Tutorials Sheet 1

Ans 1. (3) $O(N+M)$ time
$O(1)$ space

Ans 2. $T(n) = O(n)$, space $O(1)$

Ans 3.
$T(n) = O(\log_2 n)$, space $O(1)$

Ans 4.
```
int sum=0, i;
for(i=0; i*i<n; i++)
  {
    sum += i;
  }
```

$= n + (n-1) + (n-4) + (n-9) + \cdots (n-k)$

$= n + (n*k) - (1^2 + 2^2 + 3^2 + \cdots k^2)$

$= \sqrt{n}$

$i^2 < n$
$i < \sqrt{n}$

$T(n) = O(\sqrt{n})$, space $O(1)$

Ans 5.
```
int j=1, i=0
while(i<=n)
  {
    i=i+j;
    j++;
  }
```

$0 <= n$ 1
$1 <= n$ 1
$3 <= n$

$$(0, \underbrace{1, 3, 6, 10, 15, 21, \ldots n}_{K\ term})$$

$$K^{th}\ term = \frac{(K * (K+1))}{2}$$

$$n = \frac{K^2 + K}{2}$$

$$K^2 + K = 2n$$
$$K^2 + K - 2n = 0$$

$$K = \frac{-1 \pm \sqrt{1^2 + 8n}}{2}$$

$$K = \frac{\sqrt{8n+1}}{2} + 1$$

$$K = \frac{\sqrt{8n+1}}{2}$$

$$K = \frac{\sqrt{8n}}{2} = \sqrt{n}$$

$T(n) = \sqrt{n}$ Space - O(1)

Ans 6 -

```
void Recursion (int n)      ⟶ T(n)
{
   if (n == 1) return;
   recursion (n - 1)         ⟶ T(n-1)
   print (n);                ⟶ 1
}  recursion (n-1);          ⟶ T(n-1)
```

$$T(n) = \begin{cases} 1 & n = 1 \\ 2T(n-1) + 1 & n > 1 \end{cases}$$

$$T(n) = 2T(n-1) + 1 \quad - (1)$$

$$T(n-1) = 2T(n-2) + 1$$

$$T(n) = 2(2T(n-2) + 1) + 1$$
$$T(n) = 4T(n-2) + (1+2) \quad - (2)$$

$$T(n-2) = 2(T(n-3)) + 1$$

$$T(n) = 4(2T(n-3) + 1) + (1+2)$$
$$T(n) = 8T(n-3) + (1+2+4) \quad - (3)$$

$$T(n) = 8[2(T(n-4) + 1) + [1+2+4)$$
$$T(n) = 16T(n-4) + (1+2+4+8) \quad - (4)$$

$$T(n) = 2^k T(n-k) + \underbrace{(1+2+4+8+\cdots)}_{(k \text{ times})}$$

$$T(n-k) = T(1)$$
$$k = n-1$$

$$T(n) = 2^{n-1} T(1) + \underbrace{(1+2+4+8 \cdots)}_{(n-1) \text{ times}}$$

$$T(n) = \frac{2^n}{2} + \underbrace{(1+2+4+8 \cdots)}_{(n-1) \text{ times}}$$

$$S_n = \frac{a(r^n - 1)}{r-1} \quad a=1,\ r=2,\ n=n-1$$

$$T(n) = \frac{2^n}{2} + \left(\frac{2^{n-1}-1}{1}\right)$$

$$T(n) = \frac{2^n}{2} + \frac{2^n}{2} - 1$$

$$T(n) = \cancel{2}\left(\frac{2^n}{\cancel{2}}\right) - 1$$

$$T(n) = 2^{n-1}$$

$$T(n) = 2(2^{n-1}) - 1$$

$$T(n) = 2^n - 1$$

$$T(n) = O(2^n)$$ //

Ans 7- It is a Binary Search Algorithm.

