MATH 318, Assignment 3

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1. (a)

p	q	(¬	p	→	q)
$\top$	$\top$	$\bot$	$\top$	$\top$	$\top$
$\top$	$\bot$	$\bot$	$\top$	$\top$	$\bot$
$\bot$	$\top$	$\top$	$\bot$	$\top$	$\top$
$\bot$	$\bot$	$\top$	$\bot$	$\bot$	$\bot$

(b)

p	q	(p	∧	q)	∨	¬	p
$\top$	$\top$	$\top$	$\top$	$\top$	$\top$	$\bot$	$\top$
$\top$	$\bot$	$\top$	$\bot$	$\bot$	$\bot$	$\bot$	$\top$
$\bot$	$\top$	$\bot$	$\bot$	$\top$	$\top$	$\top$	$\bot$
$\bot$	$\bot$	$\bot$	$\bot$	$\bot$	$\top$	$\top$	$\bot$

(c)

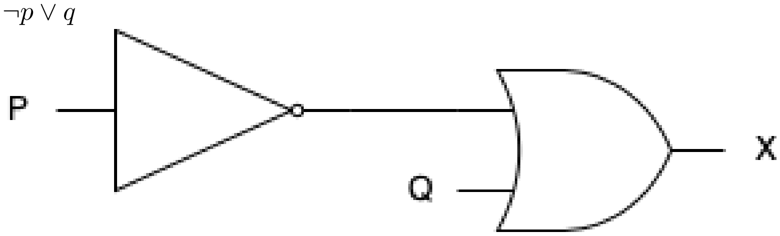
p	q	p	→	(¬	p	→	q)
$\top$	$\top$	$\top$	$\top$	$\bot$	$\top$	$\top$	$\top$
$\top$	$\bot$	$\top$	$\top$	$\bot$	$\top$	$\top$	$\bot$
$\bot$	$\top$	$\bot$	$\top$	$\top$	$\bot$	$\top$	$\top$
$\bot$	$\bot$	$\bot$	$\top$	$\top$	$\bot$	$\bot$	$\bot$

(d)

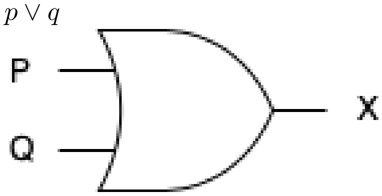
p	q	q	∨	(p	→	(q	∧	(p	→	q)))
$\top$	$\top$	$\top$	$\top$	$\top$	$\top$	$\top$	$\top$	$\top$	$\top$	$\top$
$\top$	$\bot$	$\bot$	$\bot$	$\top$	$\bot$	$\bot$	$\bot$	$\top$	$\bot$	$\bot$
$\bot$	$\top$	$\top$	$\top$	$\bot$	$\top$	$\top$	$\top$	$\bot$	$\top$	$\top$
$\bot$	$\bot$	$\bot$	$\top$	$\bot$	$\top$	$\bot$	$\bot$	$\bot$	$\top$	$\bot$

C is the only tautology.

2. (a) $\neg p \vee q$



(b) $p \vee q$



3. (a) DNF

$$p \vee r$$

(b) CNF

$$p \vee r$$

4. 1) $p \to q$ with only NAND

p	q	((p	*NAND*	q)	*NAND*	p)	↔	(p	→	q)
T	T	T	F	T	T	T	T	T	T	T
T	F	T	T	F	F	T	T	T	F	F
F	T	F	T	T	T	F	T	F	T	T
F	F	F	T	F	T	F	T	F	T	F

2) $(p \wedge q) \vee \neg p$ with only NOR

p	q	(((p	NOR	p)	NOR	q)	NOR	((p	NOR	p)	NOR	q))	$\leftrightarrow$	((p	$\wedge$	q)	$\vee$	$\neg$	p)
T	T	T	F	T	F	T	T	T	F	T	F	T	T	T	T	T	T	F	T
T	F	T	F	T	T	F	F	T	F	T	T	F	T	T	F	F	F	F	T
F	T	F	T	F	F	T	T	F	T	F	F	T	T	F	F	T	T	T	F
F	F	F	T	F	F	F	T	F	T	F	F	F	T	F	F	F	T	T	F

5. The formula is even when $n \geq 2$ and n is an even number. We will proove this by induction on n.
Base Case: take n = 2. This follows from the truth table.

p	p	$\rightarrow$	p
T	T	T	T
F	F	T	F

Induction Step: Taking that this holds for any other n, we will show that it holds for $n + 2$. Since we can assume that any formula with an even n number of p's, we need to show that $(T \rightarrow p) \rightarrow p$ is also true. This clearly follows from the truth table.

6.

7. Drawing the karnaugh map as follows:

pq \ rs	00	01	11	10
00	1	1	1	1
01	1	1	0	0
11	1	1	0	0
10	1	1	1	1

Giving the following: $f = \neg r \vee (\neg p \wedge \neg q) \vee (p \wedge \neg q)$

8. Take for example the formula $p \vee \neg p$. If we attempt to replicate this with the set $\{\oplus\}$. Considering the operations available, it's clear that outside of the identity function of $p = T$ any formulas constructed with combinations of p and $\oplus$ such as $p \oplus p$ and so on will be logically false. This makes replicating the formula $p \vee \neg p$ and many others impossible. Thus $\{\oplus\}$ is not functionally complete.