Homework #5

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May 23, 2025

Question 3.

Solve the following sections from the Discrete Math zyBook:

(a): Exercise 4.1.3

B.

Not a function and fails for $x = 2$ and $x = -2$.

C.

Is a function for all of $\mathbb{R}$. The range of the function is $[0, \infty)$.

(b): Exercise 4.1.5

B.

$\{4, 9, 16, 25\}$

D.

$\{0, 1, 2, 3, 4, 5\}$

H.

$A \times A = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)\}$.

I.

$A \times A = \{(1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,2), (3,3), (3,4)\}$.

L.

$\{\emptyset, \{2\}, \{3\}, \{2,3\}\}$.

Question 4.

I - Solve the following sections from the Discrete Math zyBook

(a): 4.2.2

C.

One to one but not onto.
For $h(x) = 2$, there does not exist an $x \in Z$ such that $h(x) = 2$.

G.

One to one but not onto.
For $f(x, y) = (0, 1)$, there does not exist a pair of $(x, y) \in Z \bigtimes Z$ such that $f(x, y) = (0, 1)$.

K.

Neither one to one or onto.
For $f(1, 3) = 5$ and $f(2, 1) = 5$, there exist a value in the range such that its values of the domain are different.
For $f(x, y) = 1$ or $f(x, y) = 2$, there does not exist a pair of $(x, y) \in Z^+ \bigtimes Z^+$ such that $f(x, y) = 1$ or $f(x, y) = 2$.

(b): 4.2.4

B.

Neither one to one or onto.
For $f(000) = 100$ and $f(100) = 100$, there exist a value in the range such that its values of the domain are different.
For $f(x) = 000$, there does not exist a $x \in \{0, 1\}^3$ such that $f(x) = 000$.

C.

One to one and onto.

D.

One to one but not onto.
For $f(x) = 0001$, there does not exist a $x \in \{0, 1\}^3$ such that $f = 0001$.

G.

Neither one to one or onto.
For $X_1 = \{2\}$ and $X_2 = \{1, 2\}$ where $F(X_1) = F(X_2) = \{2\}$, there exist a value in the range such that its values of the domain are different.
There does not exist a X in the domain such that the value in the range is $\{1\}$.

II - Give an example of a function from the set of integers to the set of positive integers that is:

A:

$f(x) = 2x$ for $x \geq 0$ and $2|x| + 1$ for $x < 0$.

B:

$f(x) = |x|$.

C:

$f(x) = 2x$ for $x \geq 0$ and $2|x| - 1$ for $x < 0$.

D:

$f(x) = 99$.

Question 5.

Solve the following sections from the Discrete Math zyBook:

(a): Exercise 4.3.2

C.

This function is well defined.
$f^{-1}(x) = \frac{x-3}{2}$.

D.

$f(x)$ is not one to one.
$f^{-1}(x)$ is not well defined.

G.

This function is well defined.
Both reverse the input bits.

I.

This function is well defined.
$f^{-1}(x, y) = (x - 5, y + 2)$.

(b): Exercise 4.4.8

C.

$f \circ h(x) = 2x^2 + 5$

D.

$h \circ f(x) = (2x + 3)^2 + 1$
$h \circ f(x) = 4x^2 + 12x + 10$

(c): Exercise 4.4.2

B.

$f \circ h(52) = (\lceil 52/5 \rceil)^2 = 11^2 = 121$

C.

$g \circ h \circ f(4) = 2^{(\lceil (4^2)/5 \rceil)} = 2^{(\lceil (16/5 \rceil)} = 2^4 = 16$

D.

$h(f(x)) = \left\lceil \frac{x^2}{5} \right\rceil$

(d): Exercise 4.4.6

C.

$h(f(010)) = 111$

D.

Range is $\{101, 111\}$.

E.

Range is $\{001, 011, 101, 111\}$.

