An Interactive Classroom Demonstration of Propositional and Analogical Representation

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In a cognitive psychology demonstration, students see a rat's-eye view of a maze, projected from a computer, and vocally vote for moves through the maze. The class takes false paths in the first run, but it avoids them in the second. The learning can be explained in terms of analogical imagery or in terms of propositions and rules for modifying them. The demonstration achieves three goals: It actively engages students, effectively explains the concepts, and provides a memorable referent for explaining other topics such as algorithms versus heuristics, production systems, and phrase-structure grammar.

Theories in cognitive psychology specify representations of mental states and processes that operate on the representations. Competing theories posit different representations and processes. A classic debate in theories of mental imagery juxtaposes propositional representation of spatial knowledge against analogical representation. In a propositional representation, spatial relations are explicitly encoded by symbols. In an analogical representation, spatial relations are only implicit in the medium itself and can be extracted only with additional processing. (For additional discussion of analogical and propositional representations, see Kosslyn, 1983, or Palmer, 1978.) The distinction between propositional and analogical representations, and their use in theories of mental imagery, can be especially difficult for students of cognitive psychology. This article presents an interactive classroom demonstration for explaining these concepts.

The demonstration achieves three goals—it actively engages students, clearly explains the concepts, and facilitates the explanation of other topics in cognitive psychology. This article reports results from an experiment that compared the demonstration with a more traditional lecture presentation and provided evidence for success in achieving the goals of engagement and explanation. Extensions of the demonstration to other topics are discussed.

Description of the Demonstration

The Maze Learning program in the PsychSim II software package (Ludwig, 1989) is projected onto a large screen at the front of the class. The program shows a rat's-eye view of a maze (see Figure 1) and allows the user to move through the maze by pressing keys. The maze is divided into cells (marked on the walls in Figure 1), and the user may move forward one cell (F), turn 90° to the left without moving from the cell (L), or turn 90° to the right (R).

The class votes vocally, "pandemonium" style (Selfridge, 1959), for each move, and the instructor enters the loudest vote. When the class is equivocal, the instructor moves whichever way is a false path. Before each move, the instructor clearly states what move will be made and, after the move, allows students ample time to reorient themselves. Two volunteers record the sequence of moves in their notebooks, using the simple codes of L, R, and F.

The first time through the maze, false paths are taken (top panel, Figure 2), but the program gives the user a second chance through the maze. The major result to be explained is that the class's second route avoids false paths (bottom panel, Figure 2).

Analogical Explanation

One explanation for the improvement on the second run is that people piece together a bird's-eye view of the maze in their mind's eye as they traverse the maze the first time. On the second attempt, they scan their mental image and determine which paths are false before mistakenly moving into them. This explanation is reinforced by the computer program, which displays a bird's-eye view of the two routes after both are completed, much as in Figure 2 but without the letter labels on the paths. The representation posited by this explanation is analogical because spatial relations among positions in the maze are encoded implicitly in the image, without explicit symbols.

Propositional Explanation

Improvement in running the maze can be explained without imagery. One of the volunteers calls out the sequence of moves for the first run, and the instructor copies it onto

