Sec 5.2- Mathematical Induction, Day 01

1. Demo of Dominos falling.

2. Students get in groups and set up Dominos.
   a. Conditions for the "last" domino fall if only the "first" domino is knocked over.
   b. What are some configurations or conditions where the "last" domino won't fall?

3. What is a predicate for this problem? Don't forget to quantify.
   P(n): the n^{th} domino falls, $\forall$ $\boldsymbol{int\ n \geq 1.}$

4. So, for the "last" domino to fall, we would need:
   a. The "first" domino to be knocked over in the correct direction.
   b. That if a particular domino is knocked over, it will knock over the "next" domino.

5. We can state this mathematically, using the predicate above:
   a. ***P(1)***: the 1^{st} domino falls.
   b. We can assume that if ***P(k)*** is true, the ***k^{th}*** domino falls, then the ***P(k+1)*** is true, the ***$(k+1)^{st}$*** domino will fall.

6. This is the Principle of Mathematical Induction. The predicate in #3 above is quantified, so it actually means there are infinitely many statements to prove. But Mathematical Induction says that if (1) ***P(1)*** can be shown to be true, which is the ***<u>Basis Step</u>***, and (2) the ***<u>Inductive Step</u>***, where we assume ***P(k)*** is true and show ***P(k+1)*** is true, then we have proven that the original predicate is true for all its infinitely many inputs.

7. Let's consider another predicate we can prove with induction, with quantification:

$$P(n): 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}, \forall \text{ int n} \geq 1$$

8. First, let's write a few of the statements, writing both the Left-Hand Side (LHS) and Right-Hand Side (RHS), with no need of simplifying:

$$\begin{aligned} P(1)&: 1 &&= \frac{1(1+1)}{2} \\ P(2)&: 1+2 &&= \frac{2(2+1)}{2} \\ P(3)&: 1+2+3 &&= \frac{3(3+1)}{2} \end{aligned}$$

   if we continue, we can consider the ***k^{th}*** and ***$(k+1)^{st}$*** statements, for a particular, but arbitrarily chosen ***k***

$$\begin{aligned} P(k)&: \quad 1+2+3+\cdots+k &&= \frac{k(k+1)}{2} \\ P(k+1)&: 1+2+3+\cdots+k+(k+1) &&= \frac{(k+1)(k+2)}{2} \end{aligned}$$

9. So, to prove ***P(n)*** in a proof by Mathematical Induction, we would have:

   <u>Basis Step</u> [Show *P(1)* is true]

   <u>Inductive Step</u> [Assume *P(k)* is true and show *P(k+1)* is true]