MC125 Project Report

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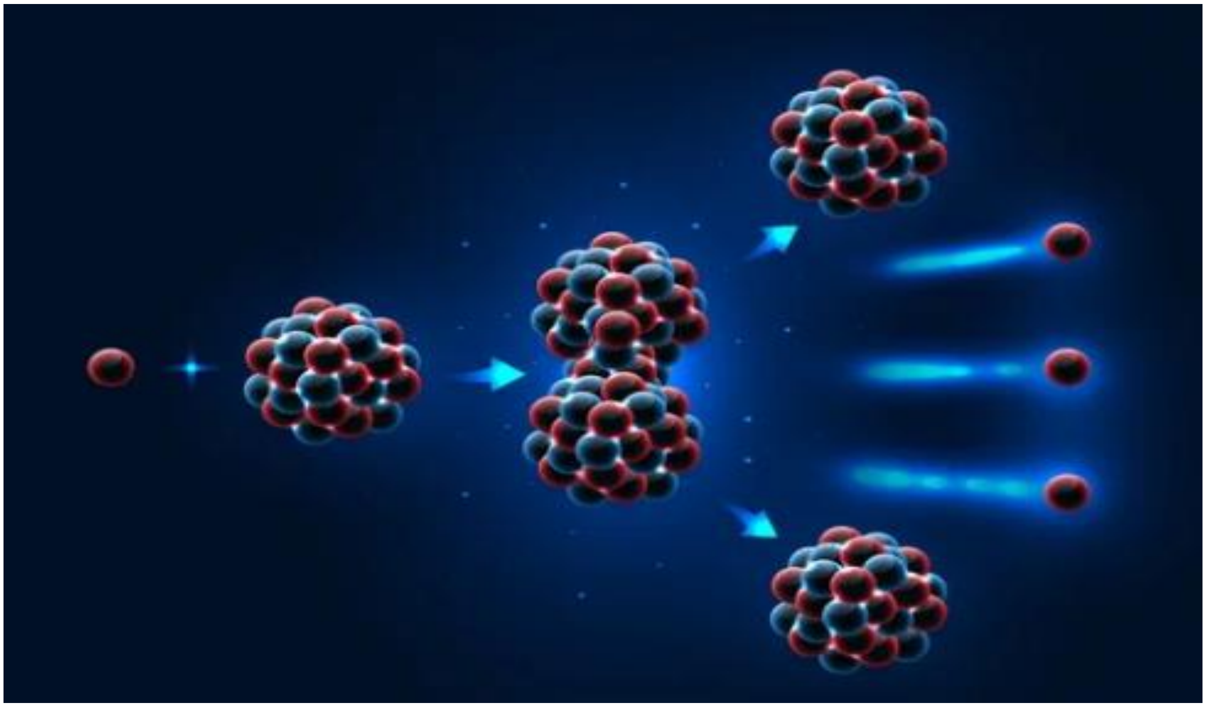
1.Nucleur Fission



1.1 Problem Statement

Nuclear Fission is a process involving radioactive material. Then how to know that how many nucleuses will undergo fission reaction in given amount of time? What is use of this process in real life?

1.2 Solution

First we want to explain what is nuclear fission. In world, some material is unstable so it will do some process to become a stable material. Radioactivity is such type of process. Radioactivity refers to the spontaneous emission of radiation from the unstable atomic nuclei of certain elements. Atom consist of a nucleus containing protons and neutrons, surrounded by electrons. In stable atoms the forces within the nucleus hold it together. However, in some atoms, the nucleus is unstable.

Radioactivity occurs naturally in the Earth’s crust, with elements such as uranium, thorium, and potassium being naturally radioactive.

- LOW OF RADIOACTIVE DECAY

Any Radioactive sample which undergo decay, it is found that the number of nuclei undergoing the decay is proportional to the total number of nuclei in the sample. If N is the number of nuclei in the sample, ΔN is a number of nuclei will undergo decay in time Δt then,

At time t-Δt,we have N nuclei,and after Δt time, we have N- ΔN nuclei.

$\therefore \frac{\Delta N}{\Delta t} \propto N$

$\therefore \frac{\Delta N}{\Delta t} = \lambda N$

where λ is called the radioactive decay constant or disintegration constant.

The change in the number of nuclei in the sample is dN = -ΔN in time Δt. Thus the rate of change of N is (in the limit Δt → 0)

$\therefore \lim_{\Delta t \to 0} \frac{\Delta N}{\Delta t} = \lim_{\Delta t \to 0} \lambda N$

$\therefore \frac{dN}{dt} = -\lambda N$

Here, ΔN is the number of nuclei that decay, so it is always positive. dN is the change in N, which may have other sign. Here it is negative, because out of original N nuclei, ΔN have decayed, leaving (N–ΔN) nuclei and $\frac{dN}{dt}$ is rate of change of total number of nucleus in time dt.