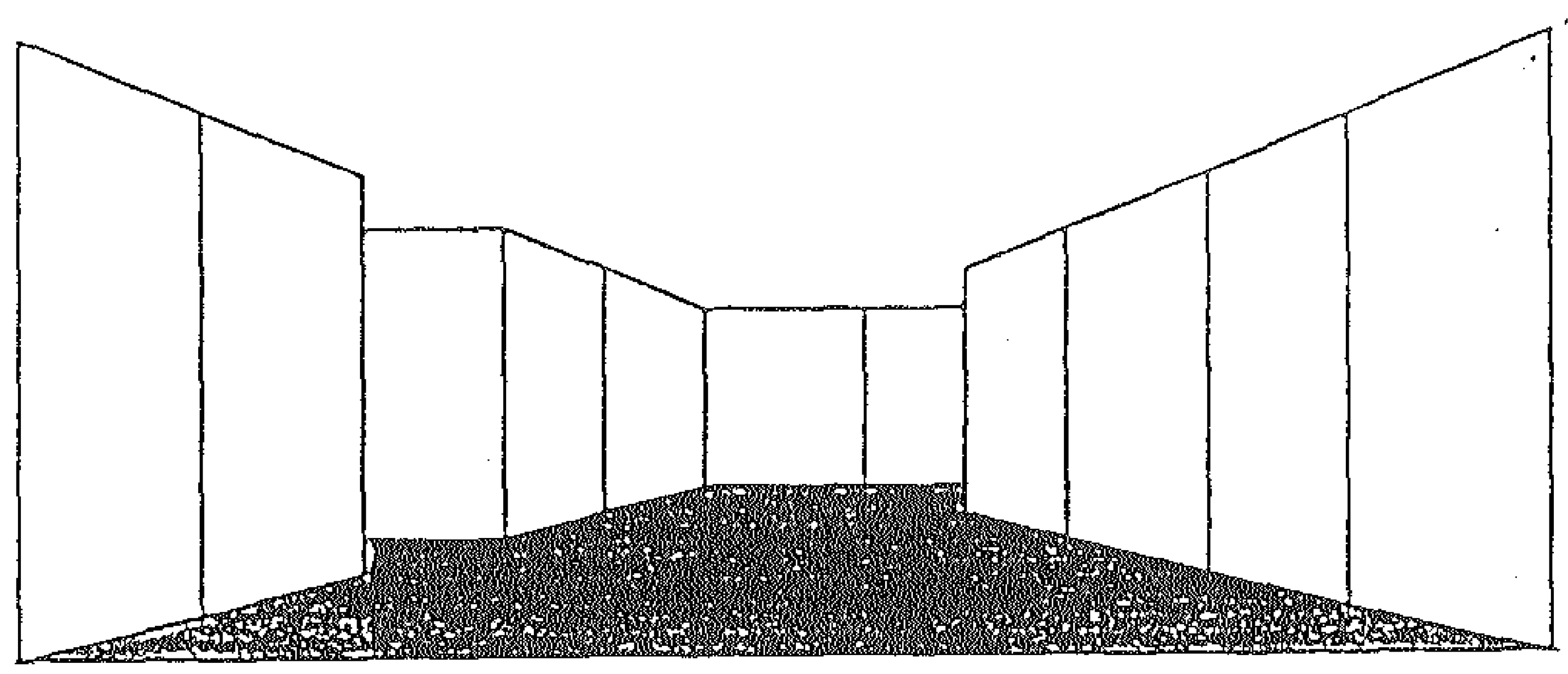

Figure 1. Rat's-eye view of a maze.

