The Macroeconomic Consequences of Government Spending (Re)Allocation

Neville Francis* Michael T. Owyang† Khushboo Thakkar‡

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Abstract

In this paper, we examine whether changes in the composition of government purchases impact macroeconomic outcomes. To do this, we employ a Factor-augmented Vector Autoregression (FAVAR) model across six spending categories, aiming to estimate the slope and curvature factors that represent the distribution of spending. Specifically, we concentrate on the slope factor, known as the reallocation factor, which captures shifts in spending proportions across categories. Our findings indicate that a reallocation from state and local government consumption to other fiscal components results in an increase in total spending and a modest rise in output, all without exacerbating the fiscal deficit. These results underscore the significance of the composition of spending in shaping fiscal policy outcomes.

*Department of Economics, University of North Carolina, Chapel Hill and NBER.
†Research Division, Federal Reserve Bank of St. Louis.
‡Department of Economics, University of North Carolina, Chapel Hill.

1 Introduction

Fiscal policy has become a focal point in policy debates since the 2008 Global Financial Crisis. The crisis caused a significant drop in aggregate demand, primarily due to the erosion of household wealth, while monetary policy constraints—such as the zero lower bound—limited the ability to stimulate recovery, necessitating substantial fiscal intervention. Effective fiscal spending required a strategic focus on areas that would most effectively enhance incomes and employment across the economy. Consequently, adjustments in spending on areas such as infrastructure, defense, and health, whether during times of crisis or stability, alter the composition of the government fiscal purchases.

In this paper, we underscore the importance of a cross-sectional shift in the composition of spending across various fiscal spending categories. Recent literature distinguishes between the fiscal multipliers arising from different categories of spending. The aggregate multiplier for the total government purchases would be approximately a weighted average of the multipliers of the components (Boehm, 2020). The effectiveness of the fiscal policy therefore does not only depend on the size but also on the composition of the basket of products purchased by the government (Ramey and Shapiro, 1998; Muratori et al., 2023).

Consider an exogenous shock impacting spending on any individual component. Such a shock also influences expenditures across other components due to inherent correlations, altering the overall spending level (size). If this change is substantial, it shifts the distribution of spending across components, thereby modifying the composition of the government's expenditure basket. For instance, a considerable rise in military expenditures driven by escalating international conflicts, such as those in the Middle East, the war in Ukraine, and other global security threats, might reduce the relative proportion of spending dedicated to education and healthcare. Such shocks to individual spending components are effectively composite shocks, which we decompose into two types: (1) a level shock that affects spending across all categories while keeping their relative proportions constant, and (2) a reallocation shock that shifts spending across components without changing the total expenditure.

We start by calculating the share of total spending allocated across each of the various fiscal components. Employing a Factor-augmented Vector Auto-regression (FAVAR), we estimate two factors that encapsulate the distribution of spending among the components. The first factor, known as the reallocation (or slope) factor, captures variations in component shares. An increase in one component's share necessarily results in a decrease in the shares of others, as these must collectively sum to one. The second factor captures the curvature of the distribution, reflecting changes in its shape. It indicates shifts in the relative significance of smaller versus larger shares within the distribution. Thus, a shock to the reallocation factor allows us to assess the macroeconomic implications of changes in the cross-sectional distribution of spending.

Examining six disaggregated components, including federal defense and non-defense consumption and investment as well as state and local government consumption and investment, we analyze the impact of a level shock and a reallocation shock as described above. A level shock, defined as a shock to the sum of spending across components without altering component shares, results in increased spending across all categories, leading to a corresponding rise in the fiscal deficit and output. These findings align with the existing literature. Furthermore, although the computed level multiplier cannot be directly compared with the government spending multiplier in the literature since it keep the distribution unchanged contemporaneously, it remains close to reported values. We find a level multiplier of 0.45 over a 20-quarter horizon, while Auerbach and Gorodnichenko (2012) finds a 0.57 multiplier over the same period. Ramey (2011) estimates multipliers ranging from 0.6 to 1.2, depending on the sample.

A reallocation in spending away from the largest component, state and local government consumption, raises the proportion of spending in other areas. This shift in distribution results in increased government spending without imposing a higher fiscal burden, as it does not significantly raise the fiscal deficit. While the output response is statistically insignificant, this reallocation results in a modest increase in output, underscoring potential gains from redistributing spending across different components.

Analyzing the reallocation of spending across multiple components provides a more comprehensive view of fiscal dynamics. An N-component approach aligns more closely with real-world policy decisions, where trade-offs are rarely binary and often involve nuanced adjustments across various sectors, such as defense, education, and healthcare. Furthermore, in a two-component system, a reduction in one component automatically increases the other proportionally, without any ambiguity about where the funds are redistributed. However, with $N-1$ additional components, a reduction in a component's share is redistributed among the remaining components based on their specific weights and interdependencies. This enables us to identify at a more granular level which component shares increase or decrease and which shares likely drive the observed outcome changes.

This paper provides empirical evidence supporting the importance of government spending composition in improving fiscal policy outcomes. Adjusting the allocation of spending across components, while keeping total spending constant, can produce varied effects on output. Such shifts in spending patterns over time may partly explain differences in aggregate multipliers across periods, offering policymakers insights on optimizing fiscal strategies within budget constraints.

The rest of the paper is organized as follows: Section 2 reviews the relevant literature that grounds our research. Section 3 details our empirical setup and section 4 covers the data utilized. Section 5 presents and discusses the results, and section 6 concludes.

2 Related Literature

This paper is related to the vast literature evaluating the factors influencing the effectiveness of the fiscal policy. Specifically, this effectiveness depends on factors including the state of the economy (Auerbach and Gorodnichenko, 2012; Laumer and Philipps, 2020), the type of financing of fiscal measures (Hagedorn et al., 2019), monetary policy response (Woodford, 2011), and country characteristics such as level of development, exchange rate regime, openness to trade, and public indebtedness (Ilzetzki et al., 2013). Additionally household heterogeneity (Flynn et al., 2022), firm size heterogeneity (Juarros, 2020), and the composition of fiscal purchases (Bouakez et al., 2020; Boehm, 2020; Muratori et al., 2023) also play significant roles. This paper introduces an empirical framework to compute dynamic responses from shifts in the distribution of spending between any number of components, in contrast to previous studies (Bouakez et al., 2020; Boehm, 2020; Muratori et al., 2023), which analyze shifts between two specific components.

Secondly, this paper contributes to the literature documenting variations in multipliers across different government spending components. Auerbach and Gorodnichenko (2012), Ellahie and Ricco (2017), Laumer and Philipps (2020) report higher multipliers for government investment as compared to government consumption, whereas Perotti (2004) and Boehm (2020) find the opposite, with higher multipliers for government consumption. Additionally, Baxter and King (1993), Auerbach and Gorodnichenko (2012), Barro and De Rugy (2013), Ellahie and Ricco (2017), Laumer and Philipps (2020) report lower multipliers for defense spending, as it does not directly influence the production or utility functions within the economy. Lastly, state and local government spending has been shown to have a higher multiplier than federal government spending components (Ellahie and Ricco, 2017; Laumer and Philipps, 2020). However, Clemens and Miran (2012) finds that state and local expenditures tend to be pro-cyclical due to balanced budget requirements, resulting in multipliers below 1. This finding aligns with Ricardian effects, where households, anticipating future taxes to offset government debt, choose to save rather than spend. At a more granular level, Ellahie and Ricco (2017) demonstrates that state and local government consumption has higher multipliers compared to federal defense and non-defense consumption. In contrast, our findings suggest that a reduction in the share of state and local government consumption contributes to increased total spending without increased fiscal burden and a modest, though insignificant, rise in output.

3 Econometric Framework

3.1 Model Specification

Let Z_t be an $(M-1)\times 1$ vector of observable macroeconomic variables such as government revenue, GDP, employment, etc on which we aim to estimate the effects of a shift in the composition of the spending components. Let us assume that there are N spending categories, denoted as $G_{1,t}$, $G_{2,t}$, $\ldots$, $G_{N,t}$, which satisfy $\sum_{i=1}^{N} G_{i,t} = G_t$ where G_t is the total government spending. When N is small, the VAR model proposed in Francis et al. (2024) can be applied to estimate the impact of reallocation between N components. For $N=2$, this model is detailed as follows:

$$\begin{bmatrix} G_t \\ s_{1,t} \\ Z_t \end{bmatrix} = C + \Psi(L) \begin{bmatrix} G_{t-1} \\ s_{1,t-1} \\ Z_{t-1} \end{bmatrix} + u_t \tag{1}$$

$$u_t \sim N(0, W)$$

Here, $s_{1,t}$ denotes the share of the component $G_{1,t}$ in G_t. C represents a constant term and a quadratic trend. $\Psi(L)$ represents the conformable lag polynomial of finite order δ and u_t is a $(M+1)\times 1$ vector of reduced form residuals that are normally distributed with a mean 0 and a non-diagonal variance covariance matrix W. An unexpected increase in $s_{1,t}$ would imply a decrease of the same magnitude in $s_{2,t}$, share of the second component $G_{2,t}$ in G_t, thereby capturing the effect of reallocation - cross-sectional shift in the distribution towards $G_{1,t}$ from $G_{2,t}$. Meanwhile, a surprise change in G_t captures a change in the total spending level which is the level effect. A composite shock to any component $(G_{i,t})$ triggers a change in total spending (level effect) and a change in the proportion of spending across components (reallocation effect). The above framework can be used to understand the macroeconomic impact from a change in the proportion of spending in a component.

Likewise, for $N = 3$, we can revise the above specification to include the shares of two out of the three components, $s_{1,t}$ and $s_{2,t}$ in place of just $s_{1,t}$. Thus, an unexpected change in either of the shares would cause adjustments in the distribution of spending across all the three components. However, as N gets large, the number of parameters to be estimated increase quadratically.[1] To conserve degrees of freedom, we extract two dynamic factors that are representative of the information contained in the spending components.

Let G_t represent the total spending, which is distributed across N components such that $\sum_{i=1}^{N} G_{i,t} = G_t$. To estimate shifts in spending across these components, we first calculate the share of each component $G_{i,t}$ as $s_{i,t} = \frac{G_{i,t}}{G_t}$. These individual shares are then successively accumulated to form a series of monotonically increasing cumulative shares: $g_{1,t} = s_{1,t}$, $g_{2,t} = s_{1,t} + s_{2,t}$, up to $g_{N,t} = 1$. We construct χ_t as an $(N-1) \times 1$ vector that comprises these cumulative shares, excluding $g_{N,t}$ since $g_{N,t} = 1$ for all t. Using these cumulative shares as our informational series, we estimate two latent factors. The first factor represents the slope, or reallocation effect, which reflects shifts in the distribution of spending across the N components. The second factor captures the curvature of the distribution, highlighting changes in the relative significance of intermediate categories. [2] The level effect continues to be represented by changes in total spending G, consistent with the model given by equation (1).

Employing a factor-augmented vector autoregression, as introduced by Bernanke et al. (2005), we assume that the cumulative shares in χ_t are related to the latent factors f_t and observable macroeconomic factors Y_t as follows:

$$\begin{bmatrix} \chi_t \\ Y_t \end{bmatrix} = \begin{bmatrix} \Lambda^f & \Lambda^Y & 0 & \dots & 0 \\ 0 & I & 0 & \dots & 0 \end{bmatrix} \begin{bmatrix} f_t \\ Y_t \\ \vdots \\ f_{t-p+1} \\ Y_{t-p+1} \end{bmatrix} + \begin{bmatrix} e_t \\ 0 \end{bmatrix} \tag{2}$$

[1] For a VAR(p) model with D variables, we need to estimate $D^2 \times p$ coefficients for the lagged variables, plus D intercepts.

[2] With only two components, it suffices to focus on the level and the share of one component, as these capture both the average spending (level) and the direction (slope/share) of the expenditure distribution.

$$e_t \sim N(0, R)$$

where f_t is a $K \times 1$ vector of latent factors, with $K = 2$ in our case. $Y_t = \begin{bmatrix} G_t & Z_t' \end{bmatrix}'$ is an $M \times 1$ vector of total government spending and other observable macroeconomic variables. Λ^f is an $(N-1) \times K$ matrix of factor loadings, and Λ^Y is an $(N-1) \times M$ matrix, which we set to 0. We assume that χ_t can be written as a linear combination of K unobservable factors. Since Y_t is observable, I is an $M \times M$ identity matrix. e_t is an $(N-1) \times 1$ vector of normally distributed error terms with mean zero and R as the diagonal covariance matrix.

Defining $F_t = \begin{bmatrix} f_t' & Y_t' \end{bmatrix}'$, the transition equation follows a VAR(1) process, expressed as:

$$\begin{bmatrix} F_t \\ \vdots \\ F_{t-p+1} \end{bmatrix} = \begin{bmatrix} \phi_1 & \phi_2 & \dots & \phi_p \\ I & 0 & \dots & 0 \\ \vdots & \ddots & & \vdots \\ 0 & \dots & I & 0 \end{bmatrix} \begin{bmatrix} F_{t-1} \\ \vdots \\ F_{t-p} \end{bmatrix} + \begin{bmatrix} v_t \\ 0 \end{bmatrix} \tag{3}$$

$$v_t \sim N(0, Q)$$

where ϕ_l is the $(K+M)\times(K+M)$ l-lag coefficient matrix for $l = 1, \dots, p$. v_t is an $(K+M)\times 1$ vector of reduced form errors with a mean of zero and covariance matrix Q, which we assume to be of full rank.

State-space models are powerful tools for analyzing dynamic systems, as they provide a flexible framework for capturing time-varying relationships. An alternative approach is to estimate a VAR using factors derived from principal component analysis (PCA). However, PCA factors can be difficult to interpret because their loadings cannot be easily restricted. In contrast, our model restricts the factor loadings, facilitates the interpretation of factors, and allows factors to be modelled as dynamic processes, capturing the evolving composition of fiscal purchases.

3.2 Factor Identification

Before we identify the structural shocks from the reduced form errors v_t in the VAR specification (3), we need to uniquely identify the factors and their corresponding factor loadings. We begin by arranging the individual shares (s_i) in ascending order, from smallest to largest, and then sequentially summing them to obtain the cumulative shares (g_i). We designate the first factor (f_1) to capture the reallocation effect across these cumulative shares. For this purpose, we set the loading on f_1 for g_1 as $\lambda^f_{1,1} = -1$ and impose that the differences between consecutive loadings satisfy $\lambda^f_{i,1} - \lambda^f_{i-1,1} > 0$ for $i = 2, \ldots, N-1$, ensuring a progressively increasing pattern for the loadings on f_1.

For the curvature factor (f_2), we assign the loading for g_1 as $\lambda^f_{1,2} = 0$. The restrictions on the factor loadings for g_1 in the measurement equation identifies the two factors against any rotations. Additionally, because the individual shares are ordered from largest to smallest, we specify that the differences between loadings on the curvature factor decrease. Specifically, we capture concavity by imposing the restriction $\lambda^f_{i,2} - \lambda^f_{i-1,2} < \lambda^f_{i-1,2} - \lambda^f_{i-2,2}$ for $i = 3, \ldots, N-1$. If the shares were instead arranged in ascending order, the differences between the loadings would need to increase to reflect convexity.

Employing cumulative shares in place of individual shares allows for capturing a reallocation effect across the entire spending distribution. For example, suppose all loadings on the first factor, f_1, are negative but follow an increasing trend. This configuration with individual shares would imply that all shares are uniformly declining, failing to capture any meaningful reallocation effect. In contrast, if cumulative shares have loadings that are negative but increasing, it signals that the first share has declined, while subsequent shares have increased, reflecting a redistribution from the first component to other components. Moreover, using cumulative shares enables the model to impose intuitive restrictions, such as increasing or decreasing differences, to capture curvature. In contrast, individual shares, even in an ordered sequence, make it challenging to capture convexity or concavity, as they lack the cumulative structure needed to highlight these differences.

3.3 Bayesian Estimation

We employ a likelihood-based Gibbs sampling approach to estimate the FAVAR model, following the methodology outlined in Bernanke et al. (2005). This iterative method provides an empirical approximation of the marginal posterior densities of both factors and parameters. As explained in Appendix B, the process involves sampling the factors based on the latest parameter draws, followed by sampling the parameters conditional on the most recent factor estimates.

