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Modelling the equilibrium exchange rate of Czech koruna

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Abstract

The Capstone project examines the issue of determining the equilibrium exchange rate of Czech koruna. Recently, the Czech National bank has made an asymmetric exchange rate commitment not to let the currency appreciate below 27 EURCZK. The commitment was effective between 2013 and 2017. The research shows that CNB interventions cannot be justified as a move towards the equilibrium exchange rate. The behavior of the Czech koruna is analyzed in the context of the economic forces driving the convergence process. Next, the exchange rate models (BEER and FEER) are modified to fit the nature of Czech economy; the convergence process and CNB interventions. Finally, the consolidated results of the models are discussed. The FEER estimation results uncovered that possibly, the Marshall-Lerner condition does not hold in the case of Czech trade balance. The BEER model results are therefore preferred.

Keywords: *Czech Republic, equilibrium exchange rate, FOREX, PPP, UIP, BEER, FEER, VECM, cointegration*

1. Introduction

The Capstone project examines the issue of determining the equilibrium exchange rate of Czech koruna. The Czech Republic is a small, open economy with a dominant sector of foreign trade with the European Union and an important role of the exchange rate. Models of exchange rate distinguish fundamentally justified exchange rate movements from non-fundamental bubbles. They are the basis for medium term and long-term forecasts of exchange rates and the European Monetary Union considers them as a source for quantifying the conversion rate when a new member joins the Eurozone [9] . Equilibrium exchange rates were recently among the most discussed topics among Czech economists when in November 2013, the Czech National bank made an asymmetric exchange rate commitment to not let the currency appreciate below 27 EURCZK. The commitment was effective until April 2017. The National bank claimed that the depreciation was a move towards equilibrium [4]. However, many market participants thought that the currency is undervalued and invested a lot of money into Czech assets. As a result, the long-term interest rates fell towards zero. In the Capstone project, the claim that interventions brought the exchange rate to equilibrium is tested using models of equilibrium exchange rates.

Models of exchange rates have a long history in economic theory. The pioneer analysis of exchange rate behavior was made by a Swedish economist, Cassel [2] who formulated the purchasing power parity in the light of the post-war evidence of exchange rate movements caused by inflation differentials between countries. According to MacDonald [21] , the role of economic fundamentals in explaining exchange rate behavior is one of the most controversial issues in international economics. By now, there are many different exchange rate models. The three popular classes of models today are the FEER by Williamson, [39] , the BEER by MacDonald, [22] and the NATREX by Stein and Allen [38]. Rather than a fixed procedure, the terms refer to a broader philosophy of how to approach exchange rate modelling and the form of the model is left to the judgment of an analyst.

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The goal of the Capstone project is to modify the individual models to best fit the Czech Republic case and assess the consolidated estimates of the models. In particular, the convergence process which started in 1989 after the Velvet revolution ended the communist era is an important fact to be considered in the models as well as a different set of macroeconomic variables than the usual models, based mainly on US data, contain.

Due to the nature of Czech economics, the focus of the project is on more complex methods, BEER and FEER rather than the basic approaches which cannot explain the exchange rate trends of CZK satisfactorily. The statistical methods of cointegration and the HP filter are used for estimating the necessary relationships for calculating the equilibrium exchange rate.

First, the theoretical approaches to modelling exchange rates are discussed. Next, the behavior of the Czech koruna is analyzed in the context of the economic forces driving the convergence process. Next, the exchange rate models (BEER and FEER) are modified to fit the nature of Czech economy. Finally, the consolidated results of the models are discussed.

2. Equilibrium exchange rate concepts

The long-term trend of an exchange rate is usually evaluated from the Purchasing power parity (PPP) perspective. However, the theory implies a constant real exchange rate (ER_R). As a result, any changes of real exchange rate from the constant level are considered as deviations from equilibrium. From the perspective of Czech koruna, PPP is not a good vehicle for determining the equilibrium exchange rate because it cannot explain the long-term appreciation trend of the currency, which can be found in all transitive economies.

According to Komárek and Motl [19], BEER, FEER and NATREX are the main concepts to calculate equilibrium exchange rate. Other approaches can be considered a derivative of these concepts. In the overview in *Table 1*, they are called sophisticated approaches, together with the BEER. The first two columns represent basic approaches PPP and UIP. Some models focus on a different time horizon than others. The UIP explains short term trends while PPP captures the long trend of an exchange rate. As a result, the estimates are not directly comparable. Also, some models are mutually exclusive. If PPP strictly holds, the ER_R is stationary and BEER does not exist. Finally, all the methods but FEER are estimated directly. The FEER is estimated indirectly by solving a current account balance equation.

2.1. Basic approaches

2.1.1. The purchasing power parity

The Purchasing Power Parity states that the equilibrium exchange rate is determined only by the ratio of domestic to foreign price levels. International goods arbitrage enforces that the prices of a similar basket of goods (P_D,P_F) in a domestic and foreign country are equalized, if converted to a single currency using the nominal exchange rate (ER_N).

As a result, if PPP holds on average, the real exchange rate (ER_R) is stationary, fluctuates around a constant level.

$$ER_R = ER_N \frac{P^F}{P^D} \qquad (1)$$

MacDonald [23] claims that PPP is not a good vehicle for exchange rate determination because of the evidence of high and persistent deviations from PPP. Mandel [25] shows that PPP systematically undervalues the currency in the case of transitive economies under the economic convergence process, such as the Czech Republic. The Czech koruna has a long-term real appreciation trend while according to PPP, the real exchange rate should fluctuate around a constant equilibrium (see *Figure 1*).

Figure 1. Real (CPI deflated) and nominal exchange rate

2.1.2. The uncovered interest parity

The uncovered interest parity is an equilibrium condition of a foreign exchange speculator. It assumes that domestic and foreign assets are perfect substitutes and a speculator compares the rate of return when investing in domestic versus foreign assets. The UIP condition is:

$$\left(1 + IR_{D,t}^{t+n}\right) \uparrow = \left(1 + IR_{F,t}^{t+n}\right) \frac{E_t(ER_{N,t+n})}{\downarrow ER_{N,t}} \qquad (2)$$

On the left-hand side, there is the return on the domestic assets. On the right-hand side, there is the return in foreign assets. Every unit of domestic currency is converted to a foreign one at the current $ER_{N,t}$, invested at current foreign interest rate $IR_{F,t}^{t+n}$and converted back to domestic currency in the current expected exchagne rate $E_t(ER_{N,t+n})$ at the maturity of the investment $t+n$. If the condition holds, the speculator is indifferent to the choice.

A usual interpretation is that a rise in the domestic interest rate attracts the import of foreign capital. As a result, the current exchange rate appreciates and resets the equilibrium condition. However according to Mandel [25] the relationship between exchange rate and interest rates is dynamic, there are many complex relationships between interest rates and the exchange rate and UIP cannot be described using a single equation model. For example, in a small, open economy with inflation targeting monetary policy, the reverse relationship from exchange rate to interest rate exists because exchange rate affects expected inflation and inflation affects the optimal monetary policy rate.

Table 1 Exchange rate concepts

Features	Basic approaches		Sophisticated approaches		
	PPP	UIP	BEER	FEER	NATREX
Name	Purchasing Power Parity	Uncovered Interest Parity	Behavioural Equilibrium Exchange Rate	Fundamental Equilibrium Exchange Rate	Natural Real Exchange rate
Research papers/ Institutions	Cassel [2], Mandel and Tomšík [26], Škop and Vejmělek [36] /OECD [29]	Durčáková et al., [8]	MacDonald [22], Komárek and Motl [19] / Czech National Bank [4]	Williamson [39], Genorio and Kozamernik, [14], Komárek and Motl [19] Rubaszek [34] / Czech National Bank [4]	Stein and Allen [38], Škop and Vejmělek [36]
Advantages	Basic approach that explains the long term trend of ER for developed countries.	Links the interest rate policy of a central bank to exchange rate	Reflects the fundamentals related to the ER in a rigorous statistical sense.	Policymakers can base assessment on their view of macroeconomic imbalances	Strong micro-founded structure. No assumptions about desirable current account.
Disadvantages	Problems in measurement of price levels. Deviations from PPP are persistent.	Difficult to empirically verify, complex and dynamic relationship.	Statistical, data mining approach. No theory guides the choice of fundamental variables in the BEER	Requires considerable parameter estimation and judgement about desirable current account.	Double dynamics of the ER_R. Difficult to isolate medium run and long run effects.
Theoretical Assumptions	Constant equilibrium E R_R	Change in ER_R or ER_N determi ned by interest differential	Real UIP with a risk premium, ER_R determin ed by fundamentals	ER_R compatible with both internal and external balance	As FEER + domes tic real IR equals the foreign rate
Relevant time horizon	Long run	Short run	Short/Mediu m run	Medium run	Medium / long run
Statistical assumptions (ER_R)	Stationary	Stationary (of change)	Non-Stationary	Non-Stationary	Non-Stationary
Dependent variable	ER_R or ER_N	ER_R or ER_N	ER_R	ER_R	ER_R
Estimation method	Direct	Direct	Direct	Underlying balance	Direct