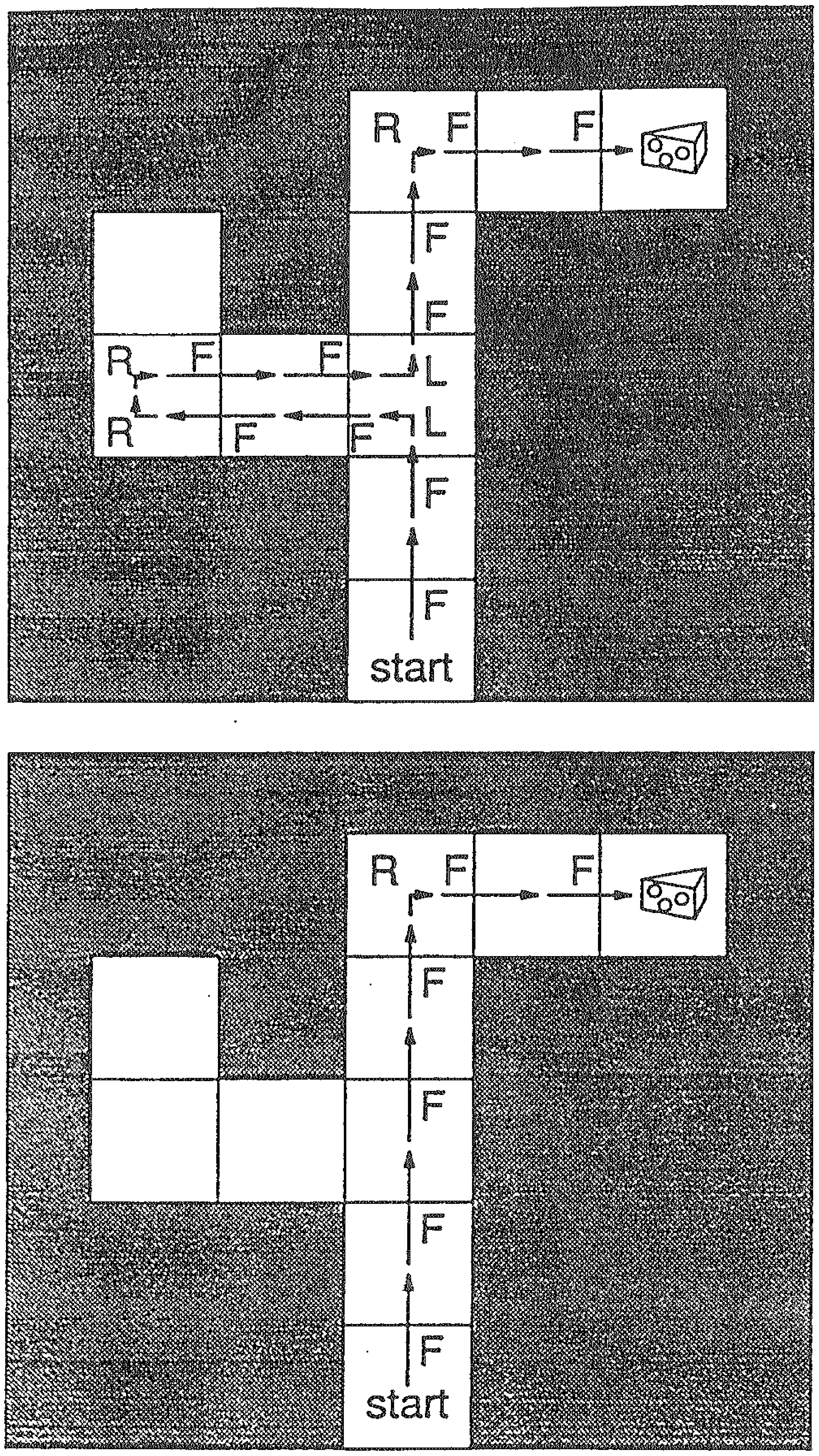

Figure 2. Top: First route through maze takes a false path. Bottom: Second route avoids the false path. (F = move forward one cell, L = turn 90° left, and R = turn 90° right.)

the top of a chalkboard (top line of Figure 3). Then, the second volunteer calls out the sequence of moves for the second run, which the instructor copies onto the bottom of the chalkboard (bottom line of Figure 3). It helps to have the two students verify each other's records.

The instructor then displays the *sequence reduction rules* shown in Table 1. The rules specify how particular subsequences can be simplified. For example, Rule 3 indicates that the subsequent LL can be replaced with —, the symbol for a 180° turn, which means that turning left and then left again has the same effect as turning around. The class searches the first-run sequence for the subsequences shown in Table 1. As students find them, the instructor replaces them with the reduced symbol on successive lines of the chalkboard, until it looks like Figure 3. The maximally reduced sequence matches the second-run sequence at the bottom of the chalkboard.

Table 1. Sequence Reduction Rules

Rule Number	Original Subsequence	Reduced Symbol
1.	LR	{ }
2.	RL	{ }
3.	LL	—
4.	RR	—
5.	L—	R
6.	R—	L
7.	—L	R
8.	—R	L
9.	——	{ }
10.	F—F	—

Note. L denotes a 90° left turn (remaining in the same cell of the maze), R is a 90° right turn, — is a 180° turn, F is a move of one cell forward, and { } is a placeholder denoting no change and is equivalent to the absence of any symbol.

The sequence reduction rules thus explain how the second route avoids false paths. This explanation uses a propositional representation, insofar as the spatial relations among consecutive positions are explicitly coded by symbols. Many students are surprised that the improvement can be explained without imagery.

Assessment of Engagement and Comprehension

The maze-learning demonstration was compared to a lecture describing an experiment by Kosslyn (1983, pp. 41–45). The presentation of Kosslyn's experiment was displayed on well-arranged, laser-printed overhead transparencies that were developed over a 5-year period—a viable competitor to the maze demonstration. In Kosslyn's experiment, participants viewed a number of line drawings until they were accurately memorized. Participants were then instructed to imagine one of the line drawings and to focus on a particular point. With the image in mind, participants had to verify the presence of a feature on the imagined drawing as quickly as possible. Response times for verification were longer for greater distances between the initial focus and the feature

