Vacuum properties and astrophysical implications

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QED's predictions that photons propagating in a magnetized vacuum should feel the vacuum birefringence are still standing. Magnetars have strong magnetic fields and may give us signals of this effect through the delay of photons travelling from this source to detectors and the polarization position or by the angle and degree of polarization of the radiation emitted. Starting from non linear electrodynamics, we analyze and discuss for weak and strong field approximations the theoretical predictions for both using a toy model of rotating neutron stars with dipolar magnetic field shape and photon trajectories that lie radial.

Keywords: Magnetic field, Neutron stars, Polarization, Magnetized vacuum.

1. Introduction

Quantum Electrodynamics is a very well-established and successful theory with great numbers of experimental test. The theory conceives that fluctuations from the virtual electron-positron pairs, the vacuum, give rise to very interesting phenomena, becoming a magnetized vacuum in non-linear interaction theory. One of these properties is birenfrigence, which means that electromagnetic waves propagating perpendicular to a constant electric or magnetic field suffer changes in their speed of propagation. This velocity is a function of the external field. As incredible as it seems, this phenomenon has not been detected experimentally yet. It is not clear the reason behind that. It could be due to the smallness of the effect of the magnetic fields generated in the lab, but it is not ruled out that it represents a manifestation of new physics.

One of the most relevant experiment designs for getting birefringence is the Polarization of Vacuum with Laser (PVLAS) experiment.[1] A magnetic field of 5.5×10^4 G was used in this experiment. The PVLAS-team blamed axions[a] as responsible for the discrepancy between the established theoretical QED birefrigence and the null experimental signal.[1]

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[a]hypothetical elementary particles with low mass, candidates for dark matter, and in this context, photons could decay in axions and a new physics may appear.

However Astrophysics could give information about birefringence and other exotic phenomena of the QED-vacuum. The Universe is our main lab, while experiments on Earth are being improved.Very exotic objects like neutron stars have huge magnetic fields that could give us signals of birefrigence of the vacuum but also the inestability of the vacuum and pair production. Neutron stars have magnetic fields of the order of Schwinger's critical field $B_c = 4.41 \times 10^{13}$G,[2] and beyond for magnetars.[3] The birefringence signal from these objects could be measured through the time delay that should undergo the emitted polarized radiation of NS to be detected.

Our work is focused on obtaining a theoretical estimate of the angle of polarization as a consequence of vacuum birefringence, considering that the pulsar magnetosphere is a magnetized vacuum. In that case, we analyze the propagation of photons in two limits, weak and strong magnetic fields, and consider the dipolar shape of the magnetic field.

We started using effective Euler Heisenberg Lagrangian[4] to obtain the modes of polarization of photons traveling perpendicular to the magnetic field ($k \perp B$). This calculation is equivalent to the previous one obtained by solving the photon dispersion equation, considering the radiative corrections given by the magnetized photon self-energy.[5]

2. Euler Heisenberg non-linear electrodynamics

The non linear Euler Heisenberg Lagrangian for the weak $B \ll Bc$,$E \ll Ec$ and strong limits $B \gg Bc$, $E \gg Ec$ is presented in this section. Despite the fact that this formulation does not cover all microscopic photon-photon interactions, it is very useful because it describes vacuum as a non-linear optical medium allowing us to investigate specific phenomena using traditional techniques and interpretation of non-linear optics in media.