The factors are drawn using the standard Kalman filter, in line with the literature (Kim and Nelson, 1999; Bernanke et al., 2005). After drawing the factors, we move on to sample the parameters in the measurement equation. Specifically, we draw the differences between successive loadings for each factor. The process of sampling the error variance-covariance matrix R follows standard procedures outlined in the literature. For f_1, these differences must be greater than zero, ensuring that the loadings on f_1 increase as the cumulative shares grow monotonically. Furthermore, the differences between loadings on f_2 should decrease (increase) when the shares are ordered in descending (ascending) order.

For the transition equation, we follow the methodology detailed in Chan and Jeliazkov (2009) to sample Q, adhering to the necessary restrictions for shock identification. Conditional on Q, the factors and the data, we then sample VAR coefficient matrix Φ. This process constitutes a single iteration, which we repeat 30,000 times, discarding the initial 15,000 samples.

3.4 Impulse Responses to Fiscal Shocks

Impulse Responses of Aggregates in Y

We turn our attention to estimating the dynamic responses of the variables in $F_t = \begin{bmatrix} f_t' & Y_t' \end{bmatrix}'$, which comprises the first $(K + M)$ equations of the VAR system, to surprise shifts in the level and reallocation factors. This requires making identifying assumptions that allow us to extract the innovations embedded in the reduced-form errors. Following Chan and Jeliazkov (2009), the $(K + M) \times (K + M)$ variance-covariance matrix (Q) of the reduced-form errors in equation (3) can be decomposed as:

$$\begin{aligned} Q &= L^{-1} D L^{-1\prime} \\ v_t &= L^{-1} D^{1/2} \varepsilon_t \\ \varepsilon_t &\sim N(0, I) \end{aligned} \tag{4}$$

ε_t is a $(K+M)\times 1$ vector of structural shocks that are mutually independent. D is a diagonal matrix with positive diagonal elements, and L is is a lower triangular matrix with ones on the main diagonal. Furthermore, we set the off-diagonal elements of L to zero for f_1, f_2 and G. These restrictions imply that a shock to G represents a pure level shock, as it does not contemporaneously alter the spending proportions across components. Conversely, a shock to f_1 can be interpreted as a pure reallocation shock, affecting the spending proportions across components without impacting total spending. This approach is particularly advantageous for analyzing exogenous shifts within distinct categories of government spending, without requiring an increase in the overall spending level.

For every $(d + 1)^{th}$ iteration, we estimate the corresponding diagonal and lower triangular matrices, $D^{(d+1)}$ and $L^{(d+1)}$, respectively, which then yields $Q^{(d+1)}$. Using the estimates of $Q^{(d+1)}$ and $\Phi^{(d+1)}$, we derive the impulse responses to both the level shock and the reallocation shock for each iteration, facilitating a comprehensive analysis of their effects on macroeconomic aggregates.

Impulse Responses of Cumulative Shares

After determining the impulse responses for the factors (f_t) and observable aggregates (Y_t), we calculate the impulse responses for the cumulative shares using the measurement equation (3), which linearly links the cumulative shares to the latent factors. For each $(d+1)^{th}$ iteration, we use the estimated factor loadings $\Lambda^{f^{(d+1)}}$ and corresponding responses of the two factors.

Impulse Responses of Individual Shares

The impulse responses of individual shares can be derived from the responses of the cumulative shares. For the first component, the impulse response is directly obtained from the first cumulative share since $g_1 = s_1$. For the remaining components, we calculate the impulse responses by subtracting the responses of consecutive cumulative shares, as $s_{i,t} = g_{i+1,t} - g_{i,t}$ for $i = 2, \ldots, N-1$. Given the imposed restrictions, a positive reallocation shock will result in a decrease in the share s_1. Since the loadings on the reallocation factors are increasing, the cumulative shares will also increase monotonically. This implies that the individual shares s_i for $i = 2, \ldots, N-1$ will rise, indicating a shift in spending distribution from G_1 to other components. Finally, because the sum of all individual shares must equal 1, the impulse response of the N^{th} component's share is obtained by taking the negative of the responses of the cumulative share $\left(g_{N-1} = \sum_{i=1}^{N-1} s_i\right)$.[3] We derive the impulse responses for the individual shares of each of the N components in every iteration.

Impulse Responses of Individual Components

Having ascertained the impulse responses for both individual shares and aggregate spending, we compute the impulse responses for spending within each components using the equation: $G_{i,t} = s_{i,t} \times G_t$

When considering all three variables in natural logs, the impulse response of spending in each component is the sum of the impulse responses of its share and total spending.

[3]The sign of the responses of s_N cannot be determined as it would depend on the sign of the responses of $g_N - 1$.

However, because the responses of individual shares represent percentage point changes, we normalize these by dividing by the average share over the sample period to convert them into percentage changes. As before, we compute the impulse responses for each component (G_i) in every iteration.

Lastly, we display the median impulse responses of all variables along with their respective 68% credible sets.

3.5 Multipliers

While impulse responses offer insight into the dynamic effects of reallocation, we also employ multipliers to quantify these effects more broadly, providing a complementary perspective on the overall impact. A fiscal multiplier measures the change in output for a unit increase in fiscal spending. The multiplier can be assessed by examining the output response at a specific point in time following an initial movement in government spending, which is also referred to as the impact multiplier (Blanchard and Perotti, 2002; Mountford and Uhlig, 2009; Ilzetzki et al., 2013). This approach does not account for any negative output responses that may develop over time. To address this limitation, the multiplier can alternatively be calculated by considering the output responses over an extended period, thereby reflecting how output adjusts to the evolving changes in government spending from the initial impact to the end of the horizon (Auerbach and Gorodnichenko, 2012; Laumer and Philipps, 2020). For comparison with the results of Ellahie and Ricco (2017), who provide multipliers for disaggregated fiscal components, we adopt their method. We calculate the multiplier in response to a shock to the level factor (f_1) by considering the net present value of the cumulative change in GDP per unit of additional government expenditure G, also discounted, over a horizon K, expressed as:[4]

$$Pure\ Level\ Multiplier(K) = \frac{\sum_{\kappa=0}^{K}(1+\bar{r})^{-\kappa} y_{\kappa}}{\sum_{\kappa=0}^{K}(1+\bar{r})^{-\kappa} G_{\kappa}} \times \frac{\bar{y}}{G} \tag{5}$$

Here, $\bar{r}$ denotes the average federal funds rate over the sample. y_{κ} and G_{κ} are the output and the aggregate government spending responses for a particular period κ. $\frac{\bar{y}}{G}$ is the sample mean of the GDP to government spending ratio. Since we consider these variables in natural logarithms, the impulse responses represent elasticities and are thus scaled by the sample mean ratio.

[4]The present value multipliers were introduced by Mountford and Uhlig (2009)

For calculating the present value multiplier resulting from a reallocation shock, specifically a shock to f_1, we obtain the impulse responses of G_1 as detailed in the earlier section. We consider the first component because an unexpected increase in the reallocation factor leads to a decrease in the share of the G_1. Meanwhile, the shares of remaining $N-1$ components increase in response to the reallocation shock, indicating a redistribution from G_1 to the others. Using $G_{1,\kappa}$ to denote the response of component G_1 in period κ to the reallocation shock, the multiplier can be computed as follows:

$$Pure\ Reallocation\ Multiplier(K) = \frac{\sum_{\kappa=0}^{K} y_\kappa}{\sum_{\kappa=0}^{K} G_{1,\kappa}} \times \frac{\bar{y}}{G_1} \tag{6}$$

In this equation, $\frac{\bar{y}}{G_1}$ represents the sample mean of the GDP to the fiscal spending component, G_1.

We compute the level and reallocation multipliers for every $(d+1)^{th}$ iteration using the impulse responses corresponding to that iteration. We report the median multiplier for different values of K along with the respective 68% credible sets in all cases.[5,6]

4 Data

The analysis covers the period from the first quarter of 1960 to the fourth quarter of 2022, utilizing quarterly data on total government purchases, which are further categorized into consumption expenditures and gross investment. Government consumption expenditures encompass the value of goods and services provided by the government free of charge, such as education and law enforcement, as well as purchases of military equipment. In cases where goods and services are provided at a subsidized cost, only the difference between the incurred costs and the revenues collected is recorded as government consumption expenditures. Government gross investment is defined as gross fixed capital formation, which includes investments made by government and government enterprises in physical assets such as structures (like highways), equipment and intellectual property products and investments made by the government on its own account.

The total government consumption expenditures and gross investments are each broken

[5] Additionally, we report multipliers with no discounting ($\bar{r} = 0$) as well.

[6] Given that the multiplier is influenced by the sample mean of the output-to-spending ratio, which can vary across different periods, we continue to use this formula to facilitate comparison with previous studies that employ the same approach.

down further into spending by the federal and state and local governments. At the federal level, expenditures are further broken down into defense and non-defense categories for both consumption and investment. Therefore, the data collected includes $N = 6$ key components: (1) Federal Defense Consumption Expenditures (DEFC), (2) Federal Defense Gross Investment (DEFI), (3) Federal Non-Defense Consumption Expenditures (NDEFC), (4) Federal Non-Defense Gross Investment (NDEFI), (5) State and Local Government Consumption Expenditures (SLGC), and (6) State and Local Government Gross Investment (SLGI). Furthermore, federal purchases do not incorporate the grants allocated to state and local governments.

The Y_t vector includes the natural logarithm of real government spending per capita (G_t), and the Z_t vector, which contains two variables as commonly used in the literature. To control for the financing side of fiscal shocks (Klein and Linnemann, 2023), we incorporate the fiscal deficit measured by the difference between government expenditures and and receipts as a percentage of GDP. Additionally, the Z_t vector contains the natural logarithm of real GDP per capita. All three variables in Y_t are detrended using a quadratic trend (Blanchard and Perotti, 2002; Ramey, 2011) before being included in the FAVAR model.

Each component is divided by total government spending to obtain individual shares. These shares are then arranged in descending order as follows: (1) $s_1 = SLGC/G$, (2) $s_2 = DEFC/G$, (3) $s_3 = SLGI/G$, (4) $s_4 = NDEFC/G$, (5) $s_5 = DEFI/G$ and (6) $s_6 = NDEFI/G$. Figure 1 shows the evolution of these shares over time from 1960 to 2022, with shaded gray bars indicating recession periods. The graph demonstrates the dominance of SLGC/G in total government spending after 1970, while NDEFI/G remains the smallest share across the entire sample period.

In summary, the informational series χ_t in the measurement equation is a 5×1 vector of cumulative shares: $g_1 = s_1,\ g_2 = s_1 + s_2,\ \ldots\ g_5 = \sum_{i=1}^{5} s_i$. The component g_6 is excluded since it equals 1 in every period t. The vector Y_t, a 3×1 vector of macroeconomic variables, includes the natural logarithm of real government spending per capita, the fiscal surplus as a percentage of GDP, and the natural logarithm of real GDP per capita. Given the quarterly frequency of the data, the number of lags is set to $p = 4$, in line with existing literature. Additional details on definitions and data sources can be found in Appendix B.

5 Results

We now comprehend the economic effects of an unexpected decrease in the share of the largest component (state and local government consumption) and the resulting adjustments in the remaining five shares.

5.1 Responses to a Level Shock

Figure 2.1 displays the median impulse responses of the two latent factors—reallocation and curvature—alongside total government spending (G), fiscal deficit, and GDP, following a shock to total government spending (G), with 16%–84% credible bands.

On impact, government spending rises by 0.8%, reaching a peak increase of 1% after four quarters, after which it gradually declines, though the increase remains statistically distinguishable from zero. Since a level shock does not contemporaneously affect the slope or curvature of the spending distribution, there is no immediate impact from the shock. As depicted in the figure, the reallocation factor begins to rise, peaks after three quarters, and then gradually declines; however, this response remains statistically insignificant throughout. The curvature factor shows no significant change, initially declining for a quarter, then peaking after three quarters before declining again.

The fiscal deficit rises on impact in response to a positive shock to government spending, reaching a peak increase of 0.3 percentage points before beginning to decline. This pattern closely mirrors the trajectory of G, indicating that short-term spending increases are financed through higher government debt. Output initially rises by 0.2%, remains near 0.2% for four quarters, and then begins to decline, with this significant increase in output dissipating after four quarters.

The output multipliers in response to this level shock are positive, diminishing over time as the increase in output declines, and are significantly different from zero only during the first two years, as shown in column (1) of Table 1. Over a five-year horizon, the discounted multiplier is 0.42 and is not statistically significant. Although this shock to G is not directly comparable to standard government spending shocks in the literature, our result is close to the 0.35 multiplier reported by Ellahie and Ricco (2017). Likewise, our non-discounted five-year multiplier of 0.56, shown in column (1) of Table 2, aligns with the 0.57 multiplier estimated by Auerbach and Gorodnichenko (2012), while Ramey (2011) finds that the government spending multiplier ranges from 0.6 to 1.2, depending on the sample analyzed.

Responses of Shares to a Level Shock

Figure 2.2 presents the median impulse responses of the five cumulative shares in response to a shock to G, accompanied by 16%–84% credible bands. Cumulative shares do not exhibit an immediate response, as the factors themselves do not react contemporaneously to a level shock. With loadings on the slope and curvature factors restricted to -1 and 0 for the first cumulative share, the impulse response of this share is equivalent to the negative of the reallocation factor's impulse response shown in Figure 2.2.

Given the imposed restrictions, the loadings on the reallocation factor are increasing, though negative, while those on the curvature factor are all positive, with the differences between consecutive loadings decreasing. Consequently, g_2, which has a relatively small loading on the curvature factor, exhibits impulse responses resembling those of the reallocation factor. In contrast, g_5-with a smaller absolute loading on f_1 and a moderately larger loading on f_2—aligns with the responses of the curvature factor, as depicted in Figure 2.1. Similar to the factors, the responses of these cumulative shares are not significantly distinct from zero. Thus, shifts in the reallocation factor are influenced by the larger shares, whereas variations in the curvature factor are driven by the smaller shares in the distribution.

As shown in Figure 2.3, the subsequent cumulative shares show progressively smaller declines, indicating that component shares must be increasing. The majority of the decline in s_1 is offset by an increase in s_2. Specifically, s_1 dips to a low of -0.5 percentage points after three quarters and then gradually returns toward zero. Conversely, s_2 peaks at an increase of 0.3 percentage points before declining back toward zero. Similarly, s_3 rises to a peak response of 0.1 percentage points before declining, while the responses of s_4 and s_5 remain close to zero. Neither of these fluctuations are statistically significant. Lastly, the response of s_6 reflects the inverse of g_5's response, as the shares collectively sum to 1.

Given the overall increase in total government spending, individual components also show an increase in spending, as shown in Figure 2.4. Each component increases by 0.8% on impact, corresponding to the response of G to its own shock. State and local government consumption (G_1) then begins to decline, reaching a trough at 0.2% before gradually rising again, mirroring the pattern of s_1's response and remaining statistically insignificant throughout.

The other components exhibit significant increases over longer horizons, following similar trajectories. Defense consumption and state and local government investment peak at approximately 2% after three quarters before beginning to decline, with the increase remaining significant from three to ten quarters. Federal non-defense consumption and defense investment increase by 1.2% and 1.4%, respectively, before gradually declining, with significance lasting sixteen quarters for the former and fourteen for the latter. The last component shows a steady but statistically insignificant increase of around 0.7–0.8%. Therefore, changes in individual shares drive spending variations across components.

5.2 Responses to a Reallocation Shock

With the constraints on the loadings for the first share ($\lambda^f_{1,1} = -1;\ \lambda^f_{1,2} = 0$), a shock to the reallocation factor translates to an exogenous reduction in the share of spending on the first component.

In Figure 3.1, a shock to the reallocation factor results in an immediate increase of 10 units, followed by a sharp decline to 2.6 units after one quarter. It subsequently rises again, peaking at 4 units before gradually trending back toward zero. This effect remains significant for up to 18 quarters. This shock induces an insignificant increase in the curvature factor. Since the curvature factor captures the relative significance of the smaller component shares, an increase would likely result from a greater influence of the smaller shares, a reduced influence of the larger shares, or a combination of both.

