Up, Up, and Away—Rocket Motion

Introduction

Human beings have long looked to the sky with a yearning to fly and have long experimented with various methods and contrivances to accomplish this goal. Around AD 1500, a mandarin named Wan-Hu attempted to fly to the moon in a wicker chair to which were attached 47 "rockets"—actually 47 bamboo tubes filled with black powder (Dvir 2003; Lethbridge 2000). Unfortunately, the innovator was unable to conduct other experiments with the potential of rockets to propel human beings into the sky. The successful launching of human beings into space would have to wait several centuries. Now, the launching of rockets with or without human beings is quite an ordinary event. The space above earth is littered with various types of satellites placed into orbit by rockets.

The Chinese generated a form of black powder, or "gun powder," during the first century AD from charcoal, saltpeter, and sulfur (Lethbridge 2000). Initially, they used this powder for fireworks; but sometime around the year AD 1000, they adapted this powder for use in fire arrows. These fire arrows were made by attaching powder-filled bamboo tubes to arrows and launching them with a bow. By 1232, they had modified these arrows by fixing tubes, open at one end and capped at the other end, to long sticks. They lit the powder, and the first true rockets were launched toward their Mongol attackers. The tips of these rockets were coated with either flammable materials or poison. How effective these rockets were as weapons is questionable, but the Chinese successfully warded off these invaders. Furthermore, the Mongols developed their own rockets and helped to spread their use to Europe. In fact, the word rocket probably originated from an Italian word "rochetta," coined by Muratori in his description of fire arrows used in medieval times (Lethbridge 2000).

From the time of the Chinese fire arrows, rockets have continued to play important military roles. During the last half of the twentieth century, however, rockets have taken on new roles in the exploration of the universe. Currently, satellites carried by rockets are providing us with three-dimensional views of polar ice sheets to give us insight into climate and its effects on life. Rockets have launched space telescopes and propelled probes to Mars, to the edge of our solar system, and beyond. More than 400 people have traveled into space borne by rockets since 1961 (NASA).

Physics Background

Before embarking on our development of a model of rocket motion, we need to consider some of the physics fundamentals. We have already worked extensively with Newton's second law, $F = ma$, where F is a force acting on an object of mass m and imparting an acceleration, a (see the section "Physics Background" from Module 3.1, "Modeling Falling and Skydiving"). In that same section, we discussed drag on an object.

With rockets, we consider another mechanical force, **thrust**. A rocket's engine generates thrust through the acceleration of a mass of gas through the bottom, propelling the rocket in the opposite direction. Thus, the forces obey **Newton's third law of motion**: "for every action, there is an equal and opposite reaction." The concept for a rocket is the same as that of a filled and released balloon, where expelled gas under pressure causes the balloon to fly around the room.

> **Definition** **Thrust** is a mechanical force caused by the acceleration of a mass of fluid and in the opposite direction to flow.

Suppose c is the velocity of the gas relative to the rocket and v is the velocity of the rocket, so that $c + v$ is the velocity of the gas in space. If m is the mass and up is positive, then the **thrust** of the engine (T) is as follows:

$$T = c\frac{dm}{dt}$$

Over a period of time Δt, we have the following discrete version:

$$T = c\frac{\Delta m}{\Delta t}$$

or

$$T\,\Delta t = c\,\Delta m$$

Quick Review Question 1

a. Select all appropriate units of measure for thrust: kg, kg m/s^2, kg/s^2, m/s^2, mi/h, N, N s, lb, lb/s^2, s.

b. With up being positive, suppose a rocket is traveling up at a speed of 500 m/s, and the speed of the downward gas is 800 m/s. Give the value of c.

c. Suppose over a period of 0.1 s, 2 kg of propellant burns. Give the engine thrust.

As rocket fuel burns, the mass of the fueled rocket decreases. From time t to time $t + \Delta t$, the **impulse** is the product of the thrust and Δt, as follows:

$$I = T\,\Delta t$$

During that period, the **specific impulse** (I_{sp}) is the impulse per newton (or pound) of fuel, or the quotient of impulse and the weight of the burned fuel, Δw.

$$I_{sp} = \frac{I}{\Delta w} = \frac{T\,\Delta t}{(\Delta m)g}$$

Solving for T, we have the following value of thrust from time t to time $t + \Delta t$:

$$T = I_{sp}g\frac{\Delta m}{\Delta t}$$

Letting Δt approach 0, we have the equivalent derivative form:

$$T = I_{sp} g \frac{dm}{dt}$$

Definitions An **impulse** is the product of the thrust and the length of time. **Specific impulse** is the impulse per unit weight of burned fuel, or the quotient of impulse and the change in the fuel's weight.

As with earlier models of motion, our model of the motion of a rocket incorporates acceleration. In this case, we wish to have a formula for acceleration due to thrust. Because thrust is a force, for acceleration a we have the following equation by Newton's second law:

$$T = ma$$

Substituting for T and solving for acceleration, the following is true:

$$I_{sp} g \frac{dm}{dt} = ma$$

or

$$a = \frac{I_{sp} g \frac{dm}{dt}}{m}$$

Quick Review Question 2

a. Select all appropriate units of measure for impulse: kg, kg m/s^2, kg/s^2, m/s^2, mi/h, N, N s, lb, lb/s^2, s.

b. Suppose a fuel burning for 2 s imparts a thrust of 75 N to a rocket. Give the impulse.

c. Select all appropriate units of measure for specific impulse: kg, kg m/s^2, kg/s^2, m/s^2, mi/h, N, N s, lb, lb/s^2, s.

d. Suppose 0.5 kg of the fuel of Part b burns during 2 s. Give the specific impulse.

