Solution to Q-1

State whether the following statement is true or false. Justify your answer.

Q-1. If a set $S \subset \mathbb{R}$ is finite, and $c := \sup S$, then $c \in S$.

It is given that the set S is finite. Suppose it contains $n \in \mathbb{N}$ elements. Since $\mathbb{R}$ is ordered, we can sort the set and represent it as
$S = \{a_1, a_2, \cdots, a_n\}$
such that $a_1 \leq a_2 \leq \cdots \leq a_n$.

Claim. $a_n = \sup S$

To prove this claim, we need to show that a_n satisfies the definition of supremum, i.e. to show that,
1. a_n is an upper bound and
2. Given $\epsilon > 0$, $\exists a \in S$ s. t. $a > a_n - \epsilon$

1. a_n is an upper bound, since for all $a_i \in S$, $a_n \geq a_i$.
2. Also, since $a_n \in S$, For every $\epsilon > 0$, $a = a_n \in S$ exists, which satisfies $a > a_n - \epsilon$

Thus, our Claim is true, hence the statement is true, as $\sup S = a_n \in S$