Description of
Deterministic Two-Phase Wake Vortex Model – D2P,
Probabilistic Two-Phase Wake Vortex Model – P2P

Deutsches Zentrum für Luft- und Raumfahrt (DLR)
Institut für Physik der Atmosphäre
82234 Oberpfaffenhofen
Germany

Documentation and User's Guide

Dr.-Ing. habil. Frank Holzäpfel
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Abstract

This document describes the design of the Deterministic (D2P) and the Probabilistic (P2P) versions of the Two-Phase wake vortex model. It contains a user's guide that details how the input and output data and formats are structured as well as how the model is compiled and executed. To make sure that the model works correctly, the data of a test case consisting of one approach of the campaign WakeMUC in 2011 is supplied. The terms and conditions for the usage of the P2P/D2P model are outlined in the Proposal-No.: Lz2022/036 "P2P model license for ZHAW" from 16 December 2022.

1. Survey on References in Scientific Journals

The design, applications, and validation activities of the Deterministic (D2P) and the Probabilistic (P2P) versions of the Two-Phase wake vortex model are described in detail in a series of Journal publications: Holzäpfel 2003, Holzäpfel and Robins 2004, Holzäpfel 2006, Holzäpfel and Steen 2007, and Frech and Holzäpfel 2008.

Further, references are provided to wake vortex systems employing D2P or P2P for wake vortex prediction:

WakeScene (Wake Vortex Scenarios Simulation Package) enables Monte Carlo simulation of departures and arrivals of aircraft pairings and estimates the frequency and severity of encounters. WakeScene can be applied for the sensitivity analysis, optimization, or elaboration of a safety case of wake vortex advisory systems. It may also be used to establish new aircraft separations as needed in RECAT or to determine wake vortex separations for new very large aircraft types. For this purpose, the wake vortex evolution is modelled with D2P in 2D planes established along the flight path of an aircraft, where meteorological conditions, traffic mix, and aircraft trajectories are modelled stochastically. The design of WakeScene and applications of the D2P model in WakeScene are documented in: Holzäpfel, Frech et al. 2009, Holzäpfel, Kladetzke et al. 2009, and Holzäpfel and Kladetzke 2011.

The Wake Vortex Prediction and Monitoring System, WSVBS, has been developed to tactically increase airport capacity for approach and landing on single runways as well as closely-spaced parallel runways. It is thought to dynamically adjust aircraft separations dependent on prevailing weather conditions and the resulting wake vortex behaviour without compromising safety. Dedicated meteorological instrumentation and short-term numerical terminal weather prediction provide the input to the prediction of wake-vortex behaviour and respective safety areas. The probabilistic envelopes of vortex position and strength are predicted with the P2P model in typically 15 gates distributed along the glide path between final approach fix and threshold. The design of the WSVBS and demonstration campaigns of the system at Frankfurt and Munich airports are described in: Gerz et al. 2009, Holzäpfel, Gerz et al. 2009, Holzäpfel et al. 2011, and Holzäpfel et al. 2021.

P2P was also used as wake vortex predictor in the ATC-Wake project which aimed to develop and build an integrated platform for ATC (Air Traffic Control) that would allow variable aircraft separation distances for arrivals and departures under favourable weather conditions (Winckelmans et al. 2005).

In the EU project FLYSAFE an airborne version of the P2P model ($P2P^a$) has been developed as part of an onboard wake vortex prediction and alerting system called Wake Encounter Prediction System (WEPS). The project reports are restricted to other programme participants and thus are not cited here. This activity was continued in SESAR P9.11 "Aircraft Systems for Wake Encounter Alleviation". DLR has further developed the onboard system now called Wake Encounter Avoidance & Advisory (WEAA, Bauer et al. 2014). WEAA flight tests were conducted in April 2014 and Nov/Dec 2016 with DLR's Falcon and ATRA research aircraft (Sölch et al. 2016). $P2P^a$ considers uncertainties of all relevant input parameters. However, comprehensive validation of the airborne version of P2P is still pending.

2. The Deterministic and the Probabilistic Two-Phase Wake Vortex Models D2P and P2P

The Probabilistic Two-Phase wake vortex decay and transport model (P2P) was developed, in the first instance, to guide the safe readjustment of aircraft separations during approach and landing. Later it was also applied successfully to departures and cruise. For this purpose, such a real-time model must be capable of reliably and fast predicting vortex positions and strengths. P2P considers all effects of the leading order impact parameters: aircraft configuration (span, weight, speed, heading, flight path angle), as well as the environmental parameters wind (cross and head components) wind shear, turbulence, temperature stratification, and proximity of the ground. The model predicts the deterministic (mean) evolution (D2P model) together with envelopes for vortex trajectories and strengths, combined with specified probabilities (P2P model) (see Figure 1).

The model design rests on four pillars:

I. dimensional analysis
II. equation for laminar decaying potential vortex
III. adjustment to LES results of different groups
IV. calibration with field experiment data

In the following, integral components including these four pillars of P2P are briefly introduced. For more details see the references cited in section 1.

I. The model is formulated in normalized form (denoted by *) where the characteristic scales are based on initial vortex separation, b_0, and circulation, Γ_0, leading to the time scale $t_0 = 2\pi\, b_0^2/\Gamma_0$, which corresponds to the time that the young vortices need to descend one vortex separation. This normalization enables the application of the wake vortex model to a large variety of different aircraft types and environmental conditions. Turbulence is characterized by the eddy dissipation rate which is normalized according to $\varepsilon^* = (\varepsilon\, b_0)^{1/3}/\, w_0$, where $w_0 = \Gamma_0/2\pi b_0$ denotes the initial wake vortex descent speed. Temperature stratification is expressed by the normalized Brunt-Väisälä frequency $N^* = (g/\theta_0\, d\theta/dz)^{1/2}\, t_0$, where θ denotes the (virtual) potential temperature.

