Section 9.1 – Introduction to Probability

It's not how to count, but ***what to count***.

Partner up

Flip a coin twice.

Either an actual coin, or use a simulation:
https://www.google.com/search?q=coin+flip&rlz=1C1CHBF_enUS957US957&oq=coin+flip&gs_lcrp=EgZjaHJvbWUyDwgAEEUYORiDARixAxiABDIKCAEQABixAxiABDIKCAIQABixAxiABDIHCAMQABiABDIHCAQQABiABDIHCAUQABiABDIHCAYQABiABDIHCAcQABiABDIHCAgQABiPAjIHCAkQABiPAtIBCTU1NTlqMGoxNagCCLACAQ&sourceid=chrome&ie=UTF-8

Record the results, which are either: 0 heads 1 head 2 heads

If you do this a bunch, flipping a coin twice, what results do you expect? How many of: 0 heads 1 head 2 heads

Go ahead and do this (flipping a coin twice) nine (9) more times, recording the results.

So, you should have ten (10) observations, total up how many of each you have: 0 heads 1 head 2 heads

0 heads	***1 head***	***2 heads***

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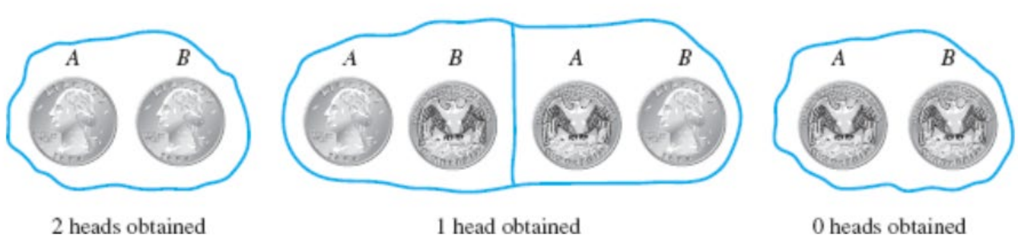
Wait, does this make sense?

Equally Likely Outcomes from Tossing Two Balanced Coins



So, we aren't surprised if the trend is: 0 heads-25% 1 head-50% 2 heads-25%

because there are two of four ways to get 1 H, and one of four ways to get each of 0H or 2 H.

Let's formalize this analysis.

To say that a process is ***random*** means that when it takes place, one outcome from some set of outcomes is sure to occur, but it is impossible to predict with certainty which outcome that will be.

A **sample space** is the set of all possible outcomes of a random process or experiment. An **event** is a subset of a sample space.

So, for our example, the ***sample space*** is: 0 heads / 1 head (either first flip and second flip) / 2 heads and an ***event*** would be one of those.

If S is a finite sample space in which all outcomes are equally likely and E is an event in S, then the **probability of E**, denoted $P(E)$, is

$$P(E) = \frac{\text{the number of outcomes in } E}{\text{the total number of outcomes in } S}.$$

For any finite set A, $N(A)$ denotes the number of elements in A

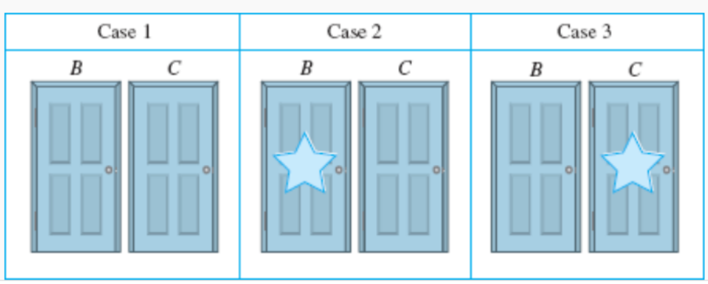
And our probability formula becomes:

$$P(E) = \frac{N(E)}{N(S)}$$

So, probabilities are really just about counting, and now we see mathematically $P(1\ head) = \frac{N(1\ head)}{N(S)} = \frac{2}{4} = 0.5$

A famous problem from a game show in the 1970's is the "Monty Hall Problem." Monty Hall was the host of a game show called "Let's Make a Deal."

There are three doors on the set for a game show. Let's call them A, B, and C. If you pick the correct door, you win the prize. You pick door A. The host of the show then opens one of the other doors and reveals that there is no prize behind it. Keeping the remaining two doors closed, he asks you whether you want to switch your choice to the other closed door or stay with your original choice of door A. What should you do if you want to maximize your chance of winning the prize: stay with door A or switch—or would the likelihood of winning be the same either way?



The probabilities favor an approach, but you might like to simulate: https://www.mathwarehouse.com/monty-hall-simulation-online/

At the point just before the host opens one of the closed doors, there is no information about the location of the prize. Thus there are three equally likely possibilities for what lies behind the doors: (Case 1) the prize is behind A (meaning it is not behind either B or C); (Case 2) the prize is behind B; or (Case 3) the prize is behind C.

Since there is no prize behind the door the host opens, in Case 1 the host could open either door and you would win by staying with your original choice: door A. In Case 2 the host must open door C, and so you would win by switching to door B. In Case 3 the host must open door B, and so you would win by switching to door C. Thus, in two of the three equally likely cases, you would win by switching from A to the other closed door. In only one of the three equally likely cases would you win by staying with your original choice. Therefore, you should switch.

Now, in reality, on the show, Monty Hall did ***not*** always offer the choice to change… Usually it was when the contestant had already chosen the winning door. The analysis to always switch only works if the host always makes the offer.

Continuing on our "counting journey:"

How many integers are ***from*** 5 to 12? $12 - 5 + 1 = 8$

How many integers are ***between*** 5 and 12? $11 - 6 + 1 = 6$

How many integers from 5 to 12 ***inclusive***? $12 - 5 + 1 = 8$

list:	5	6	7	8	9	10	11	12
	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$
count:	1	2	3	4	5	6	7	8

Notice that we need to add 1 to count all the elements.

Here's the theorem, where in our example above $m = 5$ and $n = 12$

Theorem 9.1.1 The Number of Elements in a List

If m and n are integers and $m \leq n$, then there are $n - m + 1$ integers from m to n inclusive.

How many positive 3 digit integers are divisible by 5?

We can write all these in a row, 100 to 999 inclusive, and look for the pattern of those divisible by 5:

100 101 102 103 104 105 106 107 108 109 110 111 … 995 996 997 998 999

$5 \cdot 20$ $5 \cdot 21$ $5 \cdot 22$ $5 \cdot 199$

Here, $m = 20$ and $n = 199$, so we have $199 - 20 + 1 = 180$ integers divisible by 5 from 100 to 999 inclusive.

Calculating the probability of randomly choosing an integer which is divisible by 5 from 100 to 999 inclusive is now easy.

$$\frac{N(E)}{N(S)} = \frac{180}{999 - 100 + 1} = \frac{180}{900} = \frac{1}{5}$$