PS for Chapter 10 Fluids

1. A bottle has a mass of 35.00 g when empty and 98.44 g when filled with water. When filled with another fluid, the mass is 89.22 g. What is the specific gravity of this other fluid?

To find the specific gravity of the fluid, take the ratio of the density of the fluid to that of water, noting that the same volume is used for both liquids.

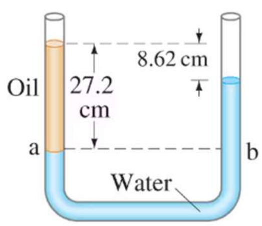
$$\mathrm{SG}_{\text{fluid}} = \frac{\rho_{\text{fluid}}}{\rho_{\text{water}}} = \frac{(m/V)_{\text{fluid}}}{(m/V)_{\text{water}}} = \frac{m_{\text{fluid}}}{m_{\text{water}}} = \frac{89.22\ \text{g} - 35.00\ \text{g}}{98.44\ \text{g} - 35.00\ \text{g}} = \boxed{0.8547}$$

2. The gauge pressure in each of the four tires of an automobile is 240 kPa. If each tire has a "footprint" of 190 cm2 (area touching the ground), estimate the mass of the car.

The sum of the force exerted by the pressure in each tire is equal to the weight of the car.

$$mg = 4PA \quad \rightarrow \quad m = \frac{4PA}{g} = \frac{4(2.40\times10^5\ \text{N/m}^2)(190\ \text{cm}^2)\left(\dfrac{1\ \text{m}^2}{10^4\ \text{cm}^2}\right)}{(9.80\ \text{m/s}^2)} = 1861\ \text{kg} \approx \boxed{1900\ \text{kg}}$$

3. Water and then oil (which don't mix) are poured into a U-shaped tube, open at both ends. They come to equilibrium as shown in the figure. What is the density of the oil? [*Hint:* Pressures at points a and b are equal. Why?]



The pressures at points a and b are equal since they are at the same height in the same fluid. If the pressures were unequal, then the fluid would flow. Calculate the pressure at both a and b, starting with atmospheric pressure at the top surface of each liquid, and then equate those pressures.

$$P_{\text{a}} = P_{\text{b}} \quad \rightarrow \quad P_0 + \rho_{\text{oil}} g h_{\text{oil}} = P_0 + \rho_{\text{water}} g h_{\text{water}} \quad \rightarrow \quad \rho_{\text{oil}} h_{\text{oil}} = \rho_{\text{water}} h_{\text{water}} \quad \rightarrow$$

$$\rho_{\text{oil}} = \frac{\rho_{\text{water}} h_{\text{water}}}{h_{\text{oil}}} = \frac{(1.00\times10^3\ \text{kg/m}^3)(0.272\ \text{m} - 0.0862\ \text{m})}{(0.272\ \text{m})} = \boxed{683\ \text{kg/m}^3}$$

4. A spherical balloon has a radius of 7.15 m and is filled with helium. How large a cargo can it lift, assuming that the skin and structure of the balloon have a mass of 930 kg? Neglect the buoyant force on the cargo volume itself.

The buoyant force of the balloon must equal the weight of the balloon plus the weight of the helium in the balloon plus the weight of the load. For calculating the weight of the helium, we assume it is at 0°C and 1 atm pressure. The buoyant force is the weight of the air displaced by the volume of the balloon.

$$F_{\text{buoyant}} = \rho_{\text{air}} V_{\text{balloon}} g = m_{\text{He}} g + m_{\text{balloon}} g + m_{\text{cargo}} g \quad \rightarrow$$

$$m_{\text{cargo}} = \rho_{\text{air}} V_{\text{balloon}} - m_{\text{He}} - m_{\text{balloon}} = \rho_{\text{air}} V_{\text{balloon}} - \rho_{\text{He}} V_{\text{balloon}} - m_{\text{balloon}} = (\rho_{\text{air}} - \rho_{\text{He}}) V_{\text{balloon}} - m_{\text{balloon}}$$

$$= (1.29\ \text{kg/m}^3 - 0.179\ \text{kg/m}^3)\tfrac{4}{3}\pi(7.15\ \text{m})^3 - 930\ \text{kg} = \boxed{770\ \text{kg}} = 7600\ \text{N}$$

5. The specific gravity of ice is 0.917, whereas that of seawater is 1.025. What percent of an iceberg is above the surface of the water?