Note: ER_R (ER_R) … real (nominal) Exchange rate **own creation based on Driver and Westaway (2004)**
IR … interest rate

2.2. Sophisticated approaches

Due to the failure of PPP in transitive countries, the Capstone project´s objectives are to estimate more sophisticated models (BEER and FEER) to better capture the economic forces in a converging economic. The BEER model was formulated by MacDonald [22] and the FEER model by Williamson [39] and Wren-Lewis [41]. The applications of the models on the Czech republic are Komárek and Motl [19] or Šmídková et al. [37]. Next possible approach to modelling exchange rate is NATREX, originally made by Stein and Allen [38] and applied on the Czech Republic by Škop and Vejmělek [36].

2.2.1. The BEER with interventions

The proposition of the BEER model is that PPP assumption of constant equilibrium real exchange rate does not hold. On the contrary, it has a trend which can be explained by real determinants. The part of the exchange rate explained by real determinants is the equilibrium rate and the model residual component represents non-fundamental bubbles (see Figure 2). The real fundamentals are typically Balassa-Samuelson effect proxy, net foreign assets of a country, terms of trade and real interest differential with a risk premium.

However, as Rojíček et al. [33] notes, BEER is a statistical, data mining approach. There is no guide how to choose the right model variables. BEER modellers often prefer to use variables with a good statistical fit but with ambiguous effect on the exchange rate in theory. According to Couharde et al, [5], the Balassa-Samuelson effect is the important economic force driving real exchange rates. In the Czech Republic case, the economic convergence proxy variable does probably have a key role in explaining the long-term real appreciation trend of Czech koruna. It is empirically documented that countries with lower real GDP per capita also have lower price level [25]. Productivity and wage growth, accompanied by price growth, in a converging economy is relatively faster. The result is real appreciation. Real GDP per capita and labour productivity grows faster in Czech Republic than in Eurozone (see Figure 3).

Further, BEER approach allows to estimate a permanent exchange rate (PEER) by calibrating explanatory variables to their trend values (see MacDonald, [23]). PEER captures the fact that the explanatory variables may not be at their long-run sustainable levels and avoids transitory movements in BEER. The PEER approach is also more comparable to FEER as it assumes economic variables are at their long run, sustainable levels.

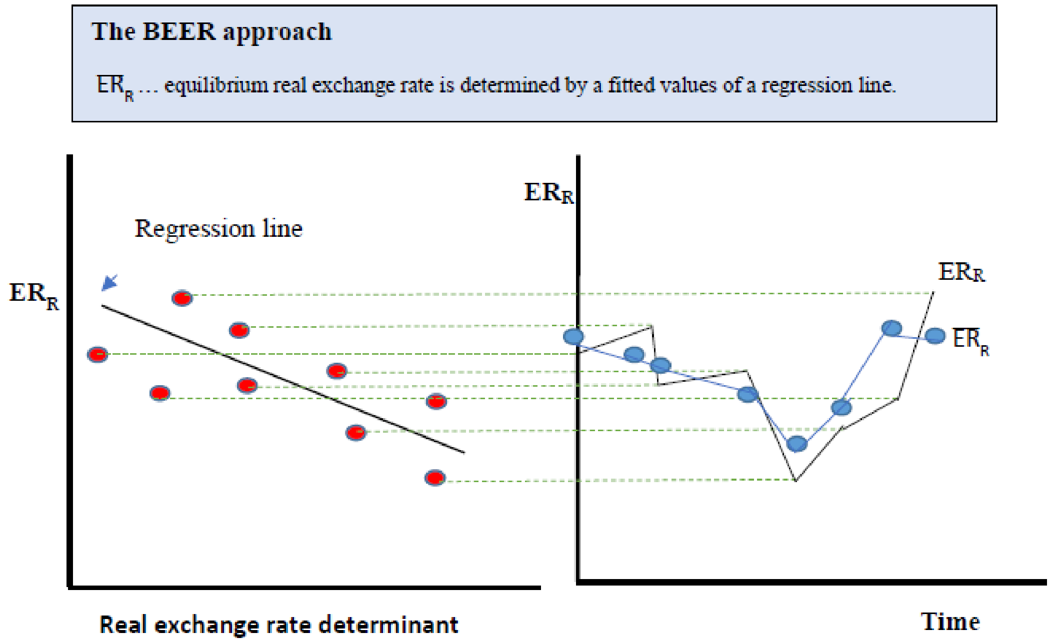

Figure 2. The BEER approach principals

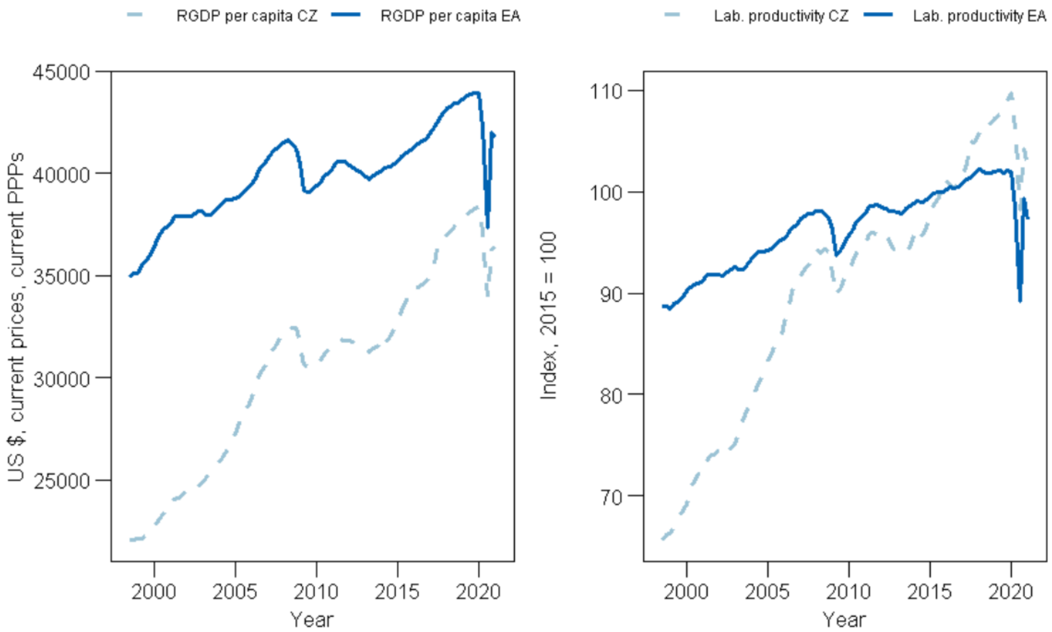

Figure 3 Real convergence

Further, BEER model is extended to account for Czech National bank interventions. Since the end of 2013, the Czech National Bank started interventions on the Czech koruna, virtually fixing its lower bound to Euro to 27 CZK to EUR. Interventions variable in the BEER model was used by Jankovič [16].

The BEER model with interventions is constructed the following way:

1. The usual BEER fundamentals are used as explanatory variables for the long-run dynamics of the exchange rate.
2. The volume of CNB interventions and other spot transactions is used as an exogenous variable in the short-run dynamics of the exchange rate.

In theory, the BEER with interventions model is able to filter out any possible noise that is made by the central bank´s interventions and not by the real determinants of the exchange rate.

The CNB market operations on the FX market are captured in Figure 4. Officially declared interventions were in the period October 2001 – September 2002 (see Lízal [20]) and more importantly, November 2013 – April 2017. In November 2013, the CNB declared a floor of 27 EURCZK.

