pract6

May 23, 2017

```
In [105]: import numpy as np
          import matplotlib.pyplot as plt
          import networkx as nx
          import seaborn
```

1 Practica 6 Redes Neuronales

1.1 Miguel Angel Carvajal

1.1.1 Ejercicio 1

Defino la matrix de covarianza Σ

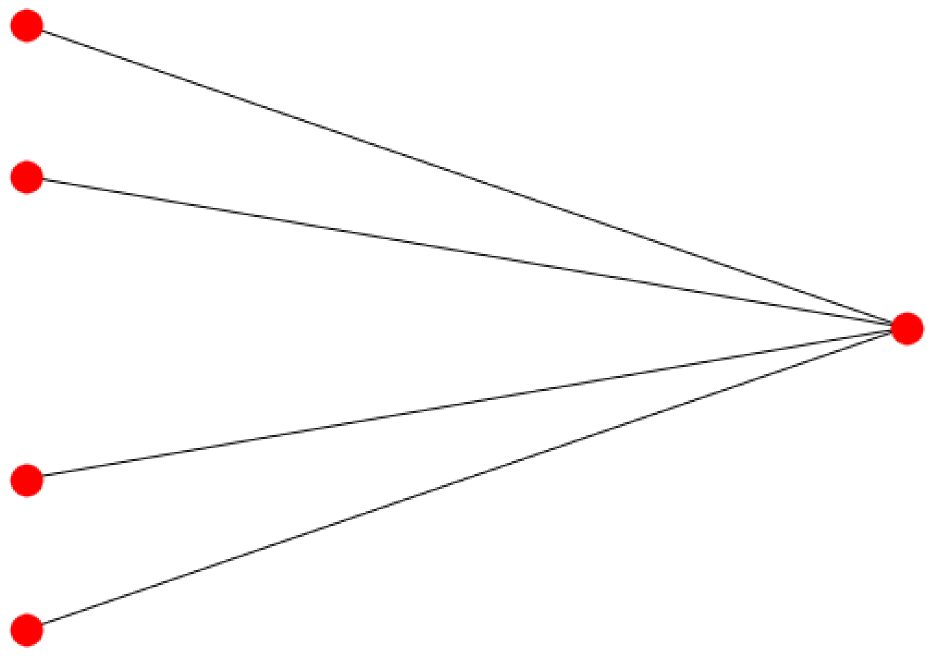
```
In [6]: cov_mat = np.ones((4,4))
        np.fill_diagonal(cov_mat,2)
        print cov_mat
```

```
[[ 2.  1.  1.  1.]
 [ 1.  2.  1.  1.]
 [ 1.  1.  2.  1.]
 [ 1.  1.  1.  2.]]
```

```
In [7]: # learning speed
        eta = 0.0001
```

Draw the network achitecture

```
In [106]: nn =  nx.Graph()
          nn.add_nodes_from([1,2,3,4,5])
          nn.add_edges_from([(1,5),(2,5),(3,5),(4,5)])
          nx.draw(nn, pos={1:(0,2),2:(0,1),3:(0,-1),4:(0,-2),5:(1,0)})
          plt.plot()
          plt.show()
```



1.2 Run the simulation

The input data is generated with a multigaussian probability given by
$P(\bar{\xi}) = \frac{1}{(2\pi)^2\sqrt{\det(\xi)}} \exp(-\frac{1}{2}\bar{\xi^T}\Sigma^{-1}\bar{\xi})$

```
In [9]: np.random.seed(1)
        N_in = 4 # input size
        N_out = 1 # output size
        # init weights
        weights = (2*np.random.rand(4,1)-1) /10
        mean_val = np.zeros(N_in)
        p = 30000
        # print weights.shape
        D_in = np.random.multivariate_normal(mean_val,cov_mat,p)
        for i in range(0,p):
            D_in_i = D_in[i:i+1,:]
        #     print D_in_i.shape
            V = np.dot(weights.T,D_in_i.T)
            error = eta * V * (D_in_i.T -  V*weights)
        #     print error.shape
        #     print np.mean(np.abs(error))
            weights += error
        print weights
```

```
[[-0.49392035]
 [-0.49219265]
 [-0.51130988]
 [-0.50250419]]
```