Since the change in the curvature factor is negligible, the increasing f_1 loadings for the subsequent cumulative shares lead to a rise in the shares of other components (s_i), indicating a reallocation of spending away from the first component toward the remaining components.

Government spending, unresponsive on impact, begins to rise, peaking at 0.6% before a slow decline, with this increase remaining statistically significant throughout. The fiscal deficit decreases by 0.5 percentage points on impact, resulting from a reallocation of spending from state and local government consumption toward other components, including government investment. Since investment spending unfolds over longer time horizons, this shift contributes to a short-term reduction in the fiscal deficit. As G rises in the first quarter, the deficit surges, then briefly dips below zero, and then stabilizes slightly above zero, remaining statistically insignificant. In contrast, GDP rises by 0.2% on impact, returning to its original level after the first quarter. Subsequently, GDP increases again, following a hump-shaped trajectory similar to that of G, though this overall increase remains statistically insignificant. The initial significant rise in GDP, despite no immediate change in total spending, can be linked to reduced spending on state and local government consumption and increased spending on other components.

The reallocation multiplier is negative, showing a downward trend across horizons, as shown in column (2) of Table 1. It is statistically significant only on impact, with a value of -0.40 over a 20-quarter horizon. This negative multiplier suggests that decreasing the share of state and local government consumption, while increasing the share of other components, could be beneficial.

When there are only two components, a multiplier above 1 suggested productivity, as an increase in one component's share would reduce the other. However, with $N = 6$ components, the output response depends on which specific component shares rise or fall in response to a reduction in the first component's share, as well as on the magnitude of these changes.

Responses of Shares to a Reallocation Shock

As shown in Figure 3.2, the median impulse responses of the five cumulative shares to a shock in f_1 are presented with 16%–84% credible intervals. The response of the first cumulative share, g_1, shows an initial drop of 10 percentage points on impact, followed by a substantial increase after one quarter, resulting in a 2.6 percentage point decline. This is followed by a gradual trend back toward zero, maintaining significance for up to sixteen quarters. With progressively larger loadings on f_1 across cumulative shares, each cumulative share exhibits a smaller initial decline but follows a similar trajectory: g_2 decreases by 4.7 percentage points, g_3 by around 2.7, g_4 by roughly 2.4, and g_5 by 1.7 percentage points. This pattern implies that component shares increase on impact, producing progressively smaller declines in the cumulative shares. Additionally, as we move from g_2 to g_5, the duration of this decline reduces: the decline in g_2 persists for approximately 12 quarters, in g_3 for around 10 quarters, in g_4 for about 8 quarters, and in g_5 for nearly 5 quarters.

As indicated in Figure 3.3, the component shares, excluding s_1, increase following the reallocation shock, in line with expectations. The federal defense consumption share initially rises by 5.3 percentage points on impact, declines to 1.3 percentage points, then increases over three quarters before gradually returning toward zero. This rise remains statistically significant for 16 quarters. The other shares exhibit similar response patterns: s_3 increases by 2 percentage point on impact, s_4 by 0.3 percentage points, and s_5 by 0.7 percentage points. The increases in these component shares remain significant for varying periods: the rise in s_3 is significantly different from zero for nearly 16 quarters, in s_4 for 2 quarters, and in s_5 for approximately 6 quarters. Lastly, federal non-defense investment mirrors the inverse response of g_5, with any decline in g_5 leading to an increase in s_6, ensuring that the component shares sum to 1. As a result, s_6 rises by 1.7 percentage points on impact and returns to baseline after six to seven quarters.

The responses in each component align directly with changes in their respective shares. The reduction in spending for state and local government consumption is outweighed by increases in other components, resulting in an overall significant rise in government spending in response to the reallocation shock.

G_1 initially declines by 23% on impact, following a pattern similar to its share. The initial sharp decline partially reverses, followed by a gradual decrease that returns to zero within 12 quarters, suggesting a potential reduction in federal support to state and local governments. On impact, defense consumption surges by 23%, state and local government investment by 17%, non-defense consumption by 3%, and defense investment by 10%, with these increases persisting for roughly 12-18 quarters. Lastly, non-defense investment by the federal government jumps by 39% on impact, returning to baseline after 10 quarters.

The shift in the spending distribution away from state and local government consumption contributed to the 0.2% increase in GDP on impact. This output increase trends back toward zero as the shares revert to their original baseline level.

In summary, reducing the share of state and local government consumption leads to increased shares across all other components. This reallocation significantly raises overall spending without a corresponding increase in the fiscal deficit, and it also boosts output. These findings suggest that a cross-sectional shift in the spending distribution can positively impact output, with outcomes depending on both the direction and magnitude of the shift.

5.3 Additional Results

In Appendix A, we arrange component shares in ascending order, defining them as follows: (1) $s_1 = NDEFI/G$, (2) $s_2 = DEFI/G$, (3) $s_3 = NDEFC/G$, (4) $s_4 = SLGI/G$, (5) $s_5 = DEFC/G$, and (6) $s_6 = NDEFC/G$. Consequently, the loadings on f_2 in equation (2) are constructed so that differences between consecutive loadings increase, reflecting the convexity of the distribution. A positive level shock elicits responses for G, the fiscal deficit, and GDP that are quantitatively similar to previous findings, with a level multiplier of 0.47 over 20 quarters (Table A1).

For the reallocation shock, the response of f_1 is relatively brief. As a result, a decrease in s_1 increases the share of spending on other components, but only for approximately one quarter. Following an initial rise, the largest of the five components (federal defense consumption) remains significantly reduced by 0.1%. This reduction in its relative importance leads to a sustained decline in the curvature factor. Consequently, s_6, representing state and local government consumption, increases. However, this rise does not fully compensate for the decrease in defense consumption and other components, resulting in an overall, though insignificant, decline in spending. Output sees an initial decline of 0.05%, primarily due to the reduced spending on federal non-defense investment (G_1).[7] The GDP response, however, becomes insignificant after the first quarter.

When shares are ordered in descending order, a reallocation shock significantly affects the reallocation factor; with ascending order, it primarily impacts the curvature factor. This variation occurs because the larger shares react more strongly to the reallocation shock, influencing the reallocation factor in descending order and the curvature factor in ascending order.

The results are influenced by which component undergoes the reallocation shock, the components that experience increases or decreases in response, and the degree of these shifts.

6 Conclusion

Optimal fiscal policy design requires strategically allocating spending across various sectors, such as education, healthcare, infrastructure, and defense, often within budget constraints. These constraints lead to trade-offs among spending categories, where an increase in spending for one area may reduce allocations for others. If output multipliers were identical across categories, changes in spending distribution would leave the aggregate multiplier unaffected. However, heterogeneous macroeconomic impacts of individual spending components result in varied multipliers across sectors.

Our study investigates how redistributing spending from one component to others impacts the overall fiscal structure, recognizing that a reduced proportion for one category does not always indicate a cut in spending. It could also reflect that spending in that category is growing at a slower rate than total government spending. Additionally, the proportions of different components shift at varying rates, and the rate of these changes also differs. Our novel framework captures both the shifts in component shares and the differing rates

[7] Ellahie and Ricco (2017) reports an impact multiplier of 4.97 for federal non-defense investment.

of change, using the slope (reallocation) factor and the curvature factor. It also reflects spending adjustments across components by considering both changes in component shares and the overall spending level.

Ordering the six main components of government spending from largest to smallest, we analyze the impact of reallocating funds from the largest—state and local government consumption—to the other five. This shift raises the shares of these five components, increasing total spending without adding to the fiscal deficit. It also leads to a significant increase in output in the first quarter, suggesting that reallocating spending away from state and local government consumption may be beneficial. Conversely, when reallocating from the smallest component, federal non-defense investment, we observe a significant fiscal deficit increase alongside a decline in output in the first quarter, implying that reducing non-defense investment may be favorable. This approach can be adapted to assess any number or type of disaggregated fiscal components.

Our findings suggest that considering the distribution and interaction of spending components can enhance fiscal policy effectiveness. Accounting for time variation in spending composition may help reconcile differences in multiplier estimates across periods. Future research should explore the use of a more flexible model that incorporates multiple spending components with fewer constraints.

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Figures and Tables

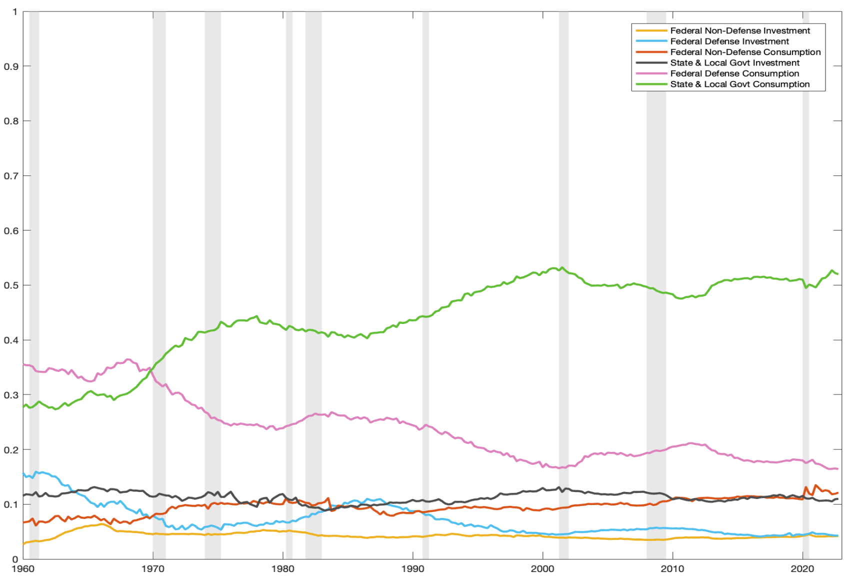


Figure 1: Components of Government Spending Fraction of Total Government Spending

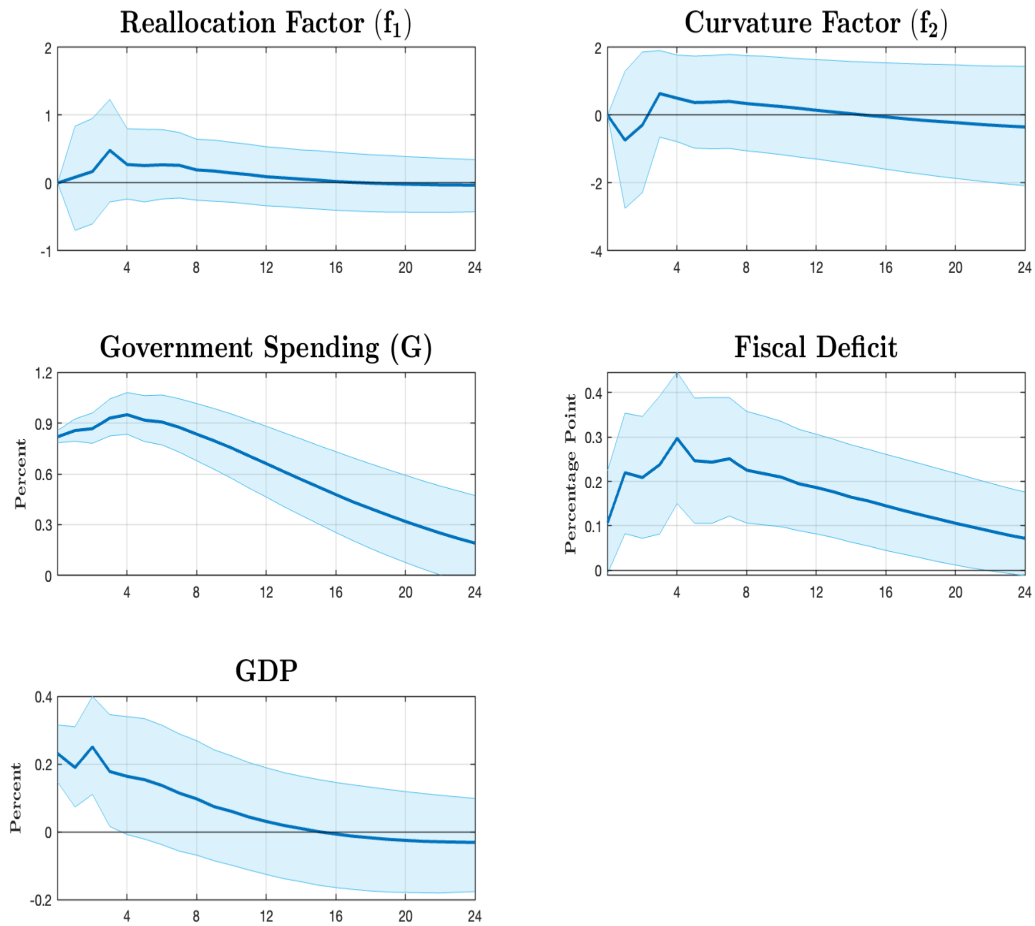


Figure 2.1: Impulse responses to a level shock (shock to G) generated from FAVAR with two factors described by equations (2) and (3). Here, the level (G) represents total government spending, which is the sum of all the six components considered. The individual shares are arranged in a descending order. Blue solid lines represent medians, and the shaded area represents the 68% credible bands.

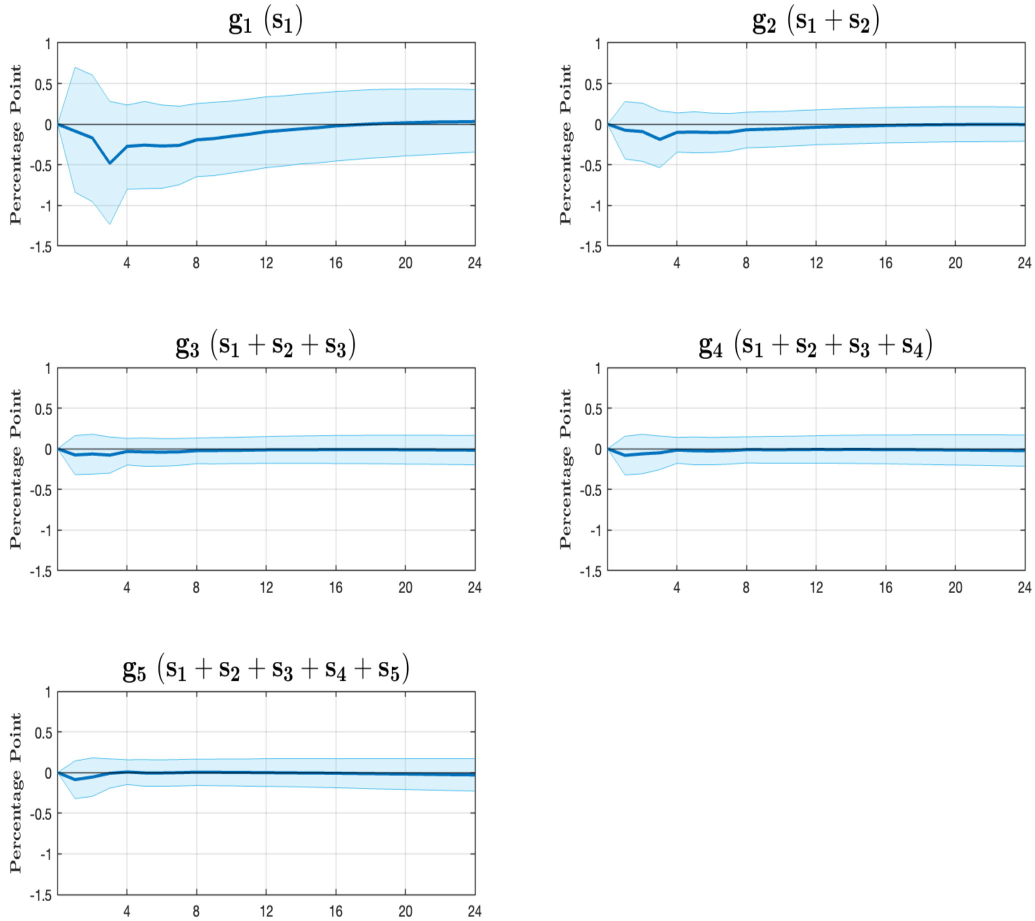


Figure 2.2: Impulse responses of cumulative shares (g_i) to a level shock (shock to G) generated from FAVAR with two factors described by equations (2) and (3). Here, the six individual shares arranged in ascending order are defined as follows - (1) $s_1 = SLGC/G$, (2) $s_2 = DEFC/G$, (3) $s_3 = SLGI/G$, (4) $s_4 = NDEFC/G$, (5) $s_5 = DEFI/G$ and (6) $s_6 = NDEFI/G$. Here, G represents total government spending, $SLGC$ is State and Local Government Consumption, $DEFC$ is Federal Defense Consumption, $SLGI$ is State and Local Government Investment, $NDEFC$ is Federal Non-Defense Consumption, $DEFI$ is Federal Defense Investment and $NDEFI$ is Federal Non-Defense Investment. The individual shares are arranged in descending order. Blue solid lines represent medians, and the shaded area represents the 68% credible bands.

