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(54) LEARNING ROBUST PREDICTORS USING GAME THEORY

(71) Applicant: **INTERNATIONAL BUSINESS MACHINES CORPORATION**, Armonk, NY (US)

(72) Inventors: **Kartik Ahuja**, White Plains, NY (US); **Karthikeyan Shanmugam**, Elmsford, NY (US); **Kush Raj Varshney**, Ossining, NY (US); **Amit Dhurandhar**, Yorktown Heights, NY (US)

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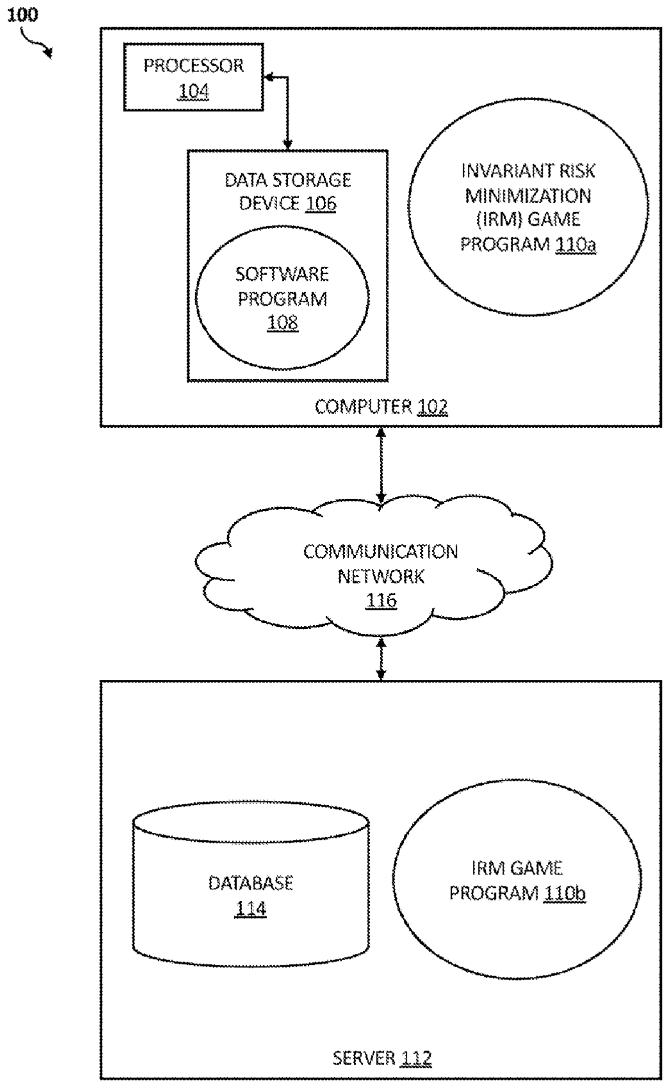
(57) **ABSTRACT**

A method, computer system, and a computer program product for invariant risk minimization games is provided. The present invention may include defining a plurality of environment-specific classifiers corresponding to a plurality of environments. The present invention may also include constructing an ensemble classifier associated with the plurality of environment-specific classifiers. The present invention may further include initiating a game including a plurality of players corresponding to the plurality of environments. The present invention may also include calculating a nash equilibrium of the initiated game. The present invention may further include determining an ensemble predictor based on the calculated nash equilibrium. The present invention may include deploying the determined ensemble predictor associated with the calculated nash equilibrium to make predictions in a new environment.



