Homework 1

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Question 1

A.

1.

$$\begin{aligned}10011011_2 &= 1 \times 2^7 + 0 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 \\ &= 128 + 0 + 0 + 16 + 8 + 0 + 2 + 1 \\ &= 155\end{aligned}$$

2.

$$\begin{aligned}456_7 &= 4 \times 7^2 + 5 \times 7^1 + 6 \times 7^0 \\ &= 196 + 35 + 6 \\ &= 237\end{aligned}$$

3.

$$\begin{aligned}38A_{16} &= 3 \times 16^2 + 8 \times 16^1 + 10 \times 16^0 \\ &= 768 + 128 + 10 \\ &= 906\end{aligned}$$

4.

$$\begin{aligned}2214_5 &= 2 \times 5^3 + 2 \times 5^2 + 1 \times 5^1 + 4 \times 5^0 \\ &= 250 + 50 + 5 + 4 \\ &= 309\end{aligned}$$

B.

1.

$$\begin{aligned}
69 \div 2 &= 34 && \mathrm{mod} 1 \\
34 \div 2 &= 17 && \mathrm{mod} 0 \\
17 \div 2 &= 8 && \mathrm{mod} 1 \\
8 \div 2 &= 4 && \mathrm{mod} 0 \\
4 \div 2 &= 2 && \mathrm{mod} 0 \\
2 \div 2 &= 1 && \mathrm{mod} 0 \\
1 \div 2 &= 0 && \mathrm{mod} 1 \\
69_{10} &= 1000101_2
\end{aligned}$$

2.

$$\begin{array}{rll}
485 \div 2 & = 242 & \mathrm{mod} 1 \\
242 \div 2 & = 121 & \mathrm{mod} 0 \\
121 \div 2 & = 60 & \mathrm{mod} 1 \\
60 \div 2 & = 30 & \mathrm{mod} 0 \\
30 \div 2 & = 15 & \mathrm{mod} 0 \\
15 \div 2 & = 7 & \mathrm{mod} 1 \\
7 \div 2 & = 3 & \mathrm{mod} 1 \\
3 \div 2 & = 1 & \mathrm{mod} 1 \\
1 \div 2 & = 0 & \mathrm{mod} 1 \\
 & & \\
485_{10} & = 111100101_2 &
\end{array}$$

3.

We translate each hexadecimal digit into a 4-bit binary equivalent.

$$\begin{aligned}
6_{16} &= 0110_2 \\
D_{16} &= 13_{10} && = 1101_2 \\
1_{16} &= 0001_2 \\
A_{16} &= 10_{10} && = 1010_2
\end{aligned}$$

Thus, combining the binary values, We get,

$$6D1A_{16} = 0110110100011010_2$$

C.

1.

$$1101011_2 = 01101011_2$$

We translate each hexadecimal digit into a 4-bit binary equivalent.

$$0110_2 = 6_{16}$$
$$1011_2 = B_{16}$$

Thus, combining the binary values, We get,

$$1101011_2 = 6B_{16}$$

2.

$$\begin{aligned} 895 \div 16 &= 55 \qquad && \text{mod } 15 \\ 55 \div 16 &= 3 && \text{mod } 7 \\ 3 \div 16 &= 0 && \text{mod } 3 \\ 15_{10} &= F_{16} \\ 895_{10} &= 37F_{16} \end{aligned}$$

Question 2

1.

$$\begin{array}{r} {}^{1\,1\,1\,1} \\ 07566_8 \\ +4515_8 \\ \hline 14303_8 \end{array}$$

$$7566_8 + 4515_8 = 14303_8$$

2.

$$\begin{array}{r} {}^{1\,1\,1\,1\,1} \\ 10110011_2 \\ +00001101_2 \\ \hline 11000000_2 \end{array}$$

$$10110011_2 + 1101_2 = 11000000_2$$

3.

$$\begin{array}{r} {}^{1\;\;1}\phantom{\;66_{16}} \\ 7\,A\;\;66_{16} \\ +4\;\;5\,C5_{16} \\ \hline C\;\;0\,2B_{16} \end{array}$$

$$7A66_{16} + 45C5_{16} = C02B_{16}$$

4.

$$\begin{array}{r} {}^{2\,4\,1} \\ 3022_5 \\ -2433_5 \\ \hline 34_5 \end{array}$$

$$3022_5 - 2433_5 = 34_5$$

Question 3

A.