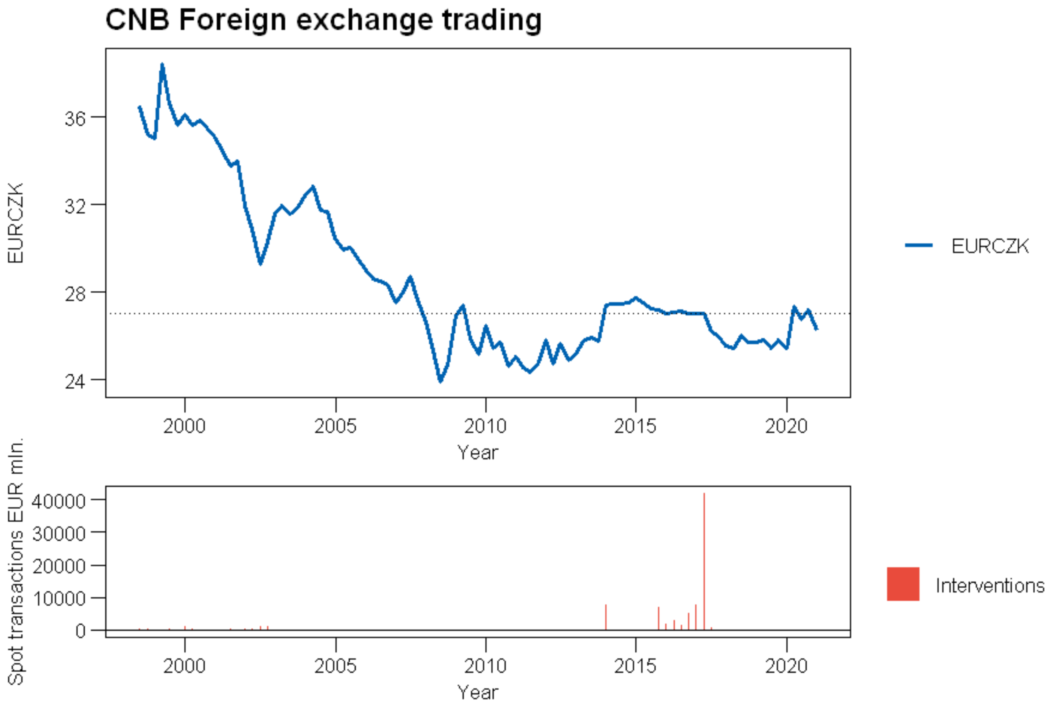

Figure 4 CNB interventions

2.2.2. The FEER

As opposed to BEER, instead of directly modelling the real exchange rate as a function of a set of variables, the FEER approach models the current account balance (CAB) and finds the equilibrium exchange rate indirectly by solving a current account balance model. The current account balance cumulates in net foreign assets. Countries with current account surpluses export capital and build foreign assets and countries with current account deficits import capital and build foreign liabilities.

The FEER is the real equilibrium exchange rate that simultaneously secures both external and internal medium-term macroeconomic equilibrium. The internal equilibrium condition is defined as domestic real output (Y^D) being at the potential output of the economy ($\bar{Y}^D$). The external balance condition is defined as a sustainable current account level (CAB^S), representing medium term desirable net flow between the countries (see MacDonald, [23]).

There are two approaches to modelling FEER. The first is a macroeconomic model of the whole economy where the internal and external balance is imposed. According to Jovanovic and Branimir [18], a more common approach is a partial equilibrium model, proposed by Wren-Lewis [41], where only the current account balance is modelled as a function of the real exchange rate and other control variables such as domestic and foreign economic activity. Exogenous estimates of potential output and sustainable current account balance are fed into the model. In the project, the simpler, partial equilibrium approach is used. The FEER model is defined as follows:

$EX = f(IM^F, ER_R)$	(3)	behavioral function of real export
$IM = f(Y^D, ER_R)$	(4)	behavioral function of real import
$CAB^S = f(CONV)$	(5)	behavioral function of CAB^S
$CAB = NX_N + BPI + BSI$	(6)	identity of current account
$NX_N = EX \times P^{EX} - IM \times P^{IM}$	(7)	identity of net exports
$CAB(\overline{IM^F}, \overline{Y^D}, \overline{ER_R}, \overline{BPI}, \overline{BSI}) = CAB^S$	(8)	equilibrium condition

Where overbar denotes equilibrium values of the variables. The $\overline{ER_R}$ represents the unknown FEER, solution to the model. The other variables at equilibrium are produced by Hodrick-Prescott filter. The estimated equations are the real exports, real imports functions in log-linear form and the sustainable current account to GDP ratio. The real exports are a positive function of foreign demand, represented by foreign real import (IM^F). Also, exports are a positive function of ER_R (a rise in ER_R denotes currency depreciation). The real imports are a positive function of domestic real output Y^D and a negative function of ER_R. The sustainable current account to nominal GDP ratio is a positive function of a convergence proxy, CONV. The convergence proxy is relative GDP per capita in PPP of the Czech Republic relative to Eurozone.

The behavioral functions (3,4 and 5 below) are estimated using cointegration methods. Then, the real exports, real imports and the sustainable current account balance are calculated using the model coefficients and observed, respectively equilibrium values (in 8) of the exogenous variables.

$$ln\, EX_t = \beta_1 + \beta_2 ln\, {IM_t}^F + \beta_3 lnER_{R,t} + \varepsilon_{1,t} \quad (3)$$
$$ln\, IM_t = \alpha_1 + \alpha_2 ln\, {Y_t}^D + \alpha_3 lnER_{R,t} + \varepsilon_{2,t} \quad (4)$$
$$\frac{{CAB_t}^S}{{Y_{N,t}}^D} = \gamma_1 + \gamma_2 CONV_t + \varepsilon_{3,t} \quad (5)$$

The other parts of the current account (export and import prices and balance of primary and secondary account) are treated as exogenously determined, see Komárek and Motl [19]. However, according to Siregar [35] if the FEER assumes the impact of ER_R on return on net foreign assets (BPI) is exogenously determined, it may overestimate the required real exchange rate appreciation because a real depreciation not only improves NX but also increases returns from foreign assets (BPI).

The model is solved by imposing the internal balance conditions – domestic and foreign real output at it´s potential value and equalizing the modelled CAB to the sustainable CAB. The real exchange rate ($\overline{ER_R}$) which satisfies the equilibrium condition (8) is the FEER.

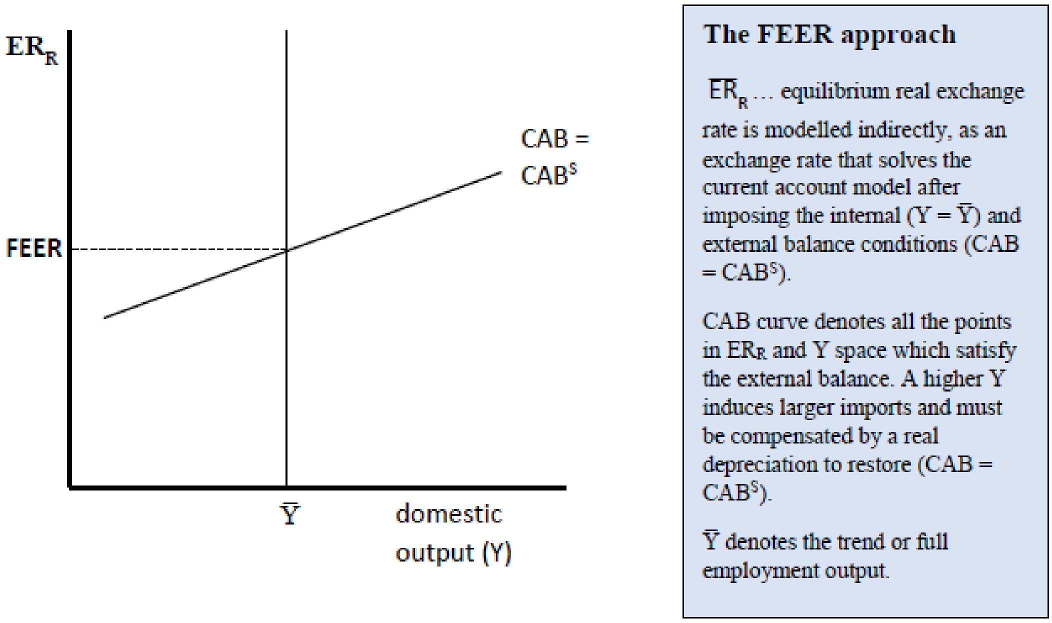

Figure 5. The FEER approach principals

The FEER is considered normative equilibrium measure (see Siregar [35]) because we need to specify ideal macroeconomic circumstances, the real GDP potential output ($\bar{Y}^D$), and a sustainable current account balance over 5 years (CAB^S). A potential GDP estimate is captured in Figure 6.

