Use the <u>formal definitions</u> to show that:

$3n^2+4 = O(n^3)$

$2n^3-3 = \Omega(n^2)$

$3n^2+5n-20 = \Theta(n^2)$

$n^3-2 = o(n^4)$

$n^2-50 = \omega(n)$

- $\underbrace{3n^2+4}_{f(n)} = O(\underbrace{n^3}_{g(n)})$

$c, n_0 = ?$ $\quad 0 \le f(n) \le c \cdot g(n)$ $\quad$ for all $n \ge n_0$

$0 \le 3n^2+4 \le c \cdot n^3$

$0 \le 3n^2+4$ ✓

$3n^2+4 \le cn^3$

$cn^3 - 3n^2 - 4 \ge 0$

<u>solution 1</u>

let $c=1$

$n^3-3n^2-4 \ge 0$

$n \ge 3.355$

$\boxed{c=1,\ n_0 = 3.355}$

<u>solution 2</u>

let $c=7$

$7n^3-3n^2-4 \ge 0$

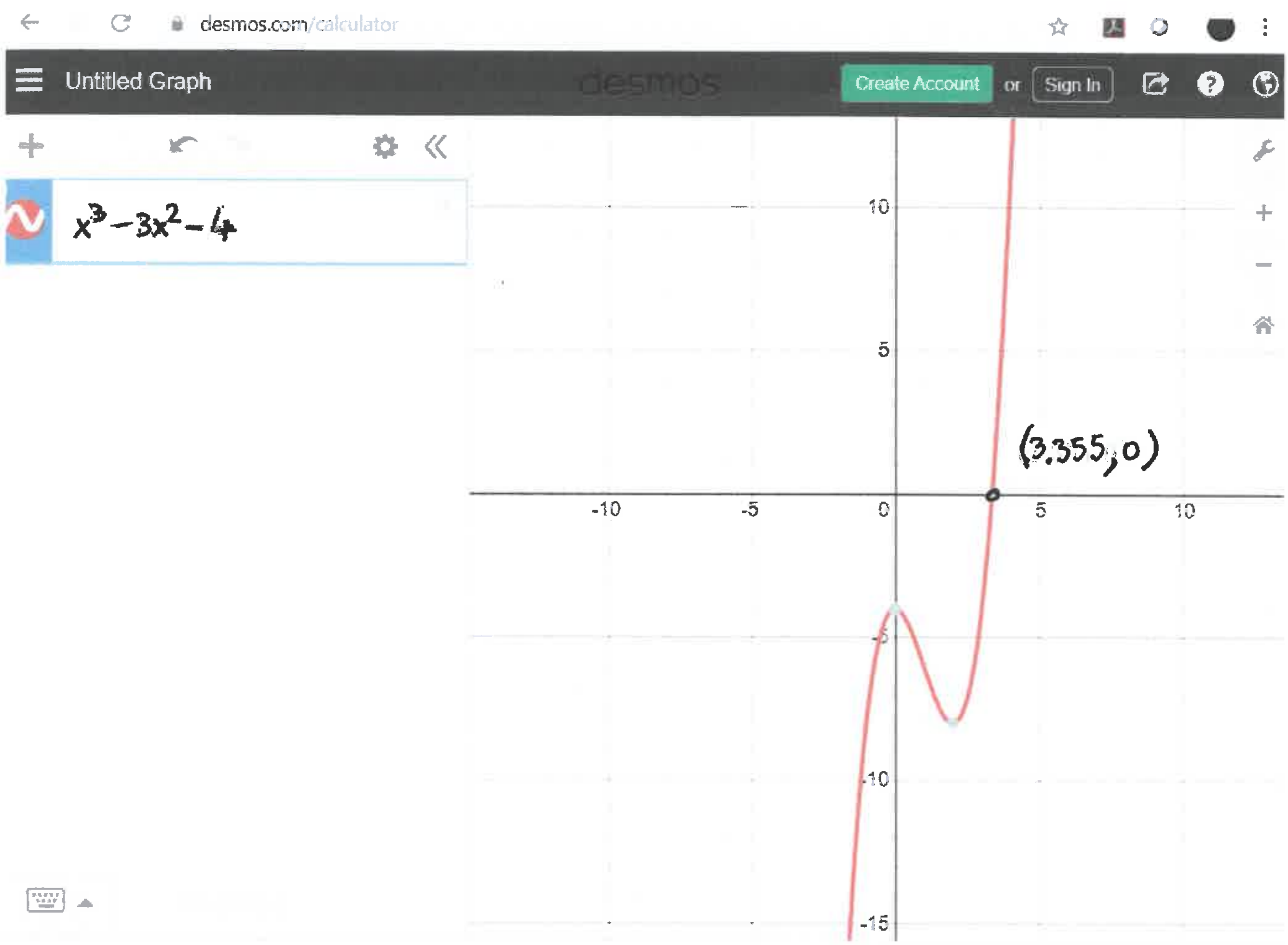
$(3n^3-3n^2)+(4n^3-4) \ge 0$

$\underbrace{3n^2(n-1)}_{+} + \underbrace{4(n^3-1)}_{+} \ge 0$

$n \ge 1$

$\boxed{c=7,\ n_0=1}$

desmos.com/calculator



• $\underset{f(n)}{2n^3-3} = \Omega(\underset{g(n)}{n^2})$

$c, n_0 = ?$ $\quad 0 \le c \cdot g(n) \le f(n)$ for all $n \geq n_0$

$0 \le c \cdot n^2 \le 2n^3 - 3$

✓ $\quad cn^2 \le 2n^3 - 3$

$2n^3 - cn^2 - 3 \geq 0$

let $c = 1$

$\underline{2n^3} - n^2 - 3 \geq 0$

$(n^3 - n^2) + (n^3 - 3) \geq 0$

$n^2(n-1) + (n^3 - 3) \geq 0$

$\underline{n \geq 1} \qquad \underline{n \geq \sqrt[3]{3}}$

$$\boxed{\begin{array}{l} c = 1 \\ n_0 = \sqrt[3]{3} \end{array}}$$

• $3n^2 + 5n - 20 = \Theta(n^2)$

$c_1, c_2, n_0 = ?$ $\quad 0 \le c_1 g(n) \le f(n) \le c_2 g(n)$ for all $n \geq n_0$

