Take-home assignment

Q1. (20pts) Consider the Insertion sort algorithm described by the following pseudo-code. The basic operation in this case is the comparison **a[j] > v**.

```
InsertionSort(a[1 .. n]) {
    for (i = 2; i ≤ n; i++) {
        v = a[i];
        j = i – 1;
        while (j ≥ 1) && (a[j] > v) {
            a[j + 1] = a[j];
            j--;
        }
        a[j + 1] = v;
    }
}
```

For each case—worst case, best case, or average case, if there is any, provide the time complexity expression and the corresponding Big-O (or Big- or Big-) asymptotic notation.

Q2. (20pts) Analyze the complexity of Mergesort algorithm (average case is not required) without using the Master Theorem in two cases.

a. Consider the comparison to be the algorithm's basic operation.

b. Consider the data movement to be the algorithm's basic operation.

Q3. (20pts) Answer the following question about Josephus problem.

a. Define the Josephus problem and give the related formula.

b. Prove the following formula by induction: $J(2k + i) = 2i + 1, \forall i \in [2k - 1]$

c. Prove that $J(n)$ can be obtained by a 1-bit cyclic shift left of n itself. For example, $J(6) = J(1102) = 1012 = 5$ and $J(9) = J(10012) = 112 = 3$.

Q4. (20pts) Present a problem that can be solved using both the Brute-force (Exhaustive search) technique and Backtracking search technique. Write two C++ code snippets: one demonstrating the Exhaustive search method and another illustrating the Backtracking search method to solve the problem.

Note: Choose a problem that is simple and clear, as a more complex problem might necessitate the use of additional techniques. Furthermore, the problem does not appear in the lecture.

Q5. (20pts) Given two sorted arrays x and y of size m and n respectively, write a C/C++ function (and auxiliary functions, if needed) to find the median of these arrays. The overall runtime complexity should be $O(\log k)$ where $k = min\{m, n\}$.

The prototype of the function is: int findMedian(int x[], int y[], int xl, int xr, int yl, int yr);

The initial values of the last four parameters are $xl = yl = 0, xr = m - 1, yr = n - 1$.

Note: Assume that the median is the element at index $\lfloor l+2r \rfloor$, where l and r are indices of the leftmost and rightmost elements of the (sub)array being considered, respectively. In addition, these two arrays contain no duplicate values.

Regulations for completing the assignment

- Each question must start with a new page in the report file.
- Plagiarism and Cheating will result in a "0" (zero) for the entire course.
- Contact: nnthao@fit.hcmus.edu.vn for more information.