
MAGIC SQUARE STAGE 2

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In stage 1 I only mentioned the Ramanujan magic square but there exist many types of a magic squares. In this project, I want to explain the types of magic squares and how to construct some of them, and also the properties of these magic squares. Let's start construction of magic square.

• GENERAL WAY TO CREATE MAGIC SQUARE

If we want to construct a magic square of any size, first we have to distribute them using some conditions.

1. Odd order magic square.

For odd-order magic square we use an algorithm published by French diplomat de la Loubère in his book.

In this method if we want to make a magic square of n order, then we use 1 to n^2 .

Magic sum = $n(n^2 + 1)/2$

• Steps of Algorithm

Let's take $n \times n$ order magic square. i denote row and j denotes column.

1. Put 1 in middle of first row.

2. Then go up and left by one position.

3. If you fall out of square then go to opposite row, as example if you at first row, then go to last row, if you at first column then go to last column.

4. If a cell is occupied, then move down by one step and continue.

example : for 3×3 The numbers which we will use in construction are 1 to n^2 . $n=3$ So, numbers are 1,2,3,4,5,6,7,8,9.

Magic sum = $3(9+1)/2 = 3(10)/2 = 15$

1.

	1	

2.

	1	
2		

3.

	1	
		3
2		

4.

	1	
		3
2		4

5.

	1	
	5	3
2		4

6.

6	1	
	5	3
2		4

7.

6	1	
7	5	3
2		4

8.

6	1	8
7	5	3
2		4

9.

6	1	8
7	5	3
2	9	4

We can create any order magic square by using this algorithm. Here, we can not use this algorithm for even order magic squares because, in an even number of rows or columns, we have two middle values $n/2$ and $(n-1)/2$. So we don't know in which row we should put 1. So this method is not useful for even ordering magic square.

2. Doubly even order magic square.

(Doubly Even number means a number which is divisible by 4.)

•Method:

1. First write numbers in square in increasing order starting from first row and first column.
2. Take another table and write numbers in decreasing order starting from the last row and last column.
3. Take the third table in which write 0,1.

Strategy to write 0 and 1:

In the first and last row write 1001 for $\frac{n}{4}$ times.

After doing this write 0110 in two rows and write 1001 in consecutive another two rows.

Repeat this step until you reached at last row.

4. Take another blank square. In which cell of the third magic square has 0, fill those cells with the first square's same cell. In which cell of the third magic square has 1, fill those cells with the second square's same cell.
Example:

1.

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48
49	50	51	52	53	54	55	56
57	58	59	60	61	62	63	64

2.

64	63	62	61	60	59	58	57
56	55	54	53	52	51	50	49
48	47	46	45	44	43	42	41
40	39	38	37	36	35	34	33
32	31	30	29	28	27	26	25
24	23	22	21	20	19	18	17
16	15	14	13	12	11	10	9
8	7	6	5	4	3	2	1

3.

1	0	0	1	1	0	0	1
0	1	1	0	0	1	1	0
0	1	1	0	0	1	1	0
1	0	0	1	1	0	0	1
1	0	0	1	1	0	0	1
0	1	1	0	0	1	1	0
0	1	1	0	0	1	1	0
1	0	0	1	1	0	0	1

4.

1	63	62	4	5	0	59	58
56	10	11	53	52	14	15	49
48	18	19	45	44	22	23	41
25	39	38	28	29	35	34	32
33	31	30	36	37	27	26	40
24	42	43	21	20	46	47	17
16	50	51	13	12	54	55	9
57	7	6	60	61	3	2	64

3. Even order magic square which order is divisible by 2 not by 4 square.

Let's take the order of magic square is $N \times N = 2 \times n$ n is an odd number.

• Method

(a) first divide order by 2.

(b) Let's take $N = 2 \times n$, Here n is an odd number.

(c) Make for a Magic square like...

1. M1 is the first magic square in which the number starts from 1 to $\frac{N^2}{4}$.
2. M2 is second magic square in which number starts from $\frac{N^2}{4} + 1$ to $\frac{2N^2}{4}$.
3. M3 is first magic square in which number starts from $\frac{2N^2}{4} + 1$ to $\frac{3N^2}{4}$.
4. M4 is first magic square in which number starts from $\frac{3N^2}{4} + 1$ to N^2 .

(d) Arrange them like the given below.

M1	M3
M4	M2

But some changes are also required in M1 and M4. So exchange elements of M1's first and second column alternately with M4's first and second column.

