Project Statement

RBE502 - Robot Control

Spring 2021

1 Problem Statement

The objective of the project is to design a controller for a quadrotor that monitors a restricted airspace. If an unidentified Unmanned Aerial Vehicle (UAV) enters the airspace, the quadrotor needs to launch from the nest, follow and capture the UAV and return to the nest. Figure 1 illustrates a sketch of the concept.

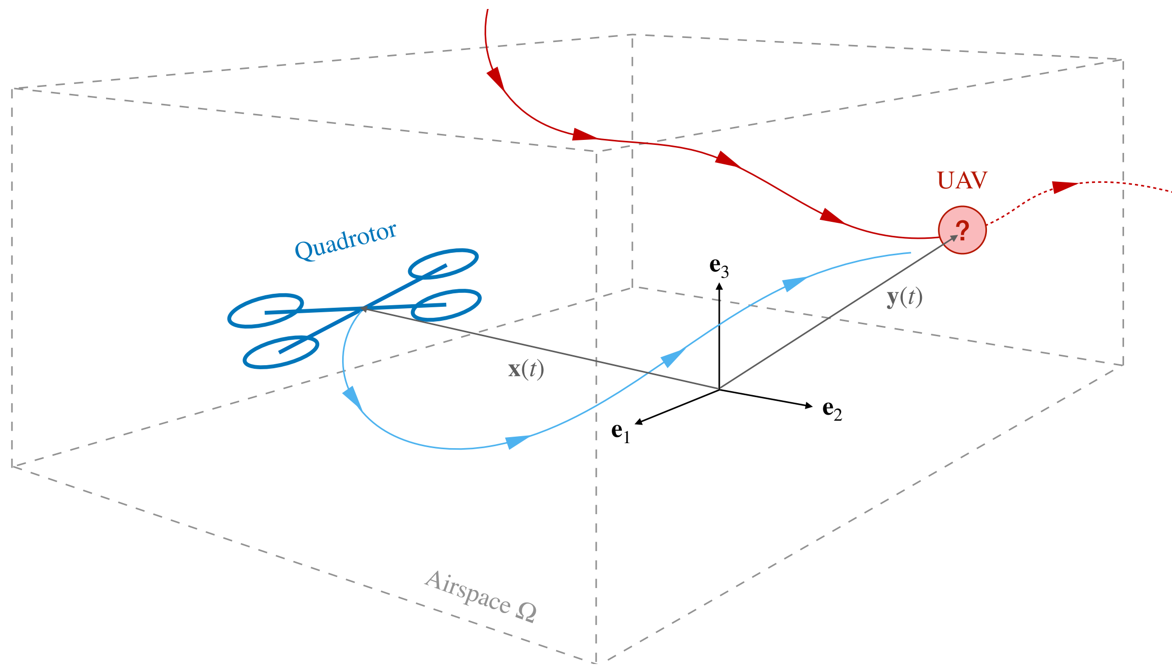

Figure 1: Conceptual illustration of the problem.

2 Problem Specifications

2.1 Quadrotor Model

A free-body diagram of the quadrotor is depicted in Figure 2 and the corresponding parameters are presented in Table 1. In order to simplify the derivations, we assume

$$\tau_i = \sigma\, u_i, \quad i \in \{1, 2, 3, 4\},$$

where $u_i \in [0, \mu]$ N, for a given $\mu > 0$, is the thrust generated by the propeller i. As depicted, the coordinate system $C = \{\mathbf{c}_1, \mathbf{c}_2, \mathbf{c}_3\}$ is attached to the body of the quadrotor and the fixed reference frame $E = \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ serves as the inertial frame, with its origin coinciding with the nest, see Figure 3. To simplify the model, it is assumed that all the rotors are located on the same plane as the center of mass

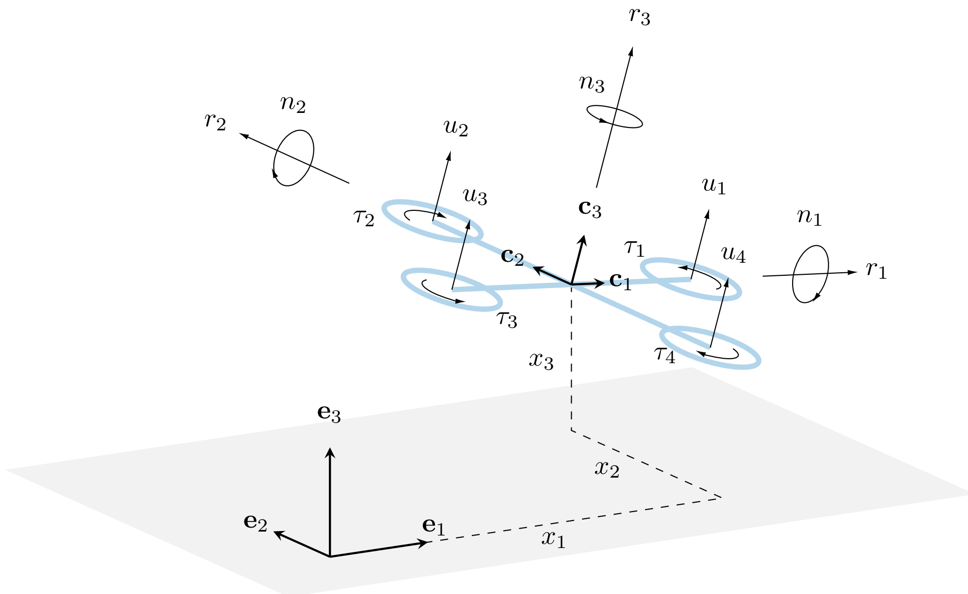

Figure 2: Free-body diagram of the quadrotor system

(that coincides with the origin of C) with the same distance $l > 0$ from the center of mass. The external force and moment vectors $\mathbf{r} = r_1\,\mathbf{c}_1 + r_2\,\mathbf{c}_2 + r_3\,\mathbf{c}_3$ and $\mathbf{n} = n_1\,\mathbf{c}_1 + n_2\,\mathbf{c}_2 + n_3\,\mathbf{c}_3$ are directly applied to the center of mass. Moreover, it is assumed that the mass moment of inertia tensor is

