COT 6405
ANLYSIS OF ALGORITHMS

Asymptotic Notations

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Asymptotic Notations

O - notation

Ω - notation

Θ - notation

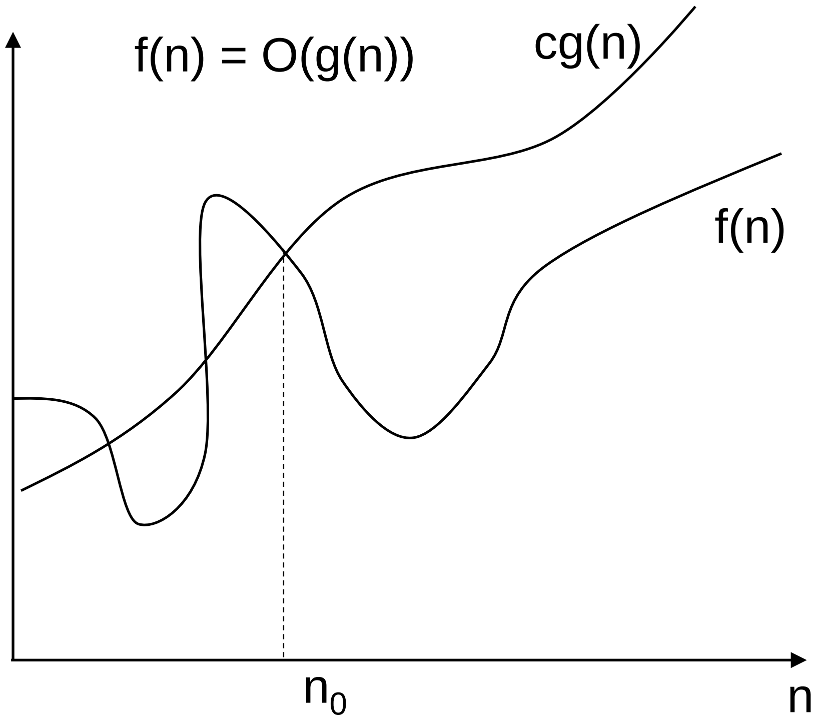
o - notation

ω - notation

Reference: *Introduction to Algorithms*, 3rd edition, by T. H. Cormen, C. E. Leiserson, R. L. Rivest, and C. Stein, The MIT Press, 2009 (chapter 3)

O - notation

$O(g(n)) = \{f(n)$: there exist positive constants c and n_0 s.t. $0 \le f(n) \le cg(n)$ for all $n \ge n_0\}$



- $g(n)$ is an asymptotic upper bound
- we write as $f(n) = O(g(n))$

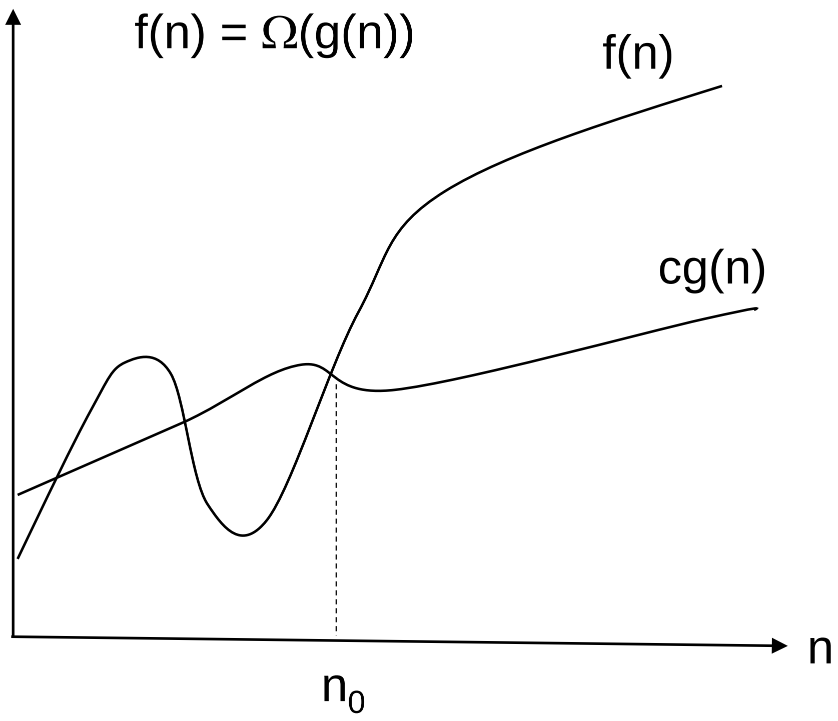
Examples:

$3n^2 + 5n – 100 = O(n^2)$

$3n^2 + 5n – 100 = O(n^4)$

Ω - notation

$\Omega(g(n))$ = {f(n): there exist positive constants c and n_0 s.t. $0 \le cg(n) \le f(n)$ for all $n \ge n_0$ }