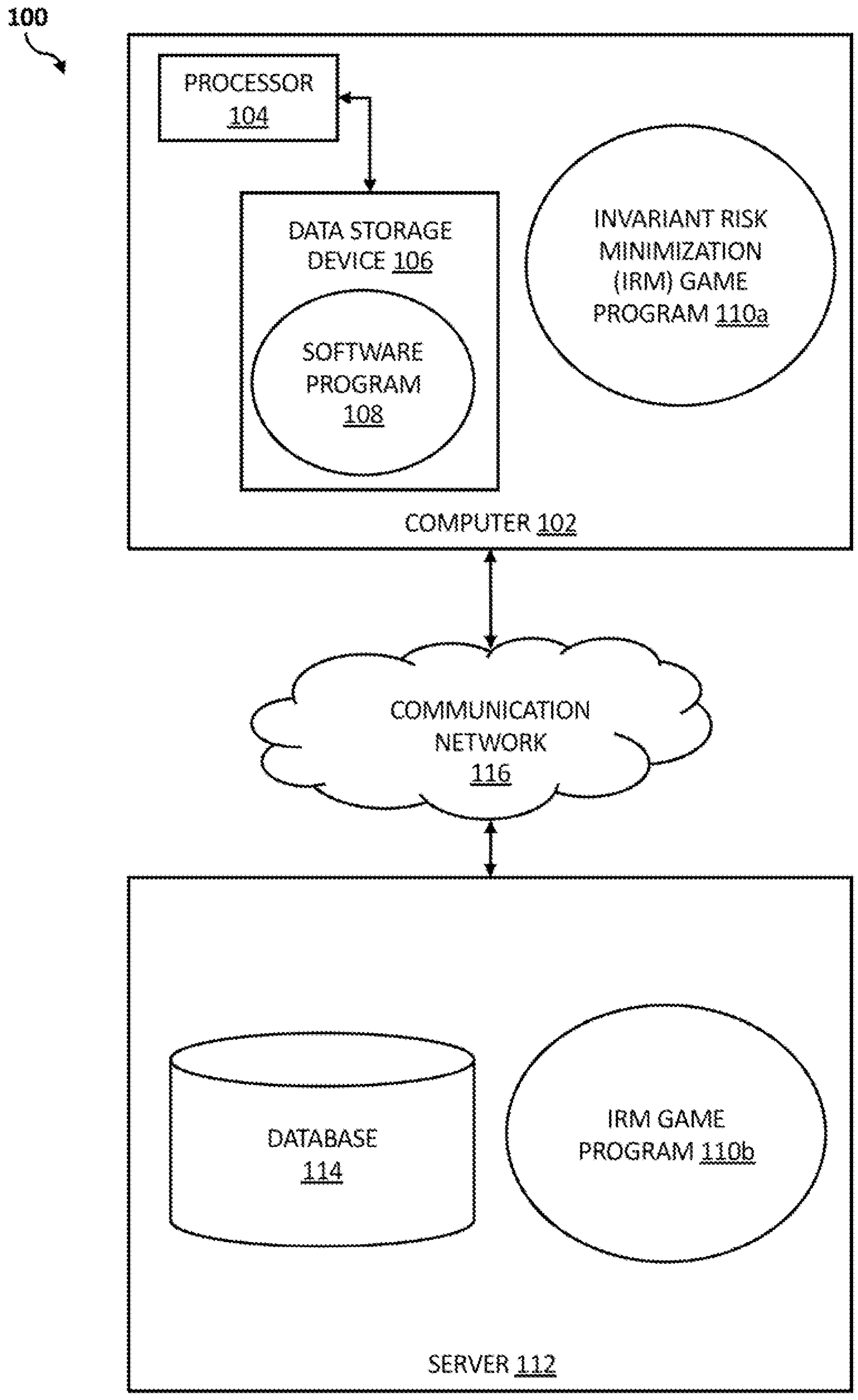

100
PROCESSOR 104
DATA STORAGE DEVICE 106
SOFTWARE PROGRAM 108
INVARIANT RISK MINIMIZATION (IRM) GAME PROGRAM 110a
COMPUTER 102
COMMUNICATION NETWORK 116
DATABASE 114
IRM GAME PROGRAM 110b
SERVER 112

