PS Chapter 13 Temperature and Kinetic Theory

1. If 3.50 m^3 of a gas initially at STP is placed under a pressure of 3.20 atm, the temperature of the gas rises to 38.0°C. What is the volume?

Assume the gas is ideal. Since the amount of gas is constant, the value of PV/T is constant.

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \quad \rightarrow \quad V_2 = V_1\frac{P_1}{P_2}\frac{T_2}{T_1} = (3.50\ \text{m}^3)\left(\frac{1.00\ \text{atm}}{3.20\ \text{atm}}\right)\frac{(273+38.0)\ \text{K}}{273\ \text{K}} = \boxed{1.25\ \text{m}^3}$$

2. What is the pressure inside a 38.0-L container holding 105.0 kg of argon gas at 21.6°C?

Assume the argon is an ideal gas. The number of moles of argon is found from the atomic weight, and then the ideal gas law is used to find the pressure.

$$n = (105.0\ \text{kg})\frac{1\ \text{mole Ar}}{39.95\times10^{-3}\ \text{kg}} = 2628\ \text{mol}$$

$$PV = nRT \quad \rightarrow \quad P = \frac{nRT}{V} = \frac{(2628\ \text{mol})(8.314\text{J/mol}\cdot\text{K})(273.15\ \text{K}+21.6\ \text{K})}{(38.0\ \text{L})(1.00\times10^{-3}\ \text{m}^3/\text{L})} = \boxed{1.69\times10^8\ \text{Pa}}$$

3. An air bubble at the bottom of a lake 41.0 m deep has a volume of 1.00cm^3. If the temperature at the bottom is 5.5°C and at the top 18.5°C, what is the radius of the bubble just before it reaches the surface?

The ideal gas law can be used to relate the volume at the surface to the submerged volume of the bubble. We assume the amount of gas in the bubble doesn't change as it rises. The pressure at the submerged location is found from Eq. 10–3c.

$$PV = nRT \quad \rightarrow \quad \frac{PV}{T} = nR = \text{constant} \quad \rightarrow \quad \frac{P_{\text{surface}}V_{\text{surface}}}{T_{\text{surface}}} = \frac{P_{\text{submerged}}V_{\text{submerged}}}{T_{\text{submerged}}} \quad \rightarrow$$

$$V_{\text{surface}} = V_{\text{submerged}}\frac{P_{\text{submerged}}}{P_{\text{surface}}}\frac{T_{\text{surface}}}{T_{\text{submerged}}} = V_{\text{submerged}}\frac{P_{\text{atm}}+\rho gh}{P_{\text{atm}}}\frac{T_{\text{surface}}}{T_{\text{submerged}}}$$

$$= (1.00\ \text{cm}^3)\frac{[1.013\times10^5 + (1.00\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(41.0\ \text{m})]}{(1.013\times10^5)\text{Pa}}\frac{(273.15+18.5)\text{K}}{(273.15+5.5)\text{K}}$$

$$= 5.198\ \text{cm}^3 = \tfrac{4}{3}\pi r^3 \quad \rightarrow \quad r = \left[\frac{3(5.198\ \text{cm}^3)}{4\pi}\right]^{1/3} = \boxed{1.07\ \text{cm}}$$

4. Estimate the number of (*a*) moles and (*b*) molecules of water in all the Earth's oceans. Assume water covers 75% of the Earth to an average depth of 3 km.

(a)Since the average depth of the oceans is very small compared to the radius of the Earth, the ocean's volume can be calculated as that of a spherical shell with surface area $4\pi R_{\text{Earth}}^2$ and a thickness Δy. Then use the density of sea water to find the mass and the molecular weight of water to find the number of moles.

$$\text{Volume} = 0.75(4\pi R_{\text{Earth}}^2)\Delta y = 0.75(4\pi)(6.38\times10^6\text{ m})^2(3\times10^3\text{ m}) = 1.15\times10^{18}\text{ m}^3$$

$$1.15\times10^{18}\text{ m}^3\left(\frac{1025\text{ kg}}{\text{m}^3}\right)\left(\frac{1\text{ mol}}{18\times10^{-3}\text{ kg}}\right) = 6.55\times10^{22}\text{ moles} \approx \boxed{7\times10^{22}\text{ moles}}$$

(b) 6.55×10^{22} moles$(6.02\times10^{23}$ molecules/1 mol$) \approx \boxed{4\times10^{46}\text{ molecules}}$

5. The lowest pressure attainable using the best available vacuum techniques is about $10^{-12}\ N/\text{m}^2$. At such a pressure, how many molecules are there per cm^3 at 0°C?