1.

$$\begin{aligned}
124 \div 2 &= 62 \quad \text{mod}0\\
62 \div 2 &= 31 \quad \text{mod}0\\
31 \div 2 &= 15 \quad \text{mod}1\\
15 \div 2 &= 7 \quad \text{mod}1\\
7 \div 2 &= 3 \quad \text{mod}1\\
3 \div 2 &= 1 \quad \text{mod}1\\
1 \div 2 &= 0 \quad \text{mod}1
\end{aligned}$$

$$124_{10} = 01111100_{\text{ 8 bit 2's comp}}$$

2.

$$124 \div 2 = 62 \quad \text{mod}0$$
$$62 \div 2 = 31 \quad \text{mod}0$$
$$31 \div 2 = 15 \quad \text{mod}1$$
$$15 \div 2 = 7 \quad \text{mod}1$$
$$7 \div 2 = 3 \quad \text{mod}1$$
$$3 \div 2 = 1 \quad \text{mod}1$$
$$1 \div 2 = 0 \quad \text{mod}1$$

Hence, the absolute binary representation of 124 is as followed:

$$124_{10} = 01111100_2$$

The inverted bits are as followed:

$$10000011$$

Add 1,

$$10000011 + 00000001 = 10000100$$

Thus,

$$-124_{10} = 10000100_{\text{ 8 bit 2's comp}}$$

3.

$$\begin{aligned}
109 \div 2 &= 54 \quad \text{mod}1 \\
54 \div 2 &= 27 \quad \text{mod}0 \\
27 \div 2 &= 13 \quad \text{mod}1 \\
13 \div 2 &= 6 \quad \text{mod}1 \\
6 \div 2 &= 3 \quad \text{mod}0 \\
3 \div 2 &= 1 \quad \text{mod}1 \\
1 \div 2 &= 0 \quad \text{mod}1
\end{aligned}$$

$$109_{10} = 01101101_{\text{ 8 bit 2's comp}}$$

4.

$$\begin{aligned}
79 \div 2 &= 39 \quad \text{mod}1 \\
39 \div 2 &= 19 \quad \text{mod}1 \\
19 \div 2 &= 9 \quad \text{mod}1 \\
9 \div 2 &= 4 \quad \text{mod}1 \\
4 \div 2 &= 2 \quad \text{mod}0 \\
2 \div 2 &= 1 \quad \text{mod}0 \\
1 \div 2 &= 0 \quad \text{mod}1
\end{aligned}$$

Hence, the absolute binary representation of 79 is as followed:

$$79_{10} = 01001111_2$$

The inverted bits are as followed:

$$10110000$$

Add 1,

$$10110000 + 00000001 = 10110001$$

Then,

$$-79_{10} = 10110001_{\text{ 8 bit 2's comp}}$$

B.

1.

$$\begin{aligned}
00011110_{\text{ 8 bit 2's comp}} &= -0 \times 2^7 + 0 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 \\
&= -0 + 0 + 0 + 16 + 8 + 4 + 2 + 0 \\
&= 30
\end{aligned}$$

2.

$$\begin{aligned}
11100110_{\text{ 8 bit 2's comp}} &= -1\times 2^7 + 1\times 2^6 + 1\times 2^5 + 0\times 2^4 + 0\times 2^3 + 1\times 2^2 + 1\times 2^1 + 0\times 2^0 \\
&= -128 + 64 + 32 + 0 + 0 + 4 + 2 + 0 \\
&= -26
\end{aligned}$$

3.

$$\begin{aligned}
00101101_{\text{ 8 bit 2's comp}} &= -0\times 2^7 + 0\times 2^6 + 1\times 2^5 + 0\times 2^4 + 1\times 2^3 + 1\times 2^2 + 0\times 2^1 + 1\times 2^0 \\
&= -0 + 0 + 32 + 0 + 8 + 4 + 0 + 1 \\
&= 45
\end{aligned}$$

4.

