Section 2.2 – Conditional Statements (Day 02)

To say "p only if q" means that p can take place *only* if q takes place also. That is, if q does not take place, then p cannot take place. Another way to say this is that if p occurs, then q must also occur (by the logical equivalence between a statement and its contrapositive).

efinition

If p and q are statements,

$$p \textbf{ only if } q \qquad \text{means} \qquad \text{"if not } q \text{ then not } p,\text{"}$$

or, equivalently,

$$\text{"if } p \text{ then } q.\text{"}$$

Partner up

Rewrite the following statement in if-then form in two ways, one of which is the contrapositive of the other.

John will break the world's record for the mile run only if he runs the mile in under four minutes.

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Version 1: If John does not run the mile in under four minutes, then he will not break the world's record.

Version 2: If John breaks the world's record, then he will have run the mile in under four minutes.

Given statement variables p and q, the **biconditional of p and q** is "p if, and only if, q" and is denoted $p \leftrightarrow q$. It is true if both p and q have the same truth values and is false if p and q have opposite truth values. The words *if and only if* are sometimes abbreviated **iff**.

The biconditional has the following truth table:

uth Table for $p \leftrightarrow q$

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Partner up

Give the truth table for $p \leftrightarrow q$

Truth Table Showing That $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

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p	q	$p \to q$	$q \to p$	$p \leftrightarrow q$	$(p \to q) \wedge (q \to p)$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T
$p \leftrightarrow q$ and $(p \to q) \wedge (q \to p)$ always have the same truth values, so they are logically equivalent					

The phrases *necessary condition* and *sufficient condition*, as used in formal English, correspond exactly to their definitions in logic.

If r and s are statements:

r is a **sufficient condition** for s means "if r then s."

r is a **necessary condition** for s means "if not r then not s."

In other words, to say "r is a sufficient condition for s" means that the occurrence of r is *sufficient* to guarantee the occurrence of s. On the other hand, to say "r is a necessary condition for s" means that if r does not occur, then s cannot occur either:

The occurrence of r is *necessary* to obtain the occurrence of s. Note that because of the equivalence between a statement and its contrapositive,

r is a necessary condition for s also means "if s then r."

Consequently,

r is a necessary and sufficient condition for s means "r if, and only if, s."

Let's take our favorite example, with Sheetz and Shmuffins, and rewrite using ***necessary*** and ***sufficient***.

p = I purchased a Shmuffin at a store. ***q*** = I went to Sheetz

$p \to q$ *If I purchased a Shmuffin at a store, then I went to Sheetz*

$p \to q \equiv p$ ***is sufficient for*** q *Purchasing a Shmuffin at a store is **sufficient** for going to Sheetz*

$p \to q \equiv q$ ***is necessary for*** p *Going to Sheetz is **necessary** for purchasing a Shmuffin at a store*

Rewrite the following statement in the form "If A then B":

Pia's birth on U.S. soil is a sufficient condition for her to be a U.S. citizen.

Solution

If Pia was born on U.S. soil, then she is a U.S. citizen.

Example 2.2.12 Converting a Necessary Condition to If-Then Form

Use the contrapositive to rewrite the following statement in two ways:

George's attaining age 35 is a necessary condition for his being president of the United States.

Solution

Version 1: If George has not attained the age of 35, then he cannot be president of the United States.

Version 2: If George can be president of the United States, then he has attained the age of 35.

Normally a parent/guardian will promise: (1) *If you finish your HW, then you can go to the movies*

or threaten: (2) *You can go to the movie only-if you finish your HW*

But in (1) the statement is vacuously true if the student doesn't finish their HW and they go to the movies, and

in (2) the statement is vacuously true if the student doesn't go to the movie even if they finished their HW.

It would be more complete for the parent to use the bi-conditional,
You can go to the movie if and only-if you finish your HW

Partner up

If <u>I purchased a Big Mac at a store</u> then <u>I was at McDonald's</u>.
p ***q***

Write symbolically: $p \to q$

Re-write using sufficient instead of if-then: *Purchasing a Big Mac is sufficient for going to McD's*
Write the contrapositive of the original statement: *If I didn't go to McD's then I didn't purchase a Big Mac*
Re-write the original statement using necessary instead of if-then:
Going to McD's is a necessary condition for buying a Big-Mac