MATH 318, Assignment 3

Due date: October 11

1. (4 points) Write truth tables of the following formulas. Which of them are tautologies?
   (A) $(\neg p) \to q$, (B) $(p \wedge q) \vee (\neg p)$,
   (C) $p \to ((\neg p) \to q)$, (D) $q \vee (p \to (q \wedge (p \to q)))$.

2. (2 points) Devise formulas (using connectives amongst $\vee, \wedge$ and $\neg$) and switching circuits (using gates amongst OR, AND and NOT) which realize each of the following Boolean functions:

(A)

$q \backslash p$	0	1
0	1	1
1	0	1

(B)

$q \backslash p$	0	1
0	0	1
1	1	1

3. (2 points) Write DNF and CNF formulas equivalent to the formula $((p \vee \neg q) \wedge (r \vee p)) \vee r$.

4. (1) (1 point) Write a formula equivalent to $p \to q$ using only the connective NAND,
   (2) (1 point) Write a formula equivalent to $(p \wedge q) \vee \neg p$ using only the connective NOR,

5. (2 points) Consider the formula

$$(\ldots((p \to p) \to p) \to \ldots) \to p,$$

where the variable p occurs n many times. For which n is the above formula a tautology? Justify your answer.

6. (4 points) Suppose φ is a formula written using only the biconditional connective $\leftrightarrow$ (besides variables and parentheses). Show that φ is a tautology if and only if every variable occurs in φ an even number of times.

7. (2 points) Using Karnaugh maps, find a formula realizing the following Boolean function f with 4 variables p, q, r, s:

p	q	r	s	$f(p,q,r,s)$
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	0
1	1	1	1	0

The following problems are for extra credit.

8*. (4 points) Show that the set $\{\vee, \wedge\}$ is not complete.

9*. (4 points) Show that the set {XOR} is not complete.