Section 2.3 – Valid and Invalid Arguments

Partner up

PRETEND to have an argument.

In Discrete Structures, we have ***mathematical arguments***. These are not a dispute. They're simply a sequence of statements ending in a conclusion.

An **argument** is a sequence of statements, and an **argument form** is a sequence of statement forms. All statements in an argument and all statement forms in an argument form, except for the final one, are called **premises** (or **assumptions** or **hypotheses**). The final statement or statement form is called the **conclusion**. The symbol $\therefore$, which is read "therefore," is normally placed just before the conclusion.

To say that an *argument form* is **valid** means that no matter what particular statements are substituted for the statement variables in its premises, if the resulting premises are all true, then the conclusion is also true. To say that an *argument* is **valid** means that its form is valid.

If Socrates is a man, then Socrates is mortal. ***hypotheses*** (note the overload on the term from the conditional!)

Socrates is a man.

$\therefore$ Socrates is mortal. ***conclusion***

has the abstract form

If p then q

p

$\therefore q$

The crucial fact about a valid argument is that the truth of its conclusion follows *necessarily* or *inescapably* or *by logical form alone* from the truth of its premises. It is impossible to have a valid argument with all true premises and a false conclusion. When an argument is valid and its premises are true, the truth of the conclusion is said to be *inferred* or *deduced* from the truth of the premises. If a conclusion "ain't necessarily so," then it isn't a valid deduction.

An argument form consisting of two premises and a conclusion is called a **syllogism**. The first and second premises are called the **major premise** and **minor premise**, respectively. The most famous form of syllogism in logic is called **modus ponens**. It has the following form:

$$\begin{array}{l} \text{If } p \text{ then } q. \\ p \\ \therefore q \end{array}$$

We have an algorithm to show that an ***argument form*** is valid

1. Identify the premises and conclusion of the argument form.
2. Construct a truth table showing the truth values of all the premises and the conclusion.
3. A row of the truth table in which all the premises are true is called a **critical row**. If there is a critical row in which the conclusion is false, then it is possible for an argument of the given form to have true premises and a false conclusion, and so the argument form is invalid. If the conclusion in *every* critical row is true, then the argument form is valid.

Let's apply this argument form to Modus Ponens (MP), which is Latin for "Method of Affirming."

		premises		conclusion
p	q	$p \rightarrow q$	p	q
T	T	**T**	**T**	**T** *
T	F	F	T	
F	T	T	F	
F	F	T	F	

Critical Row (c.r.)

Now consider another valid argument form called **modus tollens**. It has the following form:

$$\begin{array}{l} \text{If } p \text{ then } q. \\ \sim q \\ \therefore \sim p \end{array}$$

Modus Tollens (MT) is Latin for "Method of Denying." Studies have shown that nearly 100% of college students understand Modus Ponens, but only 60% understand Modus Tollens.

There are many different types of valid argument forms. We'll focus on MP and MT, as well as the algorithm above, for identifying valid argument forms.

A **fallacy** is an error in reasoning that results in an invalid argument. Three common fallacies are **using ambiguous premises**, and treating them as if they were unambiguous, **circular reasoning** (**assuming what is to be proved** without having derived it from the premises), and **jumping to a conclusion** (without adequate grounds). In this section we discuss two other fallacies, called *converse error* and *inverse error*, which give rise to arguments that superficially resemble those that are valid by modus ponens and modus tollens but are not, in fact, valid.

We'll use the algorithm to show the following example is an invalid argument form, and note that it demonstrates the Converse Error.

If Zeke is a cheater, then Zeke sits in the back row.
Zeke sits in the back row.
∴Zeke is a cheater.

$$\begin{array}{l} p \rightarrow q \\ q \\ \therefore p \end{array}$$

		premises		conclusion
p	q	$p \rightarrow q$	q	p
T	T	T	T	T
T	F	F	F	
F	T	T	T	F *
F	F	T	F	

Critical Rows (c.r.) A false conclusion in a c.r. makes the argument form invalid

We'll use the algorithm to show the following example is an invalid argument form, and note that it demonstrates the Inverse Error.

If these two vertices are adjacent, then they do not have the same color. $p \to q$

These two vertices are not adjacent. $\sim p$

$\therefore$ These two vertices have the same color. $\therefore \sim q$

p	*q*	*p* → *q*	*~p*	*~q*
T	*T*	*T*	*F*	
T	*F*	*F*	*F*	
F	*T*	*T*	*T*	*F*
F	*F*	*T*	*T*	*T*

Critical Rows (c.r.) A false conclusion in a c.r. makes the argument form invalid

Valid Forms		Invalid Forms	
Modus Ponens	***Modus Tollens***	***Converse Error***	***Inverse Error***
If p then q. p $\therefore q$	If p then q. $\sim q$ $\therefore \sim p$	$p \to q$ q $\therefore p$	$p \to q$ $\sim p$ $\therefore \sim q$

If you encounter any of these four argument forms, you can use these definitions to give their validity. Any other forms, you will need to use the algorithm above to determine validity.

Sometimes people lump together the ideas of validity and truth. If an argument seems valid, they accept the conclusion as true. And if an argument seems fishy (really a slang expression for invalid), they think the conclusion must be false. This is not correct!

Example 2.3.11 A Valid Argument with a False Premise and a False Conclusion

The argument below is valid by modus ponens. But its major premise is false, and so is its conclusion.

If Canada is north of the United States, then temperatures in Canada never rise above freezing.

Canada is north of the United States.

$\therefore$ Temperatures in Canada never rise above freezing.

Example 2.3.12 An Invalid Argument with True Premises and a True Conclusion

The argument below is invalid by the converse error, but it has a true conclusion.

If New York is a big city, then New York has tall buildings.

New York has tall buildings.

$\therefore$ New York is a big city.

An argument is called **sound** if, and only if, it is valid *and* all its premises are true. An argument that is not sound is called **unsound**.

The important thing to note is that validity is a property of argument *forms:* If an argument is valid, then so is every other argument that has the same form. Similarly, if an argument is invalid, then so is every other argument that has the same form. What characterizes a valid argument is that no argument whose form is valid can have all true premises and a false conclusion. For each valid argument, there are arguments of that form with all true premises and a true conclusion, with at least one false premise and a true conclusion, and with at least one false premise and a false conclusion. On the other hand, for each invalid argument, there are arguments of that form with every combination of truth values for the premises and conclusion, including all true premises and a false conclusion. The bottom line is that we can only be sure that the conclusion of an argument is true when we know that the argument is sound, that is, when we know both that the argument is valid and that it has all true premises.