For the approximation $B \ll B_c$ the EH lagrangian[4] has the form

$$\mathcal{L}_{EH}^{W} = \mathcal{L}_0 + \xi(\mathcal{L}_0^2 + \frac{7\epsilon_0^2 c^2}{4}(\boldsymbol{E}.\boldsymbol{B})^2), \;\; \mathcal{L}_0 = \frac{\epsilon_0}{2}(E^2 - c^2 B^2), \tag{1}$$

$\mathcal{L}_0$ is the linear Lagrangian density and ϵ_0 and c are the dielectric constant and the speed of light in the vacuum respectively. QED vacuum corrections emerge through the constant $\xi = \frac{8\alpha^2\hbar^3}{45 m_e^4 c^5}$ which encloses the interaction of virtual pairs with magnetic field and depends on charge and mass of electrons. The vacuum: electron-positron virtual pairs have an indirect photon interaction responsible of non linearity of lagrangian Eq. (1). In the limit $B \gg B_c$ we have for the EH Lagrangian the following expression[4,6]

$$\mathcal{L}_{EH}^{S} = -\frac{B^2}{2} + \frac{m^4}{24\pi^2}\frac{B}{B_c}\left(ln\left(\frac{B}{2B_c}\right) + \frac{6}{\pi^2}\xi^{i\prime}(2)\right). \tag{2}$$

where $\frac{6}{\pi^2}\xi^{i\prime}(2) = -0.5699610$ with $\zeta(x)$ the Riemann zeta-function. This limit is important in astrophysics because compact objects, such as neutron stars (magnetars), have a surface magnetic field of order $10^{13} - 10^{15}$ G[7] which is higher than the critical magnetic field of B_c.

We start off from Lagrangian in two limits: weak and strong magnetic field to study the photon propagation perpendicular to a constant magnetic field B_e (in the direction x_3) in a vacuum. The photon can be described as electromagnetic wave with electric and magnetic field E_w and B_w which are orthogonal each other. Therefore, the total electric and magnetic fields are $E_t = E_w$, $B_t = B_w + B_e$.[8] The modified Maxwell equation solutions will be determined by whether the external magnetic field B_e is aligned with or orthogonal to the wave magnetic field B_w ($B_e \perp B_w$, $B_e \parallel B_w$), resulting in two physical polarization modes.[9,10] According to the case, we label along the paper $i = \perp, \parallel$ perpendicular and parallel respectively. Assuming E_w and B_w as plane wave $\sim exp(i(kz - \omega t)$ to solve modified Maxwell equations, doing some algebra and retaining only first term corrections, we may get the dispersion equations.[11] For the weak and strong field approximation reads as

$$\omega_{\parallel}^{W} = \mid \boldsymbol{k} \mid \sqrt{(1 - \frac{\xi}{4}(\boldsymbol{B_e} \times \hat{\boldsymbol{k}})^2)}, \quad \omega_{\perp}^{W} = \mid \boldsymbol{k} \mid \sqrt{(1 - \frac{7}{4}\xi(\boldsymbol{B_e} \times \hat{\boldsymbol{k}})^2)}, \tag{3}$$

$$\omega_{\parallel}^{S} = \mid \boldsymbol{k} \mid \sqrt{(1 - \frac{\alpha}{3\pi}(\hat{\boldsymbol{b}} \times \hat{\boldsymbol{k}}))}, \quad \omega_{\perp}^{S} = \mid \boldsymbol{k} \mid \sqrt{(1 - \frac{\alpha}{3\pi}\frac{B_e}{B_c}(\hat{\boldsymbol{b}} \times \hat{\boldsymbol{k}}))}. \tag{4}$$

For photon propagation perpendicular to the external magnetic field $\theta = \pi/2$. For weak field, both modes depend on the square of the external magnetic field. However, the strong magnetic field limit (SFL) has different behavior. In that case, we obtain where only the perpendicular mode depends on the magnetic field but linearly, while the mode parallel is independent of the magnetic field.

We define $\Delta n^{S,W} \equiv n_{\perp} - n_{\parallel}$ as the difference between the refraction indices of the two polarization modes,

$$\Delta n^{S} = \frac{\alpha}{4\pi}\frac{2}{15}\left(\frac{B_e}{B_c}\right)^2, \qquad \Delta n^{W} = \frac{\alpha}{4\pi}\frac{2}{3}\left(\frac{B_e}{B_c} - 1\right). \tag{5}$$

Let us note that in a magnetized vacuum, not only do photons acquire different velocities according to their polarization, but also the polarization changes once a photon has travelled a certain distance. Initial photon linear polarization becomes elliptical and the angular rotation of the polarization plane of the electromagnetic wave travelling a path length L is $\phi = \Delta n^{S,W} \omega L$.

