Problems Set

Chapter 5

Universal Gravitational Law

1. Acceleration of a revolving ball.
A 150-g ball at the end of a string is revolving uniformly in a horizontal circle of radius 0.600 m. The ball makes 2.00 revolutions in a second. What is its centripetal acceleration?

APPROACH The centripetal acceleration is $a_R = v^2/r$. We are given r, and we can find the speed of the ball, v, from the given radius and frequency.

SOLUTION If the ball makes 2.00 complete revolutions per second, then the ball travels in a complete circle in a time interval equal to 0.500 s, which is the period $T$of its motion. This time is the circumference of the circle, $2\pi r$, where $r$is the radius of the circle. Therefore, the ball has speed

$v = \frac{2\pi(0.600\,m)}{(0.500\,s)} = 7.54\,m/s.$

The centripetal acceleration is

$$a_R = \frac{v^2}{r} = \frac{(7.54\,m/s)^2}{(0.600\,m)} = 94.7\,m/s^2$$

2. Moon's centripetal acceleration.
The Moon's nearly circular orbit around the Earth has a radius of about 384,000 km and a period T of 27.3 days. Determine the acceleration of the Moon toward the Earth.

APPROACH Again we need to find the velocity $v$in order to find a_R.

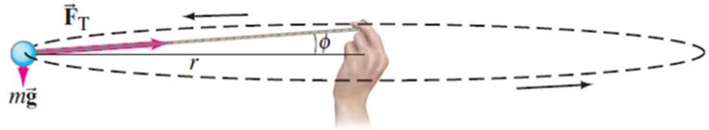
SOLUTION In one orbit around the Earth, the Moon travels a distance $2\pi r$, where $r = 3.84 \times 10^8$m is the radius of its circular path. The period of one orbit is the Moon's period of 27.3 d. The speed of the Moon in its orbit about the Earth is $v = 2\pi r/T$. The period Tin seconds is $T = (27.3\,d)(24\,h/d)(3600\,s/h) = 2.36 \times 10^6\,s$.

$a_R = \frac{v^2}{r} = \frac{(2\pi r)^2}{rT^2} = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2(3.84\times10^8\,m)}{(2.36\times10^6\,s)^2} = 0.00272\,m/s^2.$

We can write this acceleration in terms of $g = 9.80\,m/s^2$(the acceleration of gravity at the Earth's surface) as

$$a_R = 2.72 \times 10^{-3}(9.80\,m/s^2) = (2.72 \times 10^{-3})g = 0.0003\,g$$

3. ESTIMATE Force on revolving ball (horizontal).
Estimate the force a person must exert on a string attached to a 0.150-kg ball to make the ball revolve in a horizontal circle of radius 0.600 m. The ball makes 2.00 revolutions per second (T = 0.500 s). Ignore the string's mass.



APPROACH First we need to draw the free-body diagram for the ball. The forces acting on the ball are the force of gravity, mgdownward, and the tension force $F_T$that the string exerts toward the hand at the center (which occurs because the string exerts the same force on the string). The free-body diagram for the ball is shown in the figure. The ball's weight component acts vertically, but we wish to resolve it to help a ball with the correct physical laws. We estimate the force assuming the weight is small, and letting $\phi \approx 0$ in the figure. Then $F_T$will act nearly horizontally and, in any case, provides the radial acceleration that keeps the ball in its circular path.

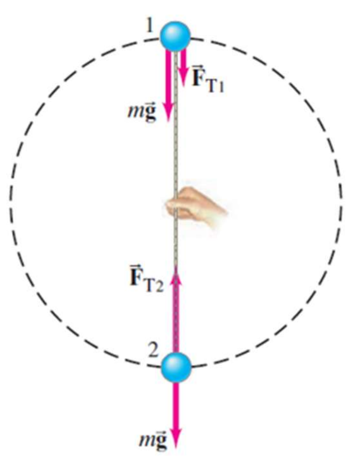
SOLUTION We apply Newton's second law to the radial direction, which we assume is horizontal:

$(\Sigma F)_r = ma_r$,

where $a_r = v^2/r$, and $v = 2\pi r/T = 2\pi(0.600\, m)/(0.500\, s) = 7.54\, m/s$. Thus

$$F_T = m\frac{v^2}{r} = (0.150\, kg)\frac{(7.54\, m/s)^2}{(0.600\, m)} \approx 14\, N$$

4. Revolving ball (vertical circle).
A 0.15-kg ball on the end of a 1.10-m-long cord (negligible mass) is swung in a vertical circle. (a) Determine the minimum speed the ball must have at the top of its arc so that the ball continues moving in a circle. (b) Calculate the tension in the cord at the bottom of the arc, assuming the ball is moving at twice the speed of part (a).