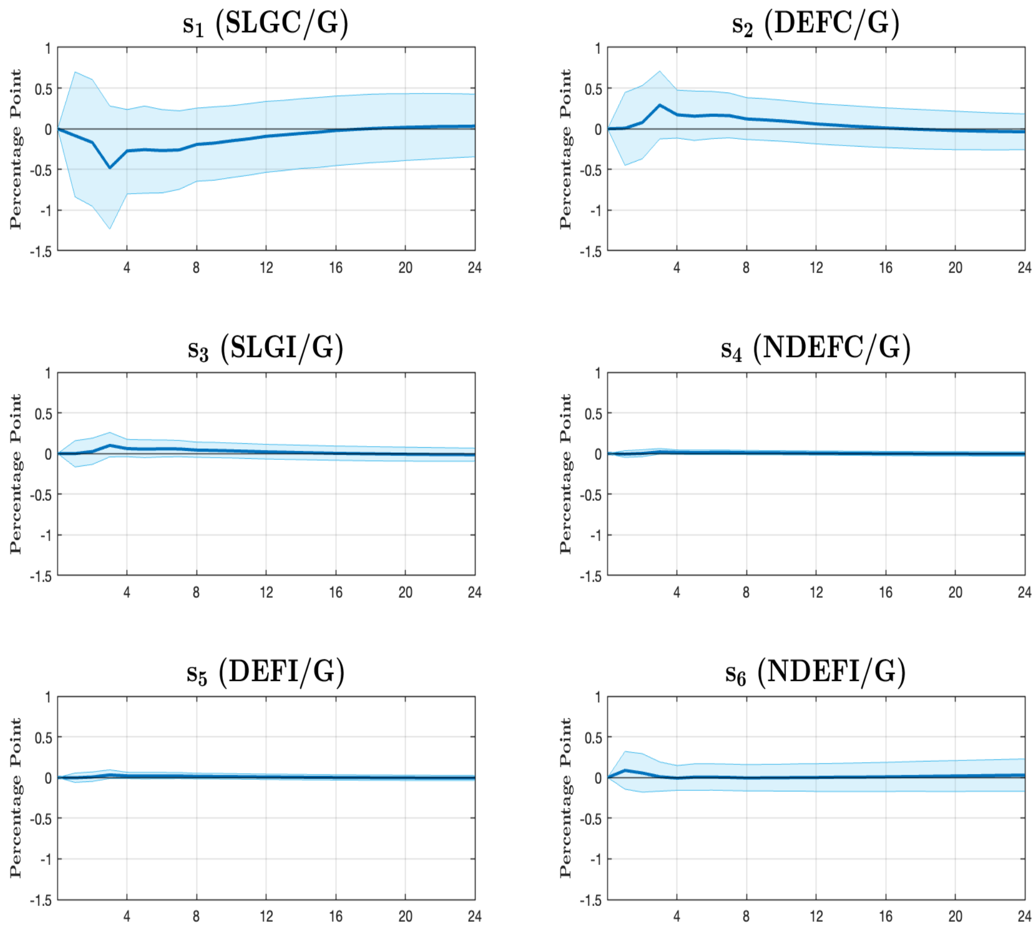


Figure 2.3: Impulse responses of individual shares (s_i) to a level shock (shock to G) generated from FAVAR with two factors described by equations (2) and (3). Here, G represents total government spending, $SLGC$ is State and Local Government Consumption, $DEFC$ is Federal Defense Consumption, $SLGI$ is State and Local Government Investment, $NDEFC$ is Federal Non-Defense Consumption, $DEFI$ is Federal Defense Investment and $NDEFI$ is Federal Non-Defense Investment. The individual shares are arranged in descending order. Blue solid lines represent medians, and the shaded area represents the 68% credible bands.

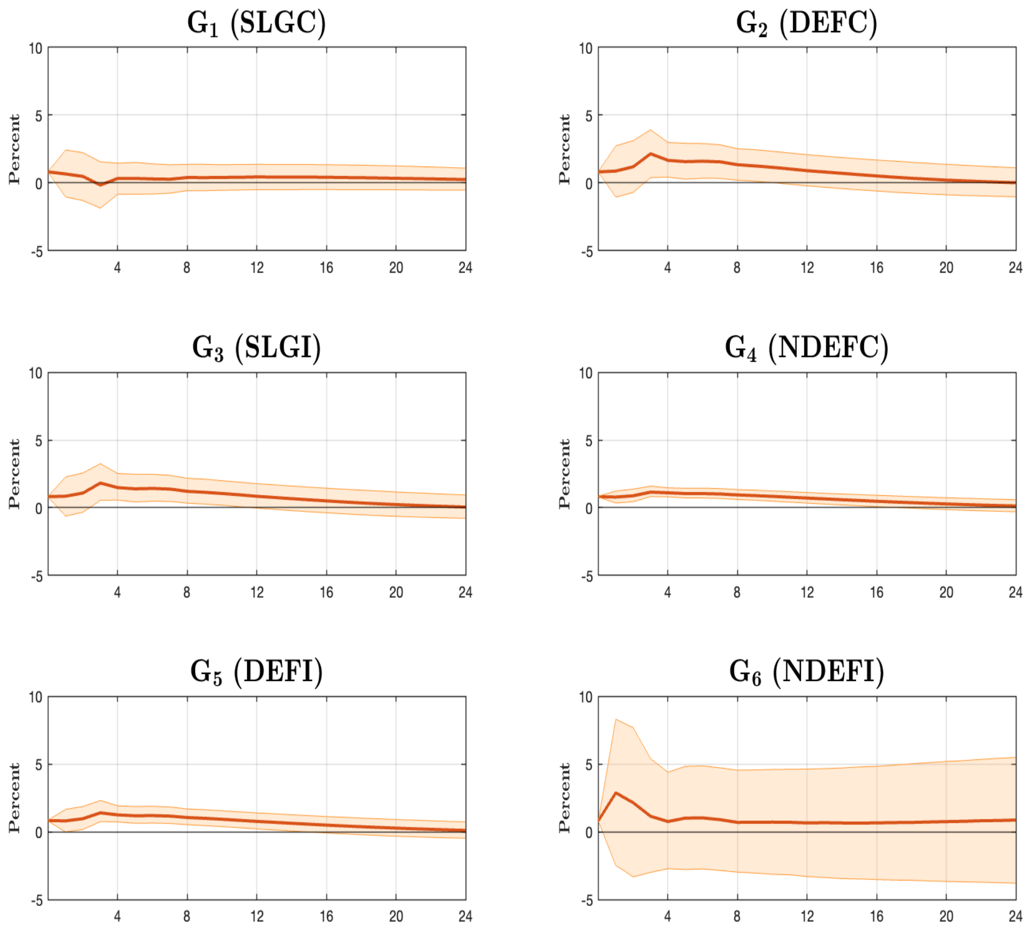


Figure 2.4: Impulse responses of spending on components (G_i) to a level shock (shock to G) generated from FAVAR with two factors described by equations (2) and (3). Here, G represents total government spending, $SLGC$ is State and Local Government Consumption, $DEFC$ is Federal Defense Consumption, $SLGI$ is State and Local Government Investment, $NDEFC$ is Federal Non-Defense Consumption, $DEFI$ is Federal Defense Investment and $NDEFI$ is Federal Non-Defense Investment. The individual shares are arranged in descending order. Orange solid lines represent medians, and the shaded area represents the 68% credible bands.

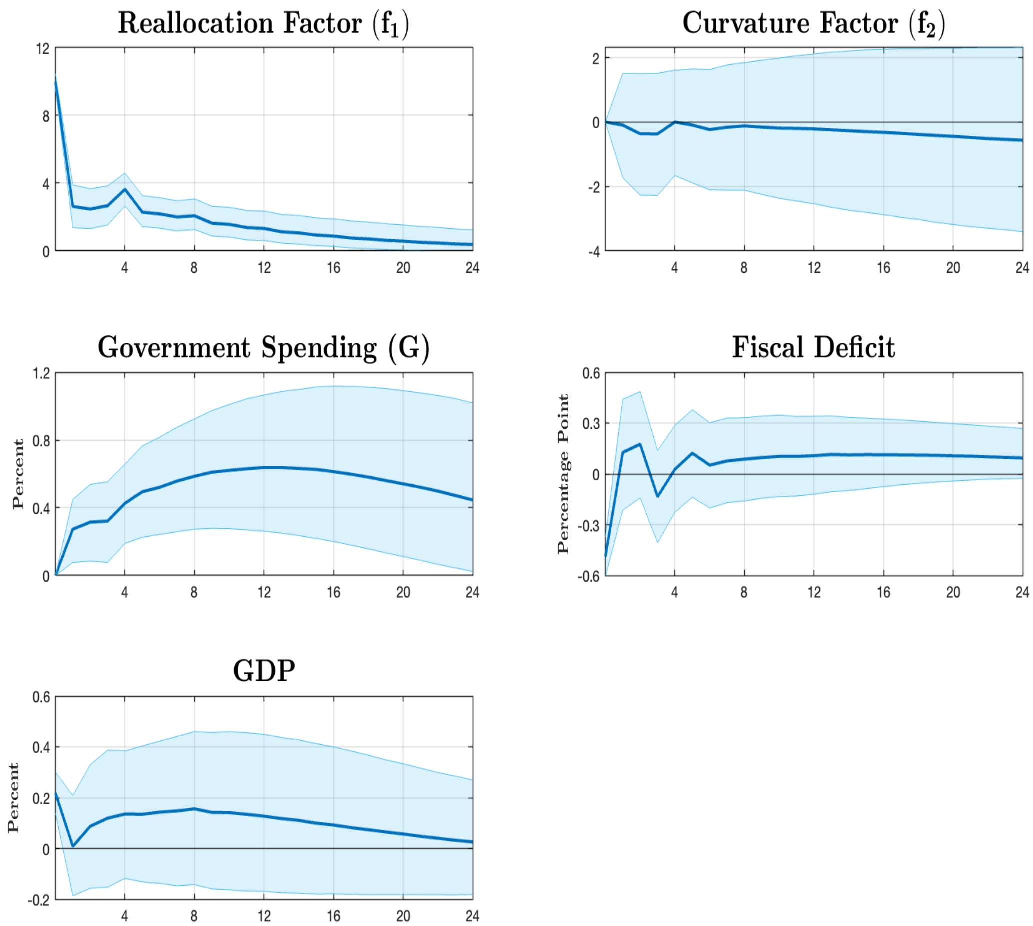
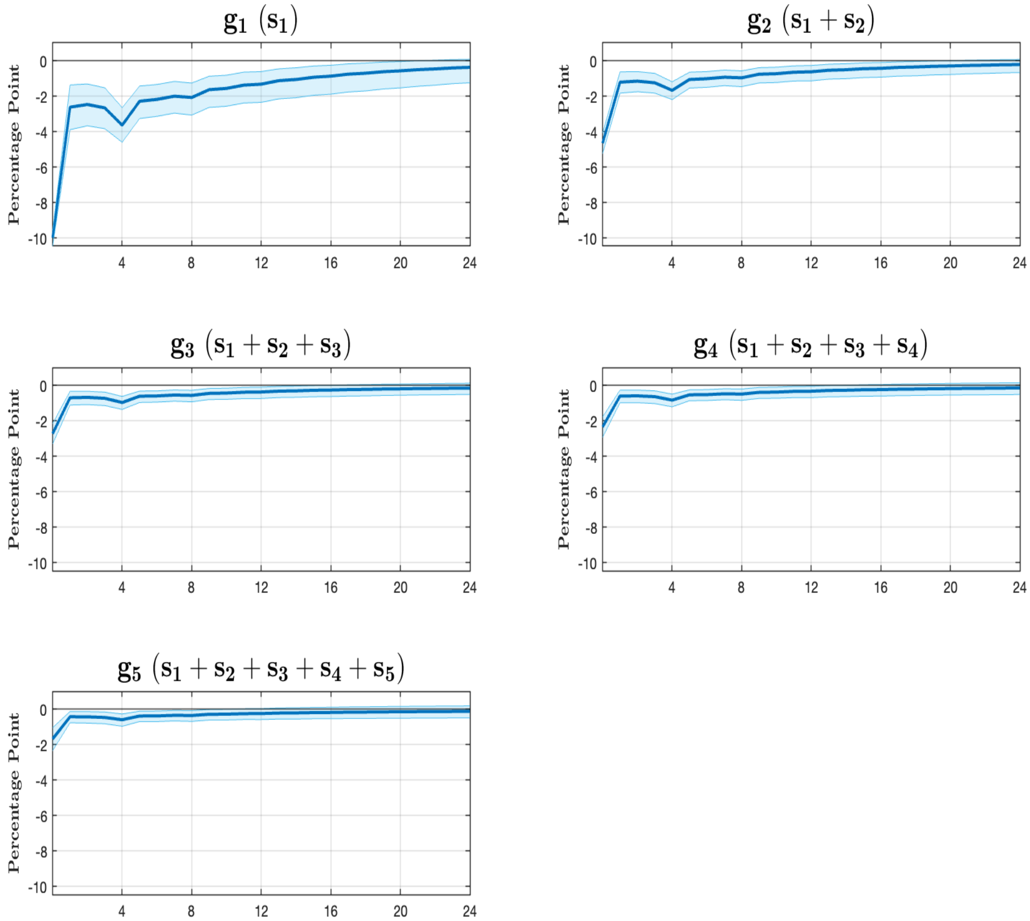


Figure 3.1: Impulse responses to a reallocation shock (shock to f_1) generated from FAVAR with two factors described by equations (2) and (3). Here, the level (G) represents total government spending, which is the sum of all the six components considered. The individual shares are arranged in a descending order. Blue solid lines represent medians, and the shaded area represents the 68% credible bands.



