Lab Report
RADIOLOCALIZATION

MATCHED FILTER

José M. González
Jordi Mateu

The matched filter

The **matched filter** provides the instantaneous highest signal to noise ratio (SNR) at its output provided that the input signal is a replica of the transmitted pulse only affected by white noise.

Being $s(t)$ the (baseband) signal transmitted by the radar, the impulse response $h(t)$ of the matched filter is:

$$h(t) = K\, s^*(t_0 - t)$$

This response is the conjugate of a scaled (K), time reversed ($-t$), and shifted (t_0) version of the signal. t_0 corresponds to the time at which the output filter response to $s(t)$ will reach its peak although it can be considered as an arbitrary, but known, quantity.

The output $s_0(t)$ of the matched filter can be found as (assuming K=1 and t t_0=0):

$$s_0(t) = h(t) * s(t) = \int_{-\infty}^{\infty} s(\gamma)\, h(t-\gamma)\, d\gamma = \int_{-\infty}^{\infty} s(\gamma)\, s^*(t-\gamma)\, d\gamma$$

The interesting point of the matched filter is that the output SNR only depends on the signal energy and not on the waveform. Then the criteria to choose the pulse shape should consider the following trade-offs: maximum energy, high resolution, and Doppler frequency performance.

Waveform coding

A way to shape the pulse is using **waveform coding**, meaning that a phase modulation $\phi(t)$ is applied to the transmitted signal, for example a pulse having a pulsewidth τ_p. Constant amplitude of the transmitted signal is assumed because amplifiers in transmitters usually operate in saturation.

$$s(t) = e^{j\phi(t)}\, rect\left[\frac{t}{\tau_p}\right]$$

There are several ways of coding the waveform. Some ways consider continuous variations of the phase across the pulse, as for example the linear frequency modulation (chirp) or the non-linear frequency modulation. Another type of coding the waveform is changing the phase in the pulse using a discrete function of time, where the phase is constant over a time period τ_c called *chip* or subpulse.

$$\phi(t) = \sum_{k=0}^{K-1} \phi_k\, rect\left[\frac{t - k\tau_c - \tau_c/2}{\tau_c}\right]$$

Though many different codes have been adapted from other disciplines of technology in this lab we will work with two classical discrete phase codes: the **Barker codes** and the **Frank polyphase codes**. They will be enough to explain the advantages of using waveform coding.

Barker codes

A pulse coded using the Barker code consists of K sub pulses of width τ. Each sub pulse has a phase of 0º or 180º and then the signal amplitude changes between +1 and -1 according to that phase. Barker codes characterize by the fact that the maximum lobe level is $1/K$ which is convenient to avoid a false target detection. Only 8 Barker codes are known (see Table 1).

Code length	Code elements
1	+
2	+ −, or − −
3	+ + −, or + − +
4	+ + − +, or + + + −
5	+ + + − +
7	+ + + − − + −
11	+ + + − − − + − − + −
13	+ + + + + − − + + − + − +

Table 1. The 8 known Barker codes.

In Figure 1 (left) we have a 7-bits baseband Barker code and (right) an example of a binary modulated signa (not a Barker code).