The buoyant force on the ice is equal to the weight of the ice, since it floats.

$$F_{\text{buoyant}} = W_{\text{ice}} \rightarrow m_{\substack{\text{seawater}\\ \text{submerged}}} g = m_{\text{ice}} g \rightarrow m_{\substack{\text{seawater}\\ \text{submerged}}} = m_{\text{ice}} \rightarrow \rho_{\text{seawater}} V_{\text{seawater}} = \rho_{\text{ice}} V_{\text{ice}} \rightarrow$$

$$(\text{SG})_{\text{seawater}} \rho_{\text{water}} V_{\substack{\text{submerged}\\ \text{ice}}} = (\text{SG})_{\text{ice}} \rho_{\text{water}} V_{\text{ice}} \rightarrow (\text{SG})_{\text{seawater}} V_{\substack{\text{submerged}\\ \text{ice}}} = (\text{SG})_{\text{ice}} V_{\text{ice}} \rightarrow$$

$$V_{\substack{\text{submerged}\\ \text{ice}}} = \frac{(\text{SG})_{\text{ice}}}{(\text{SG})_{\text{seawater}}} V_{\text{ice}} = \frac{0.917}{1.025} V_{\text{ice}} = 0.895\, V_{\text{ice}}$$

Thus, the fraction above the water is $V_{\text{above}} = V_{\text{ice}} - V_{\text{submerged}} = 0.105\, V_{\text{ice}}$ or $\boxed{10.5\%}$.

6. A 0.48-kg piece of wood floats in water but is found to sink in alcohol SG=0.79, in which it has an apparent mass of 0.047 kg. What is the SG of the wood?

The difference between the actual mass and the apparent mass is the mass of the alcohol displaced by the wood. The mass of the alcohol displaced is the volume of the wood times the density of the alcohol, the volume of the wood is the mass of the wood divided by the density of the wood, and the density of the alcohol is its specific gravity times the density of water.

$$m_{\text{actual}} - m_{\text{apparent}} = \rho_{\text{alc}} V_{\text{wood}} = \rho_{\text{alc}} \frac{m_{\text{actual}}}{\rho_{\text{wood}}} = \text{SG}_{\text{alc}} \rho_{H_2O} \frac{m_{\text{actual}}}{\rho_{\text{wood}}} \rightarrow$$

$$\frac{\rho_{\text{wood}}}{\rho_{H_2O}} = \text{SG}_{\text{wood}} = \text{SG}_{\text{alc}} \frac{m_{\text{actual}}}{(m_{\text{actual}} - m_{\text{apparent}})} = (0.79)\frac{0.48\ \text{kg}}{(0.48\ \text{kg} - 0.047\ \text{kg})} = \boxed{0.88}$$

7. What is the volume rate of flow of water from a 1.85-cm-diameter faucet if the pressure head is 12.0 m?

The pressure head can be interpreted as an initial height for the water, with a speed of 0 and at atmospheric pressure. Apply Bernoulli's equation to the faucet location and the pressure head location to find the speed of the water at the faucet, and then calculate the volume flow rate. Since the faucet is open, the pressure there will be atmospheric as well.

$$P_{\text{faucet}} + \tfrac{1}{2}\rho \upsilon_{\text{faucet}}^2 + \rho g y_{\text{faucet}} = P_{\text{head}} + \tfrac{1}{2}\rho \upsilon_{\text{head}}^2 + \rho g y_{\text{head}} \rightarrow$$

$$\upsilon_{\text{faucet}}^2 = \frac{2}{\rho}(P_{\text{head}} - P_{\text{faucet}}) + \upsilon_{\text{head}}^2 + 2g(y_{\text{head}} - y_{\text{faucet}}) = 2g y_{\text{head}} \rightarrow$$

$$\upsilon_{\text{faucet}} = \sqrt{2g y_{\text{head}}}$$

$$\text{Volume flow rate} = A\upsilon = \pi r^2 \sqrt{2g y_{\text{head}}} = \pi\left[\tfrac{1}{2}(1.85\times10^{-2}\,\text{m})\right]^2 \sqrt{2(9.80\ \text{m/s}^2)(12.0\,\text{m})}$$

$$= \boxed{4.12\times10^{-3}\ \text{m}^3/\text{s}}$$