- g(n) is an asymptotic lower bound
- we usually write $f(n) = \Omega(g(n))$

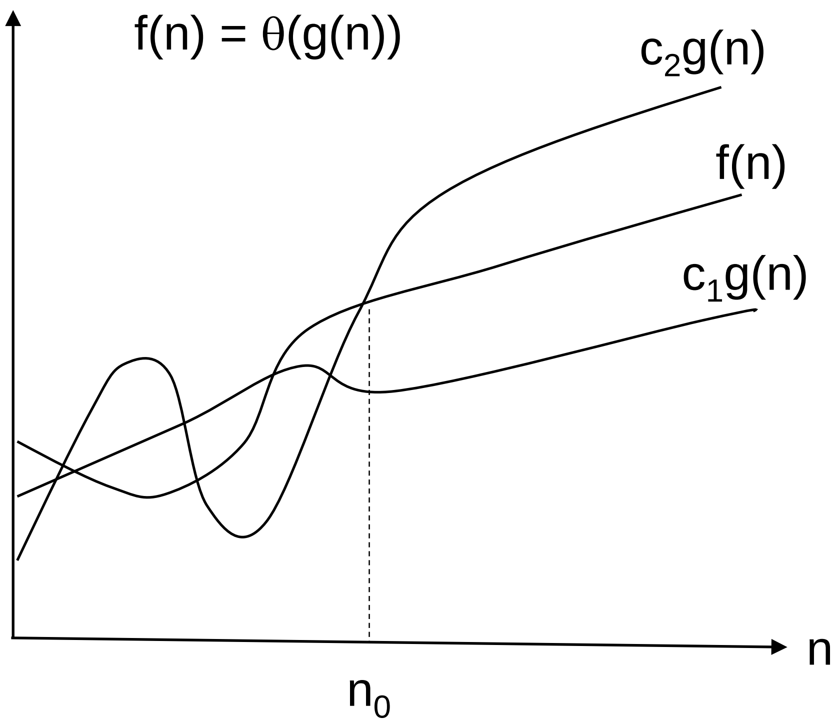
Examples:

$3n^2 + 5n - 100 = \Omega(n^2)$

$3n^2 + 5n - 100 = \Omega(n)$

Θ - notation

$\Theta(g(n)) = \{f(n)$: there exist positive constants c_1, c_2, and n_0 s.t.
$0 \le c_1 g(n) \le f(n) \le c_2 g(n)$
for all $n \ge n_0$ $\}$



- g(n) is an asymptotic tight bound
- we usually write $f(n) = \Theta(g(n))$

Theorem:

$f(n) = \Theta(g(n)$ iff $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$

Example: $3n^2 + 5n - 100 = \Theta(n^2)$

o - notation

- used to indicate an upper bound that is not asymptotically tight

$o(g(n)) = \{f(n)$: for any positive const $c > 0$, there exists a positive constant n_0 s.t. $0 \leq f(n) < cg(n)$ for all $n \geq n_0\}$

"quick" definition:

$f(n) = o(g(n))$ iff $\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0$

Example: $3n^2 - 100 = o(n^3)$

ω - notation

- used to indicate a lower bound that is not asymptotically tight

$\omega(g(n))$ = {f(n): for any positive const $c > 0$, there exists a positive constant n_0 s.t. $0 \le cg(n) < f(n)$ for all $n \ge n_0$}

"quick" definition:

$$f(n) = \omega(g(n)) \quad \text{iff} \quad \lim_{n\to\infty} \frac{f(n)}{g(n)} = \infty$$

Example: $3n^2 - 100 = \omega(n)$

Analogy between asymptotic notations and comparison of two real numbers

$f(n) = O(g(n))$	is like	$a \le b$
$f(n) = \Omega(g(n))$	is like	$a \ge b$
$f(n) = \theta(g(n))$	is like	$a = b$
$f(n) = o(g(n))$	is like	$a < b$
$f(n) = \omega(g(n))$	is like	$a > b$

Example - asymptotic notations for the expression $3n^2 + 10n - 500$

$3n^2+10n-500 = O(n^3)$

$3n^2+10n-500 = O(n^2)$

$\cancel{3n^2+10n-500 = O(n)}$

$\cancel{3n^2+10n-500 = \Omega(n^3)}$

$3n^2+10n-500 = \Omega(n^2)$

$3n^2+10n-500 = \Omega(n)$

$\cancel{3n^2+10n-500 = \Theta(n^3)}$

$3n^2+10n-500 = \Theta(n^2)$

$\cancel{3n^2+10n-500 = \Theta(n)}$

$3n^2+10n-500 = o(n^3)$

$\cancel{3n^2+10n-500 = o(n^2)}$

$\cancel{3n^2+10n-500 = o(n)}$

$\cancel{3n^2+10n-500 = \omega(n^3)}$

$\cancel{3n^2+10n-500 = \omega(n^2)}$

$3n^2+10n-500 = \omega(n)$