```html
<!DOCTYPE html>
<html>
<head>
<title>Singular Vectors</title></head>
<link rel="stylesheet" href="https://cdnjs.cloudflare.com/ajax/libs/
font-awesome/4.7.0/css/font-awesome.min.css">
  <style>
  .slidecontainer {
    width: 1200px;

  }

/* TITLE OF THE FOLDER THAT CONTAINS EVERYTHING */

  .slider {
    -webkit-appearance: none;
    width: 600px;
    height: 25px;
    background: #d3d3d3;
    outline: none;
    opacity: 0.7;
    -webkit-transition: .2s;
    transition: opacity .2s;

  }

  .slider:hover {
    opacity: 1;
  }

  .slider::-webkit-slider-thumb {
    -webkit-appearance: none;
    appearance: none;
    width: 25px;
    height: 25px;
    background: #4CAF50;
    cursor: pointer;
  }

  .slider::-moz-range-thumb {
    width: 25px;
    height: 25px;
    background: #4CAF50;
    cursor: pointer;
  }

/* MANY SHORT LABELED INFORMATIONAL AREAS. NOTE WHERE THESE ARE USED
LATER */

  .left {
  float: left;
  width: 900px;
  padding: 10px;
```

```css
  }

  .right{
    float:right;
    width: 1100px;
    padding: 10px;
  }

  h2 {
        color: #000;
        text-align: center;
        text-decoration: underline;
        text-decoration-color: black;
      }

  h3 {
        color: #000;
        text-decoration: underline;
        text-decoration-color: black;
      }
      .tab { margin-left: 20px; }
      label {
          display:inline-block;
          *display: inline;      /* for IE7*/
          zoom:1;                /* for IE7*/
          padding-top: 5px;
          text-align: left;
          width: 140px;
      }

      i {
        color: blue;
      }


      .arrow1 {
        animation: slide1 1s ease-in-out infinite;
        margin-left: -20px;
      }


      @keyframes slide1 {
        0%,
        100% {
          transform: translate(0, 0);
        }


        50% {
          transform: translate(10px, 0);
        }
      }

/* THREE DIFFERENT div.absolute AREAS THAT WILL BE REFERRED TO LATER
```

```css
*/

      div.absolute
      {
        position: absolute;
        top: 250px;
        left: 420px;
        width: 200px;
        height: 100px;
      }
      /* a */
      div.absolute2
      {
        position: absolute;
        top: 85px;
        left: 949px;
        width: 30px;
        height: 10px;

      }
      div.absolute3
      {
        position: absolute;
        top: 15px;
        left: 950px;
        width: 600px;
        height: 100px;
      }

/* TWO MORE ITEMS THAT WILL BE REFERRED TO LATER */

      library
      {
        margin: 0;
        width: 100%;
        height: 100%;
        background-color: black;
        overflow: hidden;
      }


      body
      {
        padding: 0;
        margin: 0;
      }



/* NOW THINGS BEGIN  */
/* NOTICE THE USE OF "LEFT" */
/*  SWITCHING TO A NEW DIV CLASS */

  </style>
```

```html
    <body >
<div class="left" >



    <h2>SELF-DISCOVERY OF SINGULAR VECTORS AND VALUES</h2>
    <p class="tab">A 2x2 matrix A maps the unit circle onto an
ellipse.  One can reveal this ellipse, point by point, by holding
down and moving along<br>
      the t- slidebar.  For the value of t giving the "greatest
stretch" on the ellipse, that is, the point on the ellipse that is
farthest from the<br>
      origin, the corresponding (x,y) is the first right singular
vector of A.  The first left singular vector is the normalized (u,
v), that is,<br>
      (u,v) divided by its length.  The first singular value is |(u,
v)|, the length of this greatest stretch.



<div class="absolute">
<p style="margin-left:10px">A


    <br>



<i class="fa fa-long-arrow-right arrow1" aria-hidden="true"
style="font-size:48px;"></i></div>



<canvas id="demoCanvas" width="400" height="320" style="margin-left:
5px"> canvas</canvas>
<canvas id="Canvas2" width="400" height="320" style="margin-left:
35px;" > canvas</canvas>

  <div class="slidecontainer" style="margin-left:30px">
      <div class="block">

        <h3> t - SLIDEBAR <input type="range" min="0" max="360"
step="1" value="0" class="slider" id="myRange"/><label> &nbsp &nbsp
t = <span id="demo"></span>°</label></h3 >

          <p> (t = angle of inclination in the (x,y) coordinate
system.  The left coordinate system is (x,y) and the right is
(u,v).)