FIG. 1

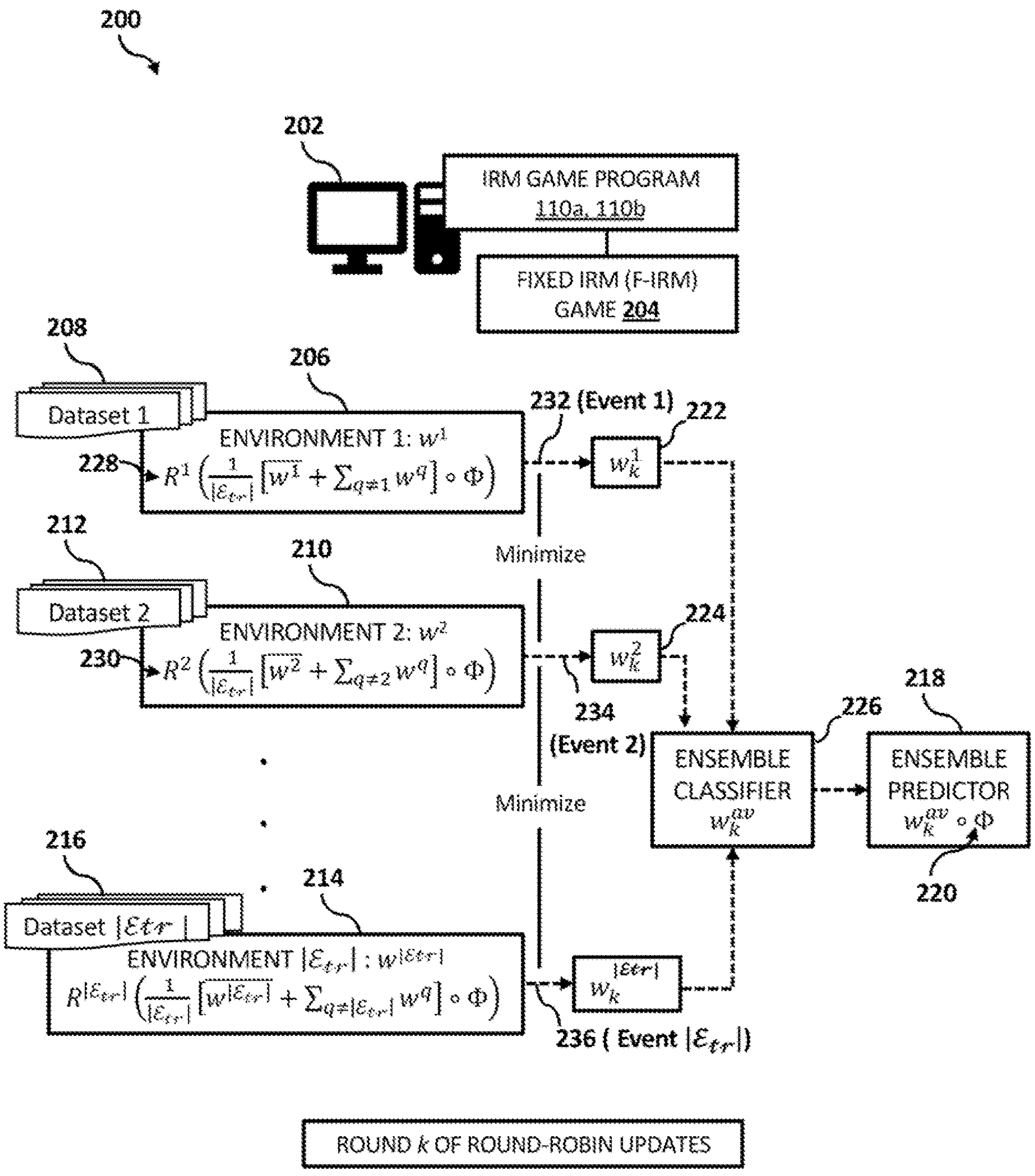

200
202
IRM GAME PROGRAM 110a, 110b
FIXED IRM (F-IRM) GAME 204
208
Dataset 1
206
ENVIRONMENT 1: w^1
228
$R^1\left(\frac{1}{|\mathcal{E}_{tr}|}\left[\overline{w^1}+\sum_{q\neq 1} w^q\right]\circ\Phi\right)$
232 (Event 1)
222
w_k^1
Minimize
212
Dataset 2
210
ENVIRONMENT 2: w^2
230
$R^2\left(\frac{1}{|\mathcal{E}_{tr}|}\left[\overline{w^2}+\sum_{q\neq 2} w^q\right]\circ\Phi\right)$
224
w_k^2
234 (Event 2)
Minimize
226
ENSEMBLE CLASSIFIER w_k^{av}
218
ENSEMBLE PREDICTOR $w_k^{av}\circ\Phi$
220
216
Dataset $|\mathcal{E}tr|$
214
ENVIRONMENT $|\mathcal{E}_{tr}|$: $w^{|\mathcal{E}tr|}$
$R^{|\mathcal{E}_{tr}|}\left(\frac{1}{|\mathcal{E}_{tr}|}\left[\overline{w^{|\mathcal{E}_{tr}|}}+\sum_{q\neq|\mathcal{E}_{tr}|} w^q\right]\circ\Phi\right)$
$w_k^{|\mathcal{E}tr|}$
236 (Event $|\mathcal{E}_{tr}|$)
ROUND k OF ROUND-ROBIN UPDATES