Check that the coverged weight has norm 1

```
In [10]: norm =np.sum(weights**2)
         print norm
```

```
1.0001591745
```

Diagonalize the covarianza matriz and find the biggest eigenvalue

```
In [11]: vals, vectors = np.linalg.eig(cov_mat)
         print vals
```

```
[ 1.   5.   1.   1.]
```

The biggest eigenvalue is 5. and the eigenvector associated with it is

```
In [12]: y  = vectors[:,1]
         print y
         print weights[:,0]
```

```
[ 0.5   0.5   0.5   0.5]
[-0.49392035 -0.49219265 -0.51130988 -0.50250419]
```

Comparando el resultado obtenido con el autovector de la matriz correspondiente al *mayor autovalor* se observa que estos tienen la misma dirección.

1.2.1 Ejercicio 2

Considere una red neuronal de Kohonen con dos neuronas de entrada. Utilice 10 neuronas de salida, dispuestas sobre una línea. Alimente a las neuronas de entrada con una distribución

Define the neighboring function

```
In [13]: def neigh(sigma):
             def neig_impl(i,k):
                 return np.exp(-(i - k )**2/(2* sigma**2))
             return neig_impl

         f1 = neigh(1)
         assert f1(1,1) == 1 # must be one for the same neurons
```

```
In [14]: def scale(initial, final):
             """ Scale an interval (x1,x2) to (y1,y2)"""
             x1, x2 = initial
             y1, y2 = final
             slope = (y2 - y1)/(x2 - x1)
             intercept  = y1 - slope * x1
             def linf(x):
                 assert x >=  x1, "invalid input value " + str(x)
                 assert x <=  x2, "invalid input value " + str(x)
                 retval = x * slope + intercept
                 assert retval >= y1
                 assert retval <= y2
                 return retval
             return linf
```