   It should be noted that initially P2P employed only a circulation, $\Gamma^*_{5\text{-}15}$, that was averaged over circles with radii from 5 m to 15 m, because it was assumed that wake vortex predictions would only be needed for large aircraft (wing spans larger than $8/\pi \cdot 15$ m). Later a circulation average from 3 m to 8 m was introduced for smaller aircraft. It is recommended to apply the most recent model for the descent speeds (used by default) which was developed for a consistently nondimensional P2P model corresponding to $\Gamma^*_{5\text{-}15}$ for an aircraft with a span of 60 m. Benefits and drawbacks of radii-averaged circulation definitions are discussed in Holzäpfel et al. (2003).

II. The equations for circulation decay and vortex descent are built on an exact solution of the Navier-Stokes equations for a laminar decaying potential vortex. This equation has been extended to mimic the behaviour of turbulent vortex pairs

III. and has been adjusted to large eddy simulations of DLR (Holzäpfel 2001) and NASA

(Proctor and Switzer 2000). For the prediction of circulation, the concept of two-phase circulation decay is pursued (see equation (1) and Figure 1). The turbulent diffusion phase described by part 1 of equation (1) is followed by a rapid decay phase that can be parameterized by the full equation

$$\Gamma^*_{5-15}(t^*) \;=\; A - \exp\frac{-R^2}{\nu_1^*(t^* - T_1^*)} - \exp\frac{-R^2}{\nu_2^*(t^* - T_2^*)} \qquad (1)$$

The onset time of rapid decay at T_2^* depends on ambient turbulence and stratification (see Figure 2). The determination of T_2^* is based on the model of Sarpkaya (2000) for the relation between the eddy dissipation rate, ε^*, and the time at which a "catastrophic demise event" takes place which has been extended to the impact of thermal stratification (Holzäpfel 2003). For wake vortex evolution in ground proximity T_2^* depends on vortex altitude above ground (Holzäpfel and Steen 2007). The respective decay rate is adjusted by the effective viscosity ν_2^* which out of ground proximity mainly depends on ambient stratification. The constant parameters T_1^* and ν_1^* control decay in the diffusion phase, R corresponds to a mean radius and A is a constant to adjust $\Gamma^*_{5\text{-}15}(t^*=0)$ to about unity.

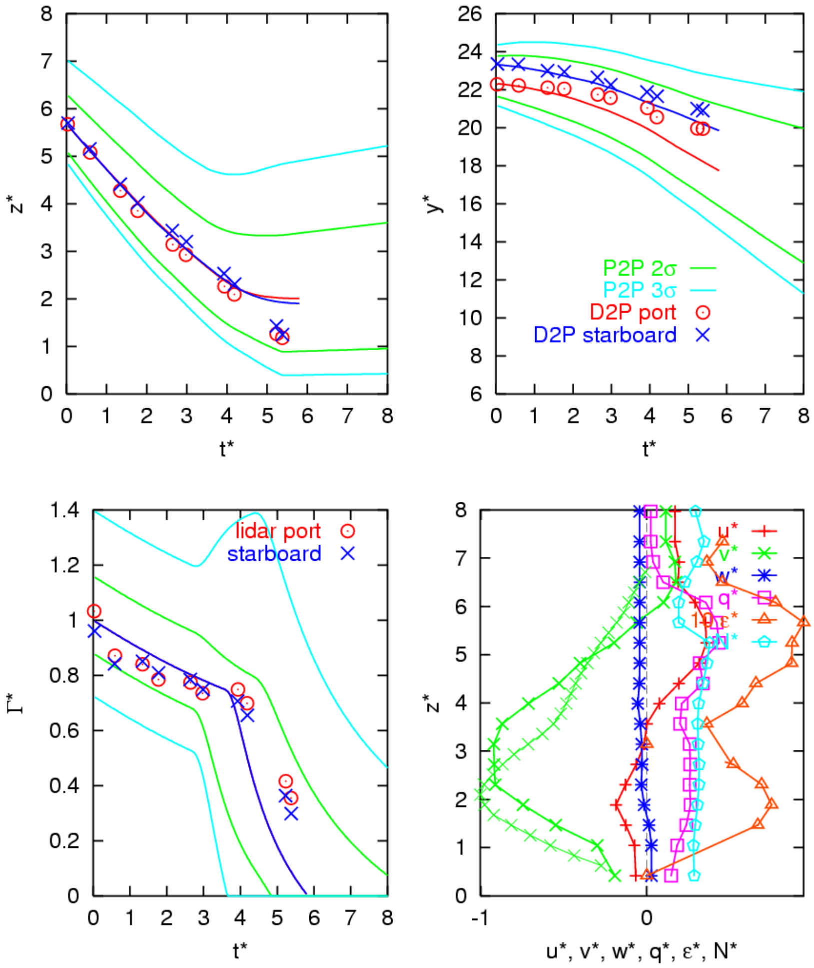

Figure 1: Example of P2P model output. Measured (symbols) and predicted (lines) evolution of normalized vertical and lateral positions and circulation. Red and blue lines denote deterministic behaviour; green and light blue lines envelopes for probabilities of 95.4% and 99.7%, respectively. Right below: vertical profiles of normalized environmental data.