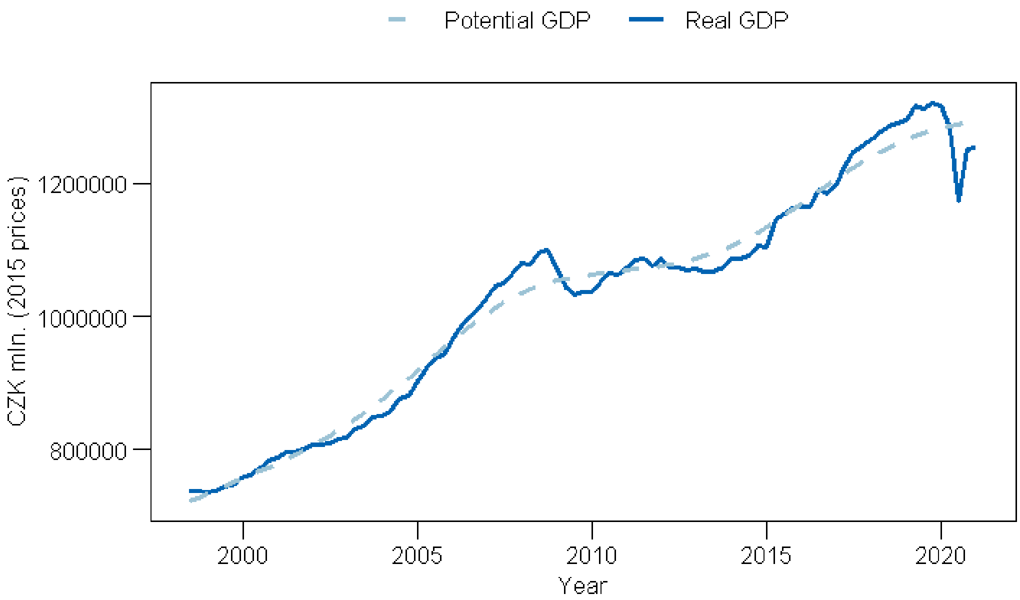

Figure 6. Czech potential GDP estimate using HP filter

The FEER assumption (3) of sustainable CAB is confronted with the theory of a life cycle of transitive economy by Mandel [25]. The main finding of the theory that the definition of external balance of a transitive economy is sequential (see Table 2). Different phases of economic development require a different definition of external balance. At first stages, the economy attracts foreign capital, imports foreign direct investments and builds foreign liabilities. As a result, there is an objective reason for the countries to have negative current account. At later stages, the economy enhances its productivity (coming from foreign know-how) and its export activity. The country stops being net importer of capital and the current account is near zero.

As Mandel [25] notes, the FEER model is interesting for application to transforming economies because it allows to define the external balance of a country as a negative and time-varying level of current account.

Table 2 Stages of development and external macroeconomic equilibrium

Economy	Saving and Investments	Net exports	Primary income	Current account	Net direct investments
Young transforming	S<I	NX<0	BPI≤0	CAB<0	NDI>0
Mature transforming	S=I	NX=0	BPI<0	CAB<0	NDI>0
Posttransforming	S>I	NX>0	BPI<0	CAB=0	NDI=0
Expandining developped	S>I	NX>0	BPI<0 BPI>0	CAB>0	NDI<0
Balanced developed economy	S=I	NX=0	BPI=0	CAB=0	NDI=0

Source: Mandel and Tomšík (2008)

The sustainable current account in (3) is the expected value conditional on a convergence proxy CONV, relative GDP per capita in PPP. This allows the current account to be negative in the first stages of the transitive economy development and close to zero in post/transforming stage of a country.

3. Econometric methodology

The notion of equilibrium in statistics is captured in cointegration methods. The methods estimate a long-term relationship between variables which hold on average and from which any deviations are only temporary (i.e. stationary). Formally, cointegration exists if there is a linear combination of variables I(q) such that the result is of order I(v) where v<q. The econometric methodology of cointegration methods was developed by Engle and Granger [10], Phillips and Ouliaris [32], Johansen [17] and Pesaran et al., [31].

3.1 Engle-Granger two step approach

The starting point of Engle-Granger approach is a unit root test on the variables. Assume all the variables are integrated of order 1, a static regression which describes the long run relationship between the variables is estimated using OLS where a correlated error term is assumed (9):

$$Y_t = \alpha + X_t\beta + u_t \qquad (9)$$
$$Where\ u_t = \rho u_{t-1} + \varepsilon_t\ , |\rho| < 1, \varepsilon_t \sim WN(0, \sigma_\varepsilon^2)$$

Y_t denotes the dependent variable, X_t is a vector of independent variables and ε_t is a white noise process. By estimating the static regression, residuals $\widehat{u}_t = Y_t - X_t\hat{\beta}$, measuring the degree of disequilibrium in the system are produced. Testing cointegration between Y_t and X_t is a test if the residuals are stationary. However, because the residuals were a subject to estimation, the standard critical values of ADF test are not valid. Critical values for cointegration test can be found in Engle and Yo

[11] or MacKinnon [24]. Standard statistical inference in the static regression is not valid because the distribution of t-statistics is not known.

The second step is to express the model in the error correction form by substituting the expression for Y_t and Y_{t-1} into the expression for u_t:

$$\Delta Y_t = \alpha(1-\rho) + \Delta X_t\beta + (\rho-1)(Y_{t-1} - X_{t-1}\beta) + \varepsilon_t \tag{10}$$

After plugging in the estimated residuals from the first step $\widehat{u}_t = Y_t - X_t\hat{\beta}$ for expression $(Y_{t-1} - X_{t-1}\beta)$, the regression can be estimated using OLS. The specification (10) includes only variables I(0) and the error term is a white noise. The standard statistical inference is valid.

The parameter ρ measures the speed at which any deviations from the long term equilibrium given by the static regression are being abolished by the adjustment of the dependent variable. Naturally, $0 < (\rho - 1) \leq -1$ since no more than the full amount of the disequilibrium can be the subject to an adjustment.

3.2 VECM estimator of Johansen

Under VECM multiple variables are considered endogenous. Otherwise, the idea of equilibrium is analogical to Engle-Granger approach. The approach was developped by Johansen [17] and describes a cointegration in VAR model.

Assume (nx1) vector of endogenous variables x_t integrated of order 1:

$$x_t^{'} = [x_1, x_2 \dots x_n] \tag{11}$$

Assume vector x_t has a VAR representation of order p:

$$x_t = \eta + \sum_{i=1}^{p} \Pi_i \, x_{t-p} + \varepsilon_t \tag{12}$$

Where ε_t is a (nx1) vector of white noise processes, η is a (nx1) vector of constants and Π_i are (nxn) matrices of model coefficients.

The model can be reparametrized to VECM representation:

$$\Delta x_t = \eta + \Pi x_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \, \Delta x_{t-1} + \varepsilon_t \tag{13}$$

Where $\Gamma_i = -\sum_{j=i+1}^{p} \Pi_j$ is (nxn) matrix of coefficients and $\Pi = \sum_{i=1}^{p} \Pi_i - \mathrm{I}$ is (nxn) matrix of coefficients (I denotes the identity matrix). The rank of matrix Π determines the number of cointegrating vectors (see MacDonald [23]). Testing the number of cointegration vectors is equivalent to testing the rank of the matrix Π. If the rank is full or zero, there is no cointegration and the variables are I(0) or I(1) respectively. If the rank of Π is r, where 0<r<n, there are matrices α and β such that $\Pi = \alpha\beta'$. The columns of matrix β represent independent cointegration vectors and matrix α contains adjustment parametres, signalizing the reaction of the system to deviation from equilibrium in the previous period.

The VECM representation has one less lag compared to the VAR representation. The first step is to estimate VAR in levels (not differenced) variables and determine the appropriate lag order p using interpolation criteria. Then, the presence of cointegration and number of cointegration vectors is tested. If the number of cointegrating vectors found is 0<r<n, it makes sence to estimate the VECM using MLE.