(a) At the top (point 1), two forces act on the ball: mg, the force of gravity, and F_{T1}, the tension force the cord exerts at point 1. Both act downward, and their vector sum acts to give the ball its centripetal acceleration a_R. We apply Newton's second law for the vertical direction, choosing downward as positive since the acceleration is downward (toward the center):

$$(\Sigma F)_R = ma_R$$

$$F_{T1} + mg = m\frac{v_1^2}{r} \text{ [at top]}$$

From this equation we can see that the tension force F_{T1} at point 1 will get larger if v_1 (ball's speed at top of circle) is made larger, as expected. But we are asked for the minimum speed to keep the ball moving in a circle. The cord will remain taut as long as there is tension in it. But if the tension disappears (because v_1 is too small) the cord can go limp, and the ball will fall out of its circular path. Thus, the minimum speed will occur if $F_{T1} = 0$ (ball at the topmost point), for which the equation above becomes:

$$mg = m\frac{v_1^2}{r}. \qquad \text{[minimum speed at top]}$$

We solve for v_1, keeping an extra digit for use in (b):

$$v_1 = \sqrt{gr} = \sqrt{(9.80\, m/s^2)(1.10\, m)} = 3.283\, m/s \approx 3.28\, m/s.$$

This is the minimum speed at the top of the circle if the ball is to continue moving in a circular path.

(b) When the ball is at the bottom of the circle (point 2 in the figure), the tension force F_{T2} upward, whereas the force of gravity, mg, still acts downward. Choosing upward as positive, Newton's second law gives for F_{T2}:

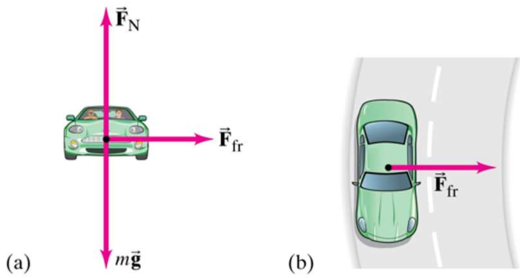
$$(\Sigma F)_R = ma_R$$

$$F_{T2} - mg = m\frac{v_2^2}{r}$$

The speed v_2 is twice that in (a), namely 6.566 m/s. We solve for F_{T2}:

$$F_{T2} = m\frac{v_2^2}{r} + mg$$
$$= (0.150\text{ kg})\frac{(6.566\text{ m/s})^2}{(1.10\text{ m})} + (0.150\text{ kg})(9.80\text{ m/s}^2) = 7.35\text{ N}.$$

5. Skidding on a curve.
A 1000-kg car rounds a curve on a flat road of radius 50 m at a speed of 15 m/s (54 km/h). Will the car follow the curve, or will it skid? Assume:
(a) the pavement is dry and the coefficient of static friction is $\mu_s = 0.60$;
(b) the pavement is icy and $\mu_s = 0.25$.



In the vertical direction (y) there is no acceleration. Newton's second law tells us that the normal force F_N on the car is equal to the car's weight as the road is flat:

$$\Sigma F_y = F_N - mg = 0$$

so

$$F_N = mg$$

In the horizontal direction the only force is friction, and we must compare it to the force needed to produce the centripetal acceleration to keep the car moving in a circle around the curve. The force required to keep the car moving in a circle around the curve is

$$(\Sigma F)_r = ma_R = m\frac{v^2}{r} = (1000\,kg)\frac{(15\,m/s)^2}{(50\,m)} = 4500\,N$$

Now we compute the maximum total static friction force (the sum of the friction forces acting on each of the four tires) to see if it is enough to provide a safe centripetal acceleration. For a flat road, the maximum static friction force attainable (recall from Section 4-8 that $F_{fr} = \mu_s F_N$) is