          <p>NOTE: The second singular vectors are also easily
visualized.  They are the unit vectors orthogonal to the first right
and first left <br> singular vectors.
           They can be found using the method of "least stretch".
The second singular value is the length of "least stretch".
      </div>
```

```
    </div>
    <script>
    var k = 0;
    var slider = document.getElementById("myRange");
    var output = document.getElementById("demo");
    output.innerHTML = slider.value;
    var canvas = document.getElementById('demoCanvas');
    var ctx = canvas.getContext('2d');
    var canvas2 = document.getElementById('Canvas2')
    var ctx2 = canvas2.getContext('2d');
    var xcen = 200;
    var ycen = 160;
    var radius = 100;
    var startAngle = 0 * Math.PI;
    var endAngle = 2 * Math.PI;
    var counterClockwise = false;
    var u = 0
    var v = 0
    var g =0

/*  CANVAS   */
  ctx.moveTo(xcen,ycen)
  ctx.strokeStyle = "#000000";
  ctx.font = "15px Arial";
  ctx2.font = "15px Arial";
  ctx.clearRect(0,0,600,600);
  ctx.fillText("1",xcen+10,45);
  ctx.fillText("−1",xcen+10,280);
  ctx.fillText("1",318,ycen+15)
  ctx.fillText("−1",67,ycen+15)
  ctx.moveTo(50,ycen)
  ctx.lineTo(350,ycen)
  ctx.stroke();
  ctx.moveTo(xcen,10)
  ctx.lineTo(xcen,310)
  ctx.stroke();

  /* CANVAS2 */
  ctx2.beginPath();
  ctx2.moveTo(xcen,ycen)
  ctx2.moveTo(50,ycen)
  ctx2.lineTo(350,ycen)
  ctx2.strokeStyle = "#000000";
  ctx2.stroke();
  ctx2.moveTo(xcen,10)
  ctx2.lineTo(xcen,310)
  ctx2.strokeStyle = "#000000";
  ctx2.stroke();



  slider.oninput = function() {
    x = 200
```

```
y = 160
radius = 100

output.innerHTML = this.value;
k = 1
if (k!=0) {
  var ctx2 = canvas2.getContext('2d');
  ctx2.beginPath();
  ctx2.strokeStyle = "white";
  ctx2.lineWidth = 4;
  ctx2.moveTo(x,y)
  ctx2.lineTo(x-(radius/g)*1.4*u,y+(radius/g)*1.4*v)
  ctx2.stroke();
}
var ctx2 = canvas2.getContext('2d');

xcen=200
ycen=160

ctx2.beginPath()
ctx2.clearRect(xcen+3,27,40,18)
ctx2.clearRect(xcen+3,274,48,18)
ctx2.clearRect(325,ycen+2,50,16)
ctx2.clearRect(46,ycen+2,48,16)
```

/* SEE THE MATH DONE IN THE FUNCTION JACOBIAN , LATER */

```
var arr = calcJacobian()
u = arr[0]
v = arr[1]
g = arr[2]
```

/* NUMBER OF DIGITS TO DISPLAY ON THE AXES */

```
g=Number(g .toPrecision(2));
if(g<10 && g>3){g=Number(g .toPrecision(1))};
if(g<1 && g>.3){g=Number(g .toPrecision(1))};
if(g<100 && g>30){g=Number(g .toPrecision(1))};
if(g<1000 && g>300){g=Number(g .toPrecision(1))};
if(g<.1 && g>.03){g=Number(g .toPrecision(1))};

var xcen = 200
var ycen = 160

/*+g*/
ctx2.fillText(g,xcen+10,40);
ctx2.fillText(-g,xcen+10,290);
ctx2.fillText(g,325,ycen+15);
ctx2.fillText(-g,60,ycen+15);

ctx2.beginPath();
ctx2.strokeStyle = "red";
ctx2.moveTo(x,y)
ctx2.fillRect(x-(100/g)*1.4*u,y+(100/g)*1.4*v,1.5,1.5)
```

```
      ctx2.moveTo(x-(100/g)*1.4*u,y+(100/g)*1.4*v)
      ctx2.lineTo(x,y)
      ctx2.stroke();

      /* X - y graph*/
      ctx.clearRect(0,0,500,600);
      ctx.fillText("1",x+10,45);
      ctx.fillText("-1",x+10,280);
      ctx.fillText("1",318,ycen+15)
      ctx.fillText("-1",67,ycen+15)
      ctx.beginPath();
      ctx.strokeStyle = "#000000";
      ctx.lineWidth=2;
      ctx.moveTo(xcen,ycen);
      ctx.moveTo(xcen,10);
      ctx.lineTo(xcen,310);
      ctx.stroke();
      ctx.moveTo(xcen,ycen);
      ctx.moveTo(50,ycen);
      ctx.lineTo(350,ycen);
      ctx.stroke();
      ctx.moveTo(xcen,ycen);
      ctx.beginPath();