3.3 Hodrick-Prescott filter

The filter produces a smoothed trend series s of y by minimizing the variance of y around s subject to a constraint on the second difference of s. HP filter finds s to minimize the following:

$$\sum_{t=1}^{T}(y_t - s_t)^2 + \lambda \sum_{t=2}^{T-1}[(s_{t+1} - s_t) - (s_t - s_{t-1})]^2 \; Where\ \lambda \text{ is a constant} \qquad (14)$$

4. Data

Due to the fact that the majority (over 75, see [19]) of Czech foreign trade turnover is realized with eurozone countries, the exchange rate examined is EURCZK. The explanatory variables are macroeconomic variables taken from official national accounts, balance of payments statistics and other central bank and national statistical office sources. All data are in quarterly frequency 1998Q2-2020Q4. The data overview is in the following table:

Table 3 Data

Data	Label	Model(s)	Source
Exchange rate EURCZK	EURCZK	BEER, FEER	CNB [3]
Czech CPI	CPI_CZ	BEER, FEER	CNB [3]
Eurozone CPI	CPI_EUROZONE	BEER, FEER	IMF [15]
Czech real GDP per capita in PPP	RGDP_per_capita_CZ	FEER	OECD [30]
Eurozone real GDP per capita in PPP	RGDP_per_capita_EUROZONE	FEER	OECD [30]
Czech real labour productivity	PROD_CZ	BEER	OECD [30]
Eurozone real labour productivity	PROD_EU	BEER	OECD [30]
Czech real government spending	G	BEER	CZSO [7]
Czech real GDP	R_GDP	FEER	CZSO [7]
Czech nominal GDP	N_GDP	FEER	CZSO [7]
Czech real export	NA_R_X	FEER	CZSO [7]
Czech real import	NA_R_M	FEER	CZSO [7]
Czech export price deflator	P_X	FEER	CZSO [7]
Czech import price deflator	P_M	FEER	CZSO [7]
Czech current account balance	BOP_CAB	FEER	CNB [3]
Czech balance of primary income	BOP_BPI	FEER	CNB [3]
Czech balance of secondary income	BOP_BSI	FEER	CNB [3]
Eurozone real import	NA_R_M_EUROZONE	FEER	Eurostat [13]
CNB interventions	Interventions	FEER	CNB [3]

5. Results

All the variables, except CNB Interventions are integrated of order I(1) according to the ADF test. The Interventions variable is integrated of order I(0).

The variables used in the BEER model with interventions were chosen based on judgement, well-known empirical experience, compliance of model parametres with economic theory and statistical inference (backward elimination procedure). The final model is presented here.

$$ER_R = f(DPROD, GOV\_GDP\_ratio, Interventions) \tag{15}$$

The real exchange rate (ER_R) is determined by *DPROD*, the labour productivity differential of Czech Republic to Eurozone, ratio of real Czech government spending to real GDP, *GOV_GDP_ratio*, and *Interventions* of CNB. The latter two variables are used to model the short-term behavior of the exchange rate following the BEER with interventions approach. In addition, two dummy variables, *D_2020Q2* and *D_2020Q3* are used as exogenous variables to account for COVID period shocks.

The model is estimated using the Johansen approach. First, the VAR in levels is estimated in order to select lag length of the VEC model. The information criteria suggest using 1-3 lags in the VAR model so that the VEC model would use 0-2 lags.

Next, the number of cointegration vectors is tested in Table 4. The null hypothesis is that there are at most x (x = 0,1,2) cointegration vectors. The row with hypothesis r, the number of cointegrating vectors = 0 has the test statistic 27.89, higher than the critical value on 5pct significance level of 22 so that the null of no cointegration is rejected. The Trace test indicates that there is one cointegration vector.

Table 4 Significance of cointegration vectors - BEER

H_0:r ≤	Trace statistic	Trace 5% crit
0	27.89	22.00
1	10.41	15.67
2	4.45	9.24

The estimated VEC model has 3 lags (adjusted to remove serial correlation in residual tests) and one cointegration relationship. The estimated cointegration equation is:

$$lnER_{R,t} = 2.3256290\ [-5.36] - 1.4491908\ lnDPROD_t\ [5.92]\ -$$

$$0.5365669\ lnGOV\_GDP\_ratio_t\ [2.04] \tag{16}$$

Where [] denotes t-statistic. All long-run coefficients are statistically significant on 5% confidence level.

Next, the estimated loading coefficients are reported in Table 5. As expected, the real exchange rate adjusts to deviations from the long run relationship. According to the model, the productivity differential also adjusts to deviations from the long run relationship. The adjustment of government spending to GDP ratio is not statistically significant.

Table 5 Alpha adjustment matrix

Variable	Alpha	Standard error
ΔlnER$_R$	-0.1690	0.0659*
ΔlnDPROD	0.0614	0.0131***
ΔlnGOV_GDP_ratio	-0.0852	0.0502

The statistical significance of exogenous variables is tested in Table 6. Interventions are not statistically significant. Both dummy variables, indicating COVID periods are statistically significant at 5% confidence level at least at one equation.

Table 6 Exogenous variables

Note: Standard errors in ()	**Parameter (t-stat)**		
Equation (endogenous variable)	**Interventions**	**D_2020Q2**	**D_2020Q3**
ΔlnER$_R$	4.2e-07 (6.5e-07)	-0.0114 (0.0302)	0.0297 (0.0343)
ΔlnDPROD	1.5e-07 (1.3e-07)	0.0186 (0.0060)**	-0.0523 (0.0068)***
ΔlnGOV_GDP_ratio	-2.6e-07 (4.9e-07)	0.0873 (0.0231)***	-0.0340 (0.0261)

Finally, the model residual diagnosis is presented in Table 7. Brüggemann et al. [1] shows that some autocorrelation tests for VAR with cointegrated variables need to be modified compared to stationary VAR case. However, the multivariate Breush-Godfrey LM test has the same asymptotic chi-squared null distribution as in the stationary case. The Breush-Godfrey test is therefore preferred. The null hypothesis of no autocorrelation is not rejected at 5% significance level in all 4 tests using different lag specification. The normality of error term is tested using multivariate Jarque-Bera test and does not reject multivariate normality of the error term. It is assumed that the model is correctly specified.

Table 7 Residual diagnosis BEER model

Test	BG(1)	BG(2)	BG(3)	BG(4)	JB
p-value	0.5402	0.3395	0.2043	0.08047	0.8009

The BEER and it´s permanent component PEER estimates are displayed in Figure 7. The PEER was obtained by using the trend values of explanatory variables. However, there is practically no difference between the two estimates. According to the model, the Czech koruna was undervalued since the interventions started in 2013.

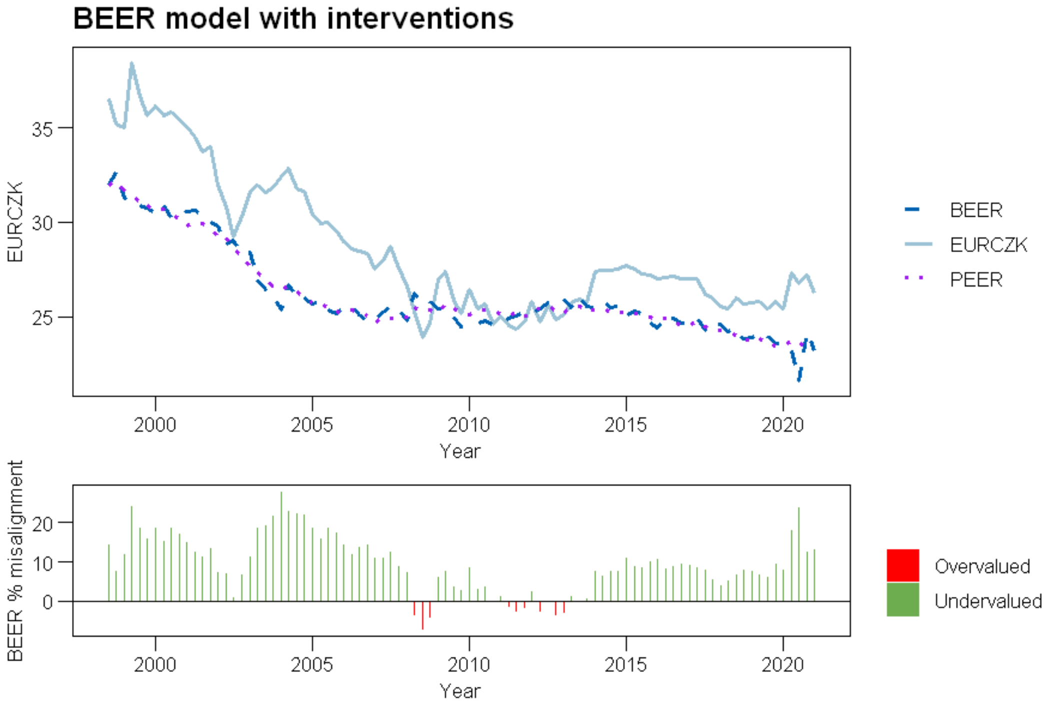

Figure 7. The BEER model results

The first equation to be estimated for the FEER model estimate is the real import function. For the purpose of robustness check, trade equations are estimated using both Engle-Granger procedure and Johansen approach.