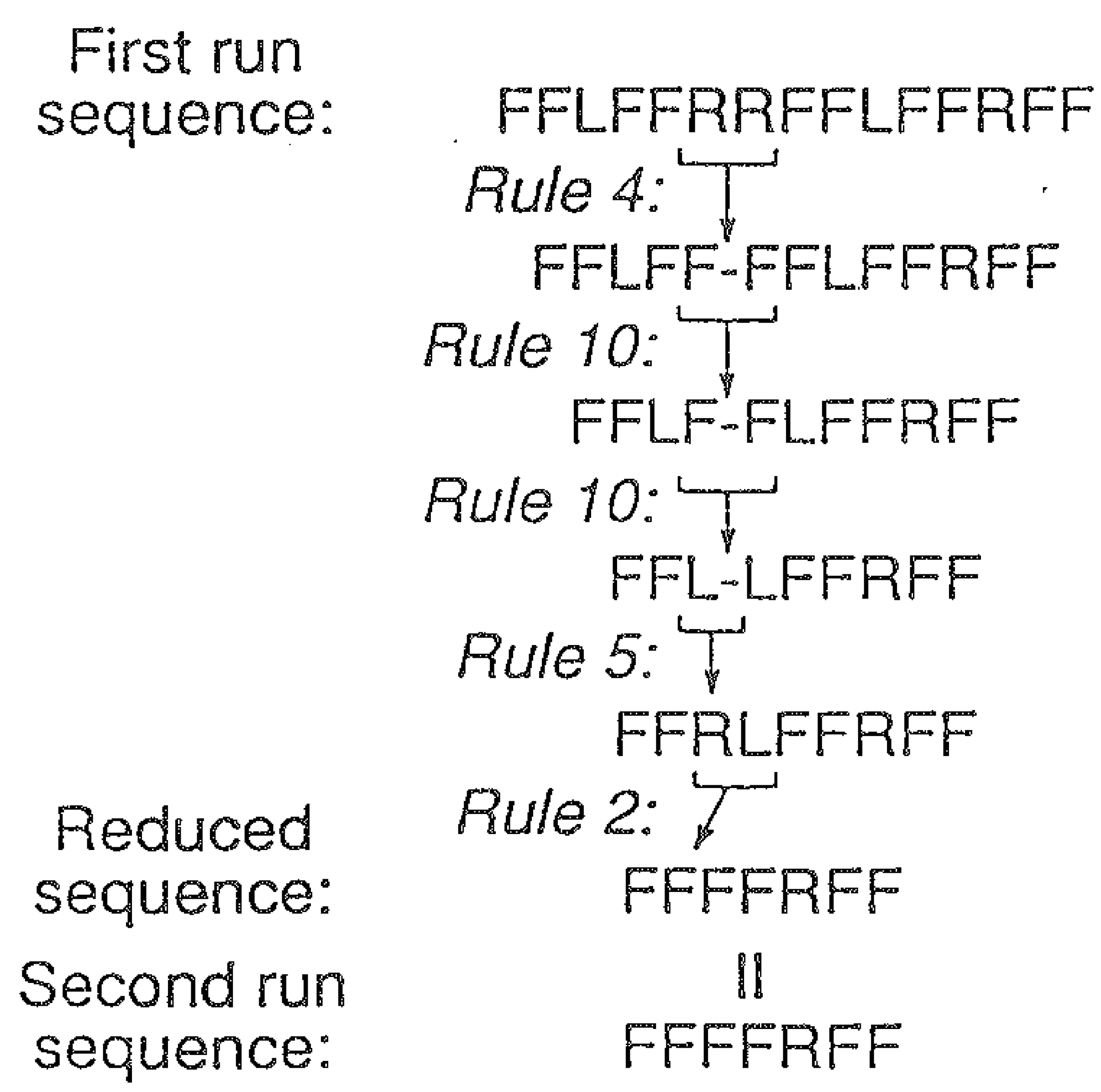

Figure 3. Application of sequence reduction rules (from Table 1) to the first run through the maze yields the second run.

to be verified. An analogical explanation posits a mental image that is scanned at a limited speed, so greater distances take longer to scan. A propositional explanation asserts that the drawing is represented by propositions that specify spatial relations among adjacent features of the drawing, and verification occurs by chaining from one proposition to another via shared arguments. Greater distances require chaining through more propositions and thereby produce longer response times.