FIG. 2

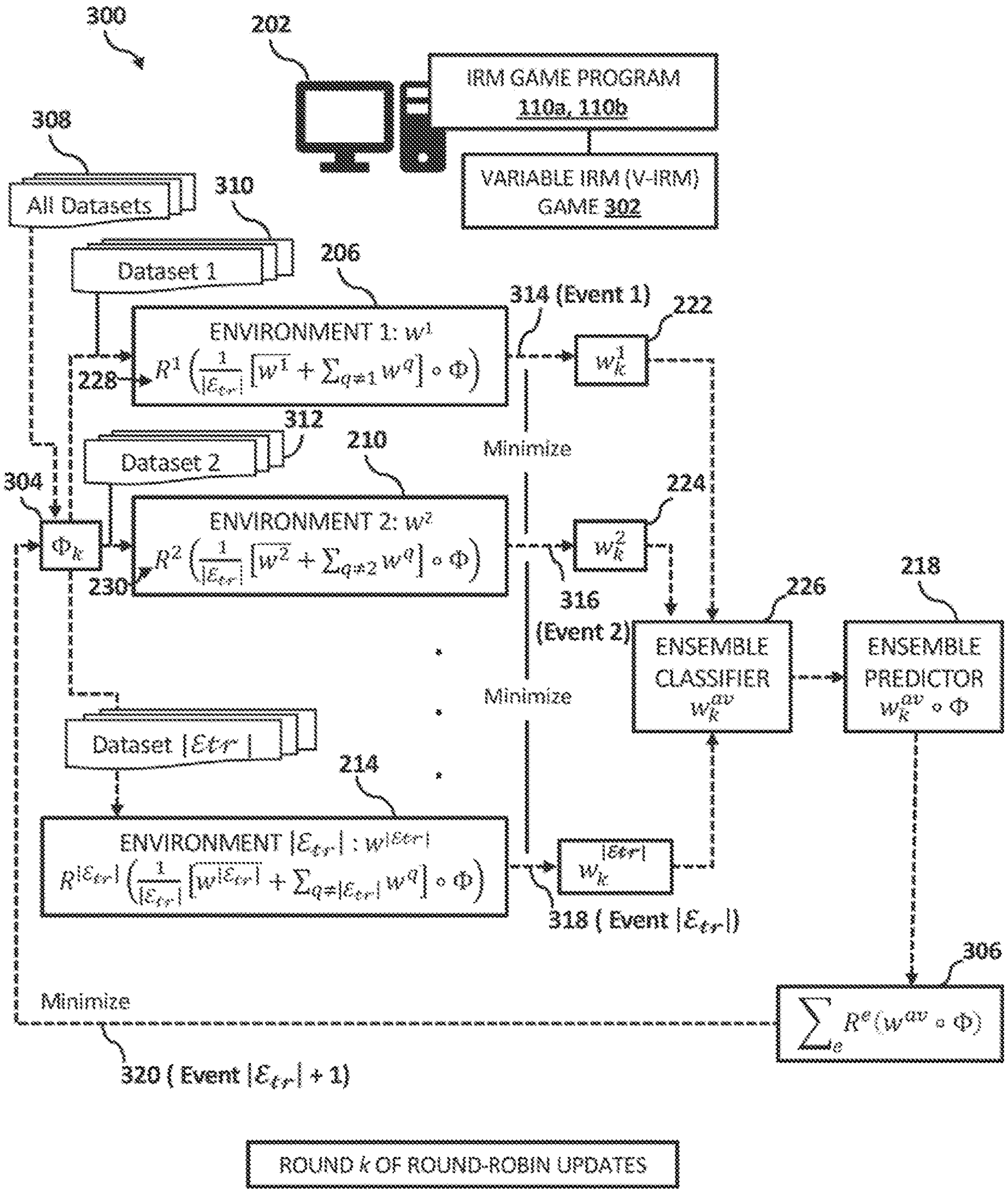

300
202
IRM GAME PROGRAM 110a, 110b
VARIABLE IRM (V-IRM) GAME 302
308
All Datasets
310
Dataset 1
206
ENVIRONMENT 1: w^1
$R^1\left(\frac{1}{|\mathcal{E}_{tr}|}\left[w^1+\sum_{q\neq 1} w^q\right]\circ\Phi\right)$
314 (Event 1)
222
w_k^1
228
312
Dataset 2
210
Minimize
304
Φ_k
ENVIRONMENT 2: w^2
$R^2\left(\frac{1}{|\mathcal{E}_{tr}|}\left[w^2+\sum_{q\neq 2} w^q\right]\circ\Phi\right)$
224
w_k^2
230
316 (Event 2)
226
ENSEMBLE CLASSIFIER w_k^{av}
218
ENSEMBLE PREDICTOR $w_k^{av}\circ\Phi$
Minimize
Dataset $|\mathcal{E}tr|$
214
ENVIRONMENT $|\mathcal{E}_{tr}|$: $w^{|\mathcal{E}tr|}$
$R^{|\mathcal{E}_{tr}|}\left(\frac{1}{|\mathcal{E}_{tr}|}\left[w^{|\mathcal{E}_{tr}|}+\sum_{q\neq|\mathcal{E}_{tr}|} w^q\right]\circ\Phi\right)$
$w_k^{|\mathcal{E}tr|}$
318 (Event $|\mathcal{E}_{tr}|$)
306
$\sum_e R^e(w^{av}\circ\Phi)$
Minimize
320 (Event $|\mathcal{E}_{tr}|$ + 1)
ROUND k OF ROUND-ROBIN UPDATES