      ctx.strokeStyle = "#000000";
      ctx.arc(x, y, radius, startAngle, endAngle, counterClockwise);
      ctx.lineWidth = 2;
      // line color
      ctx.stroke();
      var t = output.innerHTML = this.value;
      t = t * (Math.PI/180);
      ctx.moveTo(x,y)
      ctx.strokeStyle = '#ff0000'
      ctx.lineTo(x+ radius*Math.cos(t),y- radius*Math.sin(t))
      ctx.stroke()
}


function calcJacobian() {
  var myArr = document.forms.inputField;
  var myControls = myArr;
  var slider = document.getElementById("myRange");
  var angle = document.getElementById("demo1");
  angle.innerHTML = slider.value + "°"; /* t value*/
  var x1 = document.getElementById("x")
  x1.innerHTML = Math.cos(slider.value * (Math.PI/180)).toFixed(3)
  var y1 = document.getElementById('y')
  y1.innerHTML = Math.sin(slider.value * (Math.PI/180)).toFixed(3)
  var a = myControls[0].value;
  var b = myControls[1].value;
  var c = myControls[2].value;
  var d = myControls[3].value;

  var u = a*Math.cos(slider.value * (Math.PI/180)) +
```

```javascript
b*Math.sin(slider.value * (Math.PI/180)).toFixed(3)
    var v = c*Math.cos(slider.value * (Math.PI/180)) +
d*Math.sin(slider.value * (Math.PI/180)).toFixed(3)
    var rad2 = Math.sqrt(Math.pow(u,2)+Math.pow(v,2)).toFixed(3)
    var U = document.getElementById('u')
    U.innerHTML = u.toFixed(3)
    var V = document.getElementById('v')
    V.innerHTML = v.toFixed(3)
    var len1 = document.getElementById('len1')
    len1.innerHTML = Math.sqrt(Math.pow(u,2)+Math.pow(v,
2)).toFixed(3)
    var norm_u = document.getElementById('normU')
    norm_u.innerHTML = (u/Math.sqrt(Math.pow(u,2)+Math.pow(v,
2))).toFixed(3)
    var norm_v = document.getElementById('normV')
    norm_v.innerHTML = (v/Math.sqrt(Math.pow(u,2)+Math.pow(v,
2))).toFixed(3)
    var intialU = 0
    var intialV = 0
    var intialL = 0
    var temp = 0
    var highT = 0;
    for (var i = 0; i <= 360; i = i+1) {
      intialU = a*Math.cos(i * (Math.PI/180)) + b*Math.sin(i*
(Math.PI/180))
      intialV = c*Math.cos(i * (Math.PI/180)) + d*Math.sin(i *
(Math.PI/180))
      intialL = Math.sqrt(Math.pow(intialU,2)+Math.pow(intialV,2))
      if (temp < intialL) {
        temp = intialL;
        highT = i;

      }
      else {
        temp = temp
      }
    }


    /* user defined greatest stretch*/

    var uMax = a*Math.cos(slider.value * (Math.PI/180)) +
b*Math.sin(slider.value * (Math.PI/180)).toFixed(3)
    var vMax = c*Math.cos(slider.value * (Math.PI/180)) +
d*Math.sin(slider.value * (Math.PI/180)).toFixed(3)
    var g = 1.05 * Math.max(Math.sqrt(Math.pow(a,2) + Math.pow(c,
2)), Math.sqrt(Math.pow(b,2) + Math.pow(d,2)), Math.sqrt(Math.pow(a,
2) + Math.pow(b,2)), Math.sqrt(Math.pow(c,2) + Math.pow(d,2)))
    var singleValue = Math.sqrt(Math.pow(uMax,2)+Math.pow(vMax,
2)).toFixed(3)
    var T = document.getElementById('HighT')
    T.innerHTML = slider.value + "°"
    var LSVx = document.getElementById('LSVx')
```

```javascript
    LSVx.innerHTML = Math.cos(slider.value * (Math.PI/
180)).toFixed(3)
    var LSVy = document.getElementById('LSVy')
    LSVy.innerHTML = Math.sin(slider.value * (Math.PI/
180)).toFixed(3)
    var temp1 = 0
    temp1 = uMax/Math.sqrt(Math.pow(uMax,2)+Math.pow(vMax,
2)).toFixed(3)
    var temp2 = vMax/Math.sqrt(Math.pow(uMax,2)+Math.pow(vMax,
2)).toFixed(3)
    var maxNorm_u = document.getElementById('maxNormU')
    maxNorm_u.innerHTML = temp1.toFixed(3)
    var maxNorm_v = document.getElementById('maxNormV')