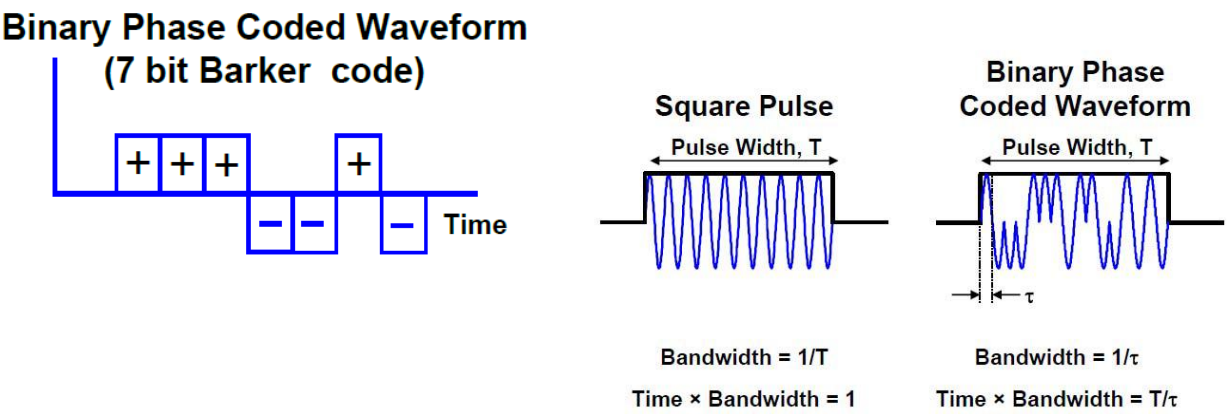

Figure 1. (*left*) 7-Bits baseband Barker code and (*right*) and an example of a binary modulated signal.

Lab Tasks

Part I

In this lab session you should plot the baseband complex signal and the output of the matched filter for a given code and a given signal to noise ratio.

4.1. Design a MATLAB function that generates a baseband signal corresponding to a given code at a given SNR level. The output of the function should be the vector of the baseband signal samples without the noise and the vector of the baseband signal samples with noise. To generate the noise, use the function ***randn*** of MATLAB.

Inputs	Pulse duration (s)
	Number of samples per chip [adim.]
	Vector of chip phases [degrees]
	Vector of chip amplitudes (1 by default) [adim.]
	Signal to noise ratio [dB]
Outputs	Samples of the baseband output signal vector without noise
	Samples of the generated noise vector
	Samples of the baseband output signal vector with noise
	Sampling Time (s)

4.2. Using the previously designed function make a plot of the pulse burst phases corresponding to Barker codes of length K=5, 7, 11, and 13. Consider that the SNR is 30 dB. Consider a pulse duration of 1 µs and 20 samples per chip. Any comment? (burst length, phases, ...)

4.3. Using ***conv*** (convolution), ***conj*** and ***fliplr*** functions of MATLAB find the output of the matched filter for the previously computed Barker codes (K=5, 7, 11, and 13). Make two plots of the filter output, one in linear scale and the other in dB. In the absence of noise (in fact SNR=30 dB), take note of the main to sidelobe level, number of lobes, and the length of the output pulse.

4.4. Make a linear plot of the matched filter output and the input signal for a Barker code of length K=13 when the SNR changes from 30 dB to -10 dB in steps of 10 dB. Make comments about the results (peak level, sidelobe level, peak width, ...).

Part II

In this session we will analyze the robustness of the matched filter to non-expected Barker signals and Doppler shifts.

Doppler effect changes the received frequency of the signal according to the radial velocity of the target with respect to the radar, and consequently the phase of the chips changes due to this frequency change from chip to chip.

Then, the received signal is:

$$s(t)=e^{j\phi(t)}\cdot e^{j\omega d\, t}\cdot rect[t/\tau p]$$

Being v_r the radial velocity of the target with respect to the radar, c the speed of light and f_0 the radar carrier frequency, the ω_d is:

$$\omega_d=2\pi\cdot f_0\cdot 2v_r/c$$

4.5. Using the function of 4.1, make a plot (linear scale) showing the output of a Barker-5 filter when the incoming signals are B-5 (Barker-5), B-7, B-11, B-13. Use a SNR of 30 dB, **chip pulse duration of 0.1 µs**, and 20 samples per chip for all the signals. Make comments about the resulting plot.

4.6. Considering a target with a radial velocity of 600 km/h and a radar carrier frequency of 2.8 GHz, plot (linear scale) the matched filter output of a radar using B-13 compared with the output of a static target (SNR of 30 dB, chip pulse length of 0.1μs, 10 samples per pulse). What happens when the radial velocity of the target is 60000 km/h.
Hint: to solve this exercise you should add the Doppler frequency shift to the (noisy) received signal $s_n(t)$ by doing: $s_n(t)\cdot\exp(j\, 2\pi\cdot f_0\cdot 2v_r/c\; t)$

4.7. Using a chip pulse length of 0.1μs, and 1 sample per pulse, check if there exists a B-6 code. Consider plotting all the matched filter outputs in the same figure.