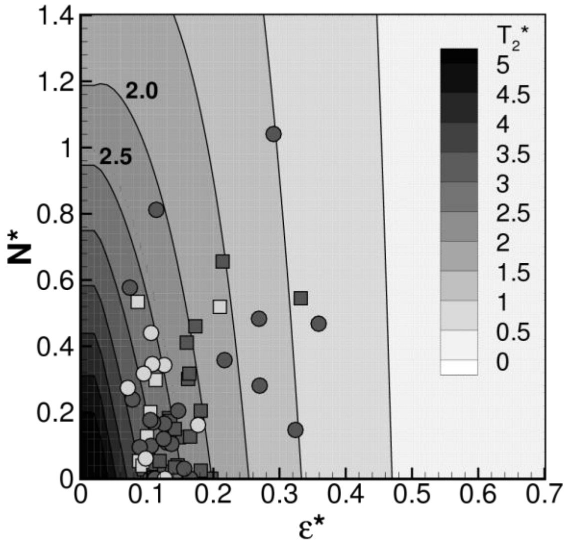

Figure 2: Onset time of rapid decay, T_2*, as a function of mean normalized eddy dissipation rate, ε*, and Brunt-Väisälä frequency, N*. Symbols denote environmental conditions during flight tests at WakeToul and AWIATOR-FT1 field measurement campaigns (Holzäpfel 2006).

The descent rate obeys a non-linear dependence on circulation, which allows for a reduction of circulation without the reduction of the descent rate during the early vortex evolution and stagnating or even rebounding vortices with non-zero circulation in strongly stably stratified environments (see Figure 3 and Holzäpfel 2003).

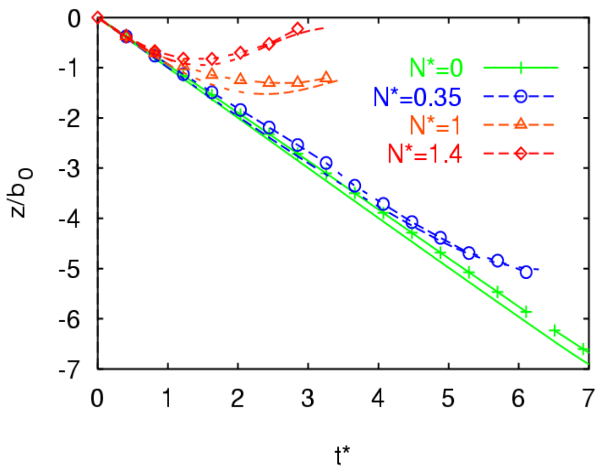

Figure 3: Comparison of vortex descent between large eddy simulations (symbols) and P2P (lines) in quiescent atmosphere with different degrees of stratification (Holzäpfel 2003).

Precise deterministic wake vortex predictions are not feasible operationally. Primarily, it is the nature of turbulence that deforms and transports the vortices in a stochastic way and leads to considerable spatio-temporal variations of vortex position and strength. Moreover, uncertainties of aircraft parameters and the temporal and spatial variability of environmental conditions must be considered. P2P is therefore designed to predict wake vortex behaviour within defined confidence intervals. For this purpose, decay parameters, T_2^* and ν_2^*, are varied in consecutive model runs and various static and dynamic uncertainty allowances are added which consider the increased scatter in turbulent and sheared environments. The deterministic model version D2P provides mean wake vortex evolutions employing intermediate decay parameters.

In the first instance, the increased scatter of vortices in turbulent and convective environments is modelled by the assumption that the RMS value of ambient turbulence serves as a superimposed propagation velocity, which widens the predicted envelopes. If wake vortices encounter a pronounced shear layer, a normalized shear velocity, $v_{sh}^* = \partial v/\partial z \, b_0/w_0$, is employed to expand the envelopes for vortex transport in a similar way as it is done for turbulence. These measures address the average increase of scatter caused by environmental conditions.

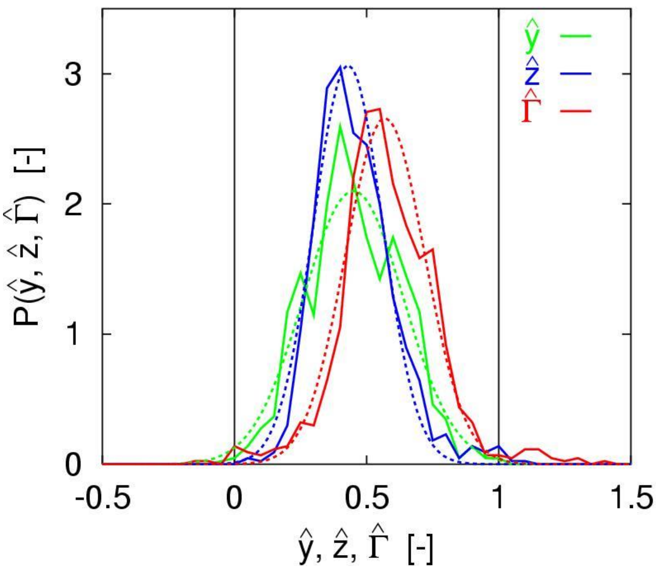

Figure 4: Probability density distributions of measured lateral position, vertical position, and circulation of wake vortices normalized with respect to the uncertainty bounds predicted by P2P. Fits of respective PDFs are denoted by dotted lines.

IV. For the calibration of the probabilistic model with field measurement data a large number of measurement data is related to the predicted probabilistic bounds such that values of zero and one denote measurements situated on the lower and upper bounds of vortex predictions, respectively (Holzäpfel 2006). As an example, the resulting statistics of 49 vortex evolutions out of ground proximity is shown in Figure 4. Clearly, the unusual cases are situated in the tails of the distributions. The second step is to fit a mathematical PDF to the empirical distributions (dotted lines in Figure 4). The validity of the fits is assessed by Kolmogorov-Smirnov goodness-of-fit tests at a significance level 0.05. This calibration has been conducted separately for wake vortex evolution out of ground effect (Holzäpfel 2006) and in ground proximity (Holzäpfel and Steen 2007). For wake vortices approaching the ground there is a smooth transition between the respective PDFs.

