Section 4.2 – Direct Proof and Counterexample II: Writing Advice

A proof is a way to communicate a convincing argument for the truth of a mathematical statement.

We don't want to impede anyone's creativity, but format for writing Direct Proofs has become fairly standard. Having said that, two correct proofs written by different people are rarely identical.

Try to be clear and complete. A reader of your proof will only see what is presented, not any underlying thoughts.

1. Copy the statement of the theorem to be proved on your paper.
2. Clearly mark the beginning of your proof with the word Proof.
3. Make your proof self-contained.
4. Write your proof in complete, grammatically correct sentences. Note, you ***can*** still use symbols.
5. Keep your reader informed about the status of each statement in your proof.
6. Give a reason for each assertion in your proof.
7. Include the "little words and phrases" (hence, therefore, etc.) that make the logic of your arguments clear.
8. Display equations and inequalities.

Try this out on the statement:

Prove that the difference of any odd integer and any even integer is odd.

Proof:

For all integers a and b, if a is odd and b is even, then $a - b$ is odd.

Suppose a is an odd integer and b is an even integer. [We must show $a - b$ is odd]

$a = 2k + 1$, for some integer k. [By definition of odd]

$b = 2l$, for some integer l. [By definition of even]

Note that k and l must be different

Then, $a - b = (2k + 1) - 2l$ [By substitution]

$= 2k - 2l + 1$ [Algebra, combining like terms]

$= 2(k - l) + 1$ [Algebra, factoring]

Since $k - l$ is the difference of two integers, which is itself an integer, the expression $2(k - l) + 1$ is of the form $2 * integer + 1$, which is odd.

So, , $a - b$ is odd.

Q.E.D. ("is as was to be shown, baby")

What are some common mistakes to avoid in Direct Proofs?

1. Arguing from examples.

   Consider the following "proof" that the sum of any two even integers is even (Theorem 4.1.1).

   This is true because if $m = 14$ and $n = 6$, which are both even, then $m + n = 20$, which is also even.

2. Using the same letter to mean two different things.

   Suppose m and n are any odd integers. Then by definition of odd, $m = 2k + 1$ and $n = 2k + 1$ where k is an integer.

3. Jumping to a conclusion.

   Suppose m and n are any even integers. By definition of even, $m = 2r$ and $n = 2s$ for some integers r and s. Then $m + n = 2r + 2s$. So $m + n$ is even.

4. Assuming what is to be proved.

   Suppose m and n are any odd integers. When any odd integers are multiplied, their product is odd. Hence mn is odd.

5. Confusion between what is known and what is still to be shown.

   Suppose m and n are any odd integers. We must show that mn is odd. This means that there exists an integer s such that

$$mn = 2s + 1.$$

6. Use of ***any*** when the correct word is ***some***.

   By definition of odd, $m = 2a + 1$ for any integer a.

7. Misuse of the word if.

   Suppose p is a prime number. If p is prime, then p cannot be written as a product of two smaller positive integers.

   The use of the word *if* in the second sentence is inappropriate. It suggests that the primeness of p is in doubt. But p is known to be prime by the first sentence. It cannot be written as a product of two smaller positive integers *because* it is prime. Here is a correct version of the fragment:

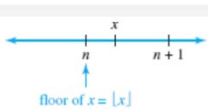
<u>From Sec 4.6:</u>

Given any real number x, the **floor of x**, denoted $\lfloor x \rfloor$, is defined as follows:

$$\lfloor x \rfloor = \text{that unique integer } n \text{ such that } n \le x < n+1.$$

Symbolically, if x is a real number and n is an integer, then

$$\lfloor x \rfloor = n \quad \Leftrightarrow \quad n \le x < n+1.$$

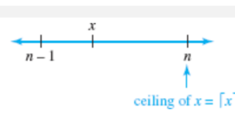


Given any real number x, the **ceiling of x**, denoted $\lceil x \rceil$, is defined as follows:

$$\lceil x \rceil = \text{that unique integer } n \text{ such that } n-1 < x \le n.$$

Symbolically, if x is a real number and n is an integer, then

$$\lceil x \rceil = n \quad \Leftrightarrow \quad n-1 < x \le n.$$



Compute: $\lfloor -1.1 \rfloor = -2$, $\left\lfloor \frac{25}{4} \right\rfloor = 6$, $\lceil -1.1 \rceil = -1$, $\left\lceil \frac{25}{4} \right\rceil = 7$