MIS 4 - End SEM Project

Team Nemesis

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AIM

- Separating the Foreground and Background
- Using Truncated SVD and Robust PCA
- Truncated SVD:-
  - Gives low rank approximation of the matrix (Background)
- Robust PCA:-
  - Low rank Matrix (Background)
  - Sparse Matrix (Foreground)

PRE-REQUISITES

- SVD
- Randomized SVD
- PCA
- Robust PCA

Singular Value Decomposition (SVD)

- Low rank approximation to matrix
- Decomposition:
  - $X_{mxn} = U_{mxm} * \Sigma_{mxn} * V^{T}_{nxn}$
  - Economical SVD:-

$$\mathrm{U}\,\Sigma\, V^T = \sigma_1 u_1 v_1 + \sigma_2 u_2 v_2 + \sigma_3 u_3 v_3 \ldots\ldots + \sigma_n u_n v_n = \hat{U}\hat{\Sigma}V^T$$

  - Truncated SVD:-

$$\mathrm{X} \sim \sigma_1 u_1 v_1 + \sigma_2 u_2 v_2 + \sigma_3 u_3 v_3 \ldots\ldots + \sigma_r u_r v_r = \tilde{U}\tilde{\Sigma}\widetilde{V^T}$$

Randomized Singular Value Decomposition (RSVD)

- Original matrix is X_{nxm}
- Steps for RSVD:-
  - $Z_{nxr} = X_{nxm} \times P_{mxr}$
  - $Z_{nxr} = Q_{nxr} \times R_{rxr}$
  - $Y_{rxm} = Q^T_{rxn} \times X_{nxm}$
  - $Y = U_y * \Sigma * V^T$
  - $U_x = Q * U_y$
  - $X = U_x * \Sigma * V^T$ (Where X is final matrix)

Principal Component Analysis (PCA)

- Dimensionality reduction process
- Finds patterns in data and compresses it
- Seeks best rank r estimate using truncated SVD
- Fragile to outliers and noise
- Applications:-
    - Data compression, Image processing, Visualization, Pattern recognition, etc.

Robust Principal Component Analysis (RPCA)

- Modification of PCA
- Decomposition of matrix → $X = L + S$
- Properties of L and S:-
  - L is a low rank matrix that is not sparse
  - S is a sparse matrix that is not low ranked

Robust Principal Component Analysis (RPCA)

- Min rank(L) + min $||S||_0$

  subject to L + S = M $\rightarrow$ Non-convex optimization problem

- Convex relaxation:-
  - rank(L) $\rightarrow$ $||L||_*$
  - $||S||_0 \rightarrow ||S||_1$
- Min ($||L||_* + \lambda ||S||_1$) subject to L+S=X
- Can be solved using ALM

PRINCIPAL COMPONENT PURSUIT (PCP)

- $(L_k , S_k) = \arg\min_{L,S} l(L, S, Y_k) \rightarrow \min_L l(L,S, Y)$ and $\min_S l(L, S, Y)$
- Estimate L and S in an alternating aspect using a shrinkage operator and singular value thresholding operator.
- For sparse matrix approximation, apply shrinkage operator on M - L while keeping track of the error.
- For low rank matrix approximation, apply truncated SVD on M - S and reconstruct the matrix.
- Alternate back to sparse matrix.

Implementation