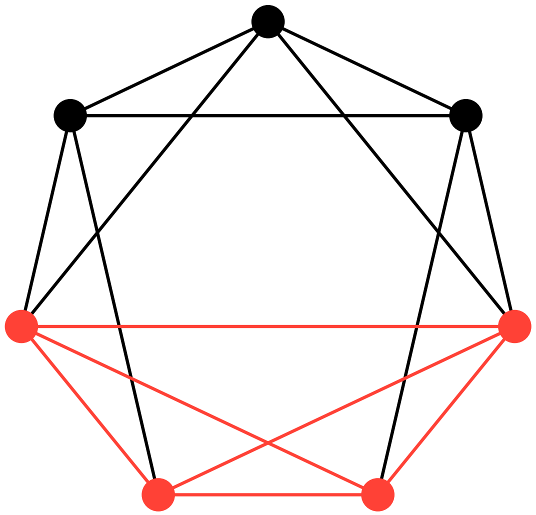

Maximum clique

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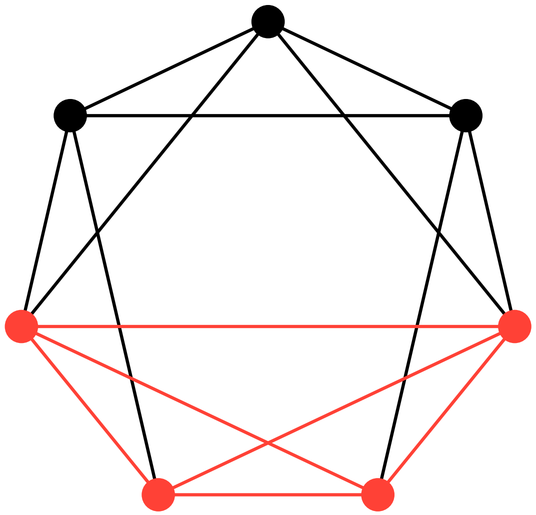
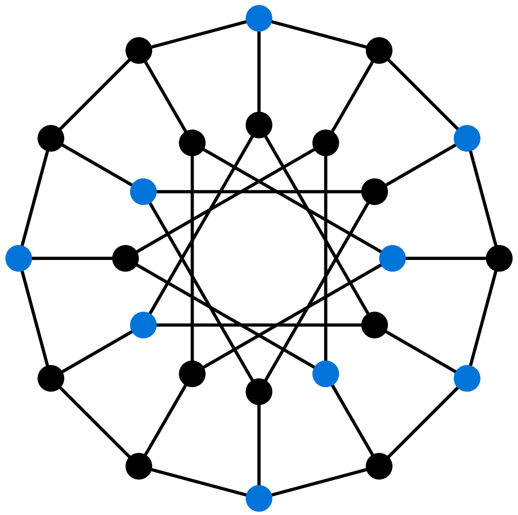


Maximum clique



Maximum independent set

Formulation

Variables and constraints

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Optimisation target: minimise the number of used colours.

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- M is a large penalty (e.g., $|E| + 1$)
- Works with heuristic search (GA, SA, PSO)

Approach

Representation

- Individual: integer vector of length $|V|$
- Gene: colour index in $\{1, ..., K\}$
- Outer loop: try $k = 1, 2, ..., K$ until a conflict-free colouring is found

Evolutionary algorithm outline

1. Initialise random population
2. Evaluate fitness
3. Select parents (tournament)
4. Crossover + mutation
5. Elitism, repeat until stop

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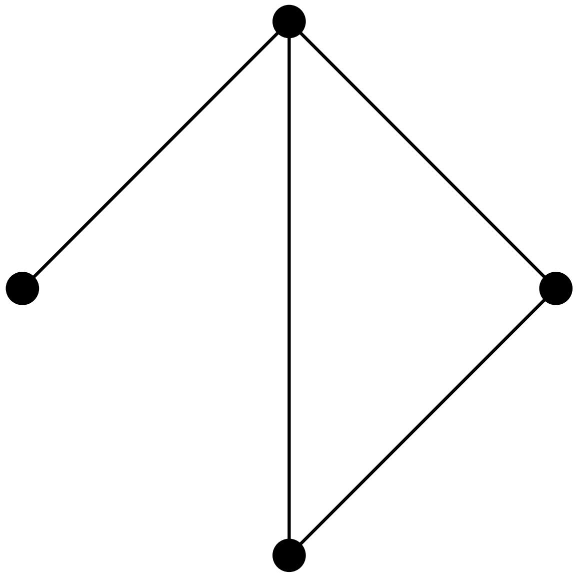

