PS Chapter 14 Heat

1. An average active person consumes about 2500 Cal a day. (*a*) What is this in joules? (*b*) What is this in kilowatt-hours? (*c*) If your power company charges about 10 ¢ per kilowatt-hour, how much would your energy cost per day if you bought it from the power company? Could you feed yourself on this much money per day

$$(a)\ 2500\ \text{Cal}\left(\frac{4.186\times10^3\ \text{J}}{1\ \text{Cal}}\right)=\boxed{1.0\times10^7\ \text{J}}$$

$$(b)\quad 2500\ \text{Cal}\left(\frac{1\ \text{kWh}}{860\ \text{Cal}}\right)=\boxed{2.9\ \text{kWh}}$$

(*c*) At 10 cents per day, the food energy costs $\boxed{\$0.29\ \text{per day}}$. It would be impossible to feed yourself in the United States on this amount of money.

2. (*a*) How much energy is required to bring a 1.0-L pot of water at 20°C to 100°C? (*b*) For how long could this amount of energy run a 60-W lightbulb?

(*a*)The heat absorbed can be calculated from Eq. 14–2. Note that 1 L of water has a mass of 1 kg.

$$Q=mc\Delta T=\left[(1.0\ \text{L})\left(\frac{1\times10^{-3}\ \text{m}^3}{1\ \text{L}}\right)\left(\frac{1.0\times10^3\ \text{kg}}{1\ \text{m}^3}\right)\right](4186\ \text{J/kg}\cdot\text{C}^\circ)(100^\circ\text{C}-20^\circ\text{C})$$

$$=3.349\times10^5\ \text{J}\approx\boxed{3.3\times10^5\ \text{J}}$$

(*b*) Power is the rate of energy usage.

$$P=\frac{\Delta E}{\Delta t}=\frac{Q}{\Delta t}\quad\rightarrow\quad \Delta t=\frac{Q}{P}=\frac{3.349\times10^5\ \text{J}}{60\ \text{W}}=5582\ \text{s}\approx\boxed{5600\ \text{s}}\approx 93\ \text{min}$$

3. How long does it take a 750-W coffeepot to bring to a boil 0.75 L of water initially at 11°C? Assume that the part of the pot which is heated with the water is made of 280 g of aluminum, and that no water boils away.

The heat must warm both the water and the pot to 100°C. The heat is also the power times the time. The temperature change is 89 C°.

$$Q=Pt=(m_{\text{Al}}c_{\text{Al}}+m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}})\Delta T_{\text{H}_2\text{O}}\quad\rightarrow$$

$$t=\frac{(m_{\text{Al}}c_{\text{Al}}+m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}})\Delta T_{\text{H}_2\text{O}}}{P}=\frac{[(0.28\ \text{kg})(900\ \text{J/kg}\cdot\text{C}^\circ)+(0.75\ \text{kg})(4186\ \text{J/kg}\cdot\text{C}^\circ)](89\ \text{C}^\circ)}{750\ \text{W}}$$

$$=402\ \text{s}\approx\boxed{4.0\times10^2\ \text{s}=6.7\ \text{min}}$$

4. The *heat capacity, C*, of an object is defined as the amount of heat needed to raise its temperature by 1 C°. Thus, to raise the temperature by *ΔT* requires heat *Q* given by

$$Q = C\,\Delta T.$$

(a)Write the heat capacity C in terms of the specific heat, c, of the material. (b) What is the heat capacity of 1.0 kg of water? (c) Of 45 kg of water?

(a) Since $Q = mc\Delta T$ and $Q = C\Delta T$, equate these two expressions for Q and solve for C.

$$Q = mc\Delta T = C\Delta T \quad \rightarrow \quad \boxed{C = mc}$$

(b) For 1.0 kg of water: $C = mc = (1.0\text{ kg})(4186\text{ J/kg}\cdot\text{C°}) = \boxed{4.2\times10^{3}\text{ J/C°}}$

(c) For 45 kg of water: $C = mc = (45\text{ kg})(4186\text{ J/kg}\cdot\text{C°}) = \boxed{1.9\times10^{5}\text{ J/C°}}$

5. What mass of steam at 100°C must be added to 1.00 kg of ice at 0°C to yield liquid water at 30°C?

The heat lost by the steam condensing and then cooling to 30°C must be equal to the heat gained by the ice melting and then warming to 30°C.

$$m_{\text{steam}}[L_{\text{V}} + c_{\text{H}_2\text{O}}(T_{\text{isteam}} - T_{\text{eq}})] = m_{\text{ice}}[L_{\text{F}} + c_{\text{H}_2\text{O}}(T_{\text{eq}} - T_{\text{iice}})]$$

$$m_{\text{steam}} = m_{\text{ice}}\frac{[L_{\text{F}} + c_{\text{H}_2\text{O}}(T_{\text{eq}} - T_{\text{iice}})]}{[L_{\text{V}} + c_{\text{H}_2\text{O}}(T_{\text{isteam}} - T_{\text{eq}})]} = (1.00\text{ kg})\frac{[3.33\times10^{5}\text{ J/kg} + (4186\text{ J/kg}\cdot\text{C°})(30\text{°C})]}{[22.6\times10^{5}\text{ J/kg} + (4186\text{ J/kg}\cdot\text{C°})(30\text{°C})]}$$

$$= \boxed{0.18\text{ kg}}$$

6. A 64-kg ice-skater moving at 7.5 m/s glides to a stop. Assuming the ice is at 0°C and that 50% of the heat generated by friction is absorbed by the ice, how much ice melts?

