Section 2.2 – Conditional Statements (Day 01)

Have you ever heard of a Shmuffin?

It's a breakfast sandwich from Sheetz.

If you purchase a Shmuffin, the only store you can purchase it from is a Sheetz.

Who has been to a Sheetz in the last week?

What did you purchase there?

So, you can go to Sheetz and purchase something other than a Shmuffin.

But, if you purchase a Shmuffin, you ***HAD*** to go to Sheetz.

Let's define two propositional variables. (***why are they propositional***?)

p = I purchased a Shmuffin at a store. ***q*** = I went to Sheetz

These ideas lead us to a new connective, the conditional.

Formally/symbolically: $p \to q$ or $if\ p\ then\ q$ or $p\ implies\ q$

Informally: If I purchased a Shmuffin at a store, then I went to Sheetz.

The first part is called the ***hypothesis*** and the second part is called the ***conclusion***.

The truth table for the conditional is ***defined*** as:

p	***q***	***p → q***
T	***T***	***T***
T	***F***	***F***
F	***T***	***T***
F	***F***	***T***

The <u>entire</u> statement, ***p → q*** is true if the hypothesis is false. This is called ***vacuously true***. Let's explore another example:

Suppose you go to interview for a job at a store and the owner of the store makes you the following promise:

If you show up for work Monday morning, then you will get the job.

Under what circumstances are you justified in saying the owner spoke falsely? That is, under what circumstances is the above sentence false? The answer is: You *do* show up for work Monday morning and you do *not* get the job.

After all, the owner's promise only says you will get the job *if* a certain condition (showing up for work Monday morning) is met; it says nothing about what will happen if the condition is *not* met. So if the condition is not met, you cannot in fairness say the promise is false regardless of whether or not you get the job.

If we negate the hypothesis and conclusion, and swap the order, we get the ***contrapositive***, $\sim q \rightarrow p$

If I did NOT go to Sheetz, then I did NOT purchase a Shmuffin at a store.

What is the truth table? ***It's ok to use intermediate columns***.

p	***q***	***~p***	***~q***	***~q→~p***
T	***T***	***F***	***F***	***T***
T	***F***	***F***	***T***	***F***
F	***T***	***T***	***F***	***T***
F	***F***	***T***	***T***	***T***

The truth values are the same, so $p \rightarrow q \equiv \sim q \rightarrow p$, the conditional and its contrapositive are equivalent.

Can we construct an expression for the conditional using only $\wedge$, $\vee$, $or \sim$? Yes!

p	***q***	***p→q***	***~p ∨ q***
T	***T***	***T***	***T***
T	***F***	***F***	***F***
F	***T***	***T***	***T***
F	***F***	***T***	***T***

Which gives us an additional equivalency, $p \rightarrow q \equiv \sim q \rightarrow p \equiv \sim p \vee q$

And it also gives us a mechanism to ***negate*** the conditional: $\sim(p \rightarrow q) \equiv \sim(\sim p \vee q) \equiv \sim\sim p \wedge \sim q \equiv p \wedge \sim q$

In informal language (no symbols or variables), the negation of:

If I purchased a Shmuffin at a store, then I went to Sheetz

is:

~~If~~ I purchased a Shmuffin at a store, ~~then~~ ***AND*** I *did* ***NOT*** *go to Sheetz*

The negation is NOT an if-then!

Finally, we introduce two more definitions, and note that they are ***NOT*** equivalent to the original conditional.

The ***converse*** of $p \rightarrow q$ is $q \rightarrow p$. *If I went to Sheetz, then I purchased a Shmuffin at a store*

The ***inverse*** of $p \rightarrow q$ is $\sim p \rightarrow \sim q$. *If I did* ***NOT*** *purchase a Shmuffin at a store, then I did* ***NOT*** *go to Sheetz*