OLS estimates of the real import and real export functions are:

$$ln(IM_{R,t}) = -18.3714225477267 + 2.3609959013236\, ln(Y_t^D) - 0.300415817846581\, ln(ER_{R,t}) \quad (17)$$

$$ln(EX_{R,t}) = -18.3714225477267 + 2.3609959013236\, ln(IM_t^F) - 0.942384460903691 * ln(ER_{R,t}) \quad (18)$$

The cointegration test results are recorded in Table 8. The test statistic of import function is equal to -9.54 and the null hypothesis of no cointegration is rejected on 5% confidence level. However, the null hypothesis of no cointegration is not rejected in the case of export function.

Table 8 EG cointegration test trade equations

Type	Lag	EG	p-value
Real import	1	-9.54	0.01
Real export	1	-2.59	≥ 0.10

In Johansen VEC specification, dummy variables are used to account for the COVID periods *(2020Q2, D_2020Q3*), the Global Financial Crisis (*2008Q4*) and

2004Q2 accounts for the accession of the Czech Republic to the European Union (May 2004) that meant a sharp rise of exports and imports.

The Johansen procedure starts by running the VAR in levels model. The information criteria suggest lag length 1 for both models, except AIC that suggests 8 lags in VAR specification for the import function and 9 lags in export function. Both models use 1 lag in VEC specification[1] .

The number of cointegration vectors using the Trace statistic is tested in Table 9. The test indicates the presence of one cointegration vector in both models.

Table 9 Significance of cointegration vectors real import

	Real import		**Real export**	
H_0:r ≤	**Trace statistic**	**Trace 5% crit**	**Trace statistic**	**Trace 5% crit**
0	49.95	34.91	60.15	34.91
1	16.73	19.96	10.67	19.96
2	3.61	9.24	3.40	9.24

The estimates of the long-run equation are:

$$ln(IM_{R,t}) \ = \ -10.748044\ [\,2.07] + \ 1.985192\ ln(Y_t^D)\ [-6.74] \ - \ 1.057715\ ln(ER_{R,t})\ [\,2.98] \quad (19)$$

$$ln(EX_{R,t}) = -6.379082\ [\,1.13834] + \ 1.768015\ ln(IM_t^F)\ [-5.82412] - 1.435043\ ln(ER_{R,t})\ [\,3.03928] \quad (20)$$

Both methods give similar results. Czech real imports are positively related to Czech GDP and negatively related to the real exchange rate. The results are as expected; a real depreciation results in decline of real import because foreign goods becomes relatively more expensive. The graphical representation of the results is captured in Figure 9. Residual diagnosis (see Table 10) indicates there is no serial correlation. However, the error term is not multivariate normal on 5% significance level.

The Czech real exports are positively related to foreign demand, represented by foreign real import. However, the results imply, that the Marshall-Lerner condition does not hold in Czech economy, real depreciation does not improve net exports. The model implies that real depreciation worsens real exports. Residual diagnosis (see Table 10) indicate that there is serial correlation. However, choosing higher order model does not improve the BG test result.

Although the EG test rejected cointegration in the export function, OLS estimates are preferred because the VEC model estimates do not have good statistical properties. Folprecht [13] shows that the EG test has small power when the loading coefficient

[1] The model was limited by the software. R functions such as ca.jo do not allow using 0 lags in VĚC model.

is low in absolute value. It may therefore falsely fail to reject the null hypothesis when the Trace test rejected.

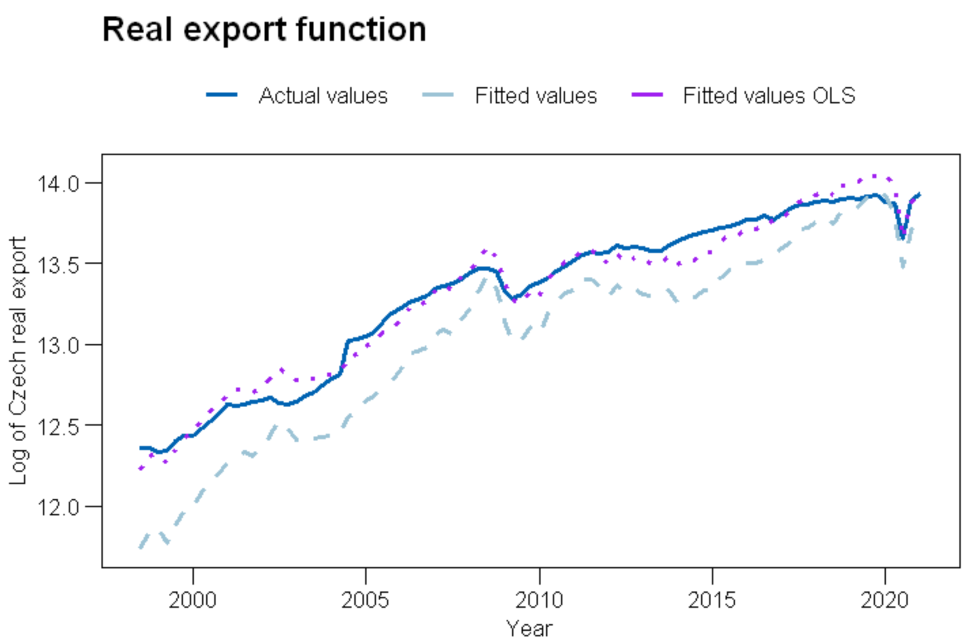

Figure 8. Czech real export

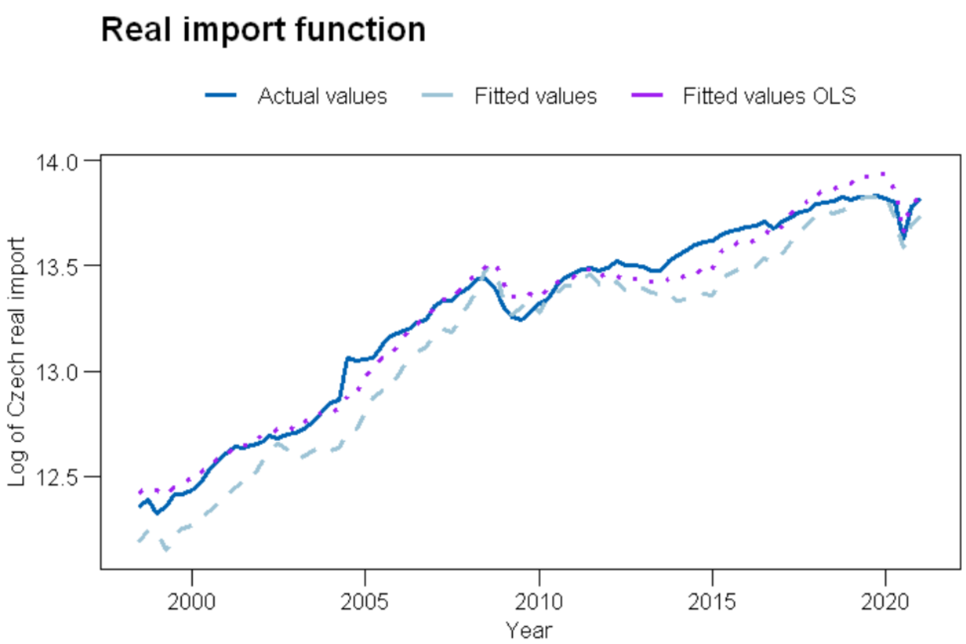

Figure 9. Czech real import

Table 10 Residual diagnosis trade equations

Test	BG(1)	BG(2)	BG(3)	BG(4)	JB
Real import p-value	0.2405	0.4346	0.5077	0.4879	0.04669
Real export p-value	0.03853	0.1251	0.08189	0.0207	0.1053

Next, the sustainable current account function is estimated. The current account balance to nominal GDP ratio is modelled as a function of relative real GDP per capita in PPP of Czech Republic and Eurozone (CONV).

$$\frac{CAB_t{}^S}{Y_{N,t}{}^D} = -19.0611737108153 + 22.9882481071494\, CONV_t \qquad (16)$$

The Engle-Granger cointegration test rejects the null hypothesis of no cointegration on 5% signifikance level (see Table 11). The resuls are es expected, a higher relative GDP per capita results is current account improvement.