$\therefore \frac{dN}{N} = -\lambda dt$

Let's apply integration on this equation.

$\therefore \int_{N_0}^{N_t} \frac{dN}{N} = \int_{t_0}^{t} \lambda dt$

$\therefore [\ln N]_{N_0}^{N_t} = -\lambda [t]_{t_0}^{t}$

$\therefore lnN_t - lnN_0 = \lambda(t - t_0)$ ……………………………….(1.1)

Here N_0 is the number of radioactive nuclei in the sample at some arbitrary time t_0 and N_t is the number of radioactive nuclei at any subsequent time t. let's put $t_0 = 0$ in equation (1.1)

$\therefore lnN_t - lnN_0 = -\lambda(t - 0)$

$\therefore lnN_t - lnN_0 = -\lambda t$

$\therefore ln\frac{N_t}{N_0} = -\lambda t$

$\therefore \frac{N_t}{N_0} = e^{-\lambda t}$

$\therefore N_t = N_0 e^{-\lambda t}$……………………………………………(1.2)

Equation (1.2) is low of radioactive decay.

- **Half-Life**

Definition: The interval of time required for one-half of the atomic nuclei of a radioactive sample to decay.

Now, we want to find that what is half-life for any radioactive substance.

Let's take

t= $T_{\frac{1}{2}}$(half-life)

N_t=$\frac{N_0}{2}$(∵after time $T_{\frac{1}{2}}$ amount of nuclei is decreased by half.)

If we put this values in equation (1.2),

$\therefore N_t = N_0 e^{-\lambda t}$

$\therefore \frac{N_0}{2}=N_0 e^{-\lambda T_{\frac{1}{2}}}$

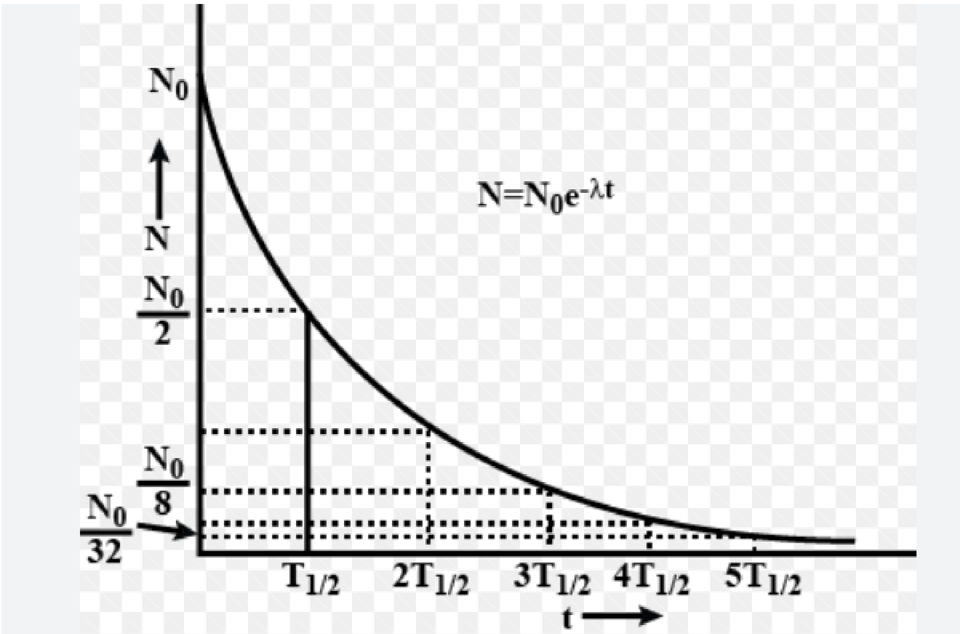
$\therefore \frac{1}{2}=e^{-\lambda T_{\frac{1}{2}}}$

$\therefore \ln\left(\frac{1}{2}\right) = \ln(e^{-\lambda T_{\frac{1}{2}}})$

$\therefore$-ln(2)=-λ$T_{\frac{1}{2}}$

$\therefore \lambda=\frac{\ln(2)}{T_{\frac{1}{2}}}$…………………………………..(1.3)

Equation (1.3) is relation between radioactive decay constant and half-life.



Exponential decay of a radioactive species. After a lapse of $T_{\frac{1}{2}}$, population of the given species drops by a factor of 2.

