PS for Chapter 11 and Chapter 12

1. The springs of a 1700-kg car compress 5.0 mm when its 66-kg driver gets into the driver's seat. If the car goes over a bump, what will be the frequency of oscillations? Ignore damping.

The spring constant is found from the ratio of applied force to displacement.

$$k = \frac{F_{\text{ext}}}{x} = \frac{mg}{x} = \frac{(66\text{ kg})(9.80\text{ m/s}^2)}{5.0\times 10^{-3}\text{ m}} = 1.294\times 10^5\text{ N/m}$$

The frequency of oscillation is found from the total mass and the spring constant.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{1.294\times 10^5\text{ N/m}}{1766\text{ kg}}} = 1.362\text{ Hz} \approx \boxed{1.4\text{ Hz}}$$

2. A mass m at the end of a spring oscillates with a frequency of 0.83 Hz. When an additional 780-g mass is added to m, the frequency is 0.60 Hz. What is the value of m?

The spring constant is the same regardless of what mass is attached to the spring.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad\rightarrow\quad \frac{k}{4\pi^2} = mf^2 = \text{constant} \quad\rightarrow\quad m_1 f_1^2 = m_2 f_1^2 \quad\rightarrow$$

$$(m\text{ kg})(0.83\text{ Hz})^2 = (m\text{ kg} + 0.78\text{ kg})(0.60\text{ Hz})^2 \quad\rightarrow\quad m = \frac{(0.78\text{ kg})(0.60\text{ Hz})^2}{(0.83\text{ Hz})^2 - (0.60\text{ Hz})^2} = \boxed{0.85\text{ kg}}$$

3. It takes a force of 91.0 N to compress the spring of a toy popgun 0.175 m to "load" a 0.160-kg ball. With what speed will the ball leave the gun if fired horizontally?

The spring constant is found from the ratio of applied force to displacement.

$$k = \frac{F}{x} = \frac{91.0\text{ N}}{0.175\text{ m}} = 520\text{ N/m}$$

Assuming that there are no dissipative forces acting on the ball, the elastic potential energy in the loaded position will become kinetic energy of the ball.

$$E_{\text{i}} = E_{\text{f}} \quad\rightarrow\quad \tfrac{1}{2}kx_{\text{max}}^2 = \tfrac{1}{2}mv_{\text{max}}^2 \quad\rightarrow\quad v_{\text{max}} = x_{\text{max}}\sqrt{\frac{k}{m}} = (0.175\text{ m})\sqrt{\frac{520\text{ N/m}}{0.160\text{ kg}}} = \boxed{9.98\text{ m/s}}$$

4. What is the period of a simple pendulum 47 cm long (*a*) on the Earth, and (*b*) when it is in a freely falling elevator?

The period of a pendulum is given by $T = 2\pi\sqrt{\ell/g}$.

(*a*) $$T = 2\pi\sqrt{\ell/g} = 2\pi\sqrt{\frac{0.47\text{ m}}{9.80\text{ m/s}^2}} = \boxed{1.4\text{ s}}$$

(*b*) If the pendulum is in free fall, there is no tension in the string supporting the pendulum bob and no restoring force to cause oscillations. Thus there will be no period—the pendulum will not oscillate, so no period can be defined.

5. A sound wave in air has a frequency of 282 Hz and travels with a speed of 343 m/s. How far apart are the wave crests (compressions)?

The distance between wave crests is the wavelength of the wave.

$$\lambda = \upsilon/f = (343\ \text{m/s})/282\ \text{Hz} = \boxed{1.22\ \text{m}}$$

6. What is the ratio of (*a*) the intensities, and (*b*) the amplitudes, of an earthquake P wave passing through the Earth and detected at two points 15 km and 45 km from the source?

(*a*) Assume that the earthquake waves spread out spherically from the source. Under those conditions, intensity is inversely proportional to the square of the distance from the source of the wave.

$$I_{45\ \text{km}}/I_{15\ \text{km}} = (15\ \text{km})^2/(45\ \text{km})^2 = \boxed{0.11}$$

(*b*) The intensity is proportional to the square of the amplitude, so the amplitude is inversely proportional to the distance from the source of the wave.

$$A_{45\ \text{km}}/A_{15\ \text{km}} = 15\ \text{km}/45\ \text{km} = \boxed{0.33}$$

7. A particular string resonates in four loops at a frequency of 240 Hz. Give at least three other frequencies at which it will resonate. What is each called?