$$T(n) = \log_2 n$$

$$T(n) = T\left(\frac{n}{2}\right) + 1$$

by using Masters meathod (can't be solved)

$$T(n) = aT\left(\frac{n}{b}\right) + f(n)$$

so $a = 1$
$b = 2$
$f(n) = 1$

~~T(n) =~~ $c = \log_b a = \log_2 1 = 0$

$\simeq 1$

$n^{0} = f(n) = 1$

$\therefore \quad n^{c} = f(n)$

$T(n) = \Theta(\log_2 n)$

Ans 8- $T(1) = 1$

1. $T(n) = T(n-1) + 1 \quad — ①$

$T(n) = T(n-2) + 2 \quad — ②$

$T(n) = T(n-3) + 3 \quad — ③$

$T(n) = T(n-k) + k \quad — ④$

$n-k = 1$

$k = n-1$

$T(n) = T(1) + n-1$

$T(n) = n$

$T(n) = O(n)$ //

2. $T(n) = T(n-1) + n \quad — ①$

$T(n-1) = T(n-2) + (n-1)$

$T(n) = T(n-2) + (n + (n-1)) \quad — ②$

$T(n) = T(n-3) + (n + (n-1) + (n-2)) \quad — ③$

$T(n) = T(n-k) + (n + (n-1) + (n-2) - \cdots (n-k-1))$

$T(n-k) = T(1)$

$n = k+1$

$k = n-1$

$T(n) = T(1) + (n + (n-1) + (n-2) - \cdots \underbrace{(n-n+1-1)}_{0})$

$T(n) = 1 + (n + (n-1) + (n-2) + \cdots 1)$

$T(n) = 1 + \frac{n(n+1)}{2} = \frac{n^2+1}{2} + 1$

$$T(n) = \frac{n^2+2}{2}$$

$$T(n) = O(n^2)$$

Ans 8
(Ans 3)- $T(n) = T(n/2) + 1 \quad \text{—} (1)$

$$T\left(\frac{n}{2}\right) = T\left(\frac{n}{4}\right) + 1$$

$$T(n) = T\left(\frac{n}{4}\right) + 2 \quad \text{—} (2)$$

$$T\left(\frac{n}{4}\right) = T\left(\frac{n}{8}\right) + 1$$

$$T(n) = T\left(\frac{n}{8}\right) + 3 \quad \text{—} (3)$$

$$T(n) = T\left(\frac{n}{2^k}\right) + k \quad \text{—} (4)$$

$$\frac{n}{2^k} = 1$$

$$2^k = n$$

$$k = \log_2 n$$

$$T(n) = T(1) + \log_2 n$$

$$T(n) = O(\log_2 n)$$

Ans 8
(Ans 4) $T(n) = 2T\left(\frac{n}{2}\right) + 1.$

$c = 1$
$n^c = n$
$f(n) = 1$

$n^c > f(n)$
$T(n) = \Theta(n)$

Ans 8
(Ans 5) $T(n) = 2T(n-1) + 1$

$T(n) = O(2^n)$ //

Ans 8
(Ans 6) $T(n) = 3T(n-1)$, $T(0) = 1$

$T(n) = 3(T(n-1)$ — ①
$T(n-1) = 3T(n-2)$
$T(n) = 9T(n-2)$
$T(n) = 3^3 T(n-3)$

$T(n) = 3^k T(n-k)$
for $n-k = 0$
$n = k$

$T(n) = 3^n T(0)$

$T(n) = 3^n$

$T(n) = O(3^n)$ //

$T(n) = \begin{cases} 1 & ; n <= 2 \\ T(n) & ; n > 2 \end{cases}$

Ans 8

(Ans 7) - $T(n) = T(\sqrt{n}) + 1$ — (1)

$T(\sqrt{n}) = T(n^{1/4}) + 1$

$T(n) = T(n^{1/4}) + 2$ — (2)

$T(n) = T(n^{1/8}) + 3$ — (3)