FIG. 3

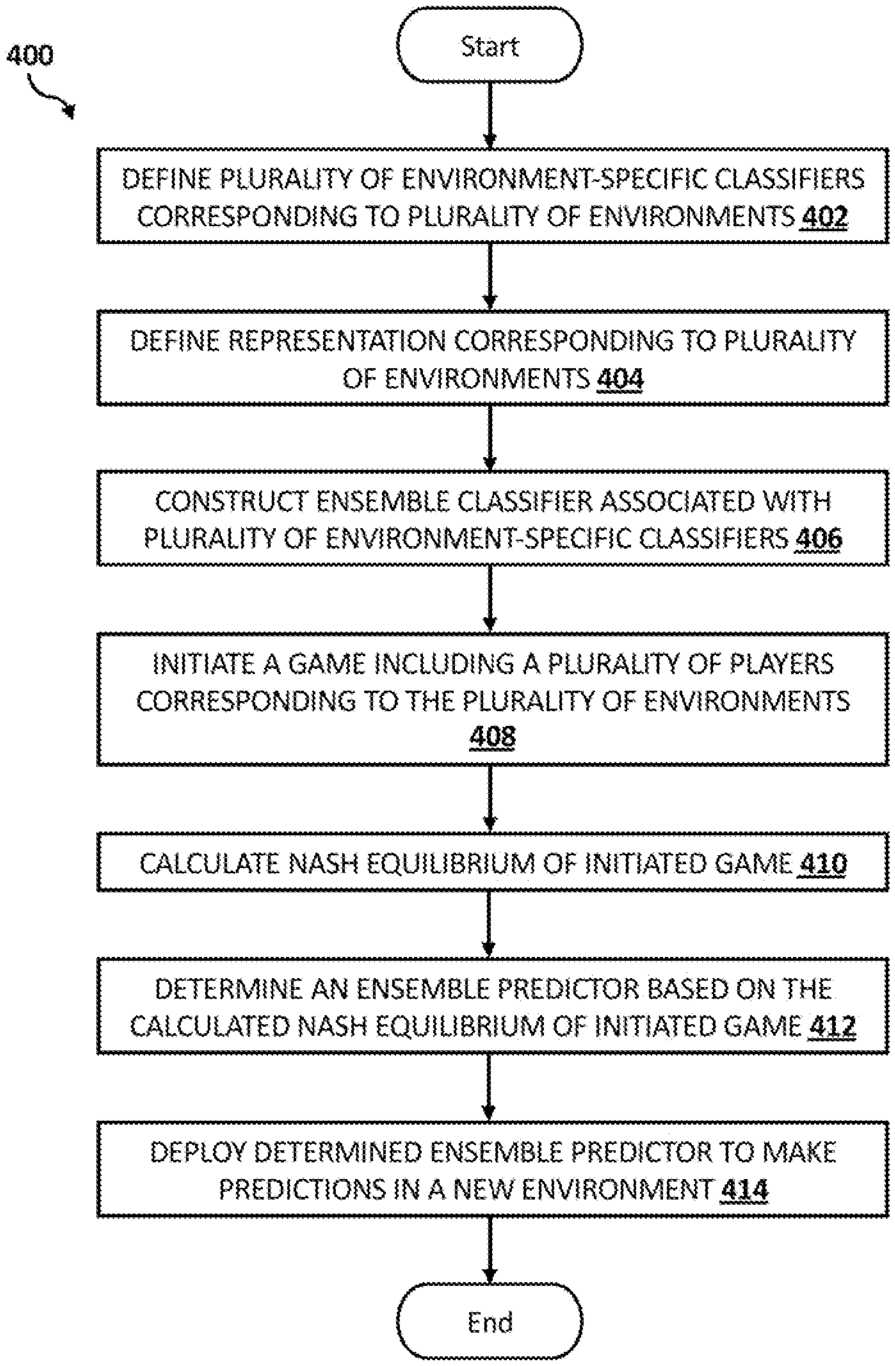

400
Start
DEFINE PLURALITY OF ENVIRONMENT-SPECIFIC CLASSIFIERS CORRESPONDING TO PLURALITY OF ENVIRONMENTS 402
DEFINE REPRESENTATION CORRESPONDING TO PLURALITY OF ENVIRONMENTS 404
CONSTRUCT ENSEMBLE CLASSIFIER ASSOCIATED WITH PLURALITY OF ENVIRONMENT-SPECIFIC CLASSIFIERS 406
INITIATE A GAME INCLUDING A PLURALITY OF PLAYERS CORRESPONDING TO THE PLURALITY OF ENVIRONMENTS 408
CALCULATE NASH EQUILIBRIUM OF INITIATED GAME 410
DETERMINE AN ENSEMBLE PREDICTOR BASED ON THE CALCULATED NASH EQUILIBRIUM OF INITIATED GAME 412
DEPLOY DETERMINED ENSEMBLE PREDICTOR TO MAKE PREDICTIONS IN A NEW ENVIRONMENT 414
End

