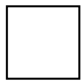
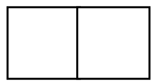
Section 5.1 – Sequences, Day 01

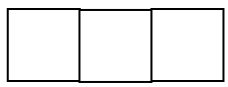
Many of you probably have previous experience with sequences and series in a mathematical sense, and in this section we will explore shortcuts to defining them, and formulas for calculating the values of series and products.

Partner up

Consider rows of square boxes.

A row with one box has 4 sides,  so $a_1 = 4$

A row with two boxes has 7 sides,  so $a_2 = 7$

A row with three boxes has 10 sides,  so $a_3 = 10$

(a) Find an explicit (not a recursive) formula to describe this, which is a function of n, where n is the number of boxes and a_n is the number of sides. Be sure to ***quantify*** n.

$a_n =$

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There are infinitely many correct answers, two are: $a_n = 3n + 1, \forall\, int\; n \geq 1$ or $a_n = 3n + 4, \forall\, int\; n \geq 0$

The only difference in these two explicit formulas are where the subscripts on the variable a begin, 0 or 1.

Sometimes we are simply given a sequence in roster notation: $4, 7, 10, 13, 16, \cdots$
The "dot, dot, dot" is called an "ellipsis."

For those of you with programming experience, you might notice an analogy between sequences and arrays in Java or lists in Python:

`a = [4, 7, 10, 13, 16]` where `a[0] = 4` `a[1] = 7` and so on.

(b) Find a recursive (***NOT*** explicit) formula to describe this, which is a function of n, where n is the number of boxes and a_n is the number of sides. Be sure to give an initial value for the sequence.

$a_1 =$

$a_n =$

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$a_1 = 4$ again, there are infinitely many correct answers: $a_0 = 4$

$a_n = a_{n-1} + 3, \forall\, int\; n \geq 2$. $a_n = a_{n-1} + 3, \forall\, int\; n \geq 1$.

This is recursive because the definition for a_n depends on previous value in the sequence, a_{n-1}.

Why does the quantification of n start at 2? ***Because in the subscript,*** *n-1*, ***if*** *n* ***was only*** *1*, ***the subscript would be*** *0*

(c) Find an explicit formula for the number of "joins" (where two or more edges intersect) as a function of n, where n is the number of boxes and b_n is the number of "joins". Note that $b_1 = 4$, $b_2 = 6$, and $b_3 = 8$. Be sure to quantify n.

$b_n = 2n + 2, \forall\ int\ n \geq 1$

(d) Finally, find a recursive formula for the number of "joins" (where two or more edges intersect) as a function of n, where n is the number of boxes and b_n is the number of "joins". Be sure to give an initial value for the sequence.

$b_1 = 4$

$b_n = b_{n-1} + 2, \forall\ int\ n \geq 2.$

In many ways, the recursive definitions are easier, or more natural, to find.

A **sequence** is a function whose domain is either all the integers between two given integers or all the integers greater than or equal to a given integer.

We typically represent a sequence as a set of elements written in a row. In the sequence denoted

$$a_m, a_{m+1}, a_{m+2}, \ldots, a_n,$$

each individual element a_k (read "a sub k") is called a **term**. The k in a_k is called a **subscript** or **index**, m (which may be any integer) is the subscript of the **initial term**, and n (which must be an integer that is greater than or equal to m) is the subscript of the **final term**. The notation

$$a_m, a_{m+1}, a_{m+2}, \ldots$$

denotes an **infinite sequence**. An **explicit formula** or **general formula** for a sequence is a rule that shows how the values of a_k depend on k.