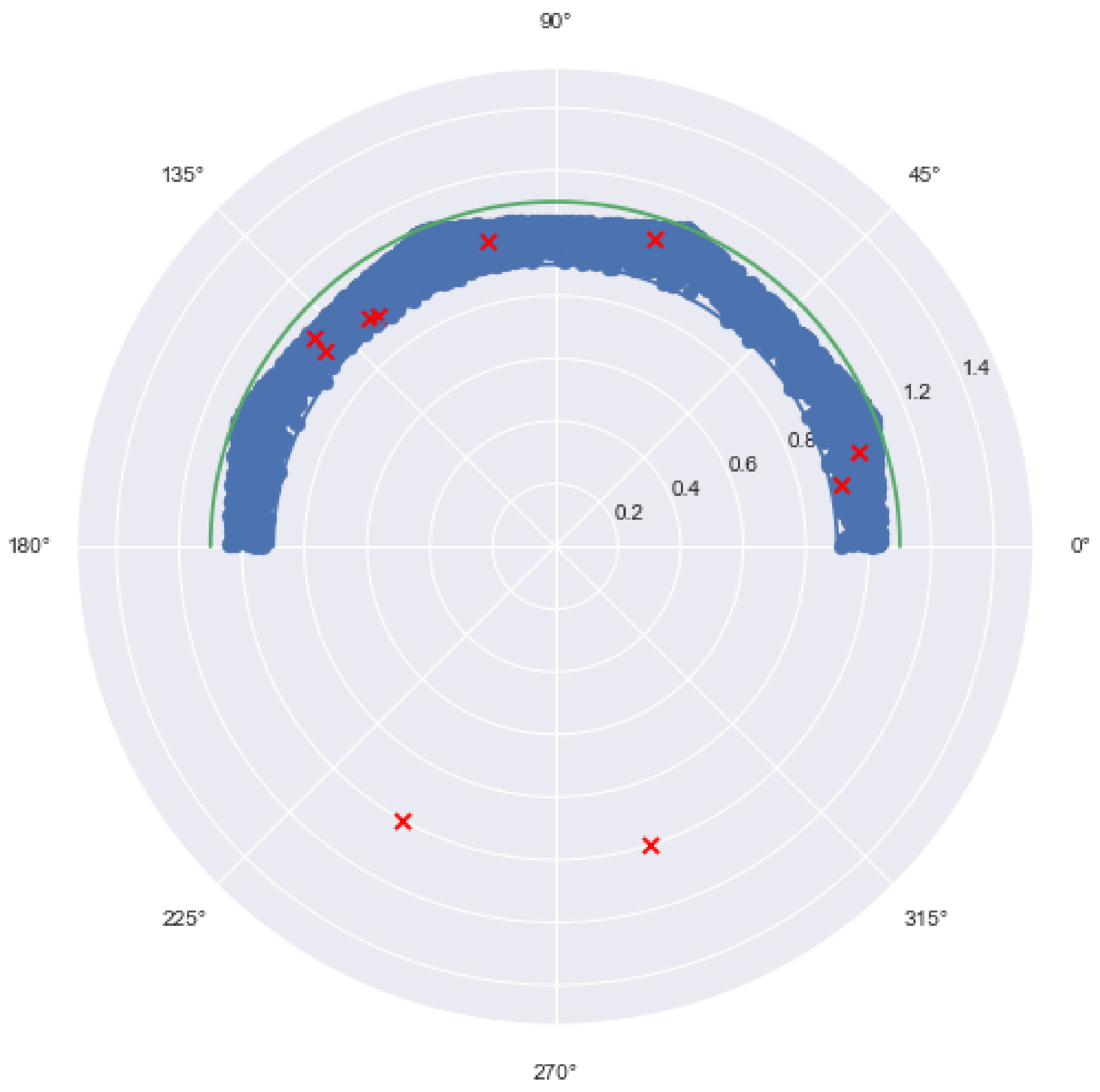
```
In [25]: def to_radial_coords(point):
             """ convert a point from linear coordiates to radial coordiantes
                 returns:  numpy array with[r,theta]
             """
             assert point.shape == (2,), "error on shape: " + str(point.shape)
             x = point[0]
             y = point[1]
             r =  np.sqrt(x**2 + y**2)
             assert r <= np.sqrt(2), point
             angle =  np.arctan2(y,x)
             return np.array([r,angle])
         def to_linear_coords(point):
             r = point[0]
             theta = point[1]
             x = r * np.cos(theta)
             y = r * np.sin(theta)
             return np.array([x,y])
```

```
In [115]: p = 3000 # num datapoints
          rmin = 0.9
          rmax = 1.1

          # define the dimensions of the input and output layer
          N1 = 2
          N2 = 10
          # initalize the weights
          weights = 2*np.random.rand(N1,N2)-1
          norms =  np.linalg.norm(weights,axis=0)
          # normalize the weights
          for i in range(0,N2):
              weights[:,i] = weights[:,i] / norms[i]

          data_points = (2*np.random.rand(N1,p) -1)
```

```
        # convert to radial coords
        radial_scale = scale((0,np.sqrt(2)), (rmin,rmax))
        angular_scale = scale((-np.pi,np.pi), (0, np.pi))
        for i in range(0,p):
            data_points[:,i] = to_radial_coords(data_points[:,i])
            data_points[0,i] = radial_scale(data_points[0,i])
            data_points[1,i] = angular_scale(data_points[1,i])

        assert np.max(data_points[0, :]) <= np.sqrt(2) ,np.max(data_points[0, :])
        assert np.min(data_points[0, :]) >= rmin
        assert np.min(data_points[1, :]) >= -np.pi
        assert np.max(data_points[0, :]) <= rmax, "error for max boundary " + str
        assert np.max(data_points[1, :]) <= np.pi

        neighf = neigh(0.1)

        for i in range(0,p):
            x = to_linear_coords(data_points[:,i])
            h = np.dot(weights.T,x)
            k = np.argmax(h) # get the pos of the max element
            for j in range(0,N2):
                error =  neighf(j,k)*(x - weights[:,j])
                weights[:,j] += error
        for i in range(0,N2):
            weights[:,i] = to_radial_coords(weights[:,i])
```