- **Radiocarbon Dating**

Radiocarbon dating (also referred to as carbon dating or carbon-14 dating) is a method for determining the age of an object containing organic material by using the properties of radiocarbon, a radioactive isotope(Isotopes are different forms of the same element that have a different number of neutrons) of carbon.

It is based on the fact that living organisms-like trees, plants, people, and animals-absorb carbon-14 into their tissue. When they die, the carbon-14 starts to change into other atoms over time by nuclear fission. Scientists can estimate how long the organism has been dead by counting the remaining carbon-14 atoms.

In this method we can use equation (1.2) and (1.3). If We know number of atoms in organism and amount of atoms when this organism dead, then we can calculate time which was taken by carbon-14 atoms to decay.

How to know λ (Radioactive decay constant)?

We can calculate it by this process.

First we calculate what is amount of nuclei at time t=0.let's assume it is N_0.After time $t_1 - t = t_1 - 0 = t_1$,again calculate amount of nuclei at time t_1,let's assume that it is N_{t_1}.

By equation (1.2)

$\therefore N_{t_1} = N_0 e^{-\lambda t_1}$

$\therefore \ln(N_{t_1}) = \ln(N_0 e^{-\lambda t_1})$

$\therefore \ln(N_{t_1}) = \ln(N_0) + \ln(e^{-\lambda t_1})$

$\therefore \ln(\frac{N_{t_1}}{N_0}) = -\lambda t_1$

$$\therefore \lambda = -\frac{\ln\left(\frac{N_{t_1}}{N_0}\right)}{t_1}$$

$$\therefore \lambda = \frac{\ln\left(\frac{N_0}{N_{t_1}}\right)}{t_1}$$

By this we can find value of λ.

$$\therefore \frac{\ln(2)}{T_{\frac{1}{2}}} = \frac{\ln\left(\frac{N_0}{N_{t_1}}\right)}{t_1} \qquad (\because \text{From equation (1.3)})$$

$$T_{\frac{1}{2}} = \frac{\ln(2)}{\ln\left(\frac{N_0}{N_{t_1}}\right)} t_1$$

Using this formula, scientists derived the half-life of carbon.

Limitations of Carbon-dating.

1.Limited age range: Carbon dating is most effective for dating materials that are less than 50,000 years old. This is because the half-life of carbon-14 is approximately 5,730 years. After several half-lives, the amount of carbon-14 remaining in a sample becomes too small to accurately measure.

2.Contamination: Contamination of the sample with modern carbon can affect the accuracy of carbon dating. For example, if a sample is contaminated with newer carbon-containing materials, it can skew the dating results, making the sample appear younger than it actually is.

3.Incomplete mixing of carbon-14: The carbon-14 in the atmosphere is not evenly distributed, and this can lead to variations in carbon dating results. It is assumed that the ratio of carbon-14 to carbon-12 in the atmosphere has remained relatively constant over time, but local fluctuations or anomalies can occur.

4.Preservation of organic material: Carbon dating requires the presence of organic material that contains carbon. This means that materials such as rocks, metals, and pottery cannot be directly dated using carbon dating. Additionally, the preservation of organic material is necessary, as decay and contamination can affect the accuracy of the dating method.

2.Filling the Container

2.1 Problem Statement

Filling the container in a tank of water, water is filled in certain way. For the 1 st second, it is increase by 1 unit of volume then in 2 nd second it is decrease by 1/2 unit of volume, in 3 rd second it is increase by 1/3 unit of volume and so on. So from n-1 second to n seconds it is changing by $\frac{(-1)^{n+1}}{n}$ unit of volume. Find whether we need an infinite tank to filled the water or not. Water will fill container till time approaches to infinite.

2.2 Solution

In first Second container has 1 unit volume water.

In next second container has $1-\frac{1}{2}$ unit volume water. (Because water reduced by half…)

In next second container has $1-\frac{1}{2}+\frac{1}{3}$ unit volume water. (Because water increased by one third to the previous volume…)

In n^{th} Second container has $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots+\frac{(-1)^{n+1}}{n}$ unit volume.

We want to find that after infinite time is container has infinite unit volume if not then what amount of water will container have?

First of all we have to determine convergence of infinite series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots+\frac{(-1)^{n+1}}{n}$.

First we determine that series S_n is convergent or divergent.