Comparison of the lecture and demonstration was conducted in a single class period that began with a brief survey to measure baseline levels of interest in the course material generally. The Kosslyn (1983) picture memory experiment and its alternative explanations were then described, followed by a second survey to assess students' interest and immediate comprehension. The maze-learning demonstration was then presented, followed by a third survey. This design confounds presentation topic with presentation order, but it was selected because all schemes to unconfound these factors were impractical or introduced other confoundings. Long-term comprehension was assessed by a midterm exam question about 3 weeks later.

Method

Participants. The 48 students attended a regular meeting of their cognitive psychology class. (Total course enrollment was 55.)

Procedure and design. Students were not informed of the purpose of the surveys. Each survey began with these instructions: "Thoughtful and sincere answers are important for improving this course. *Your answers will not affect your grade in any way.*" Students did not write their names on the surveys, but they did write identification numbers to enable a within-subjects analysis of the data.

On each of the three surveys, one question asked for a rating of interest in the material (content), and another asked for a rating of engagement by the presentation (format). Both ratings were made on a scale ranging from 1 (*extremely uninteresting/unengaging*) to 7 (*extremely interesting/engaging*). Thus, the design crossed the Interest–Engagement factor with the Baseline–Lecture–Demonstration factor within subjects. For purposes of statistical planned comparisons, I predicted that engagement in the lecture would be less than baseline and that engagement in the demonstration would be greater than baseline.

Immediate comprehension was assessed by short-essay questions on the surveys following each presentation. The survey questions asked, "What was the main empirical result of the picture memory [maze-learning] experiment?," and "Briefly describe the representation and process of the propositional theory of that result." Students were thoroughly acquainted with those types of questions from previous exams and lectures. The first question was liberally scored correct (+1) or incorrect (0), and each of the two parts (representation and process) of the second question was scored correct or incorrect. The instructor scored the survey questions.

The final item on the last survey assessed students' subjective impressions of increase in immediate comprehension by asking how much they agreed or disagreed with a statement that the maze-learning demonstration improved their understanding relative to the lecture. Ratings were made on a scale ranging from 1 (*strongly disagree*) to 5 (*strongly agree*).

Long-term comprehension was assessed on a midterm examination taken about 3 weeks later. The relevant part of the exam required students to write a short essay about either the picture memory or the maze-learning task, as they preferred. In either case, they were to describe the basic result to be explained, the analogical explanation, and the propositional explanation. Exam questions were scored by an assistant who did not know the goal of the questions before grading the exams.

Results and Discussion

Interest and engagement. Four students did not respond to all three surveys; hence, the results are based on a sample size of 44. An alpha level of .05 is assumed for all statements about statistical significance. The main effect of the Interest–Engagement factor barely missed being significant. Students rated their engagement of attention ($M = 4.92$) higher than their interest in the topics ($M = 4.76$), $F(1, 43) = 3.94$, $p = .054$. There was no significant interaction between the two factors, $F < 1$; hence, the effect of the Baseline–Lecture–Demonstration factor is reported in terms of the average rating of engagement and interest.

Planned comparisons revealed that the lecture rating ($M = 4.35$) was significantly lower than the baseline rating ($M = 4.91$), $F(1, 43) = 10.00$, $p = .003$, and the demonstration rating ($M = 5.26$) was significantly higher than baseline, $F(1, 43) = 5.71$, $p = .021$. The maze-learning demonstration made interest in the relatively abstruse topic of propositional versus analogical representation rise from below-baseline levels to significantly above-baseline levels, achieving the first goal.

Immediate comprehension. Of the 42 students who responded on both surveys to the item asking for a statement of empirical results, 12 students scored better on the maze-learning question than on the picture-memory question and 4 scored worse (26 scored the same), which is a significant difference according to results of a sign test, $p = .038$.

Of the 37 students who responded on both surveys to the item asking for a statement of the propositional explanation, the mean score on the maze-learning topic ($M = 1.19$, $SD = .877$) was not significantly higher than the picture-memory topic ($M = 1.03$, $SD = .928$), as indicated by a Wilcoxon matched-pairs signed-ranks test, $T(N = 18) = 65.5$, $p > .05$. (A t test of the difference scores yielded the same conclusion.) Students had the subjective impression, nevertheless, that the maze-learning demonstration slightly improved their understanding relative to the picture-memory presentation, as indicated by their mean rating of agreement with the statement to that effect ($M = 3.31$, $SD = .873$), which is significantly higher than 3.0 (the neutral level of the 5-point scale), $t(42) = 2.36$, $p = .023$.

