Machine Learning (IE406) Lab Assignment 1

(Linear Algebra)

January 25, 2022

Note:

- 1. This exercise is to familiarize you with the working environment (Google Colab) and the Python packages.
- 2. to solidify your understanding of some Linear Algebra concepts. Please try to correlate the results of your code with your understanding of the concepts.

Question 1.

Consider the following matrix W and answer the questions below using the Python libraries.

$$W = \begin{bmatrix} 8 & 5 & 1 & 5 & 6 \\ 5 & 3 & 8 & 7 & 5 \\ 1 & 8 & 10 & 7 & 8 \\ 5 & 7 & 7 & 8 & 9 \\ 6 & 5 & 8 & 9 & 7 \end{bmatrix}$$

- 1. Is the matrix W symmetric?
- 2. What is the maximum eigenvalue of W?
- 3. What is the square root of maximum eigenvalue of W^2 ?
- 4. What is the Frobenius norm of the matrix
- 5. What is the determinant of the matrix? What is its rank?
- 6. Perform eigendecomposition of the symmetric matrix and print the eigenvector matrix Q, and the diagonal matrix Λ with eigenvalues. Verify that matrix W equals $Q\Lambda Q^T$

Question 2.

Let $y_0 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \end{bmatrix}^T$ and $b = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix}^T$. Let there be an iterative rule such that $y_1 = Wy_0 + b$ and in general $y_n = Wy_{(n-1)} + b$ where W is given in question 1. Then answer the questions below, round off answers to 4 decimal places.

- 1. What is the value of $||y_1||_2$
- 2. What is the value of $||y_1||_2/||y_0||_2$

${\bf 3. \ Orthogonal \ matrix}$

Consider the matrix

$$A = \frac{1}{3} \left[\begin{array}{ccc} 2 & -2 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & -2 \end{array} \right]$$

and vector

$$v^T = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}$$

- 1. What is the norm of vectors v and Av?
- 2. Is the matrix A orthogonal?