Valid PDF fits are likely to provide probability estimates that can be extrapolated beyond the range of the finite number of the so far available experimental measurements. Based on these PDFs, the model output can be adjusted to a required degree of probability, at least within reasonable limits. Figure 1 exemplarily shows envelopes with probabilities of 2σ (95.4%) and 3σ (99.7%).

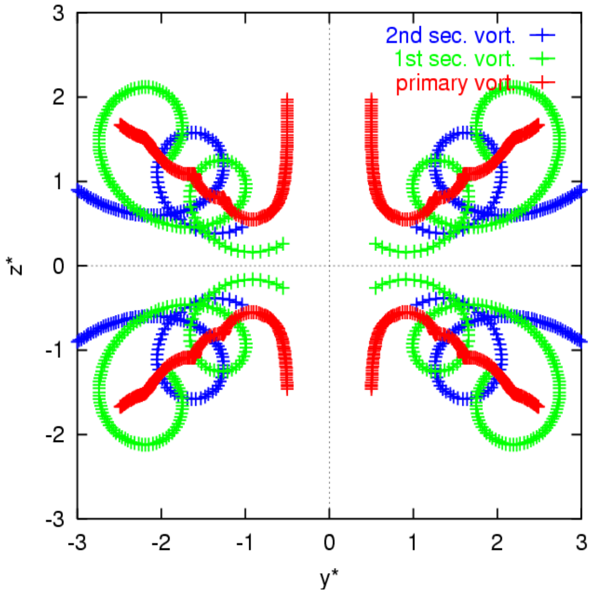

Figure 5: Trajectories of primary and secondary vortices with image vortices for previous trajectory model. Symbols plotted every 5 s.

The effect of the ground on vortex trajectories in P2P has been modelled following the approach of Robins et al. (2001) which is illustrated in Figure 5. Image vortices are introduced when the primary vortices have reached a height of 1.5 b_0 above ground. At a height of z = 0.6 b_0 counter-rotating ground-effect vortices and their respective image vortices are introduced at an angle of 45 deg inboard below the primary vortices. Another pair of secondary vortices with images is introduced when the first pair has rotated 180 deg around the primary vortices. The strength of the secondary vortices is a function of the rotation angle and reaches a maximum after being travelled 90 deg. For vortex generation altitudes below one initial vortex separation b_0, the distance between primary and secondary vortices is reduced and the decay rate is increased. The minimum vortex generation altitude amounts to 0.1 b_0.

To consider the effects of crosswind the altitude and strength of the secondary vortices have been adjusted as a function of the crosswind strength measured at an altitude of 0.6 b_0 above ground (Holzäpfel and Steen, 2007). In cases with crosswind this leads to asymmetrical wake vortex rebound as seen in Figure 6.

P2P employs optionally the headwind correction as described in Holzäpfel (2006). Compared to a calm situation, headwind (tailwind) advects younger (older) vortex segments into the lidar observation plane, which is usually spanned perpendicular to the flight direction and in which P2P predicts vortex evolution. The headwind correction modifies vortex age and vortex descent considering the flight path angle of the aircraft.

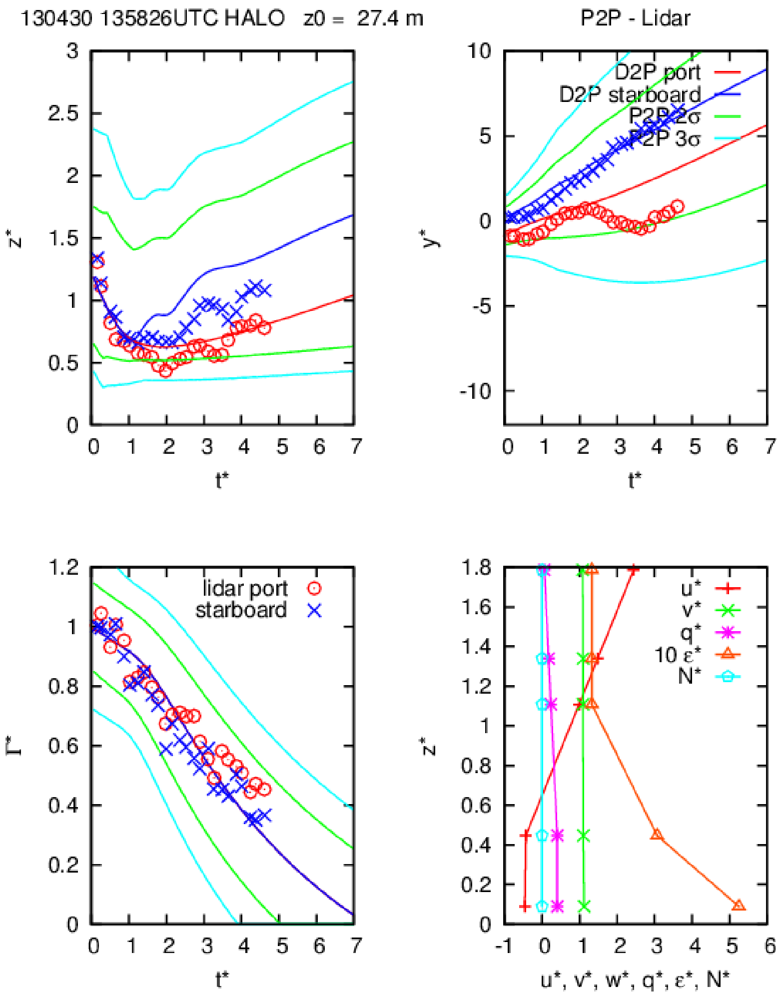

Figure 6: Example of P2P model output in ground proximity with crosswind. Measured (symbols) and predicted (lines) evolution of normalized vertical and lateral positions and circulation. Red and blue lines denote deterministic behaviour; green and light blue lines envelopes for probabilities of 95.4% and 99.7%, respectively. Right below: vertical profiles of normalized environmental data.

