What we learned last time

1. *Intelligence is the computational part of the ability to achieve goals*
    - looking deeper: 1) its a continuum, 2) its an appearance, 3) it varies with observer and purpose

2. We will (probably) figure out how to make intelligent systems in our lifetimes; it will change everything

3. But prior to that it will probably change our careers
    - as companies gear up to take advantage of the economic opportunities

4. This course has a demanding workload

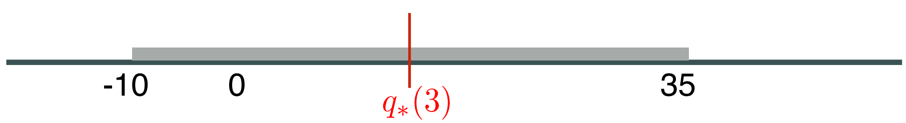
Multi-armed Bandits

Sutton and Barto, Chapter 2

The simplest
reinforcement learning
problem

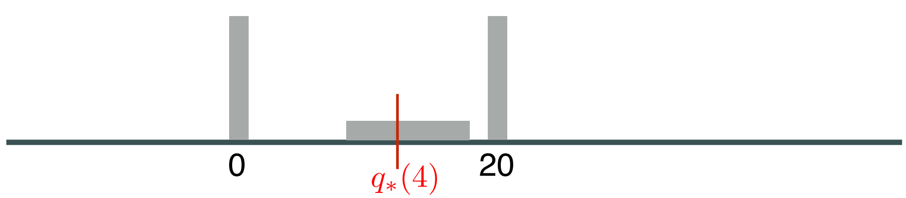
You are the algorithm! (bandit1)

- Action 1 — Reward is always 8
  - value of action 1 is $q_*(1) =$
- Action 2 — 88% chance of 0, 12% chance of 100!
  - value of action 2 is $q_*(2) = .88 \times 0 + .12 \times 100 =$
- Action 3 — Randomly between -10 and 35, equiprobable



$q_*(3) =$

- Action 4 — a third 0, a third 20, and a third from {8,9,…, 18}



$q_*(4) =$

The *k*-armed Bandit Problem

- On each of an infinite sequence of *time steps*, t=1, 2, 3, …, you choose an action A_t from k possibilities, and receive a real-valued *reward* R_t

- The reward depends only on the action taken; it is indentically, independently distributed (i.i.d.):

$$q_*(a) \doteq \mathbb{E}[R_t | A_t = a] , \quad \forall a \in \{1, \ldots, k\}$$ *true values*

- These true values are *unknown*. The distribution is unknown

- Nevertheless, you must maximize your total reward

- You must both try actions to learn their values (explore), and prefer those that appear best (exploit)

The Exploration/Exploitation Dilemma

- Suppose you form estimates

$$Q_t(a) \approx q_*(a), \quad \forall a$$

action-value estimates

- Define the *greedy action* at time *t* as

$$A_t^* \doteq \arg\max_a Q_t(a)$$

- If $A_t = A_t^*$ then you are *exploiting*
  If $A_t \neq A_t^*$ then you are *exploring*

- You can't do both, but you need to do both

- You can never stop exploring, but maybe you should explore less with time. Or maybe not.

Action-Value Methods

- Methods that learn action-value estimates and nothing else
- For example, estimate action values as *sample averages*:

$$Q_t(a) \doteq \frac{\text{sum of rewards when } a \text{ taken prior to } t}{\text{number of times } a \text{ taken prior to } t} = \frac{\sum_{i=1}^{t-1} R_i \cdot \mathbf{1}_{A_i=a}}{\sum_{i=1}^{t-1} \mathbf{1}_{A_i=a}}$$

- The sample-average estimates converge to the true values *If* the action is taken an infinite number of times