Long-term comprehension. On the midterm examination, students overwhelmingly preferred to answer the question about maze learning as opposed to picture memory, 43 to 12. The null hypothesis of equal preference can be rejected with a binomial test, $p < .0001$. The mean score of students who answered the maze-learning question (M = 13.4 out of 14 possible, SD = 1.98) was not significantly higher than the mean score of students who answered the picture-memory question (M = 12.8 out of 14 possible, SD = 2.17), as indicated by the point–biserial correlation of score with question, $r = .116$, $F(1, 53) < 1$.

In summary, comprehension of the maze-learning demonstration was at least as good as that of the picture-memory lecture. Students strongly preferred to address the maze-learning question on the midterm exam instead of the picture-memory question, and their absolute performance on the exam question was extremely high. Thus, the goal of explicating the concepts was achieved.

Discussion: Integration With Other Topics

There is no implication that lectures are ineffective teaching tools. The maze-learning demonstration cannot, by itself, supply all the important information about analogical and propositional representations in theories of mental imagery. Rather, students' high ratings of interest and engagement and their strong preference to address the maze-learning topic on the midterm exam show that the demonstration is highly effective for hooking students into the topic, so that other lecture material has a better chance of being learned. The engaging manner, comprehensibility, and memorability of the maze-learning demonstration also make it an excellent referent for explicating and integrating other topics, such as (a) the difference between an algorithm and a heuristic, (b) the concept of production systems, and (c) the concept of phrase-structure grammar.

In problem solving, an *algorithm* is a strategy that is guaranteed to find a solution, and a *heuristic* is a generally useful strategy that may sometimes fail. The difference can be illustrated by asking students to describe their strategy for getting through the maze on the first try. Typical heuristics are trying each alternative path as it is encountered, choosing randomly at forks, and hoping to remember previously taken paths. Students are intrigued to learn that one algorithm (for a maze without loops) is to close their eyes, keep their hand touching one wall, and walk (negotiating turns as needed) until they bump into the cheese. This particular algorithm is especially interesting because it requires no memory of false paths and builds no spatial representation of the maze.

The maze-learning demonstration also illustrates the concept of *productions systems*, which are sets of condition–action rules used to modify the contents of memory. Students actually experienced being a production system when they applied the sequence-reduction rules to the first path through the maze. The concept is thereby made concrete and comprehensible.

Finally, in the topic of language, phrase-structure grammars attempt to describe the structure of grammatical sentences by specifying a set of rewrite rules that recursively generate all possible grammatical sentences but no ungrammatical sentences. The goals and mechanics of phrase-structure grammars can be explained by analogy to maze learning. A participant in a succession of maze-learning experiments may want to determine regularities of the mazes. One way to capture the regularities is to invent maze-structure rules that generate the same sort of mazes as those that are run. One potential set of maze-structure rules is the sequence-reduction rules of Table 1 applied in reverse, which expand direct paths into full-fledged mazes with false paths. For example, one can begin with a direct path, such as the final sequence in Figure 3, and generate false paths by applying the rules in reverse, working up to the top sequence in Figure 3. The issue for the "mazologist" is whether the maze-structure rules generate all valid mazes and no invalid ones, just as the issue for the linguist is whether the phrase-structure rules generate all grammatical sentences and no ungrammatical ones. For instance, the experimenter's mazes might never have loops, whereas the rule-generated mazes could. If maze-structure rules adequately describe the experienced mazes, there remains the question of whether the mazes were actually generated by those rules or by some other process, just as the psychologist might ask whether phrase-structure rules reflect psychological processes. The advantages of the maze analogy are its concreteness, memorability, and additional associations for students newly introduced to the concept.

References

Kosslyn, S. M. (1983). *Ghosts in the mind's machine.* New York: Norton.

Ludwig, T. E. (1989). *Maze learning (PsychSim II)* [Computer software]. New York: Worth.

Palmer, S. E. (1978). *Fundamental aspects of cognitive representation.* In E. Rosch & B. B. Lloyd (Eds.), *Cognition and categorization* (pp. 259–303). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Selfridge, O. G. (1959). *Pandemonium: A paradigm for learning.* London: H. M. Stationery Office.

Notes

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