Example

We want to make a magic square of order 6. (6 is not divisible by 4, and it is an even number.)

M1

6	1	8
7	5	3
2	9	4

M2

15	10	17
16	14	12
11	18	13

M3

24	19	26
25	23	21
20	27	22

M4

33	28	35
34	32	30
29	36	31

After doing this combine all magic square of 3×3 to make 6×6 .

6	1	8	24	19	26
7	5	3	25	23	21
2	9	4	20	27	22
33	28	35	15	10	17
34	32	30	16	14	12
29	36	31	11	18	13

Exchange elements of the first-row first column, second-row second column, and third-row first column of M1 and M4. (Elements of M1's first and second column alternately with M4's first and second column alternately M4)

33	1	8	24	19	26
7	32	3	25	23	21
29	9	4	20	27	22
6	28	35	15	10	17
34	5	30	16	14	12
2	36	31	11	18	13

•Other tricks for some special magic square

1.Ramanujan Magic Square.

Ramanujan Magic square is special type of 4×4 . Ramanujan write first this type of magic square.

This works for any 4 number which you want to add,they can be same or different.

22	12	18	87
88	17	9	25
10	24	89	16
19	86	23	11

Properties

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In this squares when we add same color cells then their some is constant ,which is $22+12+18+87=139$.

• Method.

This works for any 4 number which you want to add,they can be same or different.

Let's take any four number a,b,c,d.(they can be same or different.)

First write these number in first row.After doing this we make such type of arrangements so that same character not repeat in same row or column or diagonal.(a,b,c,d can be equal in value but we want to make a general formula for 4×4 so we are taking a,b,c,d as different symbols but they can be equal also.)

After doing this we get this square.

a	b	c	d
d	c	b	a
b	a	d	c
c	d	a	b

It's main properties are sum of elements in same row or same column or same diagonal is same.

Some more properties are coming automatically because of symmetry. Let's take m_{ij} is element of square. i =row and j =column

1. $m_{00}+m_{03}+m_{30}+m_{33}=\mathbf{a+b+c+d.}$
2. $m_{00}+m_{01}+m_{10}+m_{11}=\mathbf{a+b+c+d.}$
3. $m_{02}+m_{03}+m_{12}+m_{13}=\mathbf{a+b+c+d}$
4. $m_{20}+m_{30}+m_{21}+m_{31}=\mathbf{a+b+c+d}$
5. $m_{22}+m_{23}+m_{32}+m_{33}=\mathbf{a+b+c+d}$
6. $m_{11}+m_{12}+m_{21}+m_{22}=\mathbf{a+b+c+d}$
7. $m_{00}+m_{01}+m_{30}+m_{31}=\mathbf{a+b+c+d}$

8. $m_{02}+m_{03}+m_{32}+m_{33}=\mathbf{a+b+c+d}$
9. $m_{10}+m_{21}+m_{13}+m_{23}=\mathbf{a+b+c+d}$
10. $m_{00}+m_{02}+m_{20}+m_{22}=\mathbf{a+b+c+d}$
11. $m_{01}+m_{03}+m_{22}+m_{23}=\mathbf{a+b+c+d}$
12. $m_{10}+m_{11}+m_{30}+m_{32}=\mathbf{a+b+c+d}$
13. $m_{11}+m_{13}+m_{31}+m_{33}=\mathbf{a+b+c+d}$
14. $m_{00}+m_{31}+m_{12}+m_{22}=\mathbf{a+b+c+d}$
15. $m_{03}+m_{33}+m_{11}+m_{21}=\mathbf{a+b+c+d}$

In this method, if we want to make a magic square of different numbers then it is also possible.

Let's take any number I and arrange them like below.

a	b	c	d
d+3i	c-i	b-3i	a+3
b-2i	a+2i	d+2i	c-2i
c+i	d-i	a+i	b-i

By using this we can create such type of magic square.

Ramanujan took a=22,b=12,c=18,d=87,i=1 in his magic square.

Ramanujan Birthdate is 22 December 1887 so he created his birthday magic square.

Let's assume my birthdate is DD/MM/CCYY

Replace a with DD,b with MM,c with CC, d with YY.

DD	MM	CC	YY
YY+3i	CC-i	MM-3i	DD+3
MM-2i	DD+2i	YY+2i	CC-2i
CC+i	YY-i	DD+i	MM-i

This is a Birthday magic square.

Reference

https://en.wikipedia.org/wiki/Magic_square

<https://youtu.be/BeWhQWYQLjU>