Plot the input data The data is distributed in an arc from 0 to π

```
In [117]: fig = plt.figure(figsize=(8,8))
          fig.clear()
          p1 = fig.add_subplot(111, projection='polar')
          p1.scatter(data_points[1,:], data_points[0,:])
          p1.scatter(weights[1,:], weights[0,:],color='r', marker='x')
          plt.plot(np.pi*np.linspace(0,1,100), rmin*np.ones(100))
          plt.plot(np.pi*np.linspace(0,1,100), rmax*np.ones(100))
          plt.show()
```



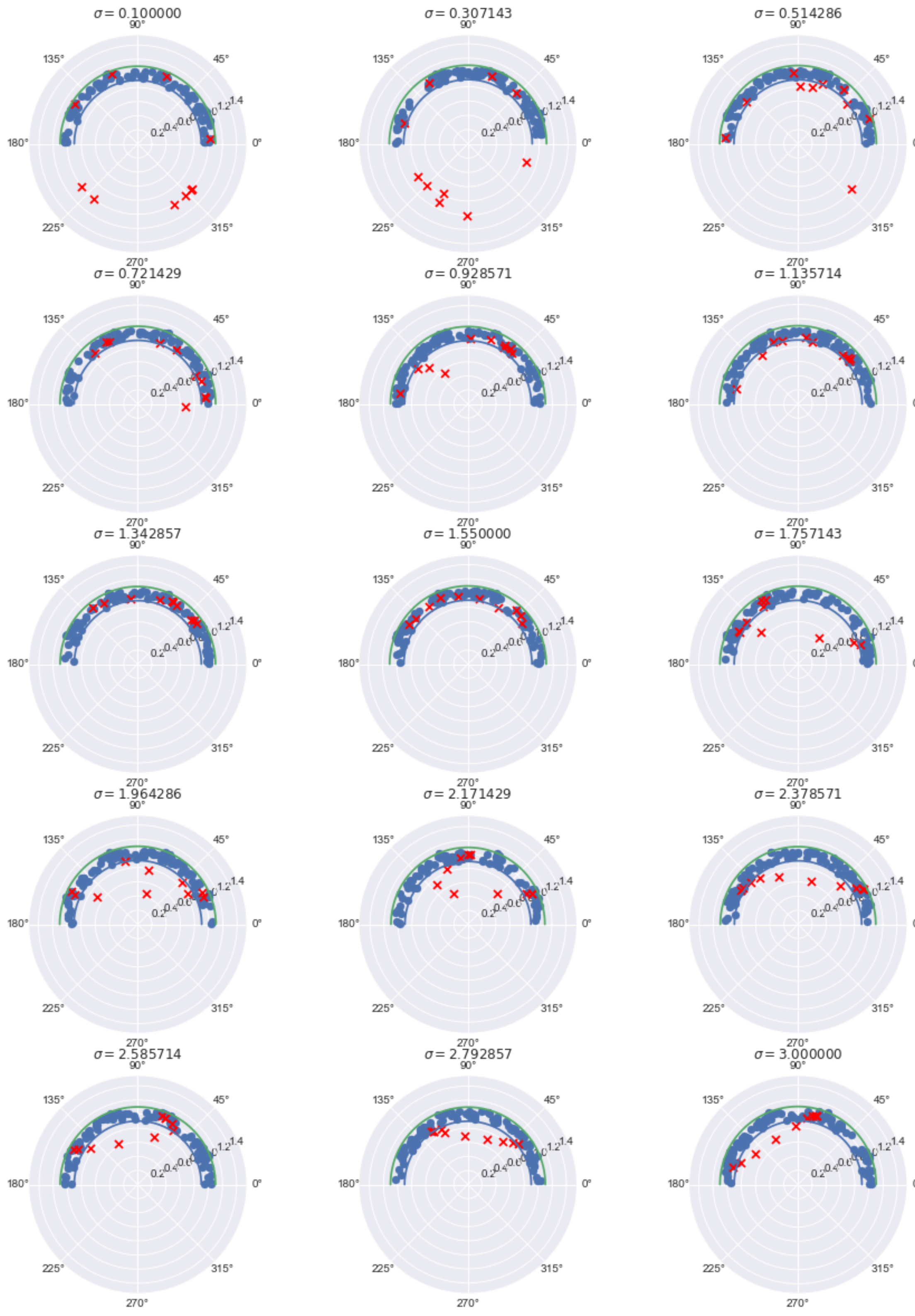
Now wrap everything inside a function with p and σ as parameters

```
In [118]: def kohonen_simulation(sigma,p):
              rmin = 0.9
              rmax = 1.1

              # define the dimensions of the input and output layer
              N1 = 2
              N2 = 10
              # initalize the weights
              weights = 2*np.random.rand(N1,N2)-1
              norms =  np.linalg.norm(weights,axis=0)
              # normalize the weights
              for i in range(0,N2):
```

```
            weights[:,i] = weights[:,i] / norms[i]

        data_points = (2*np.random.rand(N1,p) -1)
        # convert to radial coords
        radial_scale = scale((0,np.sqrt(2)), (rmin,rmax))
        angular_scale = scale((-np.pi,np.pi), (0, np.pi))
        for i in range(0,p):
            data_points[:,i] = to_radial_coords(data_points[:,i])
            data_points[0,i] = radial_scale(data_points[0,i])
            data_points[1,i] = angular_scale(data_points[1,i])

        assert np.max(data_points[0, :]) <= np.sqrt(2) ,np.max(data_points[0,
        assert np.min(data_points[0, :]) >= rmin
        assert np.min(data_points[1, :]) >= -np.pi