So far D2P / P2P have been validated against in ground effect and out of ground effect measurement data of four US and more than ten European field measurement campaigns employing over 16,000 individual cases (see Table 1). Most of the validation work is documented in peer reviewed publications and project reports. To evaluate the deterministic model's performance a scoring procedure has been applied to data from 13 campaigns. The scoring indicates that deviations between measurement and prediction have been reduced continuously, which reflects the progress in prediction, measurement and data analysis techniques. Comparisons with different versions of the AVOSS Prediction Algorithm (APA, Sarpkaya et al. 2001, Robins and Delisi 2002) and the Deterministic Vortex Model (DVM, Winckelmans et al. 2010) as well as DLR's Multi-Model Ensemble consisting of different versions of APA, Fred proctor's TDP (Proctor et al. 2006) and the D2P model (MME, Körner & Holzäpfel 2016, Körner et al. 2017, Körner et al. 2018) have been conducted and documented. Table 1 further shows that D2P / P2P have been applied to wake vortex predictions of a large variety of aircraft types and flight phases including approach, landing, takeoff, departure and cruise.

campaign	No. cases	det./prob. scoring	compared to	a/c types	flight phases	documentation
Memphis, TN (1994, 1995)	282 211	X/-	APA	23 hvy/med/li	arrival OGE/IGE	J. Aircraft 2003/2004
Dallas Fort Worth, TX (1997)	191	X/-		16 hvy/med/li	arrival OGE/NGE	J. Aircraft 2004
WakeOP (2001)	41	X/-	APA	ATTAS	level/hi-lift OGE	J. Aircraft 2004
WakeTOUL Tarbes (2002)	32	X/X	APA	A340	lev/arr/cl/hi OGE	J. Aircraft 2006
AWIATOR FT1 Tarbes (2003)	32	X/X	APA	A340	level/take-off/OGE	J. Aircraft 2006
WakeFRA Frankfurt (2004)	282 + 233	X/X	APA / DVM	25 hvy/med	arrival IGE/OGE	AIAA J. 2007
CREDOS EDDF-1 (2007)	137	X/-	DVM	12 heavy	departure OGE	CREDOS D2-3, 2008
unpublished		X/X		A380, A340, B747	cruise/arr. OGE/IGE	
CREDOS EDDF-2 (2007)	~ 9,000	X/-	DVM	28 med/hvy	departure IGE/NGE	CREDOS D2-3, 2009
WakeMUC Munich 2011	374 of 907	X/X	MME	med/hvy	landing IGE/NGE	J. Aircraft 2017
WakeOP-GE Oberpfaffenh. 2013	31	X/X	MME	HALO (G550)	fly-by at b0	J. Aircraft 2017
Denver (2003)	527 of 772	X/X	MME	med/hvy	arrival OGE	J. Aircraft 2018
Memphis (2013)	3253 of 8183	X/X	MME	med/hvy	arrival NGE/IGE	J. Aircraft 2018
Vienna (2019)	1150	X/X		med/hvy	arrival NGE/IGE	

Table 1: Survey on validation activities of the D2P / P2P models.

3. D2P and P2P User's Guide

The model executable and the source code of some necessary files as well as an input file and an output file are contained in the file P2P_ZHAW.7z that will be provided to ZHAW. The terms and conditions for the usage are outlined in the Proposal-No.: Lz2022/036 "P2P model license for ZHAW" from 16 December 2022.

Source Code, Compilation, and Linking

The functionalities of the models D2P and P2P have been integrated into one Fortran software code.

The main program termed *main_P2P_ZHAW.f* is provided as Fortran source code to allow for adaptations of the code and its interfaces. The subroutines which contain the calculation of the vortex trajectories and circulation decay are provided as object file (*subroutines_ZHAW.o*).

The compiler used by DLR is ifort version 12.1.5 (alternatively gfortran). The model is run on a linux system. The commands for compilation and for linking the main program with the object file *subroutines_ZHAW.o* read:

```
ifort main_P2P_ZHAW.f -C -save -o P2P subroutines_ZHAW.o
```

This command will generate the executable *P2P*. In order to run many cases consecutively the shell script *while_script* is used. It reads which cases are to be simulated from the file cases.dat and calls the executables. Also, the working directory is specified in this script. It is executed with the following command:

```
./while_script
```

The *while_script* reads each line of the file *cases.dat* which contains information on which cases are to be simulated and where the necessary input data can be found for each case. Furthermore, it copies the input data from the input data directory into the working directory.

In the next step *P2P* reads the input data and simulates the wake vortex behaviour. The results are stored in the output files mentioned below. Then the *while_script* calls gnuplot which generates a postscript file based on the output data (cf. Figure 6).

For dimensional output of wake vortex parameters the parameter *normalize* has to be set to 0 in main_P2P_ZHAW.f. For normalized output use *normalize = 1*. To plot dimensional output use *gnuplot P2P_dim.plo* in the *whilescript* or *gnuplot P2P_norm.plo* for normalized output.

The headwind correction described in section 2 can be deactivated by setting *dtu* = 0 and *dzu* = 0 in *main_P2P_ZHAW.f* (which is the case in the provided P2P version).