$S_n = 1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}+\cdots+\frac{(-1)^{n+1}}{n}$

$S_{2k} = \left(1-\frac{1}{2}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\left(\frac{1}{5}-\frac{1}{6}\right)+\cdots+\left(\frac{1}{2k-1}-\frac{1}{2k}\right)>0$

$S_{2k+1} = 1-\left(\frac{1}{2}-\frac{1}{3}\right)-\left(\frac{1}{4}-\frac{1}{5}\right)-\cdots-\left(\frac{1}{2k}-\frac{1}{2k+1}\right)<1$

$S_1 > S_3 > S_5 > S_7 > \ldots > 0$

$S_2 < S_4 < S_6 < S_8 < \ldots < 1$

S_{2k} is monotonic increasing sequence and S_{2k+1} is monotonic decreasing sequence.

S_{2k} and S_{2k+1} are monotone and bounded So we can say by Monotone Convergent theorem that S_{2k} and S_{2k+1} converges.

$S_{2k+1} = S_{2k} + \frac{1}{2k+1}$

$S_{2k+1} - S_{2k} = \frac{1}{2k+1}$

$\lim(S_{2k+1} - S_{2k}) = \lim(\frac{1}{2k+1})$

$\lim(S_{2k+1}) = \lim(S_{2k}) = S$ (S =any finite number)

So…

$\lim(S_n) = S$ (Because n can either odd or even)

$\lim(S_n) = \lim(S_{n+1})$ when $n \to \infty$

So, Our sequence S_n is convergent.

Let's find S.

$S_n = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots + \frac{(-1)^{n+1}}{n}$.

$S_n = \sum_{k=1}^{n} \frac{(-1)^{n+1}}{n}$

$\lim_{n\to\infty} S_n = \lim_{n\to\infty} S_{2n}$

$S_{2n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots + \frac{1}{2n-1} - \frac{1}{2n}$.

$S_{2n} = \left(1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \cdots + \frac{1}{2n-1}\right) - \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \cdots \frac{1}{2n}\right)$

We replace $-\frac{1}{2}$ by $-1 + \frac{1}{2}$, $-\frac{1}{4}$ by $-\frac{1}{2} + \frac{1}{4}$,… , Similarly $-\frac{1}{2n}$ by $-\frac{1}{n} + \frac{1}{2n}$.

Now…

$S_{2n} = \left(1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \cdots + \frac{1}{2n-1}\right) + \left(\left(-1 + \frac{1}{2}\right) + \left(-\frac{1}{2} + \frac{1}{4}\right) + \left(-\frac{1}{3} + \frac{1}{6}\right) + \cdots .. + \left(-\frac{1}{n} + \frac{1}{2n}\right)\right)$

$S_{2n} = \frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \cdots + \frac{1}{2n-1} + \frac{1}{2n}$.

$S_{2n} = \frac{1}{n}\left(\frac{1}{1+\frac{1}{n}} + \frac{1}{1+\frac{2}{n}} + \frac{1}{1+\frac{3}{n}} + \cdots + \frac{1}{2}\right)$

$\lim_{n\to\infty} S_{2n} = \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \frac{1}{1+\frac{k}{n}}$

As $n \to \infty$ $replace\ \frac{1}{n}\ by\ dx\ And\ \frac{k}{n}\ by$ x.

$$When\ k = 1\ then\ \frac{1}{n} = 0\ and\ k = n\ then\ \frac{k}{n} = 1.$$

$\lim_{n\to\infty} S_{2n} = \int_0^1 \frac{1}{1+x} dx$

$\lim_{n\to\infty} S_{2n} = \ln(1+1) - \ln(1+0)$

$\lim_{n\to\infty} S_{2n} = ln2 - ln1$

$\lim_{n\to\infty} S_{2n} = ln2 - 0$

$\lim_{n\to\infty} S_{2n} = ln2$

As $n \to \infty$ $S_n = S_{2n}$.

So, $\lim_{n\to\infty} S_n = ln2$.

When time approaches to infinity volume of water tends to ln2, So we do not need a container with infinite volume, We need a container with ln2 unit volume.

3.Destoy a Planet

3.1 Problem Statement

Gravitation force between any two particles is given by this equation.

$F = \frac{Gm_1m_2}{R^2}$

G=constant of gravitation

R = distance between 2 particles

m_1=mass of first particle

m_2=mass of second particle.

Now we want to destroy this planet. Then what amount of energy is required to destroy this planet?

3.2 Solution

First, density of planet is μ (mass per unit volume)

$\mu = \frac{M}{\frac{4}{3}\pi R^3}$.

Required energy to destroy a planet is equal to self-energy of planet.

Definition: Self-energy is energy required to take particle by particle from infinity and merge them.

If we want to destroy a planet, then we need to take all particles far away from each other, so we require same amount of work to do this. We calculate what amount of energy require to create a planet, it is same as energy require to destroy a planet.

