COT 6405
ANLYSIS OF ALGORITHMS

Growth of Functions

Computer & Electrical Engineering and Computer Science Department
Florida Atlantic University

Outline

- Rates of growth between polylogarithmic, polynomial, and exponential functions
- Using limits to determine asymptotic notation
- Common order of growth
- Summations

Rates of growth between polylogarithmic, polynomial, and exponential functions

- Any exponential function (base > 1) grows faster than any polynomial function

- Any positive polynomial function grows faster than any polylogarithmic function

Use limits to determine order of growth between functions

Limit value	Asymptotic Notation
$\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0$	f(n) = o(g(n))
$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \infty$	f(n) = ω(g(n))
$\lim_{n\to\infty} \frac{f(n)}{g(n)} < \infty$	f(n) = O(g(n))
$\lim_{n\to\infty} \frac{f(n)}{g(n)} > 0$	f(n) = Ω(g(n))
$0 < \lim_{n\to\infty} \frac{f(n)}{g(n)} < \infty$	f(n) = Θ(g(n))
$\lim_{n\to\infty} \frac{f(n)}{g(n)} = undefined$	cannot use

Common order of growth functions

Asymptotic Notation	Description
$\Theta(1)$	constant
$\Theta(\lg \lg n)$	log log
$\Theta(\lg n)$	log
$\Theta(n^c)$, $0 < c < 1$	sublinear
$\Theta(n)$	linear
$\Theta(n \lg n)$	nlogn
$\Theta(n^2)$	quadratic
$\Theta(n^3)$	cubic
$\Theta(n^k)$, $k \geq 1$	polynomial
$\Theta(c^n)$, $c > 1$	exponential
$\Theta(n!)$	factorial
$\Theta(n^n)$	

Summations

CLRS Appendix A

- Arithmetic Series

$$\sum_{k=1}^{n} k = 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2} = \theta(n^2)$$

- Sum of Squares

$$\sum_{k=1}^{n} k^2 = 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6} = \theta(n^3)$$

- Sum of Cubes

$$\sum_{k=1}^{n} k^3 = 1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4} = \theta(n^4)$$

Summations, cont.

- Geometric Series

$$\sum_{k=0}^{n} x^k = 1 + x + x^2 + x^3 + \cdots + x^n = \frac{x^{n+1} - 1}{x - 1}$$

If $|x| < 1$ and $n \to \infty$ then $\boxed{\sum_{k=0}^{\infty} x^k = \frac{1}{1 - x}}$ since $x^{n+1} \to 0$