Input Files (P2P standard format)

This section describes the standard input data format used by P2P. Any format other than that must be adjusted in the file *main_P2P_ZHAW.f* in advance. It is recommended that ZHAW adjusts the file structure and formats as required according to the particular application and data sources.

<u>cases.dat</u>

This file lists the cases which shall be simulated and the corresponding information on campaign, runway and the input data files containing the environmental parameters (atmosphere) and the aircraft data.

cases.dat
case ifirst campaign runway edr_file ac_file meteo_file case = name of lidar file ifirst = number of lidar measurements which shall be used as initial value (not used in this P2P version) campaign = name of campaign runway = runway where a/c landed (needed for lateral position if this varies within campaign) edr_file = file which contains eddy dissipation rate, if applicable ac_file = file which contains a/c information such as speed or weight meteo_file = file which contains meteo data

The *whilescript* copies the input files listed in cases.dat into the working directory and renames them to lidar.dat, meteo.dat, EDR.dat and ac_init.dat. The content of these files is described in the following.

<u>lidar.dat</u>

This file provides the dimensional lidar data which is used to compare the model forecast with the measurements and if necessary to calculate the initial conditions for the wake vortices due to the lack of aircraft data.

lidar.dat
tr zr yr gamr tl zl yl gaml tr = vortex age right vortex [s] zr = vertical position vortex center right [m] yr = lateral position vortex center right [m] gamr = averaged circulation vortex right [m²/s] tl = vortex age left vortex [s] zl = vertical position vortex center left [m] yl = lateral position vortex center right [m] gaml = averaged circulation vortex left [m²/s]

meteo.dat

The dimensional vertical profiles for wind, turbulence and temperature are contained in this file.

meteo.dat
zm um vm wm qq temp th tke
zm = height above ground [m] um = headwind [m/s] vm = crosswind [m/s] wm = vertical wind [m/s] qq = mean turbulence velocity [m/s] temp = temperature [K] th = potential temperature [K] tke = turbulent kinetic energy [m^2/s^2]

EDR.dat

If applicable the profile of the eddy dissipation rate is stored in this file.

EDR.dat
zm EDR
zm = height [m] EDR = eddy dissipation rate [m^2/s^3]

ac_init.dat

Initial conditions from the aircraft side.

ac_init.dat
z0 gam0 b0 uac x0 y0 gpa
z0 = vortex generation height [m] gam0 = initial circulation [m^2/s] b0 = initial vortex spacing [m] uac = airspeed [m/s] x0 = initial position in flight direction [m] y0 = initial lateral position [m] gpa = glide path angle [°]

Output Files

<u>TRAJEC.dat</u>

The array out_mean(j,i) which describes the predicted deterministic values of z, y and Γ is written to this file. (The data may be dimensional or normalized depending on the setting of the parameter *normalize*.)

TRAJEC.dat	
t gaml yl zl gamr yr zr	
t	= vortex age
gaml	= circulation left vortex
yl	= lateral position left vortex
zl	= vertical position left vortex
gamr	= circulation right vortex
yr	= lateral position right vortex
zr	= vertical position right vortex
x	= axial transport of both vortices in headwind direction

<u>P2P_lev_y.dat</u>

Contains the probabilistic envelopes for the predicted lateral vortex positions for the two probability levels 2σ (95.4%) and 3σ (99.7%).

P2P_lev_y.dat	
t 2σ y_l 2σ y_r 3σ y_l 3σ y_r	
t	= vortex age
2σ y_l	= probabilistic envelope left vortex (*2σ* uncertainty level)
2σ y_r	= probabilistic envelope right vortex (*2σ* uncertainty level)
3σ y_l	= probabilistic envelope left vortex (3*σ* uncertainty level)
3σ y_r	= probabilistic envelope right vortex (3*σ* uncertainty level)

<u>P2P_lev_z.dat</u>

Contains the probabilistic envelopes for the predicted vortex heights positions for the two probability levels 2σ (95.4%) and 3σ (99.7%).

P2P_lev_z.dat	
t 2σ z_lo 2σ z_up 3σ z_lo 3σ z_up	
t	= vortex age
2σ z_lo	= lower probabilistic envelope (*2σ* uncertainty level)
2σ z_up	= upper probabilistic envelope (*2σ* uncertainty level)
3σ z_lo	= lower probabilistic envelope (3*σ* uncertainty level)
3σ z_up	= upper probabilistic envelope (3*σ* uncertainty level)

P2P_lev_g.dat

Contains the probabilistic envelopes for the predicted vortex circulations for the two probability levels 2σ (95.4%) and 3σ (99.7%).

P2P_lev_y.dat	
t 2σ Γ_l 2σ Γ_r 3σ Γ_l 3σ Γ_r	
t	= vortex age
2σ Γ_l	= lower probabilistic envelope (*2σ* uncertainty level)
2σ Γ_r	= upper probabilistic envelope (*2σ* uncertainty level)
3σ Γ_l	= lower probabilistic envelope (3*σ* uncertainty level)
3σ Γ_r	= upper probabilistic envelope (3*σ* uncertainty level)

P2P.ps

As graphic output both the lidar data and the P2P model output are plotted in four diagrams which contain the height, lateral position and circulation over time as well as the meteorological conditions over height (see Figure 7). In order to plot all meteo quantities together the meteo data is always plotted in normalized form.