3. Polarization radiation of neutron stars

Our focus in this section is to revisit the polarization that would occur when photons propagate in the magnetosphere of a neutron star, considering only the effects of a magnetized vacuum. As we mentioned before, NS are very exotic astronomical objects, characterized by huge densities and magnetic fields. The surface magnetic field could be around 10^{12-15} G, but its geometry is complicated, and it is impossible to consider it constant and uniform throughout the star.

Therefore, to do our study we have to make three important approximations: photons propagate in a magnetosphere considered as a magnetized vacuum, the magnetic field shape is dipolar and photon trajectories are radial. These assumptions allow us to calculate the polarization degree (PD) of radiation using the evolution equation, $\frac{\partial \mathbf{s}}{\partial \hat{r}} = \hat{\boldsymbol{\Omega}} \times \mathbf{s}$, $|\hat{\boldsymbol{\Omega}}| = \frac{\omega}{c} \Delta n^{W,S}$, where $\hat{r}$ is the direction of propagation of the photons, $\mathbf{s} = \{S_1/S_0, S_2/S_0, S_3/S_0\}$ is the normalized Stokes vector and $\hat{\boldsymbol{\Omega}}$ is related to the refraction index (defined in Eq (5)). We estimate the polarization degree PD of photons consider the adiabatic approximation $\hat{\boldsymbol{\Omega}} \left(\frac{1}{|\hat{\boldsymbol{\Omega}}|} \frac{\partial |\hat{\boldsymbol{\Omega}}|}{\partial \hat{r}} \right)^{-1} \gtrsim 1/2$, obtained in.[13] This condition allows us to calculate the polarization-limiting radius r_L at which the degree of polarization begins to be constant and the polarization becomes elliptical. We consider a dipolar shape configuration for the magnetic field as $B_e(r) = B_0 \left(\frac{r_0}{r}\right)^3$, with B_0 and r_0 are the surface magnetic field and radius of the neutron star respectively. Because, regardless of the value of the surface magnetic field B_0, the important values of magnetic field strength are those where decoupling of polarization modes occurs, whether far or close to the neutron star, our calculations will be performed for the entire range of magnetic fields using the weak and strong field approximations. If the decoupling modes occur far enough from the star's surface, $r_L \gg r_0$, as the magnetic field goes as r^{-3}, the magnetic field will be significantly smaller. Then, $\Omega \sim \Delta n^W \sim B_e^2$. When decoupling arises for $r_0 \leq r_L < 2r_0$ the strong field approximation, $B_0/B_c \gtrsim 1$, is suitable, and $\Omega \sim \Delta n^S \sim B_e$. We assume rotating NS for both magnetic field regimes. Then, the angle ϕ is given by the relation $r_L - r_0 = \phi \frac{P}{2\pi}$ where P is the period of the pulsar.[12] The computed polarization angle is given by the equations

$$\phi(\omega, B, P) = \begin{cases} \frac{2\pi}{cP} \left(\left[\frac{\alpha}{90\pi} \frac{\omega}{c} \left(\frac{B_0}{B_c} \right)^2 r_0^6 \right]^{1/5} - r_0 \right), & B < B_c \text{ (weak)} \\ \frac{2\pi}{cP} \left(\left(\frac{\alpha}{9\pi} \frac{\omega}{c} \frac{B_0}{B_c} r_0^3 \right)^{1/2} - r_0 \right), & B > B_c \text{ (strong)} \end{cases} \tag{6}$$

and polarization degree is $\Pi(\omega, B, P) = 4\pi/cP \left((3/r_L)^2 + (4\pi/cP)^2 \right)^{-1/2}$. For higher frequencies, the radiation from neutron stars' magnetosphere is substantially polarized (fig. 1). Furthermore, neutron stars or magnetars with a higher magnetic field, release strongly polarized radiation. The polarization degree (PD) of radiation of a pulsar is inversely related to its period; millisecond pulsars have more polarization than second period pulsars. The behavior of angle and polarization degree with the magnetic field is depicted in fig. 1 for weak and strong field approximations. For magnetic fields greater than $2B_c$, strong field approximation contributes to an increase the polarization degree and polarization angle compared to weak field approximation.