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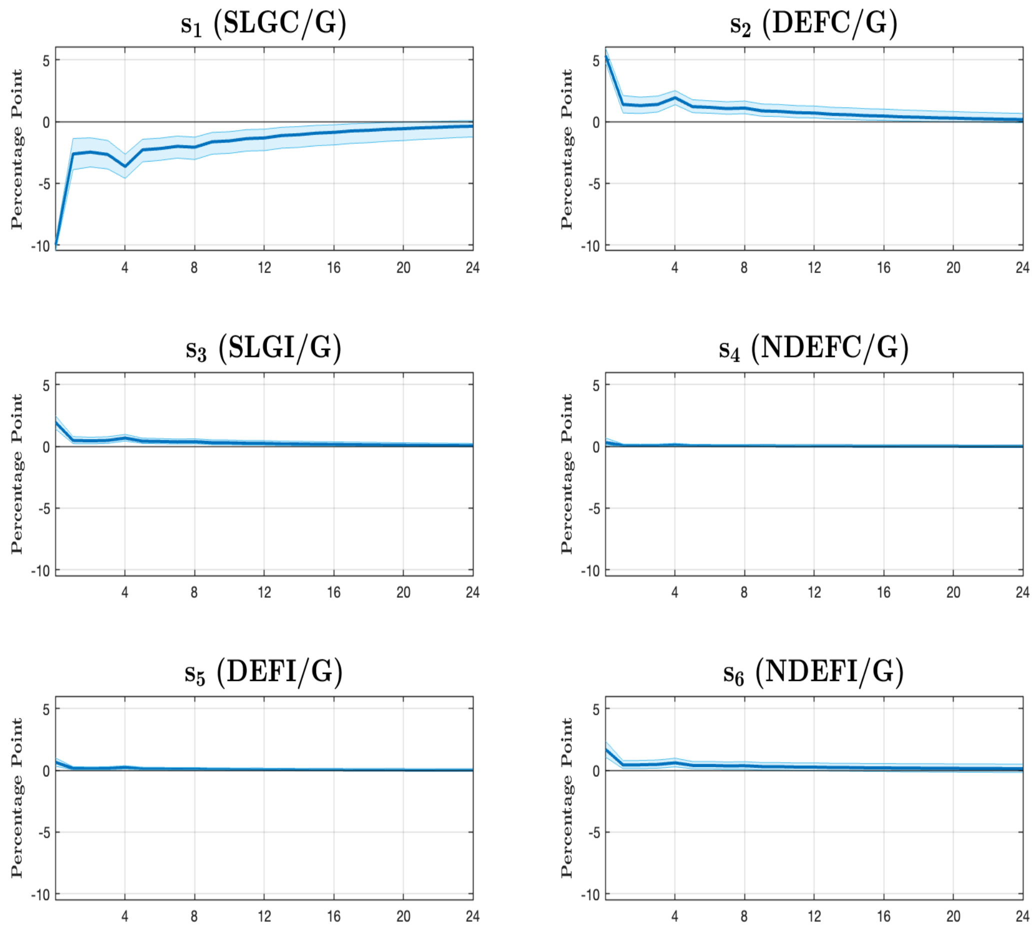

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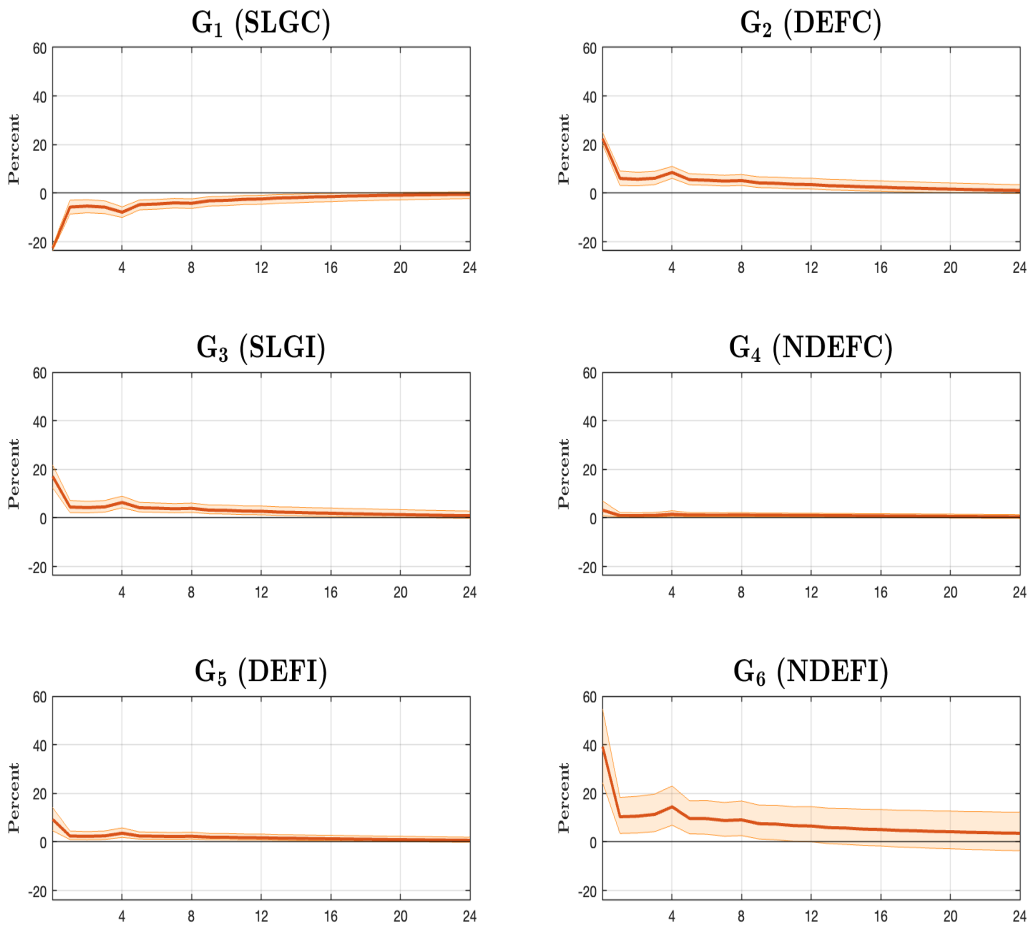

Figure 3.4: Impulse responses of spending on components (G_i) to a reallocation shock (shock to f_1) generated from FAVAR with two factors described by equations (2) and (3). Here, G represents total government spending, $SLGC$ is State and Local Government Consumption, $DEFC$ is Federal Defense Consumption, $SLGI$ is State and Local Government Investment, $NDEFC$ is Federal Non-Defense Consumption, $DEFI$ is Federal Defense Investment and $NDEFI$ is Federal Non-Defense Investment. The individual shares are arranged in descending order. Orange solid lines represent medians, and the shaded area represents the 68% credible bands.

Table 1: Discounted Cumulative Multipliers

This table presents the discounted cumulative multipliers for GDP across the full sample at different horizons with 68% credible bands in brackets. Columns (1) and (2) represent discounted multipliers in response to a level shock (shock to G) and a reallocation shock (shock to f_1) when the individual shares are arranged in descending order.

Horizon	Level multiplier (1)	Reallocation multiplier (2)
On Impact	1.40	-0.11
	[0.89, 1.91]	[-0.15, -0.07]
1 quarter	1.24	-0.09
	[0.69, 1.80]	[-0.19, 0.01]
4 quarters	1.12	-0.14
	[0.45, 1.81]	[-0.38, 0.09]
8 quarters	0.92	-0.22
	[0.14, 1.65]	[-0.61, 0.15]
12 quarters	0.74	-0.29
	[-0.12, 1.48]	[-0.87, 0.23]
16 quarters	0.57	-0.35
	[-0.39, 1.34]	[-1.15, 0.33]
20 quarters	0.42	-0.38
	[-0.73, 1.27]	[-1.46, 0.44]
24 quarters	0.30	-0.39
	[-1.14, 1.24]	[-1.77, 0.58]

Table 2: Non-discounted Cumulative Multipliers

This table presents the discounted cumulative multipliers for GDP across the full sample at different horizons with 68% credible bands in brackets. Columns (1) and (2) represent discounted multipliers in response to a level shock (shock to G) and a reallocation shock (shock to f_1) when the individual shares are arranged in descending order.

Horizon	Level multiplier (1)	Reallocation multiplier (2)
On Impact	1.40	-0.11
	[0.89, 1.91]	[-0.15, -0.07]
1 quarter	1.25	-0.09
	[0.69, 1.80]	[-0.19, 0.01]
4 quarters	1.14	-0.14
	[0.47, 1.81]	[-0.36, 0.08]
8 quarters	0.95	-0.20
	[0.19, 1.66]	[-0.56, 0.13]
12 quarters	0.80	-0.26
	[-0.02, 1.51]	[-0.75, 0.19]
16 quarters	0.66	-0.30
	[-0.21, 1.40]	[-0.92, 0.25]
20 quarters	0.56	-0.32
	[-0.41, 1.32]	[-1.08, 0.32]
24 quarters	0.48	-0.33
	[-0.62, 1.28]	[-1.22, 0.38]

Appendix A

A1 Results (Ordering - Smallest to Largest)

We now comprehend the economic effects of an unexpected decrease in the share of the smallest component (federal non-defense investment) and the resulting adjustments in the other shares.[8] Since the component shares are ordered in ascending magnitude, the differences between successive loadings on f_2 increase to reflect convexity.

A1.1 Responses to a Level Shock

Figure A1.1 displays the median impulse responses of the two latent factors—reallocation and curvature—alongside total government spending (G), the fiscal deficit, and GDP, following a shock to total government spending (G), with 16%–84% credible bands.

On impact, government spending rises by 0.2%, reaching a peak increase of 0.25% after four quarters, after which it gradually declines, though the increase remains statistically distinguishable from zero. The reallocation factor shows no significant change. The curvature, represented by factor f_2, begins to rise, peaks after three quarters, and then gradually declines; however, this response remains statistically insignificant throughout.

In response to a positive shock to government spending, the fiscal deficit rises on impact, reaching a peak increase of 0.06 percentage points before beginning to taper off. This trajectory closely aligns with that of G, indicating that the initial rise in spending is supported by an increase in government debt. Output also initially rises by 0.06%, holding steady for four quarters before declining, with this significant effect on output fading after four quarters.

The output multipliers in response to this level shock are positive, gradually diminishing as output growth slows. These multipliers are significantly different from zero only within the first two years, as indicated in column (1) of Table A1. The five-year horizon discounted multiplier is 0.47, while the non-discounted multiplier is 0.59, as shown in column (1) of Table A2, consistent with previous estimates.

[8] Instead of a unit shock, we scale the shocks by 1/4.

Responses of Shares to a Level Shock

Figure A1.2 presents the median impulse responses of the five cumulative shares in response to a shock to G, accompanied by 16%–84% credible bands. Cumulative shares do not exhibit an immediate response, as the factors themselves do not react contemporaneously to a level shock. With loadings on the slope and curvature factors restricted to -1 and 0 for the first cumulative share, the impulse response of this share is equivalent to the negative of the reallocation factor's impulse response shown in Figure A1.1.

Given the imposed restrictions, the loadings on the reallocation factor are increasing, though negative, while those on the curvature factor are all positive, with the differences between consecutive loadings also increasing. As a result, cumulative shares g_2 and g_3, with relatively small loadings on the curvature factor, have impulse responses resembling the reallocation factor's responses. In contrast, for g_4 and g_5, the smaller absolute loadings on f_1 and the larger loadings on f_2 lead their impulse responses to follow the curvature factor, as seen in Figure A1.1. Specifically, g_4 and g_5 rise, peaking at a 0.04 percentage point change before declining back to zero around 12 quarters later. As with the curvature factor, the responses of these cumulative shares do not differ significantly from zero.

For individual shares, a similar pattern is observed, as shown in Figure A1.3. For the smaller shares, s_2 and s_3, the responses align more closely with those of the reallocation factor. As the shares increase, their responses begin to resemble those of the curvature factor. Finally, the response of s_6 mirrors the negative response of g_5 since the shares must collectively sum to 1.

Given the overall increase in total government spending, individual components also show an increase in spending, as shown in Figure A1.4. Each component increases by 0.2% on impact, corresponding to the response of G to its own shock. Federal non-defense investment peaks with a 0.7% increase after three quarters, then returns to baseline after four quarters. Similarly, federal defense investment reaches a peak increase of 0.4% by the fourth quarter and subsequently declines gradually. These responses, however, are not statistically significant.

The larger components show significant increases over longer horizons. Federal non-defense consumption rises by 0.4% after four quarters, then starts to decline, with its increase staying significant from four to fourteen quarters. State and local government investment also reaches a peak increase of 0.4%, sustaining significance from the third to the fourteenth quarter. Similarly, federal defense consumption rises by 0.3%, remaining significant up to fourteen quarters before declining. The last component shows a steady but insignificant 0.2% increase. Although the share of the last component declines, spending on this component still increases, albeit less than the rise in total spending. Thus, variation in individual shares influences spending changes across components.

A1.2 Responses to a Reallocation Shock

A shock to the reallocation factor translates to an exogenous reduction in the share of spending on the first (smallest) component.

Turning to Figure A2.1, a shock to the reallocation factor is transient, returning to the baseline within a quarter. This shock, however, leads to a significant, lasting decrease in the curvature factor, with f_2 reaching its peak reduction after a quarter before gradually rising. Given that the curvature factor measures the relative significance of larger component shares, its decline implies an increase in smaller shares, a decrease in larger shares, or a mix of both. Government spending declines and stays at a reduced level of around 0.1%, though the decrease is statistically insignificant.

The fiscal deficit initially increases by 0.12 percentage points on impact, then gradually declines as G declines, returning to its original level after four quarters. This initial fiscal strain is due to increased spending on non-defense consumption, while expenditures on federal non-defense investment contract. GDP, in contrast, decreases by 0.05% on impact, where it is statistically significant, before gradually returning toward its original level and becoming statistically indistinguishable from zero in subsequent periods. This initial significant decline in GDP, without a corresponding reduction in total G on impact, reflects shifts in the distribution of spending across components.

The reallocation multiplier is positive, showing an upward trend across horizons, as shown in column (2) of Table A1. It is statistically significant only on impact, with a value of 0.83 over a 20-quarter horizon. A reallocation away from federal non-defense investment may not be beneficial for the economy.

Responses of Shares to a Reallocation Shock

Figure A2.2 displays the median impulse responses of the five cumulative shares in reaction to a shock to f_1, with 16%–84% credible intervals. The first cumulative share, g_1, decreases by

2.5 percentage points on impact and returns to zero within one quarter, reflecting the inverse response of f_1. g_2 decreases by 1.5 percentage points, g_3 by around 1, g_4 by roughly 0.6, and g_5 by about 0.5 percentage points. This pattern indicates that component shares rise on impact, resulting in increasingly smaller declines across the cumulative shares. Furthermore, g_4 and g_5 show a more sustained decline, with g_4 experiencing a drop of around 0.1 percentage points and g_5 around 0.2 percentage points. This persistence in the cumulative shares' decline contributes to the longer-lasting decline in the curvature factor seen in Figure A2.1.

As seen in Figure A2.3, component shares, other than s_1, increase in response to the reallocation shock, as expected. Federal defense investment and federal non-defense consumption increase by approximately 1 and 0.5 percentage points, respectively, on impact, then revert to their initial levels after the first quarter. The significant decline in the curvature factor is largely due to reductions in the share of federal defense consumption. Defense consumption sees an initial rise of 0.2 percentage points, followed by a slightly larger long-lasting decline of 0.1 percentage points. Lastly, the share of state and local government consumption increases by 0.5 percentage points on impact, though this increase stabilizes at around 0.15 percentage points.

The spending responses in each component directly follow the changes in their shares. The decline in spending for state and local government investment and federal government consumption is greater than the increase observed in state and local government consumption, leading to an overall decrease in government spending following the reallocation shock.

The initial reduction in federal non-defense investment contributed to the 0.1% drop in GDP. Ellahie and Ricco (2017) reports that non-defense investment components generally have higher multipliers than non-defense consumption components, with the multiplier for federal non-defense investment exceeding 2 across horizons.

In summary, a reduction in the share of federal non-defense investment leads to an increase in the shares of all other components. Although the initial decline in the first component's share returns to zero after the first period, it prompts a reallocation of resources from federal government consumption and state and local government investment towards state and local government consumption. The impact of reallocation on output not only depends on which shares adjust in response to the decline of one but also on the magnitude of these changes.