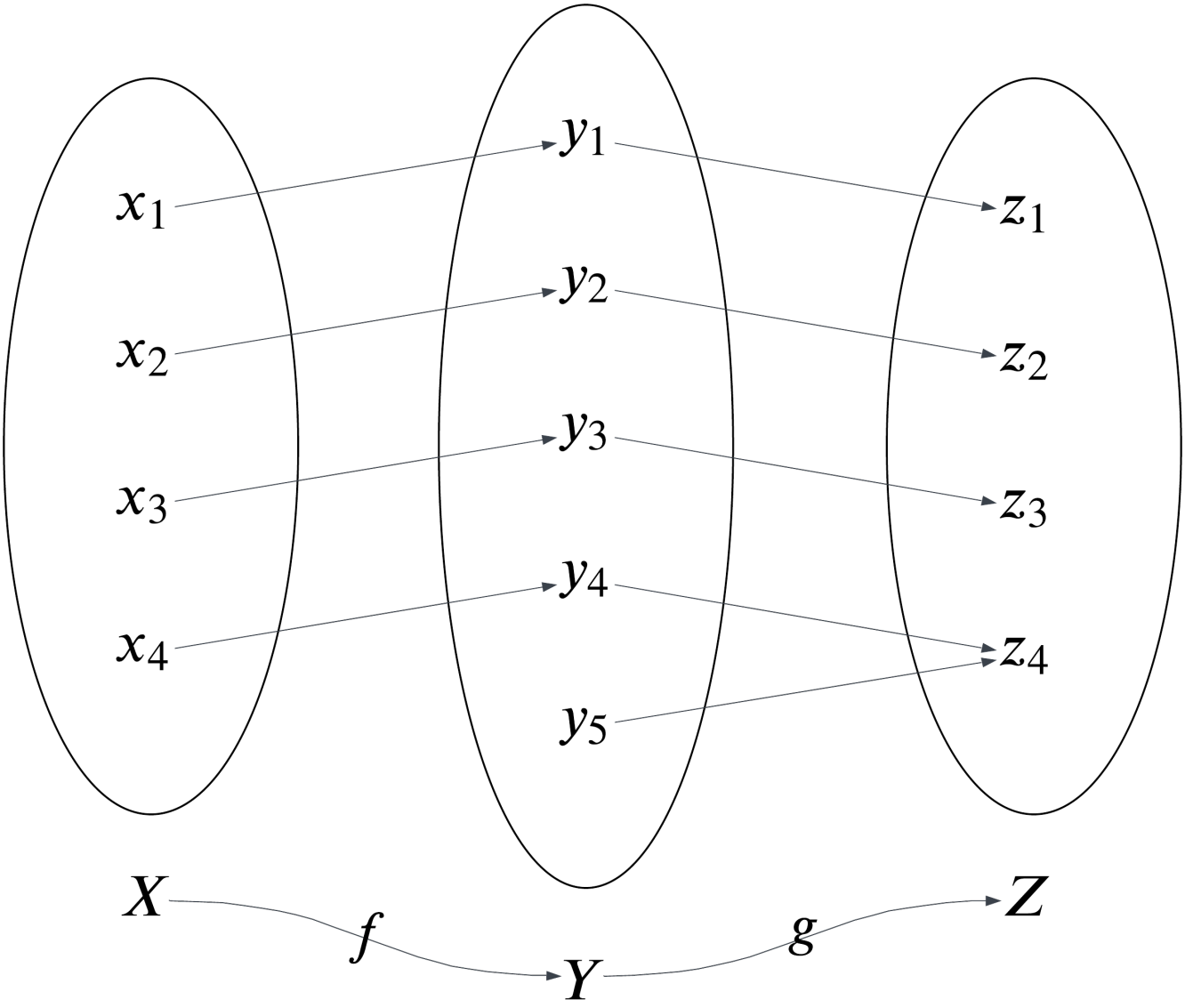

Figure 1: Exercise 4.4.4.D

(E): Exercise 4.4.4

C.

No. When f is not one-to-one, there exist a value in the range such that its values of the domain are different.
$g \circ f(x_1) = g(f(x_1)) = g(y)$
$g \circ f(x_2) = g(f(x_2)) = g(y)$
This indicate that $g \circ f$ exist a value in the range such that its values of the domain are different.

D.

Yes. See Figrue 1: Exercise 4.4.4.D.