System Dynamics Model

The model of the motion of a ball tossed into the air in Figure 3.1.1 of Module 3.1, "Modeling Falling and Skydiving," serves as a basis for the rocket-motion model. For the extension, we make several assumptions:

- The only forces acting on the rocket are gravitation and thrust derived from burning fuel.
- Acceleration due to gravity is constant.
- The earth is flat.
- The rocket is vertical.
- The rocket has only one stage.

Quick Review Question 3

This question reflects on Step 2 of the modeling process—formulating a model—for developing a model for rocket motion. We employ the preceding simplifying assumptions. After completing this question and before continuing in the text, we suggest that you develop a model for rocket motion.

a. Building on the model in the section "Acceleration, Velocity, and Position" of Module 3.1, "Modeling Falling and Skydiving," determine additional variables for the rocket-motion model and their units in the metric system.

b. Give a differential equation for change in total mass (dm/dt) as a function of the constants mass of initial unburned fuel (f) and time to burn (b). Use the simplifying assumption that dm/dt is constant.

c. Give a differential equation for acceleration, or change in velocity (dv/dt), in terms of total mass (m), change in total mass (dm/dt), specific impulse (I_{sp}), and acceleration due to gravity (g).

Extending the first model in "Modeling Falling and Skydiving," in which we assume no friction, thrust from burning fuel also impacts the motion of a rocket. The *change_in_velocity*, or acceleration, now involves acceleration due to this thrust as well as acceleration due to gravity. The extended model has an additional stock (box variable), *mass*, that contains the total mass of the fuel and rocket, which has mass *rocket_mass*. This stock has an initial value of *initial_mass*. We assume that while fuel is present, the flow out, *change_in_mass*, is constant and consists of the mass of the initial unburned fuel divided by the time for it to burn (*burnout_time*). After burnout, *change_in_mass* becomes zero. Figure 3.4.1, which is similar to Figure 3.1.1, contains a model diagram of a rocket's motion.

Quick Review Question 4

Refer to Figure 3.4.1 to give the formulas for the following:

a. The mass of the unburned fuel

b. The change in mass per unit of time of rocket with fuel

Figure 3.4.2 displays a graph of position (in color) and velocity versus time when *initial_mass* = 5000 kg, *rocket_mass* = 1000 kg, *burnout_time* = 60 s, and *specific_impulse* = 200 s. The graph shows the velocity increasing more and more until the

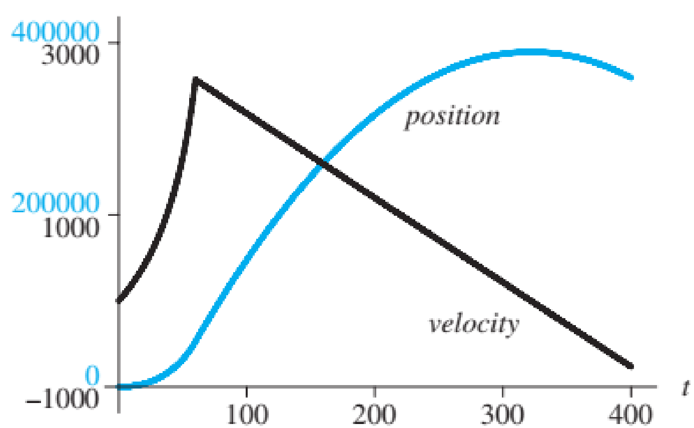

Figure 3.4.2 Graph of position (m) and velocity (m/s) versus time for a rocket

fuel completely burns. With a velocity of about 2567 m/s = 2.567 km/s at that instant, the rocket continues to rise to a height of about 388,500 m = 388.5 km before starting its descent. Various projects complete and expand upon the model in Figure 3.4.1.

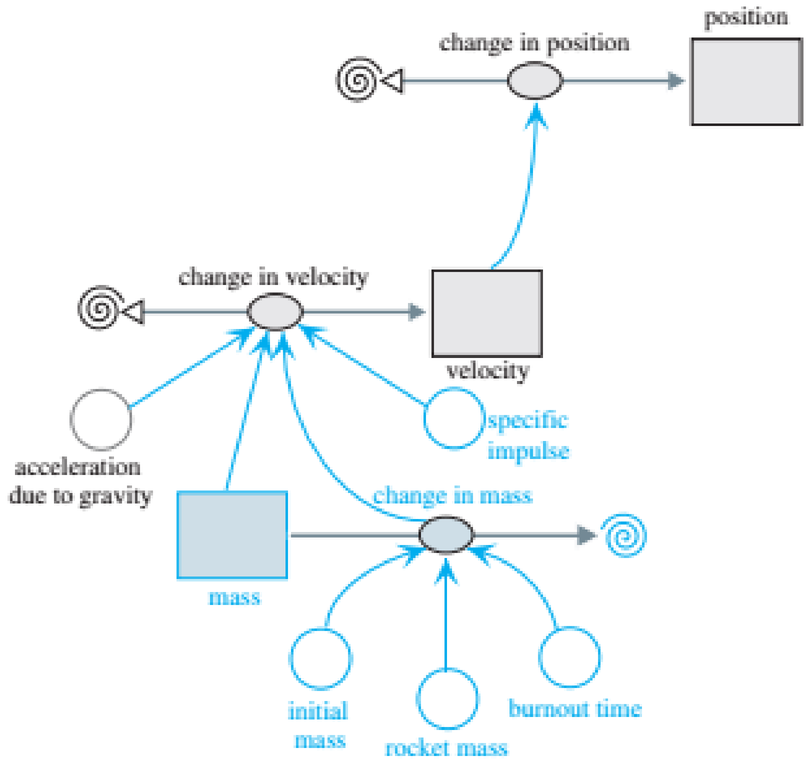

Figure 3.4.1 Model diagram of a rocket's motion