



Mathematics Club

Contingent Problem Set - 6



Challenge posed on: 19/07/2024

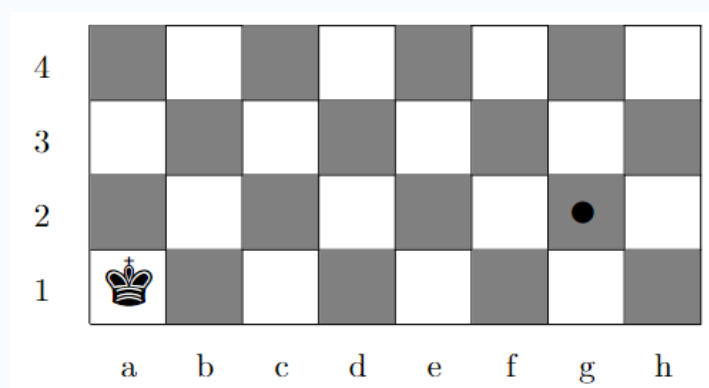
Challenge conquered by: 26/07/2024

1 Overview

- **Topics focused:**
 - Combinatorics
 - Game theory
 - Number theory
 - Mathematic Puzzles
- **Challengers:**
 - Pranjal
 - Zafir
 - Veda

2 Problems

1. **The Travelling King** In how many different ways can the king on a1 go to g2 in the shortest number of moves? (King moves in the normal manner)



2. **A coefficient game** There is a biquadratic equation as follows:

$$ax^4 + bx^3 + cx^2 + dx + e$$

Zafir and Pranjal play a game, Pranjal goes first and chooses any real number to fill the place of any one of the coefficients. Zafir goes next and in a similar fashion chooses a real number to fill any one of the other 4 coefficients. This goes on until the equation is finished.

Zafir wins if the equation so formed contains at least one real root. Pranjal wins if the equation contains no real root. Under optimal strategies who wins the game? Also a,e can't be equal to 0.

Does your answer change if it is a quadratic equation?

3. **Game 2.0** Pranjal is not satisfied with 1 game, and wishes to play some other one. In this game both of them take an 8×8 grid, and this time Zafir starts 1st by placing a horse on any square of the grid. Pranjal goes next and moves the horse previously placed, onto a square on which the horse hasn't been on before (Horse moves according to chess rules).

They play alternatively and this continues till a player is unable to place the horse on a square that it hasn't been on before.

Consider both Pranjal and Zafir play optimally, who wins the game and what is the winning strategy?

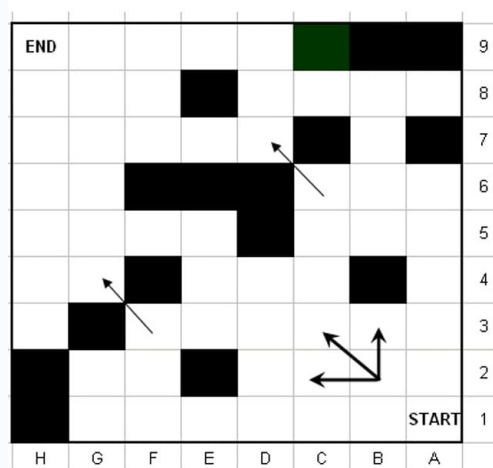
4. **Large Numbers** How many minimum numbers raised to the fourth power are required, such that their sum is 1957^{1957} . The numbers are not necessarily distinct.

5. **Enjoy the Easy one!** Find the value of

$$\sum_{n=1}^{2024} \left\lfloor \frac{5^n}{3} \right\rfloor$$

($\lfloor x \rfloor$ denotes the GIF function)

6. **Towards the End** A token is placed in the square marked 'START' (cell A1). Player 1 moves first and can move the token one square up (to A2) or one square to the left (to B1) or one square diagonally in the left-up direction (to B2). Then Player 2 moves, according to the same rules (e.g. if the token is in square B2 then the admissible moves are shown by the directed edges). The players alternate moving the token. Black squares are not accessible (so that, for example, from A3 the token can only be moved to A4 or B3 and from F3 it can only be moved to G4, as shown by the directed edge). The player who manages to place the token in the square marked 'END' wins. Is there a perfect winning strategy for any of the players? If so who?



7. **Let me get the unit square!** Zafir this time starts a game with Veda in which they take turns tearing a piece of paper in two and throwing one of the pieces. At each stage of the game the piece of paper is a rectangle with integer side lengths. On each player's turn, they tear the piece of paper into two such rectangles along a horizontal or vertical line, and throw the piece with smaller area. (If the two pieces have the same area they may throw either one.) The game ends when the piece of paper is a 1×1 square, and the winner is the last player to take their turn tearing the paper in two. At the start of the game the piece of paper is a 38×2010 rectangle. If Zafir starts the game, which player has a winning strategy?
8. **Randomly Probable** Veda was thinking something random and starts picking two random numbers a, b from $(0, 1]$ and sees the interval (a, b) . After taking many such intervals (say n intervals), he got interested in finding the probability that there is an interval that intersects all others. Help him Veda the value of $P(n)$ Veda was thinking of something random and started picking two random numbers
9. **Revenge story** Zafir wants revenge and asked Pranjal to give him the hardest challenge possible. Pranjal challenged Zafir to count the number of ways to color a 2020×2020 grid using red and blue colors such that no three consecutive squares (horizontally, vertically, or diagonally) have the same color. Help Zafir get his revenge by finding the total number of valid colorings.
10. **Betting on the World Series** You are a broker; Your job is to accommodate your client's wishes without placing any of your personal capital at risk. Your client wishes to place an even Rs.1,000 bet

on the outcome of the World Series, which is a baseball contest decided in favor of whichever of two teams first wins 4 games. That is, the client deposits his Rs.1,000 with you in advance of the series. At the end of the series he must receive from you either Rs.2,000 if his team wins, or nothing if his team loses. No market exists for bets on the entire world series. However, you can place even bets, in any amount, on each game individually. What is your strategy for placing bets on the individual games in order to achieve the cumulative result demanded by your client?