Machine Learning (IE406) Lab Assignment 2

(Gradient Descent)

February 1, 2022

Note: This exercise is to solidify your understanding of the Gradient Descent algorithm.

Question 1.

Implement the gradient descent algorithm to find the value of x which minimizes the function f(x). Note that $\mathbf{x}$ is a vector, $\mathbf{x} \in R^2$.

- Plot the function f(x) to see its shape. Is it convex or non-convex?
- Set the maximum iterations to be 1000
- For each iteration (or every 10 iterations) plot the value of $f(x^{(t)})$ to confirm that it is decreasing.
- For each function consider different values of learning rate α (you can consider 0.001, 0.1, 0.5, 1 and 5)
- For each function verify that the minima that is seen visually matches with the value of $\mathbf{x}$ obtained.
- The stopping criterion can be set to $\epsilon = 10^{-5}$
- For the 3^{rd} function consider two different initial guesses and compare the results: i) [0.5, 0.5] and ii) [-0.5, -0.5]

1. $f(x) = x_1^2 + x_2^2 + 5$
2. $f(x) = x_1^2 + x_2^2 - 6x_1 + 8x_2 + 9$
3. $f(x) = 3x_1^2 - 5x_2^2$

Note the following:

1. Depending on the nature of the function, gradient descent will give diffferent optima:
   (a) If the function is convex, there will be a unique minima which is the global minima
   (b) In case of non-convex functions there can be multiple local minima, the one found by GD is dependent on your initial guess.
2. To check if a function is convex, check if the eigenvalues of the Hessian of the function are ≥ 0.