Assume the gas is ideal at those low pressures and use the ideal gas law.

$$PV = NkT \quad\rightarrow\quad \frac{N}{V} = \frac{P}{kT} = \frac{1\times10^{-12}\text{ N/m}^2}{(1.38\times10^{-23}\text{ J/K})(273\text{ K})} = \left(3\times10^8\,\frac{\text{molecules}}{\text{m}^3}\right)\left(\frac{10^{-6}\text{ m}^3}{1\text{ cm}^3}\right)$$

$$= \boxed{300\text{ molecules/cm}^3}$$

6. What speed would a 1.0-g paper clip have if it had the same kinetic energy as a molecule at 22°C?

The average kinetic molecular energy is $\frac{3}{2}kT$. Set this equal to the kinetic energy of the paper clip.

$$\tfrac{1}{2}mv^2 = \tfrac{3}{2}kT \quad\rightarrow\quad v = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.38\times10^{-23}\text{ J/K})(273+22)\text{K}}{1.0\times10^{-3}\text{ kg}}} = \boxed{3.5\times10^{-9}\text{ m/s}}$$

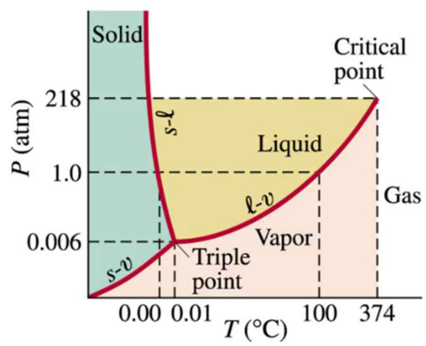
7. Show that the rms speed of molecules in a gas is given by $v_{\text{rms}} = \sqrt{3P/\rho}$, where P is the pressure in the gas and ρ is the gas density.

The rms speed is given by Eq. 13–9, $v_{\text{rms}} = \sqrt{3kT/m}$. The temperature can be found from the ideal gas law, $PV = NkT \rightarrow kT = PV/N$. The mass of the gas is the mass of a molecule times the number of molecules: $M = Nm$, and the density of the gas is the mass per unit volume, $\rho = \frac{M}{V}$. Combining these relationships gives the following:

$$v_{\text{rms}} = \sqrt{3kT/m} = \sqrt{\frac{3PV}{Nm}} = \sqrt{\frac{3PV}{M}} = \sqrt{\frac{3P}{\rho}}$$

8. Water is in which phase when the pressure is 0.01 atm and the temperature is (*a*) 90°C, (*b*) −20°C?

Figure 13–22



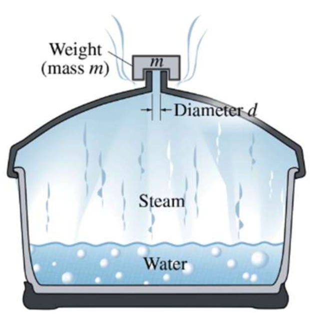
Phase diagram for water (note that the scales are not linear).

(*a*)From Fig. 13–22, water is $\boxed{\text{vapor}}$ when the pressure is 0.01 atm and the temperature is 90°C.

(*b*) From Fig. 13–22, water is $\boxed{\text{solid}}$ when the pressure is 0.01 atm and the temperature is –20°C.

9. A **pressure cooker** is a sealed pot designed to cook food with the steam produced by boiling water somewhat above 100°C. The pressure cooker in Fig. 13–32 uses a weight of mass *m* to allow steam to escape at a certain pressure through a small hole (diameter *d*) in the cooker's lid. If $d = 3.0\text{mm}$, what should *m* be in order to cook food at 120°C? Assume that atmospheric pressure outside the cooker is $1.01 \times 10^5\text{Pa}$.

Figure 13–32



For boiling to occur at 120°C, the pressure inside the cooker must be the saturated vapor pressure of water at that temperature. That value can be found in Table 13–3. For the mass to stay in place and contain the steam inside the cooker, the weight of the mass must be greater than the force exerted by the gauge pressure from the gas inside the cooker. The limiting case, to hold the temperature right at 120°C, would be with the mass equal to that force.

$$mg = F_{\substack{\text{gauge}\\\text{pressure}}} = (P_{\text{inside}} - P_{\text{atm}})A = (P_{\text{inside}} - P_{\text{atm}})\pi r^2 \quad \rightarrow$$

$$m = \frac{(P_{\text{inside}} - P_{\text{atm}})\pi r^2}{g} = \frac{(1.99\times10^5\ \text{Pa} - 1.01\times10^5\ \text{Pa})\pi(1.5\times10^{-3}\ \text{m})^2}{9.80\ \text{m/s}^2} = 0.07068\ \text{kg} \approx \boxed{71\ \text{g}}$$