
Q-3.

A function $f : [0, 1] \rightarrow \mathbb{R}$ satisfies the following properties.

1. f is continuous on $[0, 1]$.
2. $f(0) = f(1) = 0$.
3. $\forall x \in (0, 1), \exists h > 0$ such that :
 - ① $0 \leq x - h < x < x + h \leq 1$; and
 - ② $f(x) = \frac{f(x + h) + f(x - h)}{2}$

Show that $f(x) = 0 \ \forall x \in [0, 1]$.

As usual, not to be submitted or compulsory. But do try it!

Things to avoid

1. You can't assume f is a differentiable function.
2. " $\forall x \in (0, 1), \exists h > 0$ " does not mean that that equality holds for every $h > 0$, just that there is some h that satisfies it.
3. It follows that, you can't directly conclude things like $f(0.5) = \frac{f(0)+f(1)}{2}$ from what is given.

Other things

1. This is completely doable from your current MA109 knowledge, but it is a tricky question.
 2. Definitions, theorems and properties of continuous functions are your best bet here.
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Happy Mugging

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All the best!!!