Now we calculate self-energy of planet. To do this we consider planet as solid sphere of radius R and mass M.

Consider small sphere of radius r.

Now we take particles from infinity to make a layer of dr thickness.

This work dE is self-energy of this small shell, we are merging a shell with sphere.

For sphere we need to calculate the distance from its center. We are taking particles from infinity to distance r from the center of sphere.

$dE = \int_{\infty}^{r} F dr$ (F=gravitational force between shell particles and sphere)

$= \int_{\infty}^{r} \frac{\mathrm{Gmdm}}{\mathrm{r}^2} dr$ m=mass of sphere of radius r.

dm=mass of shell of thickness dr.

$= -\left[\frac{\mathrm{Gmdm}}{r}\right]_{\infty}^{r}$

$= \left[\frac{\mathrm{Gmdm}}{r}\right]_{r}^{\infty}$

$= \frac{Gmdm}{\infty} - \frac{Gmdm}{r}$

$dE = 0 - \frac{\mathrm{Gmdm}}{\mathrm{r}}$

$dE = -\frac{\mathrm{Gmdm}}{\mathrm{r}}$,but this energy is released when we take particle near to sphere, we want to calculate what amount of energy we should give to destroy a planet.

So, $dE = \frac{\mathrm{Gmdm}}{\mathrm{r}}$

Volume of shell=(Area)×(thickness)

Let's use density to make this equation of only one variable.

$Dm = \mu(4\pi r^2 dr)$

$m= \mu(\frac{4}{3}\pi r^3)$

So, Total Self energy of planet is…

$E=\int_0^R dE$

At starting, We do not have any sphere so,radius of sphere =0,and Energy is also 0,because no mass is there.

At last we have a sphere of radius R,and Energy is E.

$\int_0^E dE= \int_0^R \frac{G\left(\mu\left(\frac{4}{3}\pi r^3\right)\mu(4\pi r^2 dr)\right)}{r}$

$E-0 = \int_0^R \frac{16G\mu^2\pi^2 r^4}{3}dr$

$E=\frac{16G\mu^2\pi^2}{3\times 5}R^5$

Value of $\mu=\frac{M}{\frac{4}{3}\pi R^3}$

Putting value of μ …

$E=\frac{3}{5}\frac{GM^2}{R}$

So,we need $=\frac{3}{5}\frac{GM^2}{R}$ amount of energy to destroy a planet of mass M.

4.Position of Insects

4.1 Problem Statement

Consider x-y plane. 3 insects are at origin And they follow respective paths y=x, $y=\sin^{-1}x$, $y=\frac{x}{\sqrt{1-x^2}}$

Determine which insect will be further away from the x-axis at each point. here $x\in(0,1)$.

4.2 Solution

In this problem, 3 insects are follow respective path y=x, y=$\sin^{-1}x$, y=$\frac{x}{\sqrt{1-x^2}}$.We want to find which insect will be further away from the x-axis at each point.

To solve this problem, we need to analyze the paths of the three insects and determine their distances from the X-axis at each point. We can do this by calculating the Y-coordinate of each insect's path for different values of X and then comparing the distances.

Let's assume name of insect 1, insect 2 and insect 3 is A, B, C.

A is on curve y=x.

B is on curve y=$\sin^{-1}x$.

C is on curve y=$\frac{x}{\sqrt{1-x^2}}$.

The insect with the largest Y coordinate will be the insect furthest from the X axis.

Let's take a function f.

f(x)= $\sin^{-1}x$ $x\in(0,1)$.

$\sin^{-1}x$ is continuous on [0,1] and differentiable on (0,1).so we can apply mean value theorem.

……

Mean value theorem:

The Mean value theorem states that for any function f(x) whose graph passes through two given points (a, f(a)), (b, f(b)), there is at least one point (c, f(c)) on the curve where the tangent is parallel to the secant passing through the two given points. The mean value theorem is defined here in calculus for a function f(x): [a, b] → R, such that it is continuous and differentiable across an interval.

The function f(x) is continuous over the interval [a, b].

The function f(x) is differentiable over the interval (a, b).

There exists a point c in (a, b) such that f'(c) = $\frac{f(b)-f(a)}{(b-a)}$.

Proof of Mean value theorem:

Let's take a function g(x), which is define as

g(x)=f(x)-f(a)- $\frac{f(b)-f(a)}{(b-a)}$(x-a)……………(1).

Here, g(x) is continuous and differentiable across an interval, because it is sum of two continuous and differentiable function f(x)-f(a) and $-\frac{f(b)-f(a)}{(b-a)}$(x-a) and by equation (1) we can calculate g(a) and g(b),g(a)=g(b)=0.