        assert np.max(data_points[0, :]) <= rmax, "error for max boundary " +
        assert np.max(data_points[1, :]) <= np.pi

        neighf = neigh(sigma)

        for i in range(0,p):
            x = to_linear_coords(data_points[:,i])
            h = np.dot(weights.T,x)
            k = np.argmax(h) # get the pos of the max element
            for j in range(0,N2):
                error =  neighf(j,k)*(x - weights[:,j])
                weights[:,j] += error

        for i in range(0,N2):
            weights[:,i] = to_radial_coords(weights[:,i])
        return  data_points, weights

    def plot_simulation_results(p1,data_points, weights,sigma):
        """ Make a polar graph of the simulation results"""

        p1.scatter(data_points[1,:], data_points[0,:],marker= 'o')
        p1.set_title( r'$\sigma= %f$'%(sigma))
        p1.scatter(weights[1,:], weights[0,:],color='r', marker= 'x')

        plt.plot(np.pi*np.linspace(0,1,100), rmin*np.ones(100))
        plt.plot(np.pi*np.linspace(0,1,100), rmax*np.ones(100))
```

```
In [119]: n = 15
          sigmas = np.linspace(0.1,3,n)
          p = 100
          fig = plt.figure(figsize=(15,25))
          fig.clear()
          for k in range(0,n):
              data_points, weights = kohonen_simulation(sigmas[k],p)
```

```
    axes = fig.add_subplot(int(n/3) + 1,3,k+1, projection='polar')
    plot_simulation_results(axes,data_points, weights,sigmas[k])

plt.show()
```



En el grafico de arriba se muestra el resultado del aprendizaje para diferentes calores de σ. Se puede ver que para valores pequeños y grandes se obtiene un resultado peor, mientras que para valores cercanos a 1 se obtienen los mejores resultados.