FIG. 4

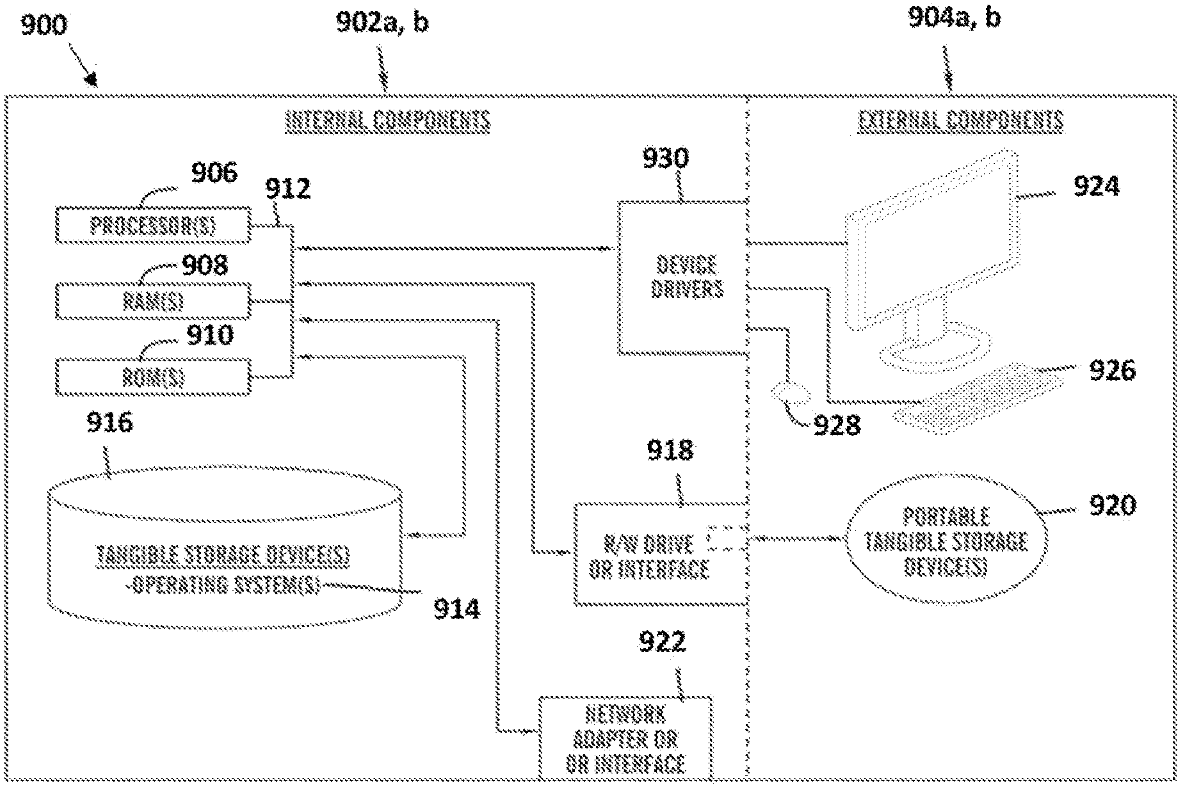

900
902a, b
904a, b
INTERNAL COMPONENTS
EXTERNAL COMPONENTS
906
912
PROCESSOR(S)
908
RAM(S)
910
ROM(S)
916
TANGIBLE STORAGE DEVICE(S)
-OPERATING SYSTEM(S)
914
930
DEVICE DRIVERS
918
R/W DRIVE OR INTERFACE
922
NETWORK ADAPTER OR OR INTERFACE
924
926
928
920
PORTABLE TANGIBLE STORAGE DEVICE(S)