By Rolle's theorem, if function is continuous and differentiable in interval [a,b],and value of function at point a and b is 0,then there exists at least one point c in (a,b) ,such that derivative of function at c is 0.

So, we can apply Rolle's Theorem on function g.

Let's differentiate g.

$g'(x)=f'(x)-\frac{f(b)-f(a)}{(b-a)}$

if c is a point where g'(x) is 0, then

$\therefore g'(c)=f'(c)-\frac{f(b)-f(a)}{(b-a)}$

$\therefore 0= f'(c)-\frac{f(b)-f(a)}{(b-a)}$

$\therefore f'(c)=\frac{f(b)-f(a)}{(b-a)}$

So, we can say that f(x) is continuous over the interval [a, b] and differentiable over the interval (a, b) then,

There exists a point c in (a, b) such that $f'(c) = \frac{f(b)-f(a)}{(b-a)}$

……

In this problem we have an interval [0,1].

Let's take interval [0, x]. (Here, $0< x \leq 1$)

$\sin^{-1}x$ is continuous on [0,1] and differentiable on (0,1) so it is also continuous on [0, x] and differentiable on (0, x). Therefore, by mean value theorem there exists a c € [0, x].

$$\therefore f'(c) = \frac{f(x)-f(0)}{x-0}$$

$$\therefore f'(c) = \frac{f(x)}{x}$$

$$\therefore f'(c) = \frac{\sin^{-1}x}{x}$$

$$\therefore \frac{1}{\sqrt{1-c^2}}=\frac{\sin^{-1}x}{x} \text{……………………………}(2.1)$$

And

$0 < c < x$……………………………………(2.2) $(\because c \in (0, x))$

$\therefore 0 < c < 1$

$\therefore 0 < c^2 < 1$

$\therefore -1 < -c^2 < 0$

$\therefore 0 < 1 - c^2 < 1$

$\therefore 0 < 1 - c^2 < 1$

$\therefore 0 < \sqrt{1 - c^2} < 1$

$\therefore \frac{1}{\sqrt{1 - c^2}} > 1$

So, $\frac{\sin^{-1} x}{x} > 1$………………………………(2.3) (From equation (2.1), $\frac{1}{\sqrt{1-c^2}} = \frac{\sin^{-1} x}{x}$)

From equation (2.2)

$c < x$

$\therefore c^2 < x^2$

$\therefore -x^2 < -c^2$

$\therefore 1 - x^2 < 1 - c^2$

$\therefore \frac{1}{\sqrt{1 - c^2}} < \frac{1}{\sqrt{1 - x^2}}$

$\frac{\sin^{-1} x}{x} < \frac{1}{\sqrt{1-x^2}}$……………(2.4) (From equation (2.1), $\frac{1}{\sqrt{1-c^2}} = \frac{\sin^{-1} x}{x}$)

From equation (2.3) and (2.4)

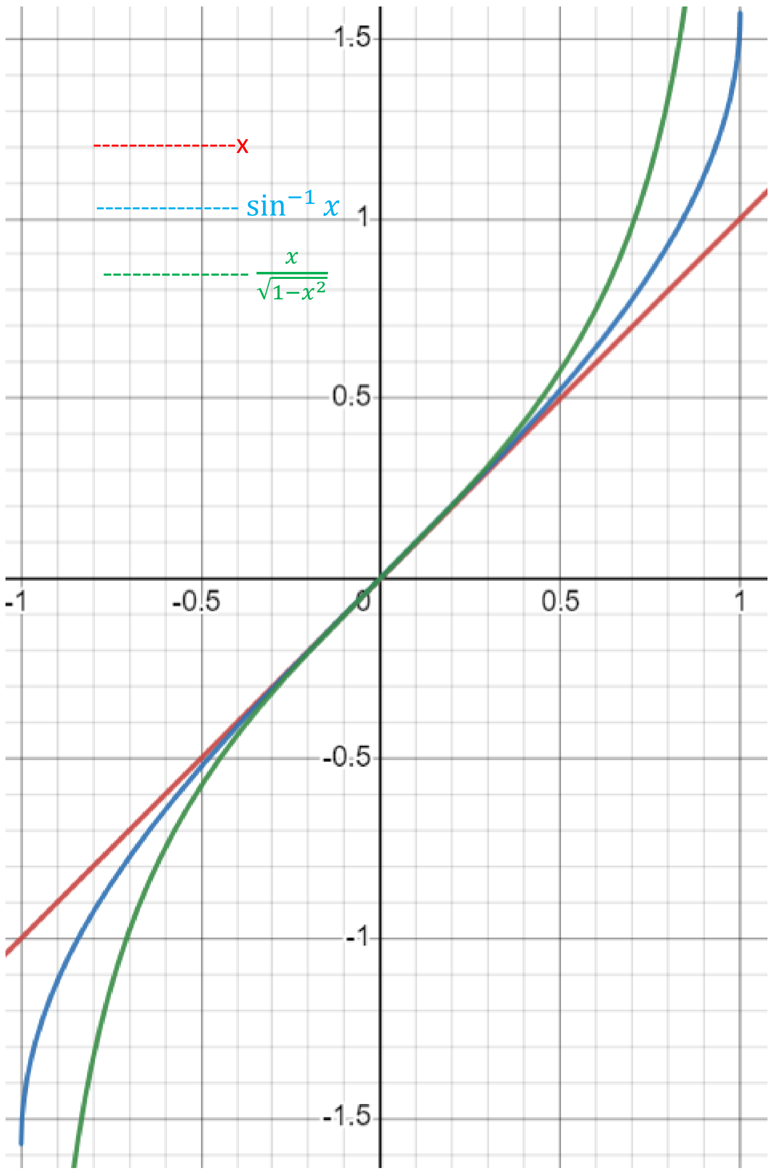
$1 < \frac{\sin^{-1} x}{x} < \frac{1}{\sqrt{1 - x^2}}$