$0 \le c_1 n^2 \le 3n^2 + 5n - 20 \le c_2 n^2$

✓

$c_1 n^2 \le 3n^2 + 5n - 20$

$(3 - c_1)n^2 + 5n - 20 \geq 0$

$\underline{\text{let } c_1 = 3}$

$5n - 20 \geq 0$

$5n \geq 20$

$\underline{n \geq 4}$

$3n^2 + 5n - 20 \le c_2 n^2$

$(c_2 - 3)n^2 - 5n + 20 \geq 0$

$\underline{\text{let } c_2 = 4}$

$n^2 - 5n + 20 \geq 0$

$n(n-5) + 20 \geq 0$

$\underline{n \geq 5}$

$$\boxed{\begin{array}{l} c_1 = 3 \\ c_2 = 4 \\ n_0 = 5 \end{array}}$$

- $n^3 - 2 = o(n^4)$

$n_0 = ?$ $\quad 0 \le f(n) < c \cdot g(n)$ for all $n \geq n_0$

$$0 \le n^3 - 2 < c \cdot n^4$$

$$0 \le n^3 - 2$$
$$n^3 \geq 2$$
$$\underline{n \geq \sqrt[3]{2}}$$

$$n^3 - 2 < cn^4$$
$$cn^4 - n^3 + 2 > 0$$
$$cn^3\left(n - \frac{1}{c}\right) + 2 > 0$$

true for $\underline{n \geq \frac{1}{c}}$

$$\boxed{n_0 = \max\left(\sqrt[3]{2}, \frac{1}{c}\right)}$$

- $n^2 - 50 = \omega(n)$

$n_0 = ?$ $\quad 0 \le c \cdot g(n) < f(n)$ for all $n \geq n_0$

$$0 \le c \cdot n < n^2 - 50$$

✓

$$cn < n^2 - 50$$
$$n^2 - cn - 50 > 0$$
$$\left(\frac{n^2}{2} - cn\right) + \left(\frac{n^2}{2} - 50\right) > 0$$
$$\frac{n}{2}(\underset{+}{n - 2c}) + \frac{1}{2}(\underset{+}{n^2 - 100}) > 0$$

$\underline{n \geq 2c}$ $\quad$ $\underline{n > 10}$

$$\boxed{n_0 = \max(2c, 11)}$$

$$\boxed{n_0 = \max(2c, 10) + 1}$$