    maxNorm_v.innerHTML = temp2.toFixed(3)
    var SV = document.getElementById('sv')
    SV.innerHTML = Math.sqrt(Math.pow(uMax,2)+Math.pow(vMax,
2)).toFixed(3)


/* The ultimate max */

    var uMax1 = a*Math.cos(highT * (Math.PI/180)) + b*Math.sin(highT
* (Math.PI/180)).toFixed(3)
    var vMax1 = c*Math.cos(highT * (Math.PI/180)) + d*Math.sin(highT
* (Math.PI/180)).toFixed(3)
    var singleValue1 =
Math.sqrt(Math.pow(uMax1,2)+Math.pow(vMax1,2)).toFixed(3)

    var con
    var dif = (Math.abs(singleValue - singleValue1))/singleValue1

/* con 1 = show
   con 0 = hidden
   */

   var alert = document.getElementById('alert')
   alert.innerHTML = con
    if (dif>0.02) {
      con = "Alert: you did not find the greatest stretch"
      alert.innerHTML = con
    }
    else {
      con = ""
      alert.innerHTML = con
    }

    return [u,v,g,con];
}



function showDiv() {
```

```
    var x = document.getElementById("welcomeDiv");
    if (x.style.display === "none") {
      x.style.display = "block";
    } else {
      x.style.display = "none";
    }

}


function refresh()
{
  location.reload();
}
  </script>
</div>
<div class="absolute2">
  A =
</div>
  <div class="absolute3" >
    <div class="well well-lg">
      <h3>Step 1. Enter matrix values</h3>
      <form id="inputField" role="form" style="margin-left:30px">

        <input type="text" name="field00" size="5" >
        <input type="text" name="field01" size="5">
        <br>
        <input type="text" name="field10" size="5" >
        <input type="text" name="field11" size="5" >
  <input type="button" onclick="calcJacobian()" value="calculate"
name="calculate" class="btn btn-info" style="margin-left:30px;" >
      </form>

        <br>
        <img src="https://i.ibb.co/cxGjpxT/note.png">
        <h3>Step 2.</h3>
        <p> Hold down and slide along the slidebar.</p>

        <h3>Step 3. Results:</h3>
        <p class="tab">t = <span id="demo1"></span> </p>
        <p class="tab"> vector (x,y) = (<span id = 'x'></span> ,
<span id='y'></span>)</p>
        <p class="tab"> vector (u,v) = (<span id = 'u'></span> ,
<span id='v'></span>)</p>

        <p class="tab"> length of (u,v) = |(u,v)| = <span
id="len1"></span></p>
        <p class="tab"> normalized (u,v) = (<span id = 'normU'></
span> , <span id='normV'></span>)</p>

        <h3>Step 4. Exploration:</h3>
        <p>Slide until the length of (u,v) given above</p>
        <p> is maximized. Then click <input type="button"
```

```
                    name="answer"  value="Done" onclick="showDiv()" /></p>
        <div id="welcomeDiv"  style="display:none;"
class="answer_list" >
            <p class="tab"> Value of t that gives the greatest stretch
= <span id="HighT"></span></p>
            <p class="tab" style="color:red;"> <span id="alert"></
span></p>
            <p class="tab"> Right Sing vec [ = the (x,y) for that t ]
= (<span id = 'LSVx'></span> , <span id='LSVy'></span>)</p>
            <p class="tab"> Left Sing vec [ = normalized (u,v) for
that t ] = (<span id = 'maxNormU'></span> , <span id='maxNormV'></
span>)</p>
            <p class="tab"> Singular Value [ = the greatest stretch ]
= <span id="sv"></span></p>
            <p class="tab"> <span id="test"></span></p>
              <p class="tab"> <span id="test1"></span></p>


        </div>
      <h3>Step 5. New Matrix
      <input type="button" name = "refresh" value="Refresh"
class="tab" onclick="refresh()"/></h3>
    </div>
  </div>


</body>
</html>
```