$T(n) = T(n^{(1/2^k)}) + k$

for $T((\sqrt{n})^{1/k}) = T(2)$

$n^{\frac{1}{2k}} = 2$

$n^{1/2^k} = 2$

$\frac{1}{2^k} \log n = 1$

$2^k = \log n$

$2^k = \log_n$

$k = \log_2 (\log n)$

$T(n) = O(\log(\log(n)))$ //

Ans 8 -

(a)- $T(n) = T(\sqrt{n}) + n$

$$T(\sqrt{n}) = T(n^{1/4}) + \sqrt{n}$$

$$T(n) = T(n^{1/4}) + (n + \sqrt{n})$$

$$T(n) = T(n^{1/8}) + (n + \sqrt{n} + n^{1/4})$$

$$T(n) = T\left(n^{\frac{1}{2^k}}\right) + (\underbrace{n + n^{1/2} + n^{1/4} + \dots}_{k \text{ terms}})$$

for $n^{\frac{1}{2^k}} = 2$

$$\frac{1}{2^k} = 1/\log(n)$$

$$2^k = \log(n)$$

$$k = \log(\log(n))$$

$$T(n) = 1 + (\underbrace{n + \sqrt{n} + \sqrt{n}\sqrt{n} + \dots}_{k \text{ terms}})$$

$$T(n) = 1 + \left(\begin{matrix} \text{G.P} \quad a = n \\ r = \sqrt{n} \\ \text{No. of terms} = k \end{matrix} \right)$$

$$T(n) = 1 + \left(\frac{n((\sqrt{n})^k - 1)}{(k-1)} \right)$$

$$T(n) = 1 + n\left(\frac{(\sqrt{n})^{\log\log(n)} - 1}{\log\log(n) - 1}\right)$$

$$T(n) = n \cdot \log\log(n)$$ {by neglecting other values

$$T(n) = O(n \cdot \log(\log(n)))$$

Ans 9 -

```
int sum = 0, i;
for (i = 0; i < n; i++)
{
    sum += i;
}
```

$0, 1, 2, \ldots n$

So $T(n) = O(n)$ //, space $O(1)$

Ans 10 -

$O(N * (N, N-1, \ldots, 1))$

$O\left(N * \left(\frac{N+1}{2}\right)\right)$

(4.) $O(N * N)$ //

Ans 11 - $O\left(\frac{n}{2} * (\log_2 N)\right)$

$O(n \log n)$ //

Ans 12 - (2) X will always be a better choice for large input.

Ans 13 - (4) $O(\log N)$

Ans 14 -

$$T(n) = 7\left(T\left(\frac{n}{2}\right)\right) + (3n^2 + 2)$$

$f(n) = 3n^2 + 2$

$a = 7$

$b = 2$

$c = \log_b a = \log_2 7 = 2.807$

$n^c = n^{2.8} \approx n^{2.8}$

$f(n) = 3n^2 + 2$

so $n^c > f(n)$

so $T(n) = \theta(n^{2.8})$ // or (c) $\theta(n^{2.8})$

(a) $O(n^{2.8})$

(d) $\theta(n3)$

Ans 15 - $f1(n) = n^{\sqrt{n}}$

$f2(n) = 2^n$

$f3(n) = (1.000001)^n$

$f_4(n) = n^{(10 * 2^{n/2})}$

a) $f_2(n) > f_4(n) > f_3(n) > f_1(n)$

Ans 16- $f(n) = 2^{2n}$

$\log f(n) = 2n \log_2 2$

$\log f(n) = 2n$

or

$f(n) = 2^n \cdot 2^n$

$\Omega(2^n)$

Ans 17- $T(n) = 2T\left(\frac{n}{2}\right) + n^2$

$c = 1$

$n^c = n$

$n^2 > n$

$f(n) > n^c$

$T(n) = \Theta(n^2)$

Ans 18- $O(\log N)$ [Its a G.C.D function where n keeps on decreasing by $n/2$]

Ans 19 -

$T(n) = O(N^2 + N)$

$T(n) = O(N^2)$