Inferential Statistics and Hypothesis Testing Assignment

Solution

1.a. For solving this problem, we'll use **a binomial distribution**, because —

i) The number of trials is fixed in this case.

ii)There are only 2 possible outcomes for each trial, i.e. success or failure.

iii)The probability of success is the same for all the trials.

b.) Let the probability of getting an unsatisfactory result be x
As per the question, the probability of getting a satisfactory result is 4x.
Thus, x + 4x =1 => x = 0.2

We are using a binomial distribution and considering getting an unsatisfactory result as a success.
Thus, n= 10 , p =0.2 and r=3
And, P(X<=r) needs to be calculated.

Hence, P(X<=3)= P(X=0)+P(X=1)+P(X=2)+P(X=3)

$= {}^{10}C_0\, 0.2^0 0.8^{10} + {}^{10}C_1 0.2^1 0.8^9 + {}^{10}C_2 0.2^2 0.8^8 + {}^{10}C_3 0.2^3 0.8^7$

= **0.879**

2.) The concept of **Central Limit Theorem** is used to solve this problem. It states that irrespective of the population distribution, the corresponding sampling distribution would follow the following properties —

i) The sampling distribution mean ($\boldsymbol{\mu_x}$) = population mean $\boldsymbol{\mu}$
ii) Sampling distribution standard deviation $\sigma_x = \sigma/\sqrt{(n)}$ — where σ is the population standard deviation, and n is the sample size.
iii) For n>30, the distribution follows a normal distribution.

b.) To solve this problem, we need to calculate the margin of error for the given sample mean.
Now, since the population standard deviation is unknown, we take $\sigma = 65$
Thus, $\sigma_x = 65/\sqrt{100} = 6.5$
For 95% confidence level, the value of $Z = 1.96$
Hence, the margin of error = $Z^* \sigma_x = 1.96*6.5 = 12.74$
The required range = (207-12.74, 207+12.74) = **(194.26,219.74)**

3.a) For the hypothesis test, we shall use 'Critical Value Method' and ' p-value ' method to solve this problem.

Considering the given claim that — the maximum time taken for the drug to affect the pain is 200 seconds, for the drug to be considered satisfactory. Hence, we formulate the following hypothesis statements —

Hypothesis statement:
H_0: $\boldsymbol{\mu} <= 200$
H_1: $\boldsymbol{\mu} > 200$

Critical Value Method
This hypothesis test would follow a one tailed test, with the critical region lying on the right of the Z critical.

The area under that region would be equal to 5%. Hence, the Z critical would correspond to the value of 0.95 on the Z table. Hence, we get the value of Z critical as 1.65.

Thus, we have $\boldsymbol{\mu} = 200$
n=100
$\sigma = 65$
$\sigma_x = \sigma/\sqrt{(n)} = 65/\sqrt{100} = 6.5$ (Here, we have assumed that the sample standard deviation is equal to the population standard deviation; since the latter information is not available in the question, and the sample size is large, i.e. more than 30)
Zc = 1.65

Critical Value =$\boldsymbol{\mu} + Zc*\sigma_x = 200 + 1.65*6.5 = 210.725$
Decision: Since the given experimental mean is 207 seconds and lies left to the Z critical, we fail to reject the null hypothesis.

P-value method

Here, we compute the Z score of the sample mean, which is given as Zs = 207-200/6.5 = 1.08. From the Z table, we find that the probability of the values on the left of 1.08 is 0.86, and hence, the p-value comes out to be 0.14 or 14%.

Decision: Since the value of p is greater than the given level of significance, we fail to reject the null hypothesis.

b) We observe that in the original method, the value of α is lesser than the newer method. The exact opposite scenario happens in the case of β for both the methods. Let's say that if we reject the null hypothesis, we might have to discard the entire batch of drugs and produce an entirely new one. This would be a costly option for the company, and hence, we need to ensure that we do not falsely reject the null hypothesis, when it, in fact, was true. Thus, the value of α needs to be less in this case. Similarly, if the drug causes severe side effects — if the time of effect is high — then, we need to ensure that we are rejecting the null hypothesis. In this case, the value of β or the probability of not rejecting a null hypothesis when it is, in fact, false — needs to be less.

Thus, if we are able to control the risk posed by a higher value of α, then the second method is preferred. Similarly, if a Type-II error poses little to no risk, i.e. there are no side effects of high time of effect, then the first method is preferred; since it has a low value of α.

4.) The situation is best solved using the A/B testing procedure. A/B testing is used in this case because the choice involved is pretty subjective and the conflict resolution can't be done using any normal /statistical procedure.

A/B testing uses the two sample proportion tests since the variables involved in this case are categorical.

Procedure

I)Two groups of people are chosen and each of them is exposed to the only type of advertising.
II.) Their conversion rates are considered.
III.) The null hypothesis and the alternate hypothesis are defined accordingly. In this case, the null hypothesis would be that — the original tagline is more effective than the alternate tagline. And the alternate hypothesis would be the vice versa.
IV)Choose the value of **α (significance level).**

V) After collecting the test data, use a suitable tool — such as XLSTAT, Optimizely, etc. — to run the hypothesis, and then calculate the p-value.
VI.) Make the decision on the basis of whether the p-value is larger than the given significance level.