$$\begin{aligned}
10011110_{\text{ 8 bit 2's comp}} &= -1\times 2^7 + 0\times 2^6 + 0\times 2^5 + 1\times 2^4 + 1\times 2^3 + 1\times 2^2 + 1\times 2^1 + 0\times 2^0 \\
&= -128 + 0 + 0 + 16 + 8 + 4 + 2 + 0 \\
&= -98
\end{aligned}$$

Question 4

1. Exercise 1.2.4

section b

p	q	$p \vee q$	$\neg(p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

section c

p	q	r	$p \wedge \neg q$	$r \vee (p \wedge \neg q)$
T	T	T	F	T
T	T	F	F	F
T	F	T	T	T
T	F	F	T	T
F	T	T	F	T
F	T	F	F	F
F	F	T	F	T
F	F	F	F	F

2. Exercise 1.3.4

section b

p	q	$p \to q$	$q \to p$	$(p \to q) \to (q \to p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

section d

p	q	$p \leftrightarrow q$	$p \leftrightarrow \neg q$	$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	T	F	T

Question 5

1. Exercise 1.2.7

section b

$$(B \wedge D) \vee (B \wedge M) \vee (D \wedge M)$$

section c

$$B \vee (D \wedge M)$$

2. Exercise 1.3.7

section b

$$(s \vee y) \to p$$

section c

$$p \to y$$

section d

$$p \leftrightarrow (s \wedge y)$$

section e

$$p \to (s \vee y)$$

3. Exercise 1.3.9

section c

$$c \to p$$

section d

$$c \to p$$

Question 6

1. Exercise 1.3.6

section b

If Joe is eligible for the honors program, then he maintains a B average.

section c

If Rajiv can go on the roller coaster, then he is at least four feet tall.

section d

If Rajiv is at least four feet tall, then he can go on the roller coaster.

2. Exercise 1.3.10

section c

p	q	r	$p \vee r$	$q \wedge r$	$(p \vee r) \leftrightarrow (q \wedge r)$
T	F	T	T	F	F
T	F	F	T	F	F

The truth table indicates that the truth value of the expression $(p \vee r) \leftrightarrow (q \wedge r)$ is false.

section d

p	q	r	$p \wedge r$	$q \wedge r$	$(p \wedge r) \leftrightarrow (q \wedge r)$
T	F	T	T	F	F
T	F	F	F	F	T

The truth table indicates that the truth value of the expression $(p \wedge r) \leftrightarrow (q \wedge r)$ is unknown.

section e

p	q	r	$r \vee q$	$p \rightarrow (r \vee q)$
T	F	T	T	T
T	F	F	F	F

The truth table indicates that the truth value of the expression $p \rightarrow (r \vee q)$ is unknown.

section f

p	q	r	$p \wedge q$	$(p \wedge q) \to r$
T	F	T	F	T
T	F	F	F	T

The truth table indicates that the truth value of the expression $(p \wedge q) \to r$ is true.

Question 7

Exercise 1.4.5

section b

The logical expression of the first sentence is as follows:

$$\neg j \to (l \vee \neg r)$$

The logical expression of the second sentence is as follows:

$$(r \wedge \neg l) \to j$$

Proof:

j	l	r	$\neg j \to (l \vee \neg r)$	$(r \wedge \neg l) \to j$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	T
F	F	T	F	F
F	F	F	T	T

The truth table indicates that the two expressions have the same truth value in all situations; therefore, they are logically equivalent.

section c

The logical expression of the first sentence is as follows:

$$j \to \neg l$$

The logical expression of the second sentence is as follows:

$$\neg j \to l$$

Proof:

j	l	$j \to \neg l$	$\neg j \to l$
T	T	F	T
T	F	T	T
F	T	T	T
F	F	T	F

The truth table indicates that the two expressions have different truth values; therefore, they are not logically equivalent.

section d

The logical expression of the first sentence is as follows:

$$(r \vee \neg l) \rightarrow j$$

The logical expression of the second sentence is as follows:

$$j \rightarrow (r \wedge \neg l)$$

Proof:

j	l	r	$(r \vee \neg l) \rightarrow j$	$j \rightarrow (r \wedge \neg l)$
T	T	T	T	F
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	F	T
F	T	F	T	T
F	F	T	F	T
F	F	F	F	T

The truth table indicates that the two expressions have different truth values; therefore, they are not logically equivalent.