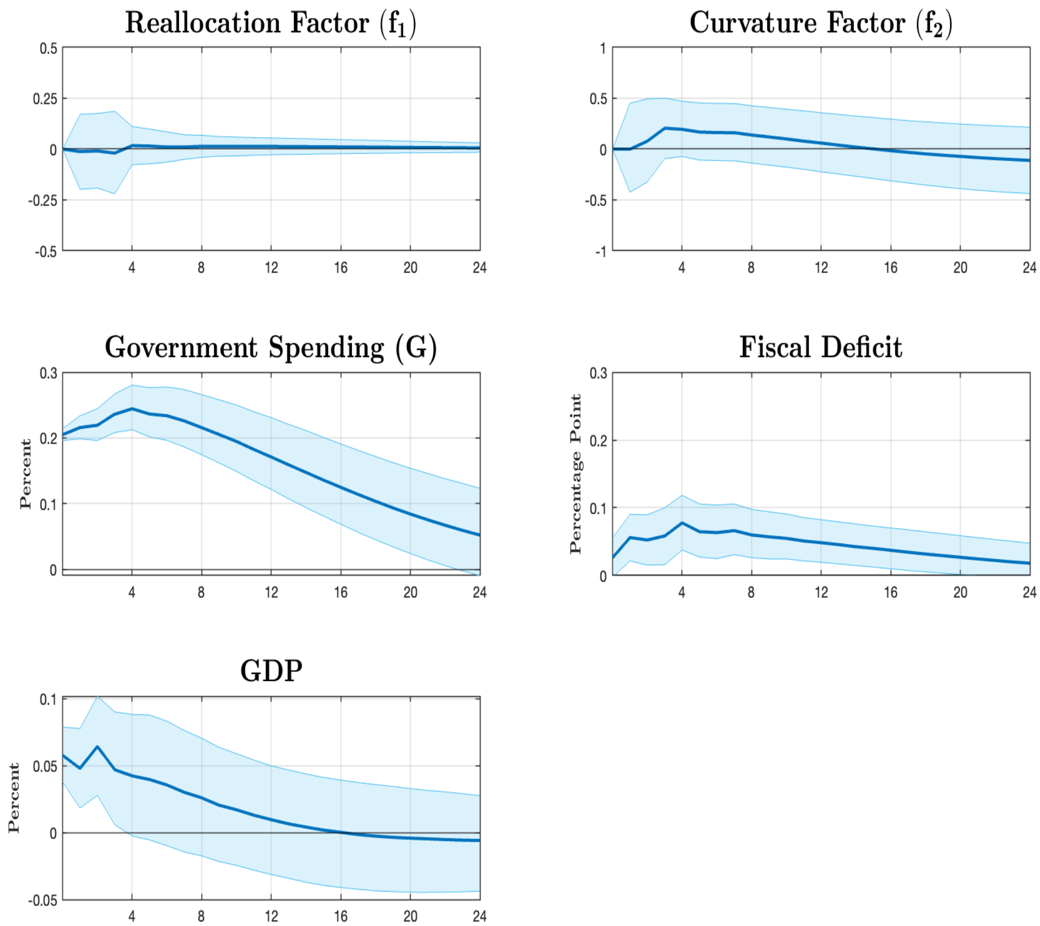

Figure A1.1: Impulse responses to a level shock (shock to G) generated from FAVAR with two factors described by equations (2) and (3). Here, the level (G) represents total government spending, which is the sum of all the six components considered. The individual shares are arranged in an ascending order. Blue solid lines represent medians, and the shaded area represents the 68% credible bands.

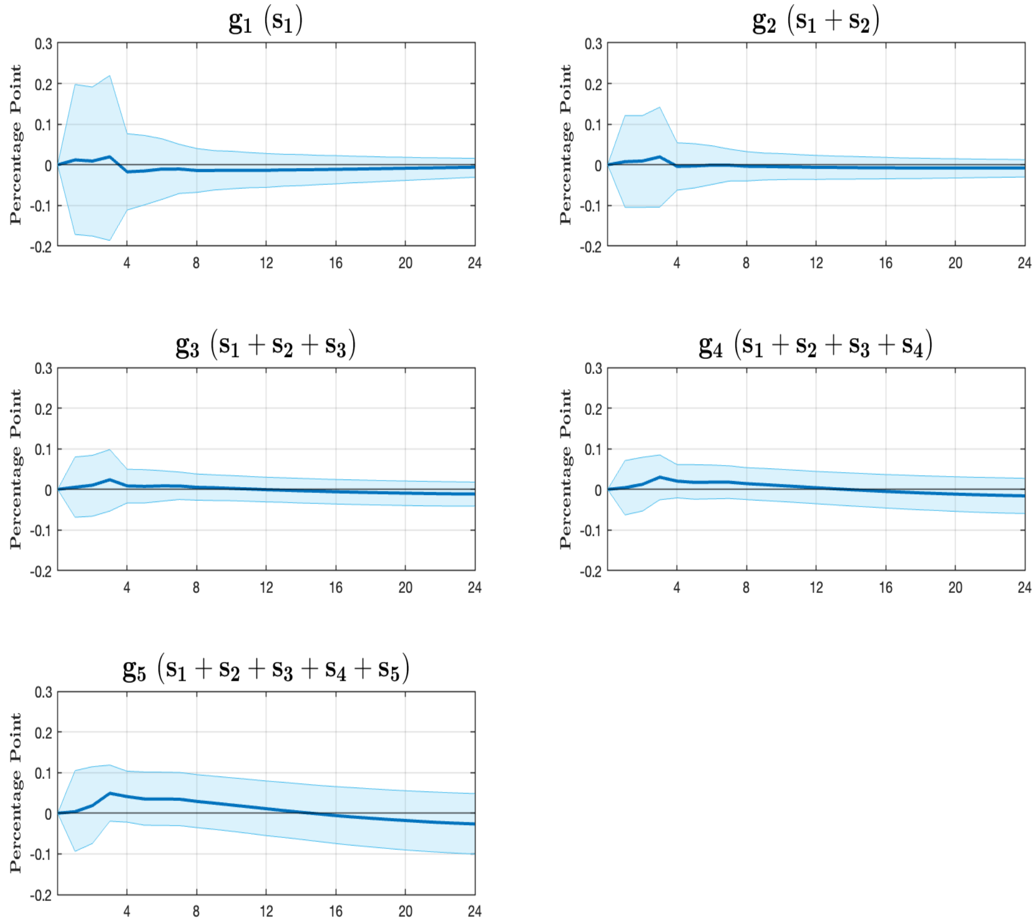

Figure A1.2: Impulse responses of cumulative shares (g_i) to a level shock (shock to G) generated from FAVAR with two factors described by equations (2) and (3). Here, the six individual shares arranged in ascending order are defined as follows - (1) $s_1 = NDEFI/G$, (2) $s_2 = DEFI/G$, (3) $s_3 = NDEFC/G$, (4) $s_4 = SLGI/G$, (5) $s_5 = DEFC/G$ and (6) $s_6 = SLGC/G$. Here, G represents total government spending; $NDEFI$ represents Federal Non-Defense Investment, $DEFI$ denotes Federal Defense Investment, $NDEFC$ is Federal Non-Defense Consumption, $SLGI$ is State and Local Government Investment, $DEFC$ is Federal Defense Consumption, and $SLGC$ is State and Local Government Consumption. The individual shares are arranged in an ascending order. Blue solid lines represent medians, and the shaded area represents the 68% credible bands.

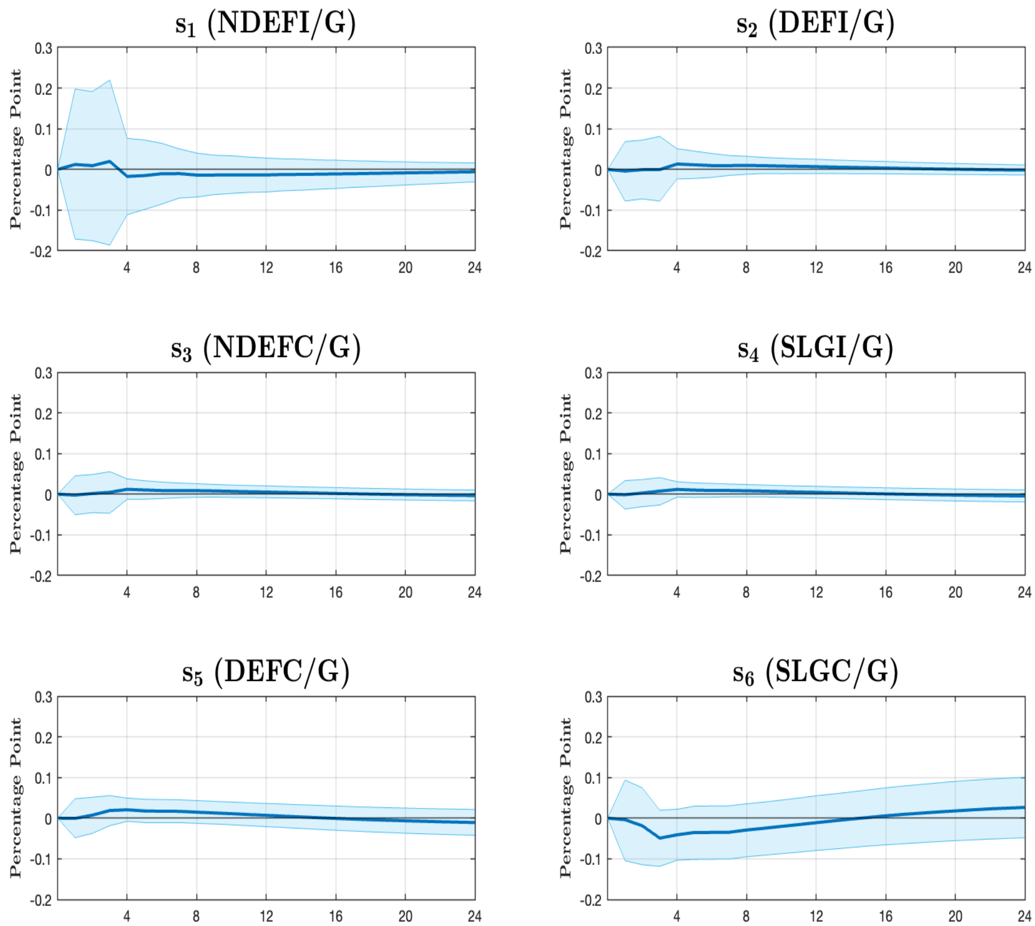

Figure A1.3: Impulse responses of individual shares (s_i) to a level shock (shock to G) generated from FAVAR with two factors described by equations (2) and (3). Here, G represents total government spending; $NDEFI$ represents Federal Non-Defense Investment, $DEFI$ denotes Federal Defense Investment, $NDEFC$ is Federal Non-Defense Consumption, $SLGI$ is State and Local Government Investment, $DEFC$ is Federal Defense Consumption, and $SLGC$ is State and Local Government Consumption. The individual shares are arranged in an ascending order. Blue solid lines represent medians, and the shaded area represents the 68% credible bands.

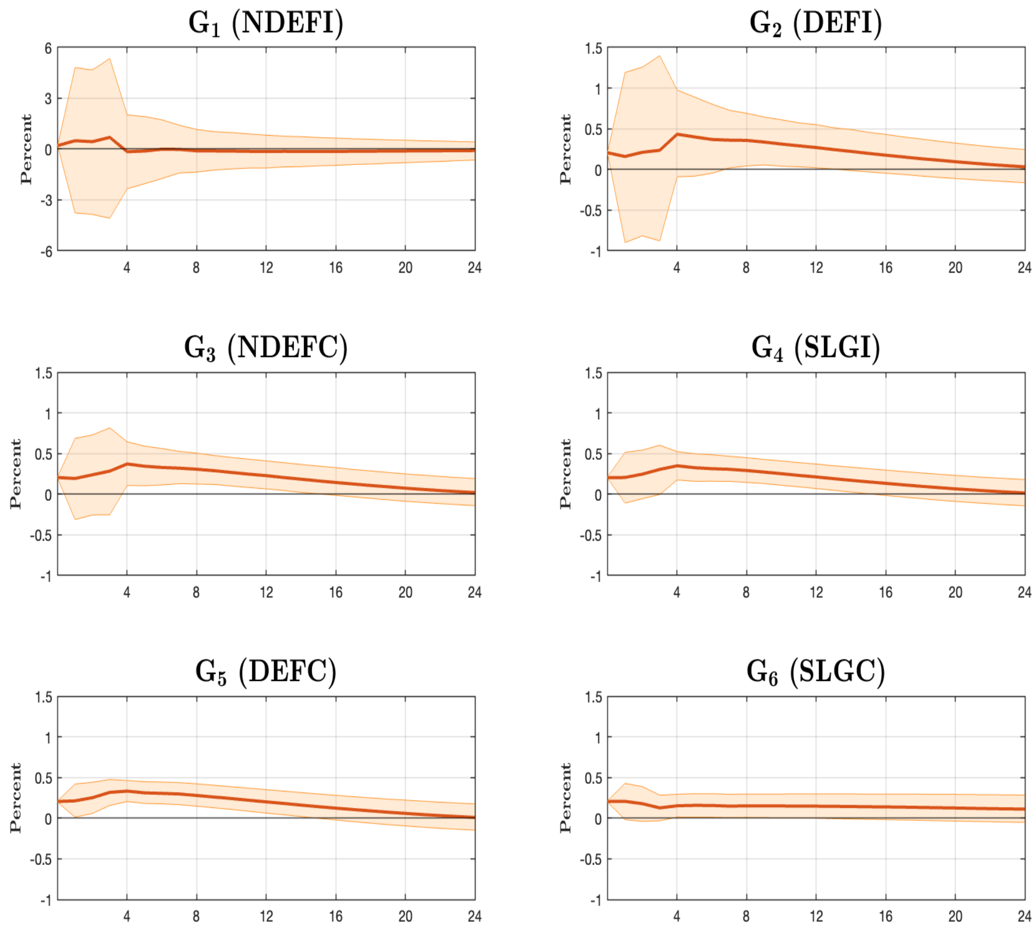

Figure A1.4: Impulse responses of spending on components (G_i) to a level shock (shock to G) generated from FAVAR with two factors described by equations (2) and (3). Here, G represents total government spending, $NDEFI$ is Federal Non-Defense Investment, $DEFI$ is Federal Defense Investment, $NDEFC$ is Federal Non-Defense Consumption, $SLGI$ is State and Local Government Investment, $DEFC$ is Federal Defense Consumption, and $SLGC$ is State and Local Government Consumption. The individual shares are arranged in an ascending order. Orange solid lines represent medians, and the shaded area represents the 68% credible bands.

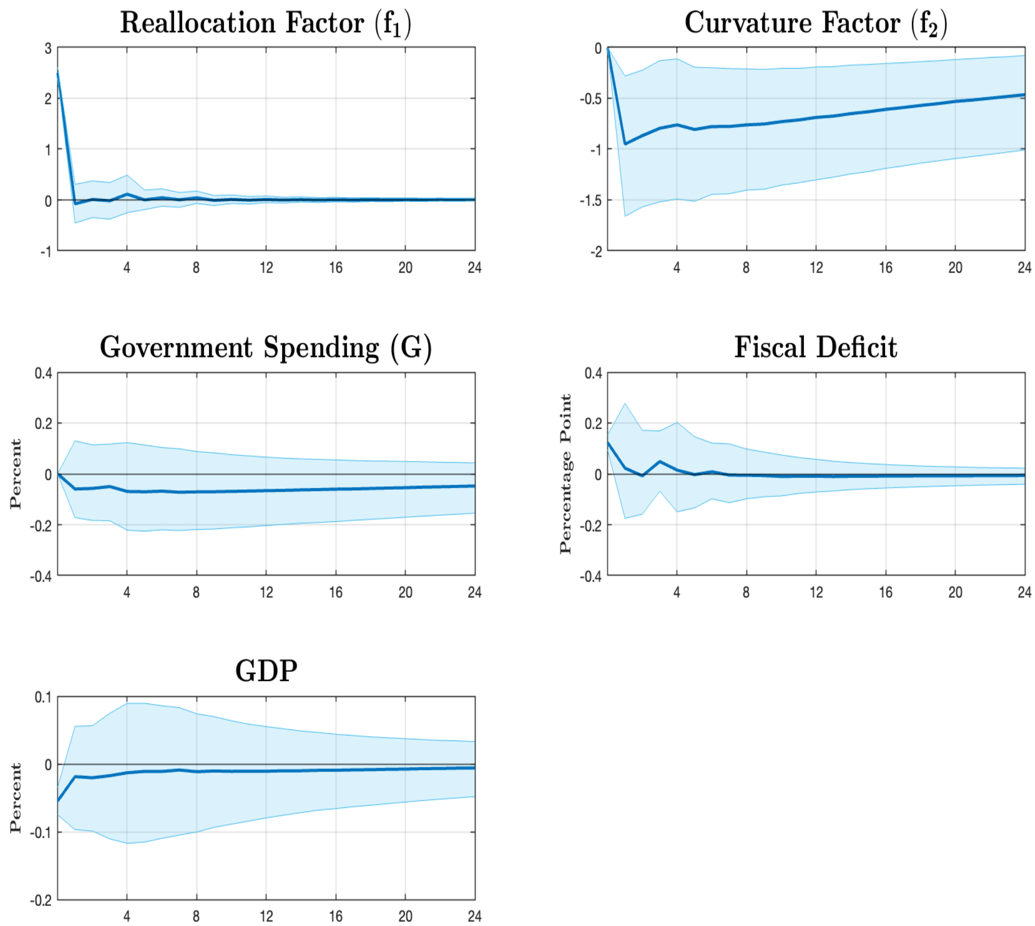

Figure A2.1: Impulse responses to a reallocation shock (shock to f_1) generated from FAVAR with two factors described by equations (2) and (3). Here, the level (G) represents total government spending, which is the sum of all the six components considered. The individual shares are arranged in an ascending order. Blue solid lines represent medians, and the shaded area represents the 68% credible bands.

