# Generating permutations

- **Decrease-by-one technique**
  - assume we have all (n-1)! permutations
  - insert n into each of the n positions of every permutation of n-1 elements

example

n = 2   12   21

n = 3   312   132   123   321   231   213

- **Bottom-up minimal-change algorithm**

each permutation can be obtained from its predecessor by exchanging 2 elements

- insert n into 12...(n-1) by moving right-to-left, then switch direction each time a new permutation {1, 2, .., n-1} is processed

example

n = 1   $\overleftarrow{1}$

n = 2   $\overleftarrow{12}$   $\overrightarrow{21}$

n = 3   $\overleftarrow{123}$   $\overrightarrow{132}$   $\overleftarrow{312}$   $\overrightarrow{321}$   $\overleftarrow{231}$   $\overrightarrow{213}$

n = 4   1234   1243   1423   4123   4132   - - - -

- **Johnson-Trotter algorithm**
  - each iteration:
    - find largest mobile element k
    - swap k with adjacent element using k's direction
    - reverse direction for element greater than k

example

n = 3

$\overleftarrow{1}\overleftarrow{2}\overleftarrow{3}$   $\overleftarrow{1}\overleftarrow{3}\overleftarrow{2}$   $\overleftarrow{3}\overleftarrow{1}\overleftarrow{2}$   $\overrightarrow{3}\overleftarrow{2}\overleftarrow{1}$   $\overleftarrow{2}\overrightarrow{3}\overleftarrow{1}$   $\overleftarrow{2}\overleftarrow{1}\overrightarrow{3}$