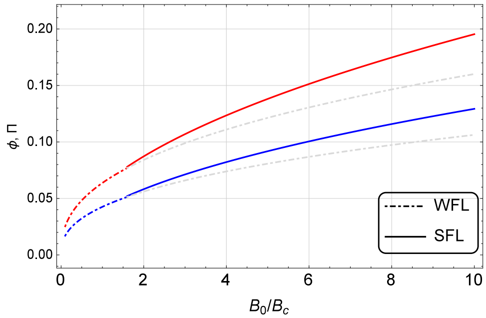

Fig. 1. Comparison of the polarization degree (Blue line) and polarization angle (Red line) for weak field limit (dotdashed line) and strong field limit (solid line) as function of the surface magnetic field of the pulsar. We take typical neutron star parameters, a radius $r_0 = 10$ km, a period of $P = 1$ s and radiation of $\omega = 2$ MeV of energy.

Our results are in agreement with previous obtained by Heyl et al[13] when weak field approximation is taken into account. Furthermore, studies on $RXJ1856.5-3764$ for polarization of visible and X radiation revealed that it is essential to include the influence of magnetized vacuum to explain the observational data of polarization of this source.[14]

For photon energy of $\omega = 1MeV$ and typical values of the parameters $B_0 = 1.5 \times 10^{13}G$, $M = 1.4M_{\odot}$, radius $r_0 = 12km$, and $P = 7.06s$, the angle and degree of polarization for $RXJ1856.5 - 3764$ owing to simply the effect of magnetized vacuum give $\phi = 6.4 \times 10^{-3}$ and $\Pi = 0.42\%$.[15,16] Obviously, when geometric factors are taken into account, this value rises.[14]

4. Conclusions

From non-linear electrodynamics we have obtained the refraction index and different phase velocity of photon propagating in strong magnetic field according to the polarization modes. We have also obtained the polarization angle that the radiation acquires as a consequence of crossing a magnetized vacuum. The calculations have been done in weak and strong field approximation for a constant magnetic field.

In addition, we investigated the influence of a magnetized vacuum on the polarization of radiation emitted by rotating NS. To do so, we considered the magnetosphere to be a magnetized vacuum with a dipolar magnetic field and photons traveling along radial trajectories.

Our findings are consistent with previous results based on weak field approximation.[13] More energetic photons have a greater polarization degree than the lower ones. Besides, the polarization degree is strongly dependent on the period of rotation of neutron stars since this can raise or decrease the polarization limiting radii, causing the couple or decouple of the propagation modes.

The authors of the work[14] claim that there is no way of explaining the polarization data from visible and X rays of the pulsar $RXJ1856.5 - 3764$ without considering the effect of magnetized vacuum. As a consequence, it might be a test of vacuum birefringence, albeit we expect it will be tested in terrestrial laboratories.

However, not all regarding vacuum birefringence has been said, and the door is opened for new physics. This work was undertaken as a starting point for developing more realistic models of polarization radiation emission of NS which will allow us to identify in robust way those linked to vacuum birefringence and may validate the QED prediction.

Acknowledgments

A.P.G and A.P.M acknowledge the support of the Agencia Estatal de Investigación through the grant PID2019-107778GB-100 from Junta de Castilla y Leon, Spanish Consolider MultiDark FPA2017-90566-REDC and PHAROS COST Actions MP1304 and CA16214. The work of E.R.Q. and A.W.R.J. were supported by the project of No. NA211LH500-002 from AENTA-CITMA, Cuba.

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