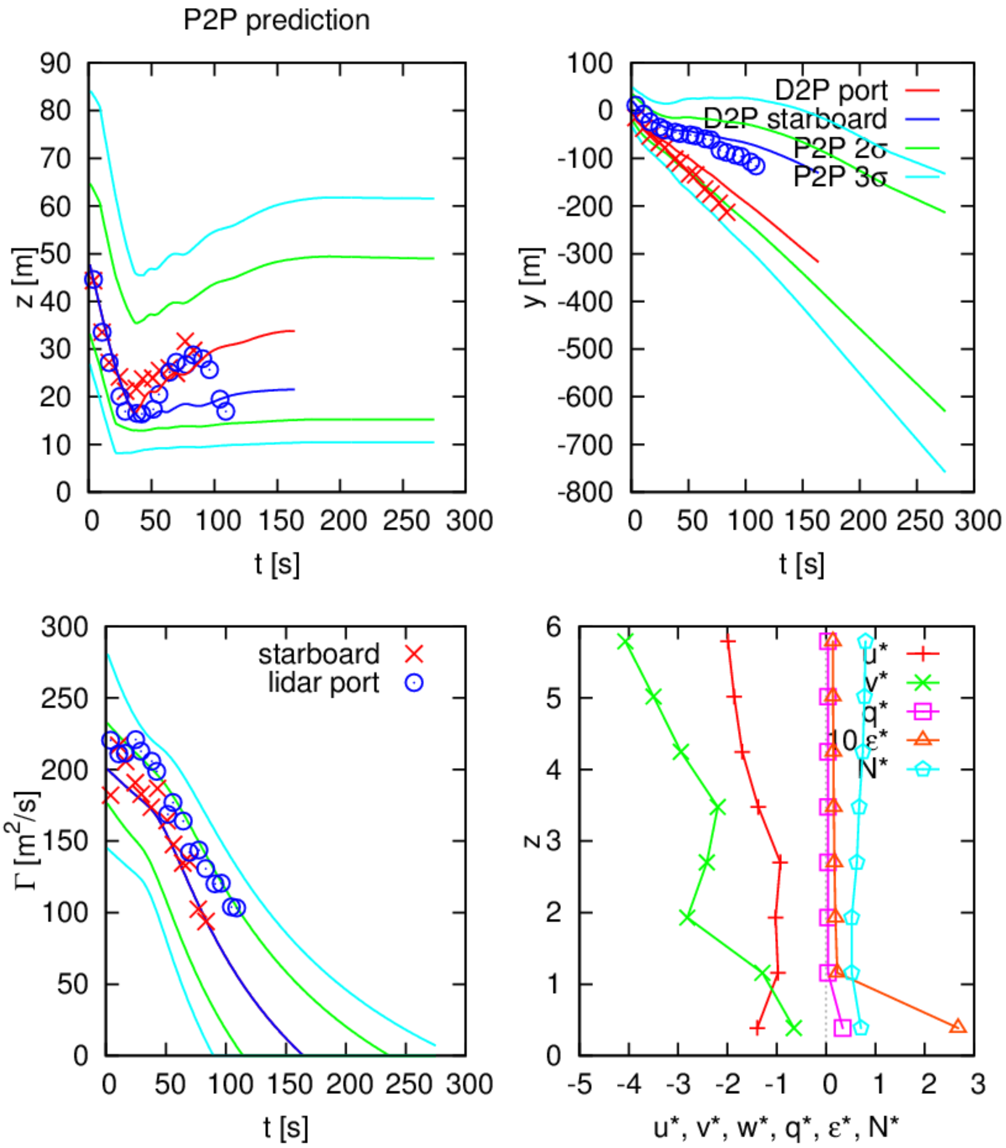

Figure 7: P2P model output of the provided test case. Measured (symbols) and predicted (lines) evolution of dimensional vertical and lateral positions and circulation. Red and blue lines denote deterministic behaviour; green and light blue lines envelopes for probabilities of 95.4% and 99.7%, respectively. Right below: vertical profiles of normalized environmental data.

References

Bauer T., Vechtel D., Abdelmoula F., Immisch T., 2014: In-Flight Wake Encounter Prediction with the Wake Encounter Avoidance and Advisory System, AIAA Paper 2014-2333.

Frech M., Holzäpfel F., 2008: Skill of an Aircraft Wake-Vortex Model Using Weather Prediction and Observation, Journal of Aircraft, Vol. 45, No. 2, pp. 461-470.

Gerz T., F. Holzäpfel, W. Gerling, A. Scharnweber, M. Frech, K. Kober, K. Dengler, S. Rahm, THE WAKE VORTEX PREDICTION AND MONITORING SYSTEM WSVBS PART II: PERFORMANCE AND ATC INTEGRATION AT FRANKFURT AIRPORT, Air Traffic Control Quarterly, Vol. 17, No. 4, 2009, pp. 323-346.

Holzäpfel F., Gerz T., and Baumann R., The Turbulent Decay of Trailing Vortex Pairs in Stably Stratified Environments," Aerospace Science and Technology, Vol. 5, No. 2, 2001, pp. 95–108.

Holzäpfel F., 2003: Probabilistic Two-Phase Wake Vortex Decay and Transport Model, Journal of Aircraft, Vol. 40, No. 2, pp. 323-331.

Holzäpfel F., Gerz T., Köpp F., Stumpf E., Harris M., Young R.I., Dolfi-Bouteyre A., Strategies for Circulation Evaluation of Aircraft Wake Vortices Measured by Lidar, J. Atmos. Ocean. Tech., Vol. 20, No. 8, 2003, pp. 1183-1195.

Holzäpfel F., Robins R.E., 2004: Probabilistic Two-Phase Aircraft Wake Vortex Model: Application and Assessment, Journal of Aircraft, Vol. 41, No. 5, pp. 1117-1126.

Holzäpfel F., 2006: Probabilistic Two-Phase Aircraft Wake-Vortex Model: Further Development and Assessment, Journal of Aircraft, Vol. 43, No. 3, pp. 700-708.