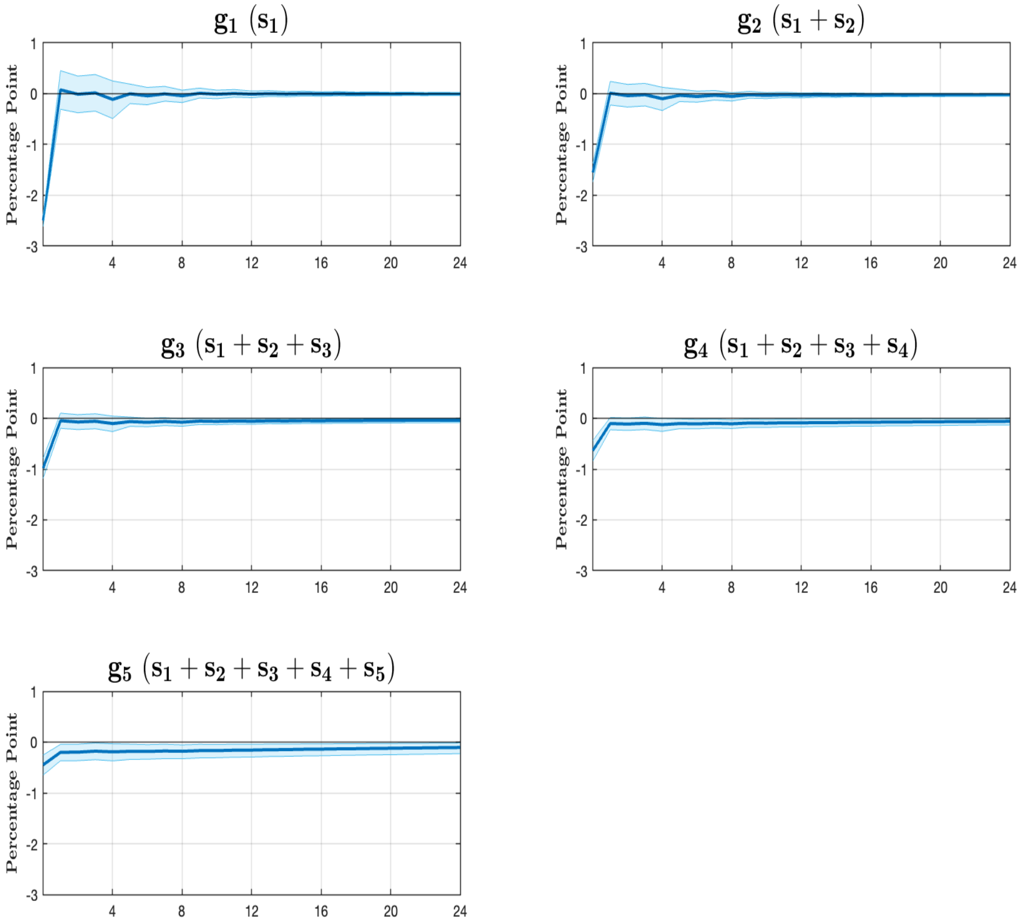

Figure A2.2: Impulse responses of cumulative shares (g_i) to aa reallocation shock (shock to f_1) generated from FAVAR with two factors described by equations (2) and (3). Here, the six individual shares arranged in ascending order are defined as follows - (1) $s_1 = NDEFI/G$, (2) $s_2 = DEFI/G$, (3) $s_3 = NDEFC/G$, (4) $s_4 = SLGI/G$, (5) $s_5 = DEFC/G$ and (6) $s_6 = SLGC/G$. Here, G represents total government spending; $NDEFI$ represents Federal Non-Defense Investment, $DEFI$ denotes Federal Defense Investment, $NDEFC$ is Federal Non-Defense Consumption, $SLGI$ is State and Local Government Investment, $DEFC$ is Federal Defense Consumption, and $SLGC$ is State and Local Government Consumption. The individual shares are arranged in an ascending order. Blue solid lines represent medians, and the shaded area represents the 68% credible bands.

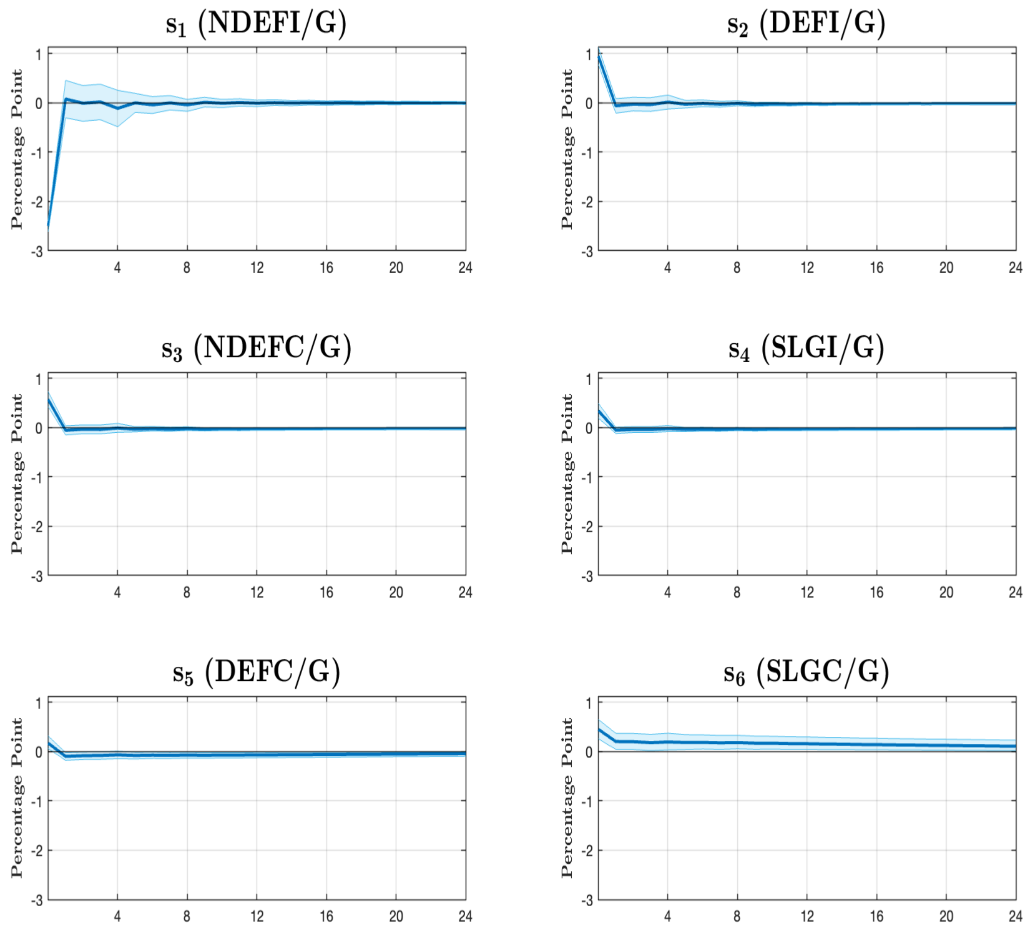

Figure A2.3: Impulse responses of individual shares (s_i) to a reallocation shock (shock to f_1) generated from FAVAR with two factors described by equations (2) and (3). Here, G represents total government spending; $NDEFI$ represents Federal Non-Defense Investment, $DEFI$ denotes Federal Defense Investment, $NDEFC$ is Federal Non-Defense Consumption, $SLGI$ is State and Local Government Investment, $DEFC$ is Federal Defense Consumption, and $SLGC$ is State and Local Government Consumption. The individual shares are arranged in an ascending order. Blue solid lines represent medians, and the shaded area represents the 68% credible bands.

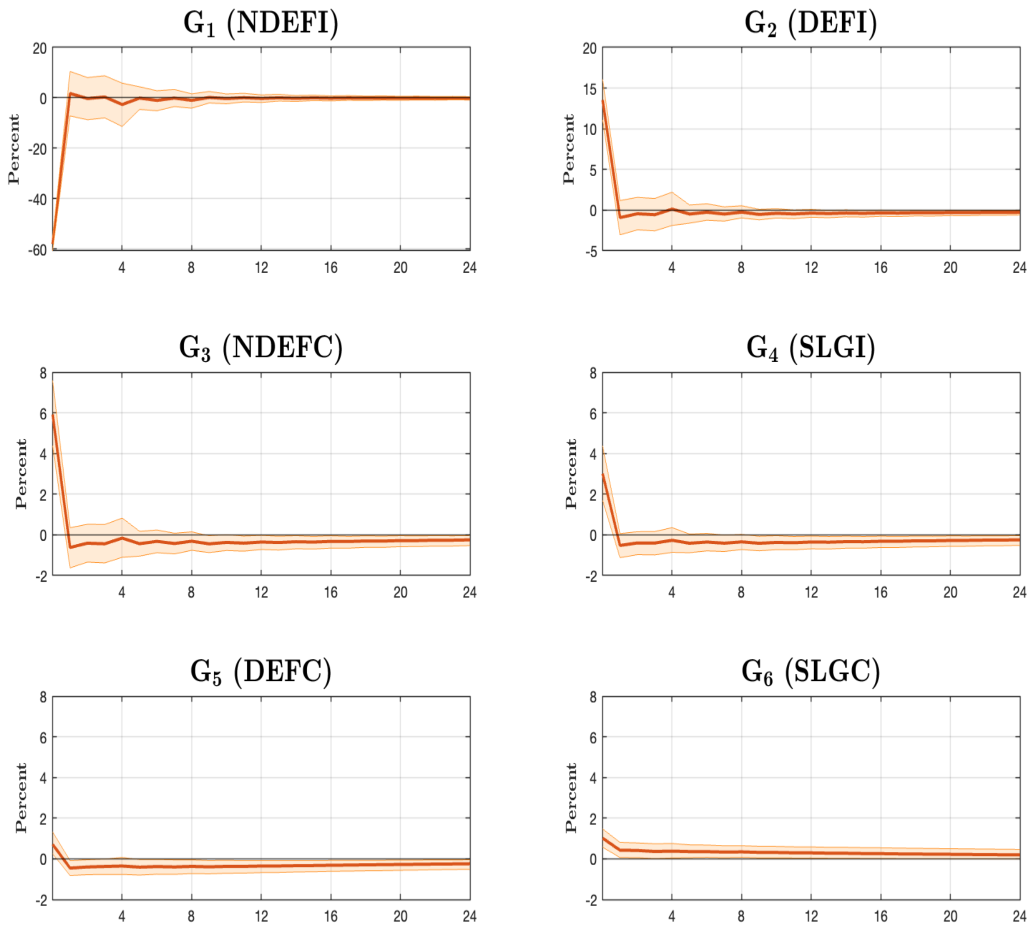

Figure A2.4: Impulse responses of spending on components (G_i) to a reallocation shock (shock to f_1) generated from FAVAR with two factors described by equations (2) and (3). Here, G represents total government spending, $NDEFI$ is Federal Non-Defense Investment, $DEFI$ is Federal Defense Investment, $NDEFC$ is Federal Non-Defense Consumption, $SLGI$ is State and Local Government Investment, $DEFC$ is Federal Defense Consumption, and $SLGC$ is State and Local Government Consumption. The individual shares are arranged in an ascending order. Orange solid lines represent medians, and the shaded area represents the 68% credible bands.

Table A1: Discounted Cumulative Multipliers

This table presents the discounted cumulative multipliers for GDP across the full sample at different horizons with 68% credible bands in brackets. Columns (1) and (2) represent discounted multipliers in response to a level shock (shock to G) and a reallocation shock (shock to f_1) when the individual shares are arranged in ascending order. Column (3) represents discounted multipliers in response to a composite shock to the component (NDEFI).

Horizon	Level multiplier (1)	Reallocation multiplier (2)
On Impact	1.40	0.11
	[0.91, 1.91]	[0.07, 0.15]
1 quarters	1.25	0.16
	[0.70, 1.79]	[-0.03, 0.35]
4 quarters	1.14	0.27
	[0.46, 1.82]	[-0.45, 1.02]
8 quarters	0.93	0.37
	[0.15, 1.68]	[-1.23, 2.05]
12 quarters	0.75	0.50
	[-0.10, 1.49]	[-1.95, 3.17]
16 quarters	0.60	0.64
	[-0.37, 1.36]	[-2.53, 4.25]
20 quarters	0.47	0.76
	[-0.68, 1.28]	[-3.10, 5.35]
24 quarters	0.36	0.83
	[-1.06, 1.26]	[-3.65, 6.23]

Table A2: Non-discounted Cumulative Multipliers

This table presents the cumulative multipliers for GDP across the full sample at different horizons with 68% credible bands in brackets. Columns (1) and (2) represent multipliers in response to a level shock (shock to G) and a reallocation shock (shock to f_1) when the individual shares are arranged in ascending order. Column (3) represents multipliers in response to a composite shock to the component (NDEFI).

Horizon	Level multiplier (1)	Reallocation multiplier (2)
On Impact	1.40	0.11
	[0.91, 1.91]	[0.07, 0.15]
1 quarters	1.26	0.15
	[0.71, 1.79]	[-0.02, 0.34]
4 quarters	1.15	0.25
	[0.48, 1.82]	[-0.39, 0.92]
8 quarters	0.96	0.33
	[0.20, 1.69]	[-0.95, 1.67]
12 quarters	0.81	0.41
	[-0.01, 1.53]	[-1.39, 2.33]
16 quarters	0.69	0.49
	[-0.21, 1.41]	[-1.67, 2.85]
20 quarters	0.59	0.56
	[-0.38, 1.34]	[-1.86, 3.28]
24 quarters	0.52	0.60
	[-0.57, 1.30]	[-2.01, 3.62]

Appendix B

Bayesian Estimation

In this appendix, we elaborate on the use of the Gibbs sampling procedure in estimating our FAVAR model. For simplicity, let $X_t = \begin{bmatrix} \chi_t' & Y_t' \end{bmatrix}'$, $\xi_t = \begin{bmatrix} F_t' & \dots & F_{t-p+1}' \end{bmatrix}'$, $e_t^X = \begin{bmatrix} e_t' & 0 \end{bmatrix}'$ and $v_t^\xi = \begin{bmatrix} v_t' & 0 \end{bmatrix}'$. Therefore, equations (2) and (3) can be written as

$$X_t = \Lambda \xi_t + e_t^X \tag{B1}$$

$$\xi_t = \Phi(L)\xi_{t-1} + v_t^\xi \tag{B2}$$

where Λ is the loadings matrix from equation (2), $R^X = cov(e_t^X e_t^{X\prime})$ represents the covariance matrix R of the measurement equation, extended with zeros beyond the $(N-1) \times (N-1)$ block. Similarly, Q^ξ is the variance-covariance matrix Q of the VAR(1) process augmented with zeros beyond the $(K+M) \times (K+M)$ block.

All of the model's parameters can be treated as random variables and are collectively referred to as θ where $\theta = (\Lambda, R^X, vec(\Phi), Q^\xi)$ and $vec(\Phi)$ is the column vector of the elements of the stacked Φ of the lag operator of the transition equation. Further, let $\tilde{X}_T = (X_1, X_2, \dots, X_T)$ denote the history of X from periods 1 to T. Similarly, define $\tilde{\xi}_T = (\xi_1, \xi_2, \dots, \xi_T)$. The aim is to obtain the empirical marginal posterior densities of $\tilde{\xi}_T$ and θ.

The multi-move Gibbs sampler begins by selecting a set of initial values θ^0. Second, conditional on θ^0 and data $\tilde{X}_T$, a set of values for $\tilde{\xi}_T$ say $\tilde{\xi}_T^1$, is drawn from the conditional density $p(\tilde{\xi}_T | \tilde{X}_T, \theta^0)$. Then, conditional on the sampled value of the state vector $\tilde{\xi}_T^1$ and the data, a set of values of the parameters θ i.e. θ^1, is drawn from the conditional distribution $p(\theta | \tilde{X}_T, \tilde{\xi}_T^1)$. These two steps constitute one iteration, and a total of 30,000 such iterations are performed, with the first 15,000 discarded as burn-in.