$$\lim_{N_t(a)\to\infty} Q_t(a) \;=\; q_*(a)$$

The number of times action a
has been taken by time t

ε-Greedy Action Selection

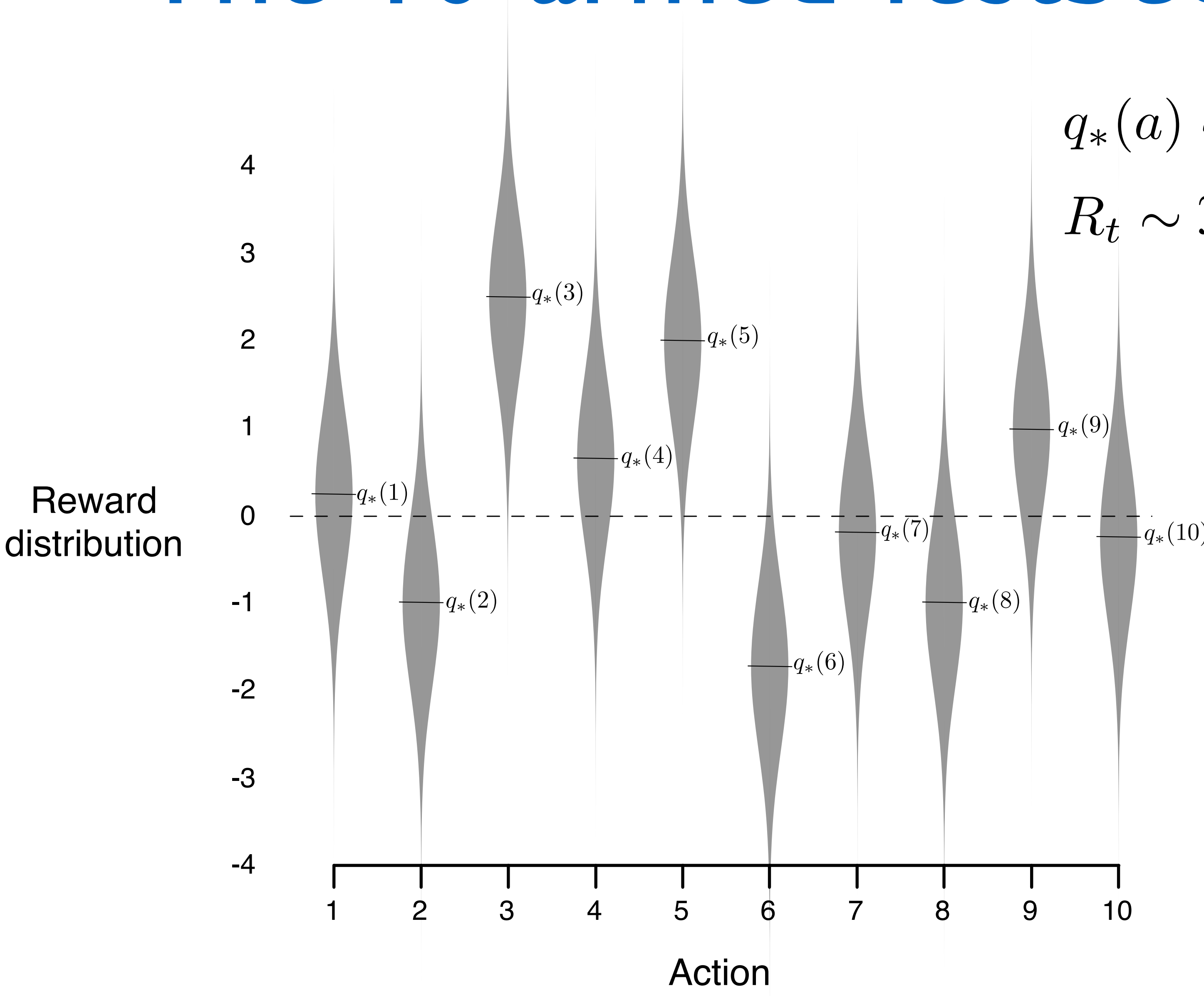
- In greedy action selection, you always exploit
- In ε-greedy, you are usually greedy, but with probability ε you instead pick an action at random (possibly the greedy action again)
- This is perhaps the simplest way to balance exploration and exploitation

Bandits: What we learned so far

1. *Multi-armed bandits* are a simplification of the real problem
    1. they have action and reward (a goal), but no input or sequentiality
2. A fundamental *exploitation-exploration tradeoff* arises in bandits
    1. *ε-greedy action selection* is the simplest way of trading off
3. *Learning action values* is a key part of solution methods

One Bandit Task from

The 10-armed Testbed



$$q_*(a) \sim \mathcal{N}(0,1)$$

$$R_t \sim \mathcal{N}(q_*(a),1)$$

Run for 1000 steps

Repeat the whole thing 2000 times with different bandit tasks

ε-Greedy Methods on the 10-Armed Testbed