$x < \sin^{-1} x < \frac{x}{\sqrt{1 - x^2}}$

A’s Y-coordinate <B’s Y-coordinate <C’s Y-coordinate.

So, Y-coordinate of insect C is the largest.

By this we can say insect C is further away from the x-axis at each point. (x$\in (0,1)$)



5.

5.1 Problem Statement

f(x)= $\lim\limits_{n\to\infty} e^{x\tan\left(\frac{1}{n}\right)\ln\left(\frac{1}{n}\right)}$ and $\int \frac{f(x)}{\left(\sqrt[3]{sinx}\right)^{11}\sqrt[3]{cosx}}dx$=g(x)+c. a. Discuss continuity and differentiability of g(x).

5.2 Problem Statement

First of all, we find f(x).

Replace n by $\frac{1}{y}$.

Now…

$$f(x)=\lim_{y\to 0} e^{x\,\tan y\,\ln y}$$

$$f(x)=\lim_{y\to 0} e^{x\,\frac{\ln y}{\cot y}}$$

$$f(x)=\lim_{y\to 0} e^{\frac{-x\sin y\sin y}{y}} \qquad (\because \text{By Applying L-Hospital low…})$$

$$f(x)=\lim_{y\to 0} e^{-x\,\sin y} \qquad (\because \text{Using Standard Limit…}\lim_{y\to 0}\frac{\sin y}{y}=0)$$

$$f(x)=e^{0}$$

$$f(x)=1$$

$$g(x)=\int \frac{1}{\left(\sqrt[3]{\sin x}\right)^{11}\sqrt[3]{\cos x}}dx$$

$$g(x)=\int \frac{1}{\left(\sqrt[3]{\sin x}\right)^{12}\sqrt[3]{\cot x}}dx$$

$$g(x)=\int \frac{\operatorname{cosec}^4 x}{\sqrt[3]{\cot x}}dx \quad …(1)$$

Now we take cot x=m…

So $\frac{dm}{dx}=-\operatorname{cosec}^2 x$

$dx=-\frac{1}{\operatorname{cosec}^2 x}dm;$

Put value of m and dx in eq. (1) ...

$g(x)=\int \frac{-\mathrm{cosec}^2 x}{\sqrt[3]{m}}\,dm$

$g(x)=\int \frac{-(m^2+1)}{\sqrt[3]{m}}\,dm$ ($\because$ using $\mathrm{cosec}^2 x = \cot^2 x + 1$ and putting $\cot x = m$ ...)

$g(x)=\int -\left(m^{\frac{5}{3}} + m^{-\frac{1}{3}}\right)dm$

$g(x)=-\frac{3}{8}m^{\frac{8}{3}} - \frac{3}{2}m^{\frac{2}{3}} + c$ ($\because$c is any arbitrary constant...)

$g(x)=-\frac{3}{8}(\cot x)^{\frac{8}{3}} - \frac{3}{2}(\cot x)^{\frac{2}{3}} + c$

$g(x)= -\frac{3}{8}(f(x))^{\frac{8}{3}} - \frac{3}{2}((f(x))^{\frac{2}{3}} + c$ ($\because$f(x)=cotx)

Continuity of g(x)

Here, f(x) is polynomial of cotx, so first we need to check where cotx is discontinuous.