Question 8

1. Exercise 1.5.2

section c

$$\begin{aligned}
(p \to q) \land (p \to r) &\equiv (\neg p \lor q) \land (p \to r) && \text{(Conditional identity)} \\
&\equiv (\neg p \lor q) \land (\neg p \lor r) && \text{(Conditional identity)} \\
&\equiv \neg p \land (q \lor r) && \text{(Distributive law)} \\
&\equiv p \to (q \lor r) && \text{(Conditional identity)}
\end{aligned}$$

section f

$$\begin{aligned}
\neg(p \lor (\neg p \land q)) &\equiv \neg((p \lor \neg p) \land (p \lor q)) && \text{(Distributive law)} \\
&\equiv \neg(T \land (p \lor q)) && \text{(Complement law)} \\
&\equiv \neg(p \lor q) && \text{(Identity law)} \\
&\equiv \neg p \land \neg q && \text{(De Morgan's law)}
\end{aligned}$$

section i

$$\begin{aligned}
(p \land q) \to r &\equiv \neg(p \land q) \lor r && \text{(Conditional identity)} \\
&\equiv (\neg p \lor \neg q) \lor r && \text{(De Morgan's law)} \\
&\equiv r \lor (\neg p \lor \neg q) && \text{(Commutative law)} \\
&\equiv (r \lor \neg p) \lor \neg q && \text{(Associative law)} \\
&\equiv (\neg p \lor r) \lor \neg q && \text{(Commutative law)} \\
&\equiv (\neg p \lor \neg\neg r) \lor \neg q && \text{(Double negation law)} \\
&\equiv \neg(p \land \neg r) \lor \neg q && \text{(De Morgan's law)} \\
&\equiv (p \land \neg r) \to \neg q && \text{(Conditional identity)}
\end{aligned}$$

2. Exercise 1.5.3

section c

$$\begin{aligned}
\neg r \lor (\neg r \to p) &\equiv \neg r \lor (\neg\neg r \lor p) && \text{(Conditional identity)} \\
&\equiv \neg r \lor (r \lor p) && \text{(Double negation law)} \\
&\equiv (\neg r \lor r) \lor p && \text{(Associative law)} \\
&\equiv T \lor p && \text{(Complement law)} \\
&\equiv T && \text{(Domination law)}
\end{aligned}$$

section d

$$
\begin{aligned}
\neg(p \to q) \to \neg q &\equiv \neg\neg(p \to q) \lor \neg q && \text{(Conditional identity)} \\
&\equiv (p \to q) \lor \neg q && \text{(Double negation law)} \\
&\equiv (\neg p \lor q) \lor \neg q && \text{(Conditional identity)} \\
&\equiv \neg p \lor (q \lor \neg q) && \text{(Associative law)} \\
&\equiv \neg p \lor T && \text{(Complement law)} \\
&\equiv T && \text{(Domination law)}
\end{aligned}
$$

Question 9

1. Exercise 1.6.3

section c

$$\exists x \left(x = x^2\right)$$

section d

$$\forall x \left(x \leq x^2 + 1\right)$$

2. Exercise 1.7.4

section b

$$\forall x(\neg S(x) \land W(x))$$

section c

$$\forall x(S(x) \to \neg W(x))$$

section d

$$\exists x(S(x) \land W(x))$$

Question 10

1. Exercise 1.7.9

section c

True.
The expression indicates that there exists an x such that either $x = c$ is false or $P(x)$ is true. The truth table indicates that if $x = a, x = c$ is false; thus, the expression is true.

section d

True.
The truth table indicates that if $x = e$, then both $Q(x)$ and $R(x)$ are true so that $Q(x) \wedge R(x)$ is true. Therefore, there exists an x such that $Q(x) \wedge R(x)$ is true.

section e

True.
The truth table indicates that both $Q(a)$ and $P(d)$ is true. Therefore, $Q(a) \wedge P(d)$ is true.

section f

True.
The truth table indicates that for all x, where $x \neq b, Q(x)$ is always true.

section g

False.
The truth table indicates when $x = c$, both $P(x)$ and $R(x)$ are false, so that $P(x) \vee R(x)$ is false. Therefore, for all $x, P(x) \vee R(x)$ is false.

section h

True.
According to the truth table, for all x, either $P(x)$ is true or $R(x)$ is false. If $P(x)$ is true, then $R(x) \to P(x)$ is true. If $R(x)$ is false, then $R(x) \to P(x)$ is also true. Therefore, for all $x, R(x) \to P(x)$ is always true.

section i

True.
The truth table indicates that when $x = a$, $Q(x)$ is true and $R(x)$ is false so that $Q(x) \vee R(x)$ is true. Therefore, there exists an x such that $Q(x) \vee R(x)$ is true.