$$I = \begin{bmatrix} I_{11} & 0 & 0 \\ 0 & I_{22} & 0 \\ 0 & 0 & I_{33} \end{bmatrix}, \tag{1}$$

where I_{11}, I_{22} and I_{33} denote the mass moment of inertia of the quadrotor about $\mathbf{c}_1$, $\mathbf{c}_2$ and $\mathbf{c}_3$ axes, respectively.

Parameter	Value	Units	Description
l	0.2	m	Distance from the center of mass to the center of each rotor
m	0.5	kg	Total mass of the quadrotor
I_{11}	1.24	kg·m^2	Mass moment of inertia about c_1 axis
I_{22}	1.24	kg·m^2	Mass moment of inertia about c_2 axis
I_{33}	2.48	kg·m^2	Mass moment of inertia about c_3 axis
g	9.81	m/s^2	The gravitational acceleration
σ	0.01	m	The proportionality constant relating u_i to τ_i
μ	3.0	N	Maximum thrust of each rotor

Table 1: The quadrotor parameters

2.2 Mission Specifications

The control algorithm needs to honor the following mission specifications.

- The thrust generated by each rotor, u_i, must satisfy the range condition $[0, \mu]$.
- The quadrotor is not allowed to leave the airspace. If the UAV escapes the airspace, then the quadrotor must return back to the nest;

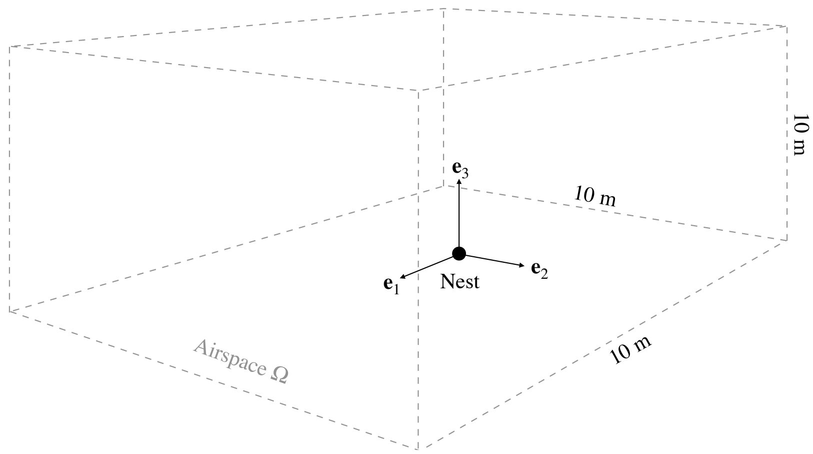

Figure 3: The airspace Ω, inertial reference frame $E = \{\mathbf{e}_1,\, \mathbf{e}_2,\, \mathbf{e}_3\}$ and the nest

- The quadrotor does not know the trajectory of the intruding UAV, $\mathbf{y}(t)$. However, the quadrotor has the capability of tracking the UAV's position. That is, at every time instance $t_i \geq t_0$, we may assume the quadrotor knows $\mathbf{y}(t)$ for all $t \in [t_0,\, t_i]$ where t_0 denotes the time that the intruding UAV has initially entered the airspace.

- The quadrotor can only capture the UAV if it gets to a certain neighborhood of the UAV denoted by S. At every time instance t, for some $\epsilon > 0$, the set $S(t)$ is defined as

$$S(t) := \big\{\mathbf{p} \in \mathbb{R}^3 \,|\, \|\mathbf{p} - \mathbf{y}(t)\|_2 \leq \epsilon\big\},$$

  where $\|\cdot\|_2$ denotes the Euclidean norm. Based on the trapping system of the quadrotor we have $\epsilon = l/2$.

- If the quadrotor captures the UAV, that is

$$\exists\, t^* \geq t_0 \quad \text{such that} \quad \mathbf{x}(t^*) \in S(t^*) \cap \Omega,$$

  then the quadrotor will experience *varying* disturbance force, $\mathbf{r} = r_1\,\mathbf{c}_1 + r_2\,\mathbf{c}_2 + r_3\,\mathbf{c}_3$, and moment, $\mathbf{n} = n_1\,\mathbf{c}_1 + n_2\,\mathbf{c}_2 + n_3\,\mathbf{c}_3$, applied by the UAV for every $t \geq t^*$. Based on the provided statistics on the intruding UAVs, we expect

$$\|\mathbf{r}\|_2 \leq 2\,\mathrm{N} \quad \text{and,} \quad \|\mathbf{n}\|_2 \leq 1\,\mathrm{Nm}.$$

  The proposed controller must be robust enough to handle such disturbances.

- For the safety concerns, the quadrotor must always land smoothly and $x_3(t) \geq 0$ for all t.

2.3 Assumptions

In order to simulate case studies, you may assume:

- The airspace is a $10{\times}10{\times}10$ m^3 cube with the nest located on the center of the bottom surface. The airspace Ω and the nest location are depicted in Figure 3.

- The nest is located at the origin and the quadrotor is stationary in the nest, $\mathbf{x}(t_0) = \mathbf{0}$.

- The intruding UAV enters the airspace at time $t_0 = 0$.

- The weight of the UAV is included in the disturbance force and moment vectors $\mathbf{r}$ and $\mathbf{n}$.