# MATH 318, Assignment 4

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1. 1) The atoms of B are as follows:

   $p \wedge q \wedge r$ | $p \wedge q \wedge \neg r$
   $\neg p \wedge \neg q \wedge r$ | $\neg p \wedge \neg q \wedge \neg r$
   $p \wedge \neg q \wedge r$ | $p \wedge \neg q \wedge \neg r$
   $\neg p \wedge q \wedge r$ | $\neg p \wedge q \wedge \neg r$

   2) From the 8 atoms above, we have that there are $2^8$ elements in B.

2. 1) Take $B = \mathcal{P}$. Each of the atoms in B makeup the singleton set with every non-empty set in B containing a singleton. This means that for every nonzero $b \in B$, there is an atom $a \in B$ with $a \leq b$.

   2) Yes, take B as any boolean algebra without atoms. Additionally let B' be the algebra 2. From this, $B \times B'$ has exactly one atom, $(0, 1)$.

3. Question 3

4. 1) To show that $|$ is a partial order on $\mathbb{N}$ we must show that it is transitive. We know that it is reflexive since $n = 1 \cdot n$ so $n|n$. Next, suppose that $n|m$ and $m|n$. For some $i$ and $j$, we can then see that $m = in$ and $n = jm$ giving that $m = in = ijm$. If m = 0, $n = j \cdot 0 = 0$ and $n = m$. Alternatively, $ij = 1$ and $i = j = 1$ giving that $n = 1 \cdot m = m$ showing it is antisymmetric. Now, if $n|m$ and $m|k$ then $m = in$ and $k = jm$ for some $i$ $j$ therefore $k = jm = ijn$. Giving that $n|p$ is transitive and a partial order.

   2) We have that $n = n \cdot 1$ for any n, therefore $1|n$ and 1 a least element.

   3) We have that $0 = 0 \cdot n$ for any n, therefore $n|0$ and 0 a greatest element.

5. We need two chains to cover P since P is not itself a chain ($2 \nmid 3$ and $3 \nmid 2$). Allow $C_1 = \{1, 3\}$ and $C_2 = \{2, 4\}$. Both $C_1$ and $C_2$ are chains and $P = C_1 \cup C_2$. This means the smallest number of chains that covers P is 2.