2. Exercise 1.9.2

section b

True.
The truth table indicates that when $x = 2, Q(x, y)$ is true for all y.

section c

True.
The truth table indicates that when $y = 1$, $P(x, y)$ is true for all x.

section d

False.
The truth table indicates that $S(x, y)$ is always false. Therefore, there are no x and a y such that $S(x, y)$ is true.

section e

False.
The truth table indicates that when $x = 1$, there does not exist a y such that $Q(x, y)$ is true. Therefore, the statement "for all x, there exists a y such that $Q(x, y)$ is true" is false.

section f

True.
The truth table indicates that for all x, there exists a y such that $P(x, y)$ is true. When $x = 1$, if $y = 1$, then $P(x, y)$ is true. When $x = 2$, if $y = 1$, then $P(x, y)$ is true. When $x = 3$, if $y = 1$, then $P(x, y)$ is true.

section g

False.
The truth table indicates that when $x = 1$ and $y = 2$, $P(x, y)$ is false. Therefore, the statement that "for all x and $y, P(x, y)$ is true" is false.

section h

True.
The truth table indicates that when $x = 2$ and $y = 1, Q(x, y)$ is true. Therefore, there exist an x and a y such that $Q(x, y)$ is true.

section i

True.
The truth table indicates that for all x and $y, S(x, y)$ is always false. Therefore, $\neg Q(x, y)$ is always true.

Question 11

1. Exercise 1.10.4

section c

$$\exists x \exists y (x + y = xy)$$

section d

$$\forall x \forall y \left(((x > 0) \land (y > 0)) \rightarrow \left(\frac{x}{y} > 0 \right) \right)$$

section e

$$\forall x \left(((x > 0) \land (x < 1)) \rightarrow \left(\frac{1}{x} > 1 \right) \right)$$

section f

$$\forall x \exists y (y < x)$$

section g

$$\forall x \exists y \left((x \neq 0) \rightarrow \left(y = \frac{1}{x} \right) \right)$$

2. Exercise 1.10.7

section c

$$\exists x (N(x) \land D(x))$$

section d

$$\forall y (D(y) \rightarrow P(\mathrm{Sam}, y))$$

section e

$$\exists x \forall y (N(x) \land P(x, y))$$

section f

$$\exists x \forall y((N(x) \wedge D(x)) \wedge ((N(y) \wedge (y \neq x)) \rightarrow \neg D(y)))$$

3. Exercise 1.10.10

section c

$$\forall x \exists y(T(x,y) \wedge (y \neq \text{ Math 101 }))$$

section d

$$\exists x \forall y((y \neq \text{ Math 101 }) \rightarrow T(x,y))$$

section e

$$\forall x \exists y_1 \exists y_2 \left((x \neq \text{Sam}) \rightarrow \left(T\left(x, y_1\right) \wedge T\left(x, y_2\right) \wedge \left(y_1 \neq y_2\right)\right)\right)$$

section f

$$\exists y_1 \exists y_2 \forall y_3 \left(T\left(\text{Sam}, y_1\right) \wedge T\left(\text{Sam}, y_2\right) \wedge \left(y_1 \neq y_2\right) \wedge \left(y_3 \neq y_1\right) \wedge \left(y_3 \neq y_2\right)\right) \rightarrow \neg T\left(\text{Sam}, y_3\right))$$

Question 12

1. Exercise 1.8.2

section b

Logical Expression:

$$\forall x(D(x) \lor P(x))$$

Negation:

$$\neg\forall x(D(x) \lor P(x))$$

Applying De Morgan's law:

$$\exists x\neg(D(x) \lor P(x))$$

Applying De Morgan's law:

$$\exists x(\neg D(x) \land \neg P(x))$$

English: There is a patient who received neither the medication nor the placebo.