Table 11 EG cointegration test trade equations

Type	Lag	EG	p-value
Sustainable CAB	1	-7.08	0.01

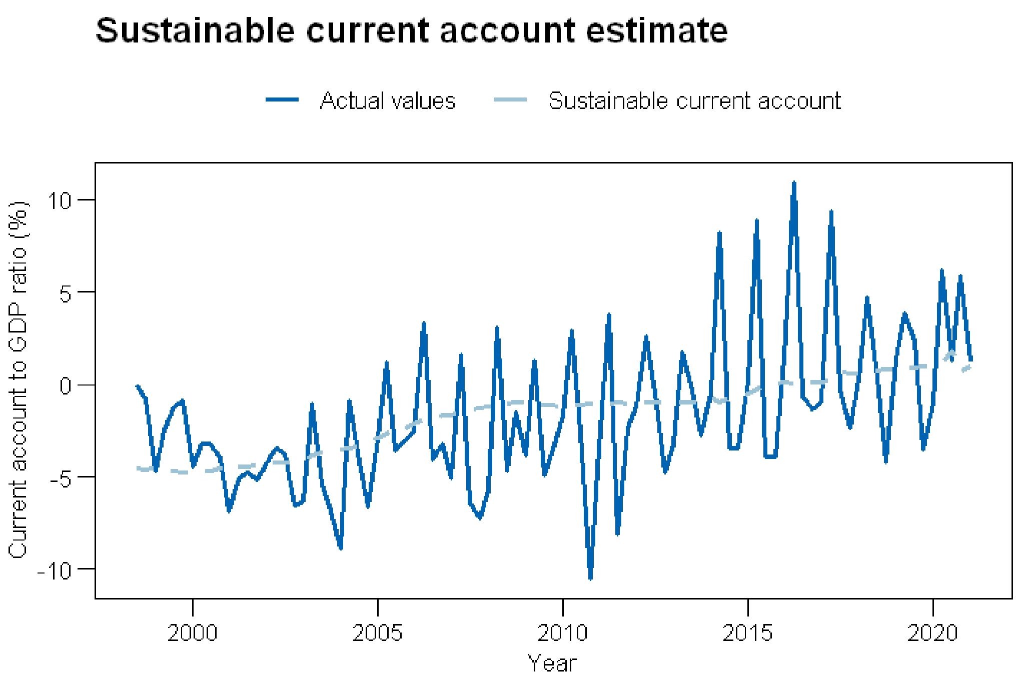

Figure 10. Sustainable current account estimate

The FEER model (equation 8) is solved using the method of interval halving. OLS estimates of the trade equations were used. The results are in Figure 11. The model indicates that the Czech koruna was undervalued during the first half of the intervention period declared in 2013. However, the FEER model results should be interpreted with caution.

The first reason is economic theory. The model implies that the Marshall-Lerner condition does not hold in Czech economy. Although it may be the case, in would indicate that the Czech foreign exchange market is deformed. It is usually assumed that the condition holds in the long run. The model assumes that causality goes from exchange rate to export. But also the opposite causality exists – a rise in export causes higher supply of foreign currency and domestic currency depreciation.

The second reason is that the statistical properties of the trade equations were not good. The EG test suggested there is no cointegration in the export function. However, the result can be attributed to low power of the test. The Johansen VEC specification, which was no used in the FEER estimate, involved many structural breaks in order to account for the accession of the Czech Republic to EU, the Global Financial Crisis and the COVID periods. Moreover, the residual diagnosis indicated there is serial correlation in the error term which could not be eliminated by choosing higher order model. Next, the export and import data was probably a subject to many methodological changes as the Czech and European methodic of measuring national accounts changed (ESA 2010 etc.) so that the data may not be reliable.

Finally, the model assumes that the foreign demand is represented by Euro area demand. Although it represents the current situation good because most export of the Czech Republic is oriented to European Union, it was not the case in the 1990´s and early 2000´s. The Czech export was oriented more towards eastern post-communist countries in the former Council for Mutual Economic Assistance.

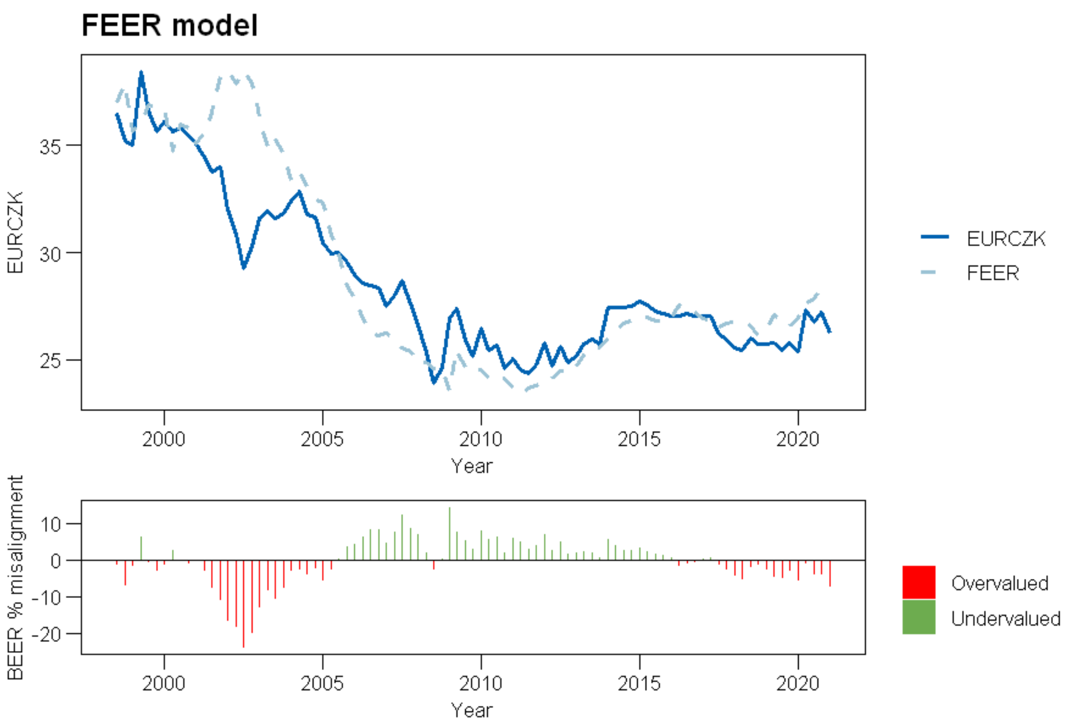

Figure 11. The FEER model results

The consolidated results of the models, represented by an equilibrium band, are captured in Figure 12. Both models indicate that the claim of the Czech National Bank that the interventions declared in 2013 moved the exchange rate to equilibrium was not true. On the contrary, market participants who invested to Czech koruna were right because the Czech koruna was undervalued in that period. Currently, the band indicates that the exchange rate is in the equilibrium zone. However, the BEER results are preferred. The BEER indicates that the currency is still undervalued.