$cotx = \frac{cosx}{sinx}$

cosx and sinx are continuous ∀x in R, so cot x is also continuous ∀x € R except x=nπ (n€Z) because sinx becomes 0 at any integral multiple of π . (R=set of real numbers, N = set of integers numbers)

cotx is continuous ∀x € R except x=nπ,So,g(x) is also continuous ∀x € R except x=nπ.

Differentiability of g(x)

g(x) is discontinuous at x=nπ, so it is not differentiable at x=nπ.

g'(a)=$\lim\limits_{x\to a} \frac{g(x)-g(a)}{x-a}$ (x,c ≠ nπ cotx is defined at that point)

=$\lim\limits_{x\to a} \frac{-\frac{3}{8}(\cot x)^{\frac{8}{3}}-\frac{3}{2}(\cot x)^{\frac{2}{3}}+c+\frac{3}{8}(\cot a)^{\frac{8}{3}}+\frac{3}{2}(\cot a)^{\frac{2}{3}}-c}{x-a}$

Here, we can apply L'Hospital's rule,

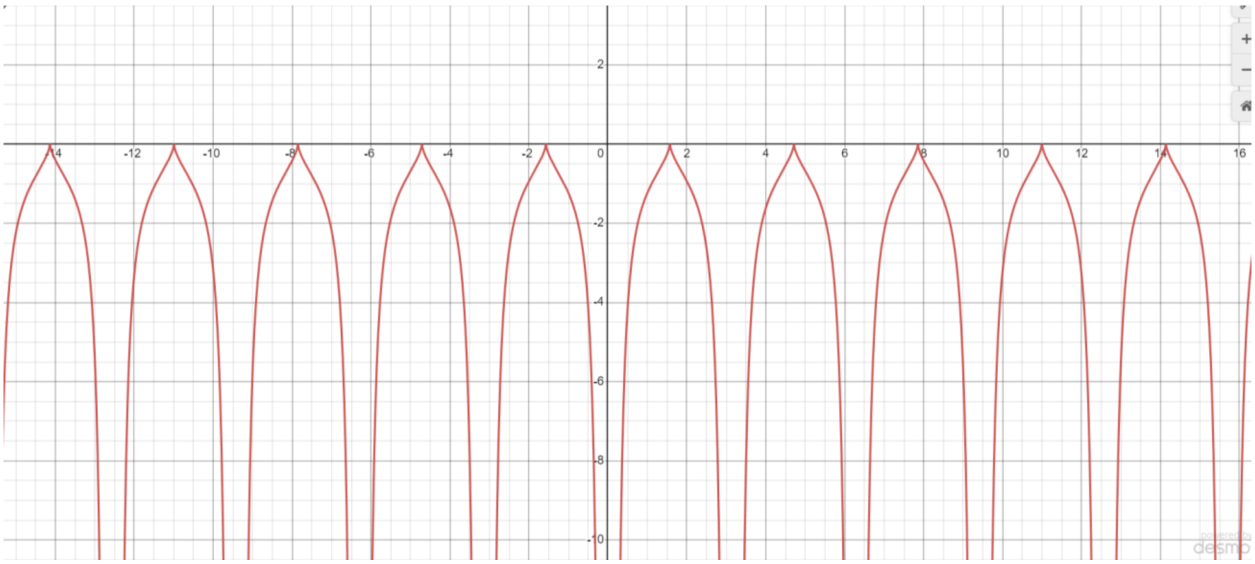
=$\lim\limits_{x\to a} \frac{(\cot x)^{\frac{5}{3}}(\mathrm{cosec}\,x)^{2} \quad +(\cot x)^{\frac{-1}{3}}(\mathrm{cosec}\,x)^{2}}{1}$

=$\lim\limits_{x\to a} \frac{(\cot x)^2-1}{(\cot x)^{\frac{1}{3}}(\sin x)^2}$

This limit does not exist when denominator becomes 0. $(\cot x)^{\frac{1}{3}}(\sin x)^2$ is zero at x= nπ and x=(2n+1)$\frac{\pi}{2}$ ($\because$cot((2n+1)$\frac{\pi}{2}$)=0, sin(n π)=0)

So, this function is differentiable ∀x € R except x=nπ, $(2n + 1)\frac{\pi}{2}$.

This is graph of g(x) for c=0.



Contribution of each member

1.Darshil Radadiya:

He worked on problem 2,3 and 5.

2.Krushi Sutariya

She worked on problem 1 and 4.