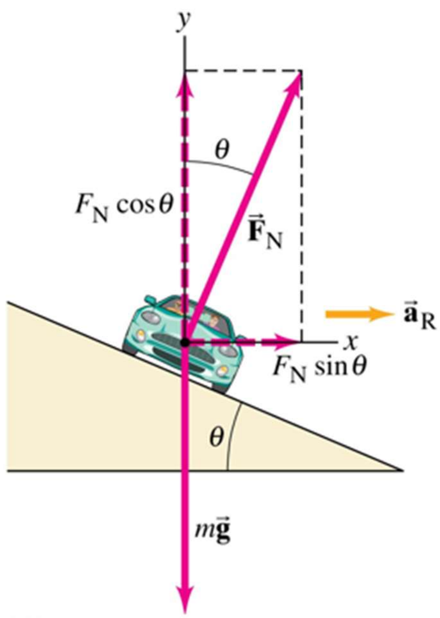
$$(F_{fr})_{max} = \mu_s F_N = \mu_s mg$$

(a) If at least 4500 N is needed, and that is, in fact, how much will be exerted by the road as a static friction force, the car can follow the curve. But in (b) if the maximum static friction force can be only 2450 N, the car will skid.

$$(F_{fr})_{max} = \mu_s F_N = (0.25)(9800\,N) = 2450\,N$$

The car will skid because the ground cannot exert sufficient force (4500 N is needed) to keep it moving in a curve of radius 50 m at a speed of 54 km/h.

6. Banking angle.
(a) For a car traveling with speed $v$around a curve of radius r, determine a formula for the angle at which a road should be banked so that no friction is required.
(b) What is this angle for a road which has a curve of radius 50 m with a design speed of 50 km/h?



(a) Since there is no vertical motion, $a_y = 0$and $\Sigma F_y = ma_y$ gives

$$F_N \cos\,\theta - mg = 0$$

or

$$F_N \cos\,\theta = mg$$

[Note in this case that $F_N \geq mg$because $\cos\,\theta \leq 1$.]
We substitute this relation for $F_N$into the equation for the horizontal motion, which becomes

$$F_N \sin\,\theta = m\frac{v^2}{r}$$

or

$$\frac{mg}{\cos\theta}\sin\,\theta = m\frac{v^2}{r}$$

or

$$\tan\ \theta = \frac{v^2}{rg}$$

This is the formula for the banking angle θ: no friction needed at this speed v.

(b) For $r = 50\ m$and $v = 50\ km/h = 14\ m/s$:

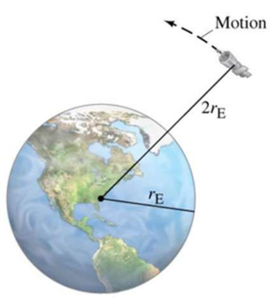
$$\tan\ \theta = \frac{(14\ m/s)^2}{(50\ m)(9.80\ m/s^2)} = 0.40$$

so

$$\theta = \tan^{-1}(0.40) = 22°.$$

7. Spacecraft at $2r_E$.

What is the force of gravity acting on a 2000-kg spacecraft when it orbits two Earth radii from the Earth's center (that is a distance $r_E = 6380\ km$above the Earth's surface, the figure? The mass of the Earth is $m_E = 5.98 \times 10^{24}\ kg$.



APPROACH We could plug all the numbers into the equation, but there is a simpler approach. The spacecraft is twice as far from the Earth's center as when it is at the surface of the Earth. Therefore, since the force of gravity $F_G$decreases as the square of the distance (and $\frac{1}{2^2} = \frac{1}{4}$), the force of gravity on the satellite will be only one-fourth its weight at the Earth's surface.

SOLUTION At the surface of the Earth, $F_G = mg$. At a distance from the Earth's center of $2r_E$, $F_G$is $\frac{1}{4}$ as great:

$$F_G = \tfrac{1}{4}mg = \tfrac{1}{4}(2000\ \text{kg})(9.80\ \text{m/s}^2) = 4900\ \text{N}.$$

8. ESTIMATE Gravity on Everest.

Estimate the effective value of g on the top of Mt. Everest, 8850 m (29,035 ft) above sea level. That is, what is the acceleration due to gravity of objects allowed to fall freely at this altitude? Ignore the mass of the mountain itself.

With $r = R_E + h$, where $R_E = 6.38 \times 10^6\ m$ is the radius of the Earth and $h = 8850\ m$ is the height of Mt. Everest. r_E is replaced by r=6380 km + 8.9 km = 6389 km =6.389 X10^6 m:

$$g = G\frac{m_E}{r^2} = \frac{(6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})(5.98 \times 10^{24}\ \mathrm{kg})}{(6.389 \times 10^6\ \mathrm{m})^2} = 9.77\ \mathrm{m/s^2}$$