Initial values θ^0

In place of the factors, we use principal components corresponding to the largest K eigenvalues, as principal components provide a good starting value for the factors. The identification of factors using principal components is standard. Following Stock and Watson (2002), we restrict the factor loadings by setting $\Lambda'\Lambda/N = I_K$. This normalization identifies the factors up to a change of sign.

We impose the normalization restrictions for g_1 as outlined in the previous section where we set $\lambda^f_{1,1} = -1$ and $\lambda^f_{1,2} = 0$. Using the principal components in place of f_t, we get the OLS estimates of Λ and R with these restrictions in place. Similarly, we use the OLS estimates for $\Phi(L)$ and Q.

Drawing the latent factors

We follow the exact methodology described in Kim and Nelson (1999, Chapter 8) in order to sample from $p(\tilde{\xi}_T|\tilde{X}_T, \theta)$, assuming that the data and the hyperparameters of the model are given. By leveraging the Markov property in state space models,[9] the conditional distribution from which the state vector is generated can be expressed as a product of conditional distributions (equation 8.8 in Kim and Nelson, 1999), as follows.

$$p(\tilde{\xi}_T|\tilde{X}_T, \theta) = p(\xi_T|\tilde{X}_T, \theta) \prod_{t=1}^{T-1} p(\xi_t|\xi_{t+1}\tilde{X}_T, \theta) \tag{B3}$$

The state space model is linear and Gaussian, hence we have:

$$\xi_T|\tilde{X}_T, \theta \sim N(\xi_{T|T}, P_{T|T}) \tag{B4}$$

$$\xi_t|\xi_{t+1}, \tilde{X}_t, \theta \sim N(\xi_{t|t,\xi_{t+1}}, P_{t|t,\xi_{t+1}}) \quad for \quad t = T-1, \ldots, 1 \tag{B5}$$

where

$$\begin{gathered} \xi_{T|T} = E(F_T|\tilde{X}_T, \theta) \\ P_{T|T} = Cov(F_T|\tilde{X}_T, \theta) \\ \xi_{t|t,\xi_{t+1}} = E(\xi_t|\xi_{t+1}, \tilde{X}_T, \theta) = E(\xi_t|\xi_{t+1}, \xi_{t|t}, \theta) \\ P_{t|t,\xi_{t+1}} = Cov(\xi_t|\xi_{t+1}, \tilde{X}_T, \theta) = Cov(\xi_t|\xi_{t+1}, \xi_{t|t}, \theta) \end{gathered} \tag{B6}$$

Here, $\xi_{t|t}$ and $P_{t|t}$ refer to the expected value (mean) and uncertainty (variance) of the latent variables given the current parameters and the observed data dated t. We run the Kalman filter to generate $\xi_{t|t}$ and $P_{t|t}$ for $t = 1, \ldots, T$, starting with the initial values, $\xi_{1|0} = 0_{p(K+M)\times 1}$

[9] $p(\xi_t|\xi_{t+1}, \ldots, \xi_T, X_T, \theta) = p(\xi_t|\xi_{t+1}, X_t, \theta)$

and $P_{1|0} = I_{p(K+M)}$ (Bernanke et al., 2005).

$$\begin{aligned} \xi_{t|t} &= \xi_{t|t-1} + K_t \eta_{t|t-1} \\ P_{t|t} &= P_{t|t-1} - K_t \Lambda P_{t|t-1} \end{aligned} \tag{B7}$$

where $\eta_{t|t-1} = X_t - \Lambda \xi_{t|t-1}$ is the conditional forecast error and its covariance is given by $H_{t|t-1} = \Lambda P_{t|t-1} \Lambda' + R$. The Kalman gain, denoted by $K_t = P_{t|t-1} \Lambda' H_{t|t-1}^{-1}$, determines the weight to be assigned to new information contained in the conditional forecast error. Additionally,

$$\xi_{t|t-1} = \Phi \xi_{t-1|t-1}$$

$$P_{t|t-1} = \Phi P_{t-1|t-1} \Phi' + Q$$

The last iteration of the Kalman filter provides $\xi_{T|T}$ and $P_{T|T}$, which can be used to draw ξ_T using equation (7). Once ξ_T is obtained, ξ_t, $t = T-1, \ldots, 1$ can be generated by running the Kalman smoother backwards, following equation (8). This process is akin to updating an estimate of ξ_t by combining $\xi_{t|t}$ with the additional information provided by ξ_{t+1}. The updating equations are as follows:

$$\begin{aligned} \xi_{t|t,\xi_{t+1}} &= \xi_{t|t} + P_{t|t} \Phi^{*\prime} (\Phi^* P_t | t \Phi^{*\prime} + Q)^{-1} (\xi^*_{t+1} - \Phi^* \xi_{t|t}) \\ P_{t|t,\xi_{t+1}} &= P_{t|t} - P_{t|t} \Phi^{*\prime} (\Phi^* P_t | t \Phi^{*\prime} + Q)^{-1} \Phi^* P_{t|t} \end{aligned} \tag{B8}$$

where Q refers to the upper $(K+M) \times (K+M)$ block of the variance-covariance matrix Q^ξ. Similarly, Φ^* and ξ^*_t refer to the first $(K+M)$ rows of Φ and ξ_t, respectively. This situation arises when Q^ξ is singular, which occurs when the number of lags in the transition equation exceeds 1.[10]

Drawing the parameters

Conditional on the factors drawn and the data for the observable macroeconomic series, we begin by drawing the parameters related to the measurement equation (4) and the transition equation (5) from the conditional distribution $p(\theta | \tilde{X}_T, \tilde{\xi_T})$. The choice of the priors for the parameters and the resulting posterior distributions follows from Bernanke et al. (2005). With known factors, equations (4) and (5) amount to standard regression equations. Starting with the measurement equation, recall that the factor loadings on the f_1 and f_2 for the first

[10] In line with literature (Blanchard and Perotti, 2002), we set the number of lags equal to 4. The details and the derivations of the equations (6) to (11) can be found in Chapters 3 and 8 of Kim and Nelson (1999).

cumulative share are set to -1 and 0. We draw the differences ($\Delta\Lambda_i^f$) between the loadings such that the differences are positive for f_1 and the differences are increasing for f_2. For $i = 2, \ldots, N-1$, we have,

$$\begin{aligned} g_{it} &= \Lambda_i^f f_t + e_{it} \\ g_{it} &= \left(\Lambda_{i-1}^f + \Delta\Lambda_i^f\right) f_t + e_{it} \\ g_{it} - \Lambda_{i-1}^f f_t &= \delta_i^f f_t + e_{it} \end{aligned} \tag{B9}$$

Since the errors are uncorrelated ($R_{ij} = 0, i \neq j$), the OLS estimates of $\hat{\delta}_i^f$ and $\hat{e}_i$ for $i = 2, \ldots, N-1$ are obtained for each equation separately. Following Bernanke et al. (2005), we assume conjugate priors:

$$\begin{aligned} R_{ii} &\sim iG\left(R_0, t_0\right) \\ \delta_i^f | R_{ii} &\sim N\left(\delta_{i0}^f, R_{ii} M_0^{-1}\right) \end{aligned} \tag{B10}$$

These priors conform to the following posterior distribution:

$$\begin{aligned} R_{ii} | \tilde{X}_t, \tilde{\xi}_t &\sim iG\left(\bar{R}_{ii}, \bar{T}\right) \\ \delta_i^f | \tilde{X}_t, \tilde{\xi}_t, R_{ii} &\sim N\left(\bar{\delta}_i^f, R_{ii} \bar{M}_i^{-1}\right) \end{aligned} \tag{B11}$$

with

$$\begin{aligned} \bar{T} &= t_0 + T \\ \bar{R}_{ii} &= R_0 + \hat{e}_i' \hat{e}_i + (\hat{\delta}_i^f - \delta_{i0}^f) \left[M_0^{-1} + \left(\tilde{f}_T' \tilde{f}_T\right)^{-1}\right]^{-1} (\hat{\delta}_i^f - \delta_{i0}^f) \\ \bar{\delta}_i^f &= \bar{M}_i^{-1} \left(M_0 \delta_{i0}^f + (\tilde{f}_T' \tilde{f}_T) \hat{\delta}_i^f\right) \\ \bar{M}_i &= M_0 + \tilde{f}_T' \tilde{f}_T \end{aligned} \tag{B12}$$

Here, M_0^{-1} denotes variance parameter in the prior for the second factor's coefficient in the i^{th} equation. The regressor for each of the i^{th} equation is denoted by $\tilde{f}_T$, where $\tilde{f}_T = (f_1, f_2, \ldots, f_T)$ is the history of the factors from period 1 to period T. We use a diffuse prior specification for R_{ii}, where $R_0 = 1$; $t_0 = 10^{-3}$. Similarly, we set the priors on the coefficients of the i^{th} equation, $\delta_{i0}^f = 0$ for $i = 2, \ldots, N-1$ and $M_0 = 1$. For each i^{th} equation, we accept draws if $\delta_{i,1}^f > 0$ and $\delta_{i,2}^f > \delta_{i-1,2}^f$.

Next, we draw $vec(\Phi^*)$ and Q conditional on the draws of the factors and the data. Φ^* refers to the first $(K+M)$ rows of the Φ matrix, and Q is the upper $(K+M) \times (K+M)$ block of Q^ξ. Since Q is a positive definite, it can be decomposed into unique matrices L

and D such that $Q^{-1} = L'D^{-1}L$ (Chan and Jeliazkov, 2009). L is a lower triangular matrix with ones on the diagonal and D is a diagonal matrix with positive diagonal elements. Let the diagonal elements in D be denoted as γ_w, $nn = 1, \ldots, (K+M)$ and let $a_{nn,mm}$, $1 \leq nn < mm \leq (K+M)$ denote the elements of L.

$$D \equiv \begin{bmatrix} \gamma_1 & 0 & \cdots & 0 \\ 0 & \gamma_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \gamma_{(K+M)} \end{bmatrix}; \ L \equiv \begin{bmatrix} 1 & 0 & \cdots & 0 \\ a_{21} & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ a_{(K+M)1} & a_{(K+M)2} & \cdots & 1 \end{bmatrix} \tag{B13}$$

The priors are given as follows:

$$\begin{gathered} \gamma_{nn} \overset{Ind}{\sim} IG(\frac{1+nn}{2}, \frac{q_{nn}}{2}), \quad nn = 1, \ldots, (K+M) \\ a_{nn} \overset{Ind}{\sim} N(0, A_0), \quad nn = 4, \ldots, (K+M) \\ vec(\Phi^*)|Q \sim N(vec(\Phi_0^*), Q \otimes \Omega_0) \end{gathered} \tag{B14}$$

Following the Minnesota prior, the priors are set assuming parameters on longer lags are more likely to be zero. The diagonal elements of Q_0, denoted as q_{nn}, are set to the residual variances of the corresponding p-lag univariate autoregressions, σ^2_{nn} (Kadiyala and Karlsson, 1997). For the off-diagonal elements a_{nn}, where $nn = 4, \ldots, (K+M)$,[11] the prior mean is set to 0, with a large variance specified as $A_0 = 10^3 \cdot I_{(K+M)}$, ensuring that these elements are not restricted to zero.

Ω_0 is an $(1+p(K+M)) \times (1+p(K+M))$ matrix, with its diagonal elements set such that the prior variances of the parameter of l lagged mm^{th} variable in the nn^{th} equation equal $\sigma^2_{nn}/l\sigma^2_{mm}$. Also, we set $\Phi^*_0 = 0$.

The posterior is given as:

$$\begin{gathered} \gamma_{nn}|\tilde{X}_t, \tilde{\xi}_t \overset{Ind}{\sim} IG\left(\frac{1+nn+T}{2}, \frac{\bar{q}_{nn}}{2}\right) \\ a_{nn}|\tilde{X}_t, \tilde{\xi}_t, \gamma_{nn} \overset{Ind}{\sim} N\left(\bar{a}_{nn}, \bar{A}_{nn}\right) \\ vec(\Phi^*)|\tilde{X}_t, \tilde{\xi}_t, Q \sim N\left(vec(\bar{\Phi}^*), Q \otimes \bar{\Omega}\right) \end{gathered} \tag{B15}$$

[11] We require $q_{21} = 0$, $q_{31} = 0$ and $q_{32} = 0$, which implies setting $a_{21} = 0$, $a_{31} = 0$ and $a_{32} = 0$ with probability 1. See Chan and Jeliazkov (2009) for additional details.

where $\bar{q}_{nn}$ is the nn^{th} diagonal element of $\bar{Q}$.

$$\begin{aligned}
\bar{Q} &= Q_0 + \hat{v}'\hat{v} + (\hat{\Phi}^* - \Phi_0^*)' \left[\Omega_0 + \left(\tilde{\xi}'_{T-1} \tilde{\xi}_{T-1} \right)^{-1} \right]^{-1} (\hat{\Phi}^* - \Phi_0^*) \\
\bar{A}_{nn} &= \gamma_{nn} \left(A_0^{-1} + \hat{v}'_{nn} \hat{v}_{nn} \right)^{-1} \\
\bar{a}_{nn} &= \bar{A}_{nn} \left(\gamma_{nn}^{-1} \hat{v}'_{nn} \hat{V}_{nn} \right) \\
\hat{v}_{nn} &= [\hat{V}_1 : \cdots : \hat{V}_{nn}] \\
\hat{V}_{nn} &= (\hat{V}_{1nn}, \hat{V}_{2nn}, \ldots, \hat{V}_{Tnn})'
\end{aligned} \tag{B16}$$

Also, we have,

$$\begin{aligned}
\bar{\Phi}^* &= \bar{\Omega} \left(\Omega_0^{-1} \Phi_0^* + \tilde{\xi}'_{T-1} \tilde{\xi}_{T-1} \hat{\Phi}^* \right) \\
\bar{\Omega} &= \left(\Omega_0^{-1} + \tilde{\xi}'_{T-1} \tilde{\xi}_{T-1} \right)^{-1}
\end{aligned} \tag{B17}$$

Here, $\hat{\Phi}^*$ and $\hat{v}$ refer to the OLS estimates of the VAR coefficients and the reduced-form residuals for the first $(K+M)$ equations of the VAR model. We discard draws of Φ with roots outside the unit circle, as the VAR model would be unstable.

The process of drawing the factors, followed by drawing the parameters, constitutes one iteration in the multi-move Gibbs sampling. We preserve the $\theta = (\Lambda, R^X, vec(\Phi), Q^\xi)$ estimates from the latter half of the 30,000 iterations, after discarding the initial 15,000 iterations.

Appendix C

Variable Definitions

1. **Government Spending Components**: Quarterly data for each of the components of government purchases are sourced from Table 3.9.5 of the National Income and Product Accounts (NIPA), provided by the Bureau of Economic Analysis. Details for each component are as follows:

   - Federal Defense Consumption Expenditures (DEFC) - Row 18
   - Federal Defense Gross Investment (DEFI) - Row 19
   - Federal Non-Defense Consumption Expenditures (NDEFC) - Row 26
   - Federal Non-Defense Gross Investment (NDEFI) - Row 27
   - State and Local Government Consumption Expenditures (SLGC) - Row 34,
   - State and Local Government Gross Investment (SLGI) - Row 35

   All components are adjusted to real per capita terms by dividing the nominal values by the GDP deflator and the population measure. The sum of these components constitutes total government spending. To calculate total federal spending, we aggregate the federal defense and federal non-defense components. Similarly, state and local government spending is computed by summing the respective consumption and investment figures.

2. **Population**: Quarterly population data, including resident population plus armed forces overseas from the Federal Reserve Board of St Louis website (B230RC0Q173SBEA).

3. **Nominal GDP**: This metric is available from Line 1 of Table 1.1.5 in the National Income and Product Accounts (NIPA). Real per capita values are calculated by dividing nominal values by both the GDP deflator and the population figure.

4. **Implicit Price Deflator for GDP**: This is sourced from Row 1 of Table 1.1.9 of National Income and Product Accounts (NIPA).

5. **Fiscal Deficit**: Fiscal deficit is defined as difference between federal government expenditures and federal government receipts as a percentage of nominal GDP. The

data on receipts (W018RC1Q027SBEA) and expenditures (W019RCQ027SBEA) is obtained from the Federal Reserve Board of St Louis website.