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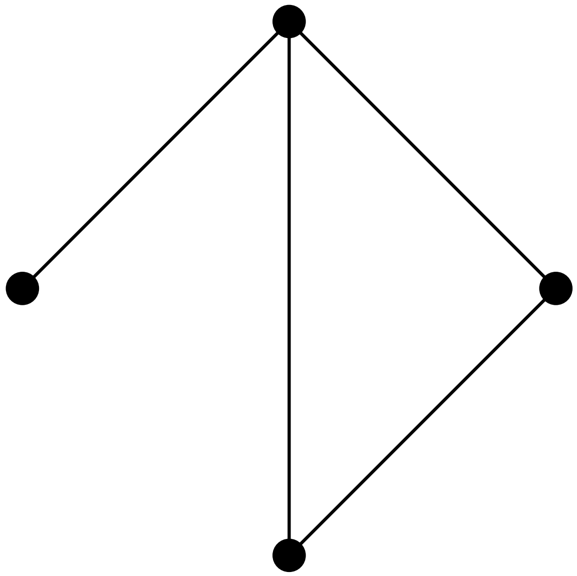

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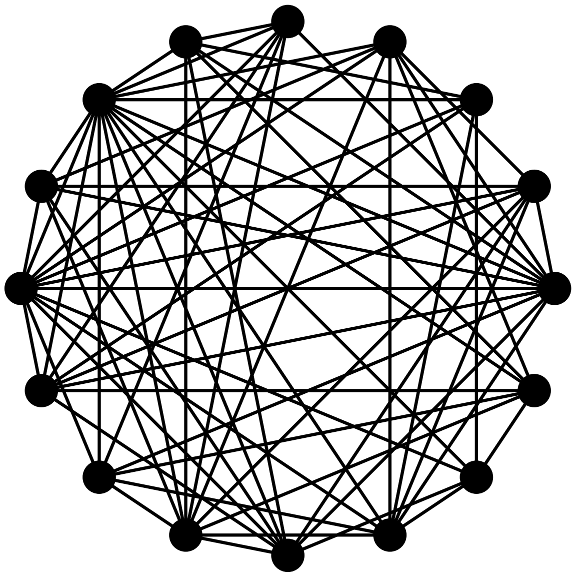

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Example

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Petersen graph (10 vertices, chromatic number 3):