Holzäpfel F., Steen M., 2007: Aircraft Wake-Vortex Evolution in Ground Proximity: Analysis and Parameterization, AIAA Journal, Vol. 45, No. 1, pp. 218-227.

Holzäpfel F., M. Frech, T. Gerz, A. Tafferner, K.-U. Hahn, C. Schwarz, H.-D. Joos, B. Korn, H. Lenz, R. Luckner, G. Höhne, Aircraft Wake Vortex Scenarios Simulation Package - WakeScene, Aerospace Science and Technology, Vol. 13, Issue 1, 2009, pp. 1-11, doi:10.1016/j.ast.2007.09.008

Holzäpfel F., J. Kladetzke, S. Amelsberg, H. Lenz, C. Schwarz, I. De Visscher, Aircraft Wake Vortex Scenarios Simulation for Takeoff and Departure, Journal of Aircraft, Volume 46, Number 2, March-April 2009, pp. 713-717.

Holzäpfel F., T. Gerz, M. Frech, A. Tafferner, F. Köpp, I. Smalikho, S. Rahm, K.-U. Hahn, C. Schwarz, THE WAKE VORTEX PREDICTION AND MONITORING SYSTEM WSVBS - PART I: DESIGN, Air Traffic Control Quarterly, Vol. 17, No. 4, 2009, pp. 301-322.

Holzäpfel F., J. Kladetzke, Assessment of Wake Vortex Encounter Probabilities for Crosswind Departure Scenarios, Journal of Aircraft, Volume 48, Number 3, May-June 2011, pp. 812-822. DOI 10.2514/1.C000236

Holzäpfel F., K. Dengler, T. Gerz, C. Schwarz, Prediction of Dynamic Pairwise Wake Vortex Separations for Approach and Landing, AIAA Paper 2011-3037, 3rd AIAA Atmospheric Space Environments Conference, 27-30 June 2011, Honolulu, Hawaii, 15 pages.

Frank Holzäpfel, Lukas Strauss, Carsten Schwarz, Assessment of Dynamic Pairwise Wake Vortex Separations for Approach and Landing at Vienna Airport, Aerospace Science and Technology, Vol. 112, May 2021, Article 106618, https://doi.org/10.1016/j.ast.2021.106618.

Körner S., Holzäpfel F., "Multi-Model Ensemble Wake Vortex Prediction", Aircraft Engineering and Aerospace Technology, Vol. 88, Issue 2, 2016, pp. 331-340, http://dx.doi.org/10.1108/AEAT-02-2015-0068.

Körner S., Ahmad N.N., Holzäpfel F., Van Valkenburg R.L.: Multimodel Ensemble Methods for Prediction of Wake-Vortex Transport and Decay, Journal of Aircraft, Vol. 54, No. 5 (2017), pp. 1849-1859. http://dx.doi.org/10.2514/1.C034287.

Körner S., Holzäpfel F., Sölch I., Probabilistic Multimodel Ensemble Wake-Vortex Prediction employing Bayesian Model Averaging, subm. to J. Aircraft 2018.

Proctor F. H., and Switzer G. F., "Numerical Simulation of Aircraft Trailing Vortices," 9th Conf. on Aviation, Range and Aerospace Meteorology, Paper 7.12, American Meteorology Society, Orlando, Florida, Sept. 2000, pp. 511–516.

Proctor, F. H., Hamilton, D. W., and Switzer, G. F., "TASS Driven Algorithms for Wake Prediction," 44th AIAA Aerospace Sciences Meeting and Exhibit, AIAA Paper 2006-1073, Jan. 2006.

Robins R. E., Delisi D. P., and Greene G. C., Algorithm for Prediction of Trailing Vortex Evolution, Journal of Aircraft, Vol. 38, No. 5, 2001, pp. 911–917.

Robins, R.E., and Delisi, D.P., NWRA AVOSS Wake Vortex Prediction Algorithm Version 3.1.1, NASA/CR-2002-211746, 2002, and NWRA-CR-00-R229A.

Sarpkaya T., New Model for Vortex Decay in the Atmosphere, Journal of Aircraft, Vol. 37, No. 1, 2000, pp. 53–61.

Sarpkaya T., Robins R. E., and Delisi D. P., Wake-Vortex Eddy-Dissipation Model Predictions Compared with Observations, Journal of Aircraft, Vol. 38, No. 4, 2001, pp. 687–692.

Sölch I., Holzäpfel F., Abdelmoula F., Vechtel D., Performance of on-board wake vortex prediction systems employing various meteorological data sources, J. Aircraft, Vol. 53, Issue 5, pp. 1505-1516, 2016, DOI: : http://arc.aiaa.org/doi/abs/10.2514/1.C033732.

Winckelmans G., Desenfans O., Barbaresco F., Deltour J.-C., Pham K., Frech M., Gerz T., Holzäpfel F., van Baren G., Speijker L., Verhoogt T., Vidal A., The ATC-Wake Predictor system and its potential use to increase the capacity at airports, Joint International Symposium on Sensors and Systems for Airport surveillance June 20-21 2005, DGAC, Paris, France, 10 pages.

Winckelmans, G., De Visscher, I., Lonfils, T., Desenfans, O., Bricteux, L., Duponcheel, M., and Bourgeois, N., "UCL operational tools for predicting aircraft wake vortex transport and decay: The Deterministic/Probabilistic wake Vortex Models (DVM/PVM) and the WAKE4D platform", Internal report of the UCL Group for Research in Turbulence and Vortical Flows, v2.2, UCL, January 2010 (https://comete.meca.ucl.ac.be/vortex).