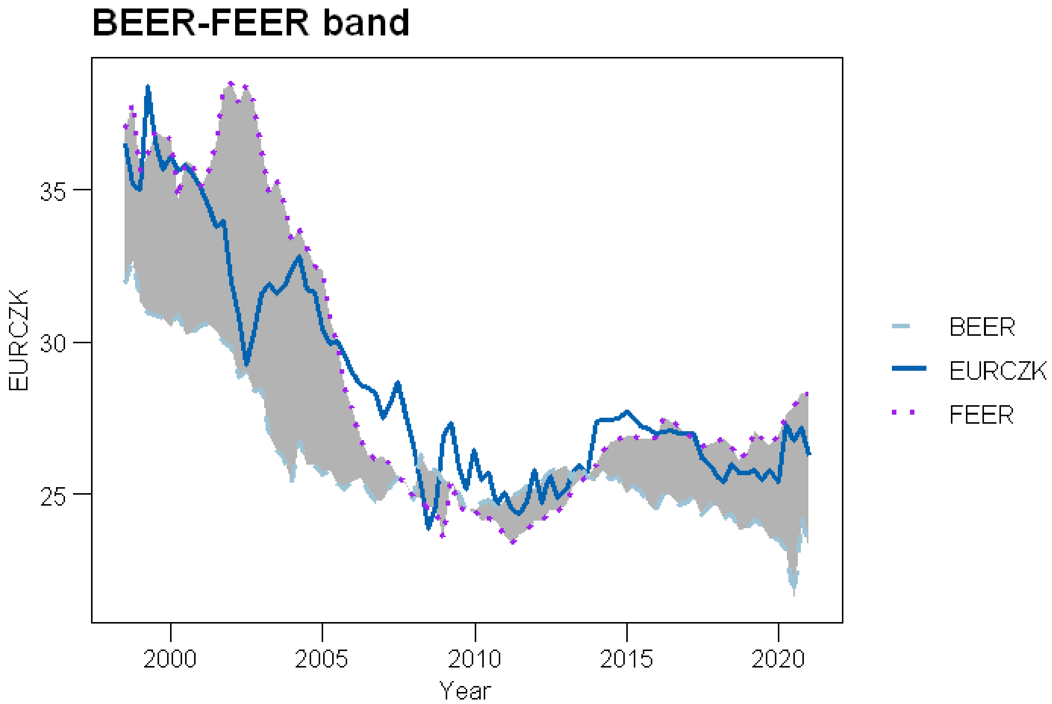

Figure 12. The consolidated results

6. Discussion

Finally, Problems with estimating the Czech export function are documented also in Obešlo [28].

For further research, it would be beneficial to uncover the puzzle concerning the role of exchange rate in the Czech export function and to formally test the Marshall-Lerner condition using the trade demand elasticities.

As for financial engineers, it could be beneficial to test the predictive power of equilibrium band. It is documented that the foreign exchange market is very efficient and the predictive power of the (often overparametrized) econometric models is equivalent to a random walk forecast (the hypothesis was claimed by Meese and Rogoff [27]). The band is a robust estimate because it uses multiple models and divides the space into three zones, equilibrium zone, overvaluation zone and undervaluation zone. It may therefore be used to indicate the future direction of the exchange rate, instead of the exact value.

7. Conclusion

The research dealt with the modification of equilibrium exchange rate models for the Czech economy. The BEER model was extended by using the interventions of Czech National Bank as an exogenous shock. However, the variable was not statistically significant. The FEER model was modified to respect the transforming nature of Czech economy in the so-called convergence process. The estimated sustainable current account level allowed the current account to be negative in the first stages of the economy development and zero in the post-transitive stage of the economy. The negative current account balance was in a medium-term equilibrium because the economy needed to import foreign capital and know how in order to rebuild the economy devastated from the communist era and converge to developed countries. The research uncovered that the FEER model has a very important drawback, its strong assumptions regarding the role of the exchange rate in determining the foreign trade volume. The econometric methods indicated that there is no long-term relationship of the real export and the real exchange rate or that the real depreciation worsens real exports, the violation of Marshall-Lerner condition. The BEER model results are therefore preferred. Both models indicated that the Czech National Bank interventions declared in 2013 could not be justified as a move towards equilibrium exchange rate. The currency was undervalued in that period. However, the Czech National Bank´s interest was different. The Czech economy was threatened by deflation and the exchange rate depreciation was supposed to increase consumer prices.

Appendix

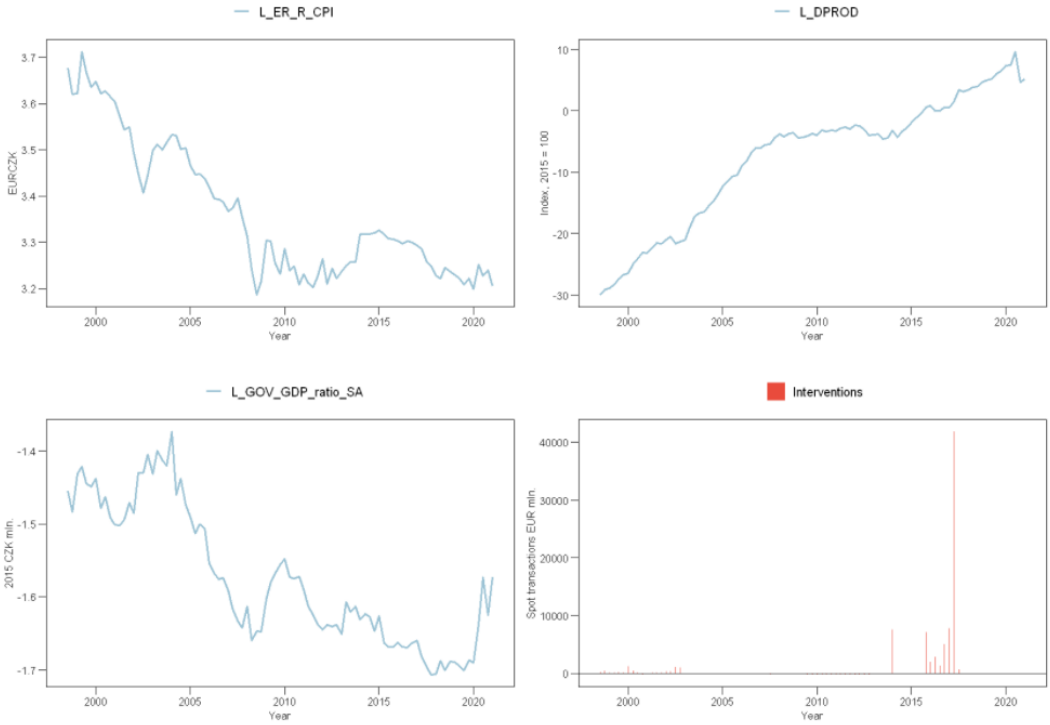

Figure 13 BEER model variables

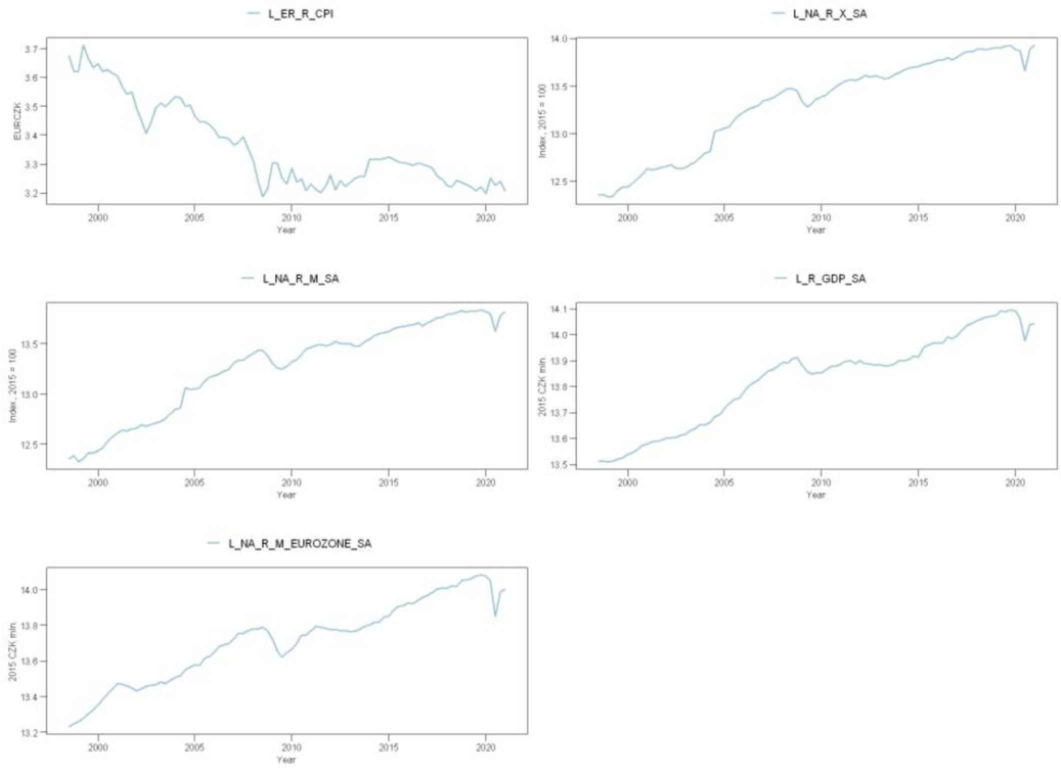

Figure 14 FEER model variables

An appendix, if needed, should appear before the acknowledgments.

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