section c

Logical Expression:

$$\exists x(D(x) \land M(x))$$

Negation:

$$\neg\exists x(D(x) \land M(x))$$

Applying De Morgan's law:

$$\forall x\neg(D(x) \land M(x))$$

Applying De Morgan's law:

$$\forall x(\neg D(x) \lor \neg M(x))$$

English: Not every patients were given the medication nor did they have migraines or both.

section d

Logical Expression:

$$\forall x(P(x) \to M(x))$$

Negation:

$$\neg \forall x(P(x) \to M(x))$$

Applying De Morgan's law:

$$\exists x \neg (P(x) \to M(x))$$

Applying conditional identity:

$$\exists x \neg (\neg P(x) \lor M(x))$$

Applying De Morgan's law:

$$\exists x(\neg\neg P(x) \land \neg M(x))$$

Applying double negation law:

$$\exists x(P(x) \land \neg M(x))$$

English: There was a patient who was given a placebo and did not have migraines.

section e

Logical Expression:

$$\exists x(M(x) \land P(x))$$

Negation:

$$\neg \exists x(M(x) \land P(x))$$

Applying De Morgan's law:

$$\forall x \neg (M(x) \land P(x))$$

Applying De Morgan's law:

$$\forall x(\neg M(x) \lor \neg P(x))$$

English: Every patient either did not have migraines or was not given a placebo.

2. Exercise 1.9.4

section c

$$
\begin{aligned}
\neg\exists x\forall y(P(x,y) \to Q(x,y)) &\equiv \forall x\neg\forall y(P(x,y) \to Q(x,y) && \text{(De Morgan's law)}\\
&\equiv \forall x\exists y\neg(P(x,y) \to Q(x,y) && \text{(De Morgan's law)}\\
&\equiv \forall x\exists y\neg(\neg P(x,y) \lor Q(x,y) && \text{(Conditional identity)}\\
&\equiv \forall x\exists y(\neg\neg P(x,y) \land \neg Q(x,y) && \text{(De Morgan's law)}\\
&\equiv \forall x\exists y(P(x,y) \land \neg Q(x,y)) && \text{(Double negation law)}
\end{aligned}
$$

section d

$$
\begin{aligned}
\neg\exists x\forall y(P(x,y) \leftrightarrow P(y,x)) \quad &\equiv \forall x\neg\forall y(P(x,y) \leftrightarrow P(y,x)) && \text{(De Morgan's la}\\
&\equiv \forall x\exists y\neg(P(x,y) \leftrightarrow P(y,x)) && \text{(De Morgan's la}\\
&\equiv \forall x\exists y\neg((P(x,y) \to P(y,x)) \land (P(y,x) \to P(x,y))) && \text{(Conditional identi}\\
&\equiv \forall x\exists y(\neg(P(x,y) \to P(y,x)) \lor \neg(P(y,x) \to P(x,y))) && \text{(De Morgan's la}\\
&\equiv \forall x\exists y(\neg(\neg P(x,y) \lor P(y,x)) \lor \neg(\neg P(y,x) \lor P(x,y))) && \text{(Conditional identi}\\
&\equiv \forall x\exists y((\neg\neg P(x,y) \land \neg P(y,x)) \lor (\neg\neg P(y,x) \land \neg P(x,y))) && \text{(De Morgan's la}\\
&\equiv \forall x\exists y((P(x,y) \land \neg P(y,x)) \lor (P(y,x) \land \neg P(x,y))) && \text{(Double negation la}
\end{aligned}
$$

section e

$$
\begin{aligned}
\neg(\exists x\exists y P(x,y) \land \forall x\forall y Q(x,y)) &\equiv \neg\exists x\exists y P(x,y) \lor \neg\forall x\forall y Q(x,y) && \text{(De Morgan's law)}\\
&\equiv \forall x\neg\exists y P(x,y) \lor \exists x\neg\forall y Q(x,y) && \text{(De Morgan's law)}\\
&\equiv \forall x\forall y\neg P(x,y) \lor \exists x\exists y\neg Q(x,y) && \text{(De Morgan's law)}
\end{aligned}
$$