Assume that all of the melted ice stays at 0°C, so that all the heat is used in melting ice and none in warming water. The available heat is half of the original kinetic energy.

$$\tfrac{1}{2}\left(\tfrac{1}{2}m_{\text{skater}}\upsilon^2\right) = Q = m_{\text{ice}}L_{\text{F}} \quad \rightarrow$$

$$m_{\text{ice}} = \frac{\frac{1}{4}m_{\text{skater}}\upsilon^2}{L_{\text{F}}} = \frac{\frac{1}{4}(64\text{ kg})(7.5\text{ m/s})^2}{3.33\times10^{5}\text{ J/kg}} = \boxed{2.7\times10^{-3}\text{ kg}} = 2.7\text{ g}$$

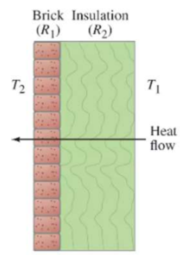
7. *Heat conduction to skin*. Suppose 150 W of heat flows by conduction from the blood capillaries beneath the skin to the body's surface area of $1.5\,\text{m}^2$. If the temperature difference is 0.50 C°, estimate the average distance of capillaries below the skin surface.

The distance can be calculated from the heat conduction rate, given by Eq. 14–5. The rate is given as a power (150 W = 150 J/s).

$$\frac{Q}{t} = P = kA\frac{T_1 - T_2}{\ell} \quad \rightarrow \quad \ell = kA\frac{T_1 - T_2}{P} = (0.2\text{ J/s}\cdot\text{m}\cdot\text{C°})(1.5\text{ m}^2)\frac{0.50\text{ C°}}{150\text{ W}} = \boxed{1.0\times10^{-3}\text{ m}}$$

8. Suppose the insulating qualities of the wall of a house come mainly from a 4.0-in. layer of brick and an R-19 layer of insulation, as shown in Fig. 14–19. What is the total rate of heat loss through such a wall, if its total area is 195ft^2 and the temperature difference across it is 35 F°?

Figure 14–19



Problem 45. Two layers insulating a wall.

The conduction rates through the two materials must be equal. If they were not, the temperatures in the materials would be changing. Call the temperature at the boundary between the materials T_x.

$$\frac{Q}{t} = k_1 A \frac{T_1 - T_x}{\ell_1} = k_2 A \frac{T_x - T_2}{\ell_2} \quad \rightarrow \quad \frac{Q}{t}\frac{\ell_1}{k_1 A} = T_1 - T_x ; \frac{Q}{t}\frac{\ell_2}{k_2 A} = T_x - T_2$$

Add these two equations together, and solve for the heat conduction rate.

$$\frac{Q}{t}\frac{\ell_1}{k_1 A} + \frac{Q}{t}\frac{\ell_2}{k_2 A} = T_1 - T_x + T_x - T_2 \quad \rightarrow \quad \frac{Q}{t}\left(\frac{\ell_1}{k_1} + \frac{\ell_2}{k_2}\right)\frac{1}{A} = T_1 - T_2 \quad \rightarrow$$

$$\frac{Q}{t} = A\frac{(T_1 - T_2)}{\left(\frac{\ell_1}{k_1} + \frac{\ell_2}{k_2}\right)} = A\frac{(T_1 - T_2)}{(R_1 + R_2)}$$

The R-value for the brick needs to be calculated, using the definition of R given on page 402 of the textbook.

$$R = \frac{\ell}{k} = \frac{4\text{ in.}\left(\frac{1\text{ ft}}{12\text{ in.}}\right)}{0.84\frac{\text{J}}{\text{s}\cdot\text{m}\cdot\text{C}°}\left(\frac{1\text{ Btu}}{1055\text{ J}}\right)\left(\frac{1\text{ m}}{3.281\text{ ft}}\right)\left(\frac{5\text{ C}°}{9\text{ F}°}\right)\left(\frac{3600\text{ s}}{1\text{ h}}\right)} = 0.69\text{ft}^2\cdot\text{h}\cdot\text{F}°/\text{Btu}$$

$$\frac{Q}{t} = A\frac{(T_1 - T_2)}{(R_1 + R_2)} = (195\text{ ft}^2)\frac{(35\text{ F}°)}{(0.69 + 19)\text{ft}^2\cdot\text{h}\cdot\text{F}°/\text{Btu}} = 347\text{ Btu/h} \approx \boxed{350\text{ Btu/h}}$$

This is about 100 watts.

9. Approximately how long should it take 8.2 kg of ice at 0°C to melt when it is placed in a carefully sealed Styrofoam ice chest of dimensions $25\text{cm} \times 35\text{cm} \times 55\text{cm}$ whose walls are 1.5 cm thick? Assume that the conductivity of Styrofoam is double that of air and that the outside temperature is 34°C.

This is an example of heat conduction. The heat conducted is the heat released by the melting ice, $Q = m_{\text{ice}}L_{\text{F}}$. The area through which the heat is conducted is the total area of the six surfaces of the box, and the length of the conducting material is the thickness of the Styrofoam. We assume that all of the heat conducted into the box goes into melting the ice and none into raising the temperature inside the box. The time can then be calculated by Eq. 14–5.

$$\frac{Q}{t}=kA\frac{T_1-T_2}{\ell} \quad \rightarrow \quad t=\frac{m_{\text{ice}}L_{\text{F}}\ell}{kA\Delta T}$$

$$=\frac{(8.2\text{ kg})(3.33\times10^5\text{ J/kg})(1.5\times10^{-2}\text{ m})}{2(0.023\text{ J/s}\cdot\text{m}\cdot\text{C°})[2(0.25\text{ m})(0.35\text{ m})+2(0.25\text{ m})(0.55\text{ m})+2(0.35\text{ m})(0.55\text{ m})](34\text{ C°})}$$

$$=3.136\times10^4\text{ s}\approx\boxed{3.1\times10^4\text{ s}}\approx 8.7\text{ h}$$