Four loops is the standing wave pattern for the fourth harmonic, with a frequency given by $f_4 = 4f_1 = 240$ Hz. Thus, $\boxed{f_1 = 60\ \text{Hz},\ f_2 = 120\ \text{Hz},\ f_3 = 180\ \text{Hz, and}\ f_5 = 300\ \text{Hz}}$ are all other resonant frequencies, where f_1 is the fundamental or first harmonic, f_2 is the first overtone or second harmonic, f_3 is the second overtone or third harmonic, and f_5 is the fourth overtone or fifth harmonic.

8. The speed of waves on a string is 97 m/s. If the frequency of standing waves is 475 Hz, how far apart are two adjacent nodes?

Adjacent nodes are separated by a half-wavelength,

$$\lambda = \frac{\upsilon}{f} \rightarrow \Delta x_{\text{node}} = \tfrac{1}{2}\lambda = \frac{\upsilon}{2f} = \frac{97\ \text{m/s}}{2(475\ \text{Hz})} = 0.10211\ \text{m} \approx \boxed{0.10\ \text{m}}$$

9. (*a*) Calculate the wavelengths in air at 20°C for sounds in the maximum range of human hearing, 20 Hz to 20,000 Hz. (*b*) What is the wavelength of an 18-MHz ultrasonic wave?

(*a*) $\lambda_{20\ \text{Hz}} = \dfrac{\upsilon}{f} = \dfrac{343\ \text{m/s}}{20\ \text{Hz}} = \boxed{17\ \text{m}}$ $\quad \lambda_{20\ \text{kHz}} = \dfrac{\upsilon}{f} = \dfrac{343\ \text{m/s}}{2.0\times10^4\ \text{Hz}} = \boxed{1.7\times10^{-2}\ \text{m}}$

The range is from 1.7 cm to 17 m.

(*b*) $\lambda = \frac{\upsilon}{f} = \frac{343 \text{ m/s}}{18\times10^6 \text{ Hz}} = \boxed{1.9\times10^{-5} \text{ m}}$

10. A stone is dropped from the top of a cliff. The splash it makes when striking the water below is heard 2.7 s later. How high is the cliff?

The total time T is the time for the stone to fall (t_{down}) plus the time for the sound to come back to the top of the cliff (t_{up}): $T = t_{\text{up}} + t_{\text{down}}$. Use constant-acceleration relationships for an object dropped from rest that falls a distance h in order to find t_{down}, with down as the positive direction. Use the constant speed of sound to find t_{up} for the sound to travel a distance h.

$$\text{down: } y = y_0 + \upsilon_0 t_{\text{down}} + \tfrac{1}{2}at_{\text{down}}^2 \rightarrow h = \tfrac{1}{2}gt_{\text{down}}^2 \qquad \text{up: } h = \upsilon_{\text{snd}}t_{\text{up}} \rightarrow t_{\text{up}} = \frac{h}{\upsilon_{\text{snd}}}$$

$$h = \tfrac{1}{2}gt_{\text{down}}^2 = \tfrac{1}{2}g(T - t_{\text{up}})^2 = \tfrac{1}{2}g\left(T - \frac{h}{\upsilon_{\text{snd}}}\right)^2 \rightarrow h^2 - 2\upsilon_{\text{snd}}\left(\frac{\upsilon_{\text{snd}}}{g} + T\right)h + T^2\upsilon_{\text{snd}}^2 = 0$$

This is a quadratic equation for the height. This can be solved with the quadratic formula, but be sure to keep several significant digits in the calculations.

$$h^2 - 2(343 \text{ m/s})\left(\frac{343 \text{ m/s}}{9.80 \text{ m/s}^2} + 2.7 \text{ s}\right)h + (2.7 \text{ s})^2(343 \text{ m/s})^2 = 0 \rightarrow$$

$$h^2 - (25862 \text{ m})h + 8.5766\times10^5 \text{ m}^2 = 0 \rightarrow h = \frac{25862 \pm 25796}{2} = 25{,}829 \text{ m}, \boxed{33 \text{ m}}$$

The larger root is impossible since it takes more than 2.7 s for the rock to fall that distance, so $\boxed{h = 33 \text{ m}}$.

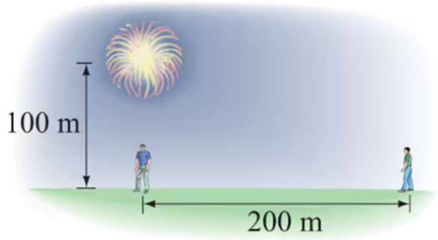
11. You are trying to decide between two new stereo amplifiers. One is rated at 75 W per channel and the other is rated at 120 W per channel. In terms of dB, how much louder will the more powerful amplifier be when both are producing sound at their maximum levels?