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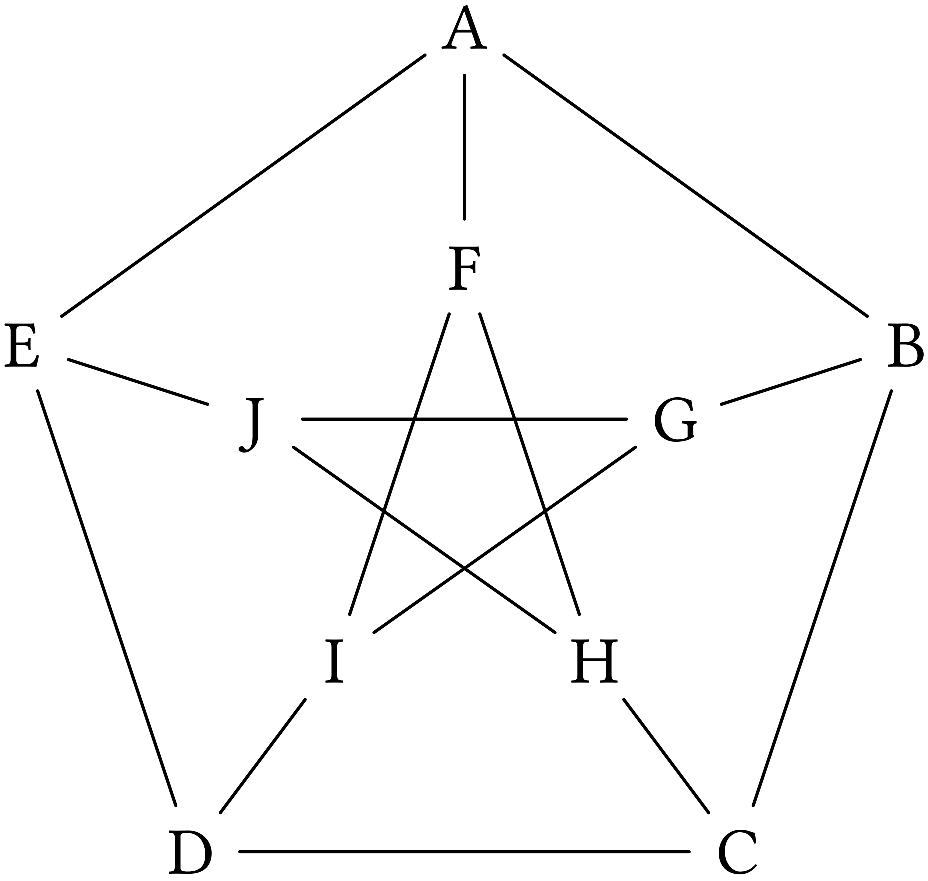
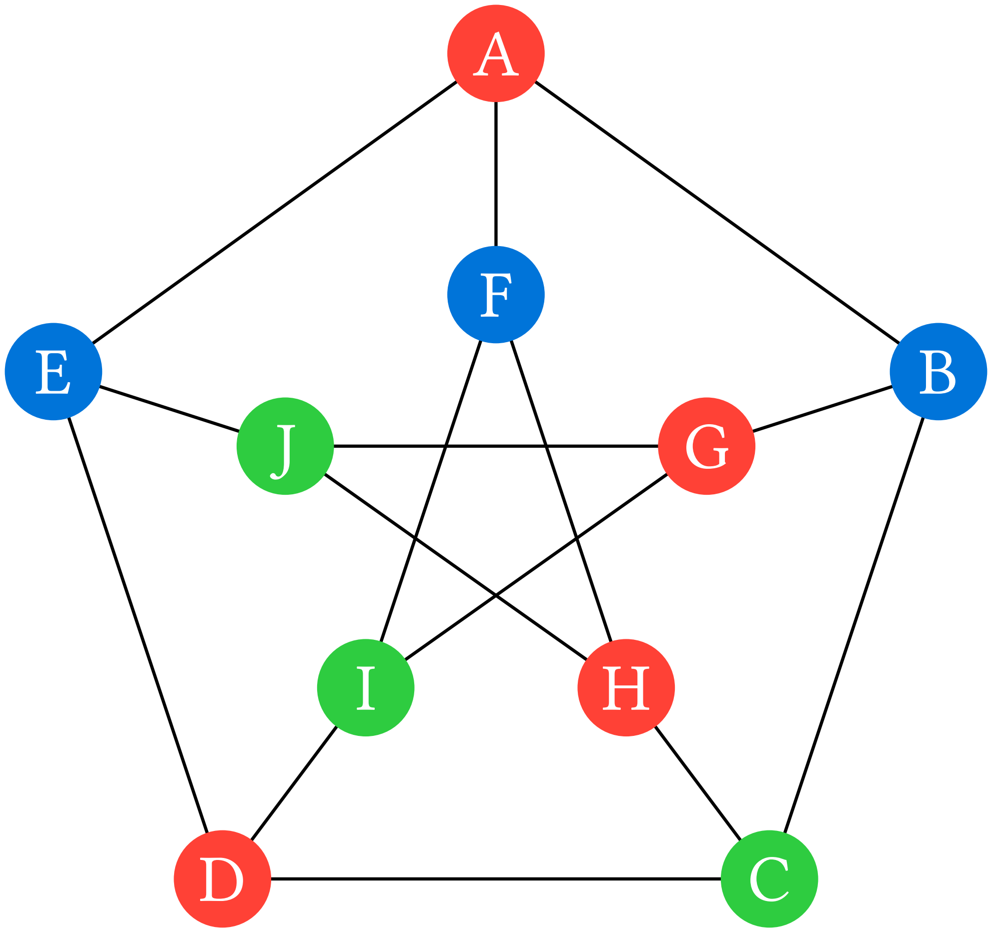


Example colouring

One optimal 3-colouring (minimum possible):



Implementation

Python formulation highlights

- Graph as edge list
- Fitness = conflicts penalty + colour count
- GA with tournament selection and mutation

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```py
@dataclass
class Graph:
    n: int
    edges: list[tuple[int, int]]
…
    def fitness(self, colouring: Sequence[int], penalty: int) -> int:
        conflicts = self.conflicts(colouring)
        colours = type(self).used_colours(colouring)
        return penalty * conflicts + colours
```

Output format

- Best colouring found
- Found k, conflict count, and colours used
- Ready for inspection or further refinement

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```
$ python 6814001748_midterm.py
```

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$ python 6814001748_midterm.py

Best colouring: [1, 3, 2, 1, 2, 2, 1, 1, 3, 3]
Found k: 3
Conflicts: 0
Colours used: 3
Fitness: 3
```

Discussion

Existing approaches

- Exact: branch-and-bound, ILP, SAT/CP
- Heuristic: DSATUR, greedy with reorder
- Metaheuristics: GA, SA, tabu search, ACO

Strengths and limitations

- Strength: flexible, easy to adapt
- Limitation: no optimality guarantee without enough search

Conclusion

Takeaways

- Graph colouring is NP-hard and practically important
- Evolutionary search offers a scalable path to good solutions
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All code and slides are available at:

https://github.com/krtchnt/optimisation-midterm

Thank you!

Any questions?