Compare the two power output ratings using the definition of decibels.

$$\beta = 10 \log\frac{P_{150}}{P_{100}} = 10 \log\frac{120 \text{ W}}{75 \text{ W}} = \boxed{2.0 \text{ dB}}$$

This would barely be perceptible.

12. A fireworks shell explodes 100 m above the ground, creating colorful sparks. How much greater is the sound level of the explosion for a person at a point directly below the explosion than for a person a horizontal distance of 200 m away?



The first person is a distance of $r_1 = 100\,\text{m}$ from the explosion, while the second person is a distance $r_2 = \sqrt{5}(100\text{ m})$ from the explosion. The intensity detected away from the explosion is inversely proportional to the square of the distance from the explosion.

$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2} = \left[\frac{\sqrt{5}(100\text{ m})}{100\text{ m}}\right]^2 = 5; \quad \beta = 10\log\frac{I_1}{I_2} = 10\log 5 = 6.99\text{ dB} \approx \boxed{7\text{ dB}}$$

13. If you were to build a pipe organ with open-tube pipes spanning the range of human hearing (20 Hz to 20 kHz), what would be the range of the lengths of pipes required?

For a pipe open at both ends, the fundamental frequency is given by $f_1 = \frac{v}{2\ell}$, so the length for a given fundamental frequency is $\ell = \frac{v}{2f_1}$.

$$\ell_{20\text{ Hz}} = \frac{343\text{ m/s}}{2(20\text{ Hz})} = \boxed{8.6\text{ m}} \qquad \ell_{20\text{ kHz}} = \frac{343\text{ m/s}}{2(20{,}000\text{ Hz})} = \boxed{8.6\times10^{-3}\text{ m}}$$

14. What is the beat frequency if middle C (262 Hz) and C# (277 Hz) are played together? What if each is played two octaves lower (each frequency reduced by a factor of 4)?

The beat frequency is the difference in the two frequencies, or $277\text{ Hz} - 262\text{ Hz} = \boxed{15\text{ Hz}}$. If both frequencies are reduced by a factor of 4, then the difference between the two frequencies will also be reduced by a factor of 4, so the beat frequency will be $\frac{1}{4}(15\text{ Hz}) = 3.75\text{ Hz} \approx \boxed{3.8\text{ Hz}}$.

15. The predominant frequency of a certain fire truck's siren is 1650 Hz when at rest. What frequency do you detect if you move with a speed of 30.0 m/s (*a*) toward the fire truck, and (*b*) away from it?

(*a*)Observer moving toward stationary source:

$$f' = \left(1 + \frac{v_{\text{obs}}}{v_{\text{snd}}}\right) f = \left(1 + \frac{30.0\text{ m/s}}{343\text{ m/s}}\right)(1650\text{ Hz}) = \boxed{1790\text{ Hz}}$$

(*b*) Observer moving away from stationary source:

$$f' = \left(1 - \frac{v_{\text{obs}}}{v_{\text{snd}}}\right) f = \left(1 - \frac{30.0\text{ m/s}}{343\text{ m/s}}\right)(1650\text{ Hz}) = \boxed{1510\text{ Hz}}$$

16. A bat at rest sends out ultrasonic sound waves at 50.0 kHz and receives them returned from an object moving directly away from it at 27.5 m/s. What is the received sound frequency?

The moving object can be treated as a moving "observer" for calculating the frequency it receives and reflects. The bat (the source) is stationary.

$$f'_{\text{object}} = f_{\text{bat}}\left(1 - \frac{\upsilon_{\text{object}}}{\upsilon_{\text{snd}}}\right)$$

Then the object can be treated as a moving source emitting the frequency f'_{object} and the bat as a stationary observer.

$$f''_{\text{bat}} = \frac{f'_{\text{object}}}{\left(1 + \frac{\upsilon_{\text{object}}}{\upsilon_{\text{snd}}}\right)} = f_{\text{bat}} \frac{\left(1 - \frac{\upsilon_{\text{object}}}{\upsilon_{\text{snd}}}\right)}{\left(1 + \frac{\upsilon_{\text{object}}}{\upsilon_{\text{snd}}}\right)} = f_{\text{bat}} \frac{(\upsilon_{\text{snd}} - \upsilon_{\text{object}})}{(\upsilon_{\text{snd}} + \upsilon_{\text{object}})}$$

$$= (5.00 \times 10^4 \text{ Hz})\left(\frac{343 \text{ m/s} - 27.0 \text{ m/s}}{343 \text{ m/s} + 27.0 \text{ m/s}}\right) = \boxed{4.27 \times 10^4 \text{ Hz}}$$