FIG. 5

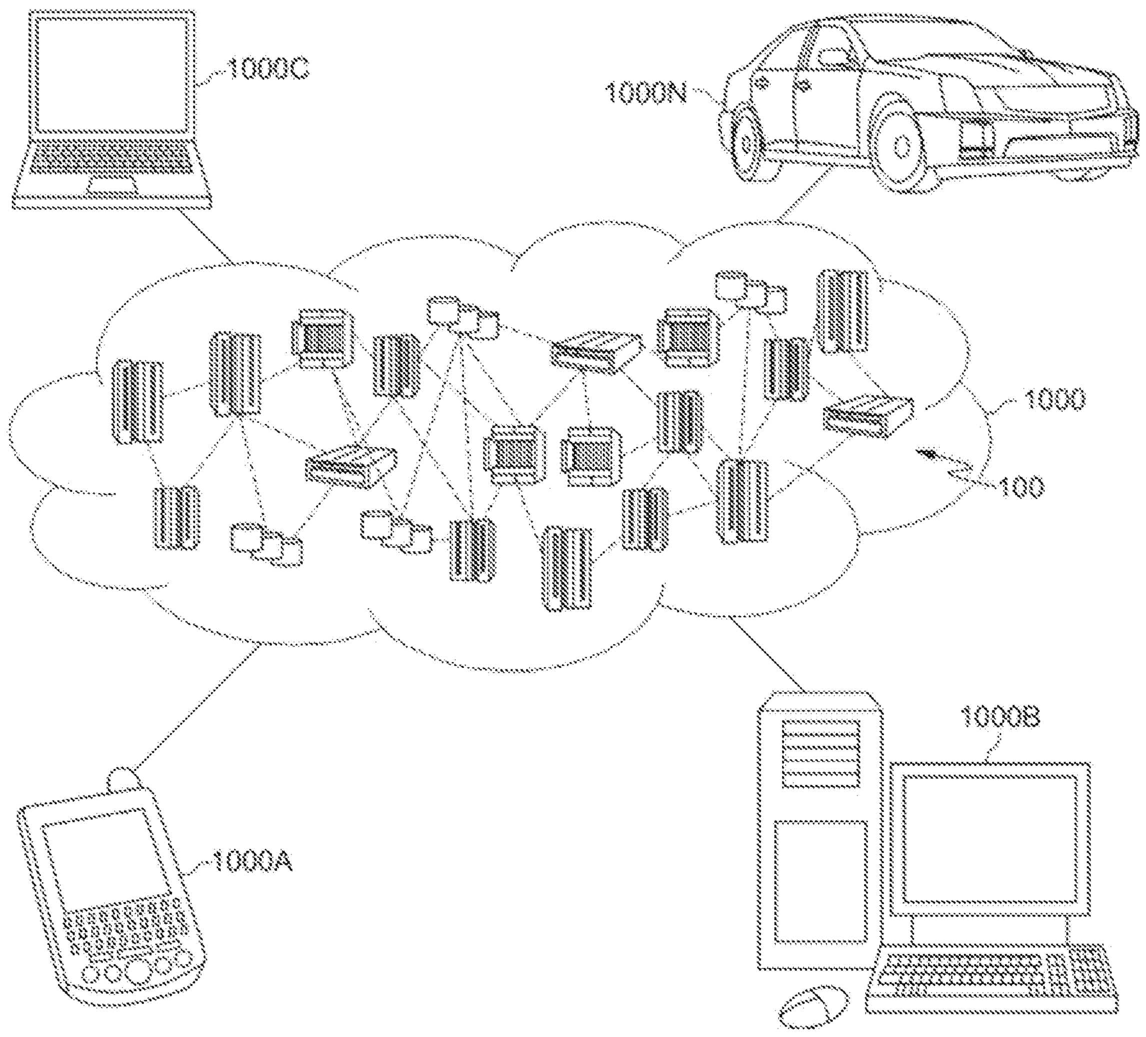


FIG. 6

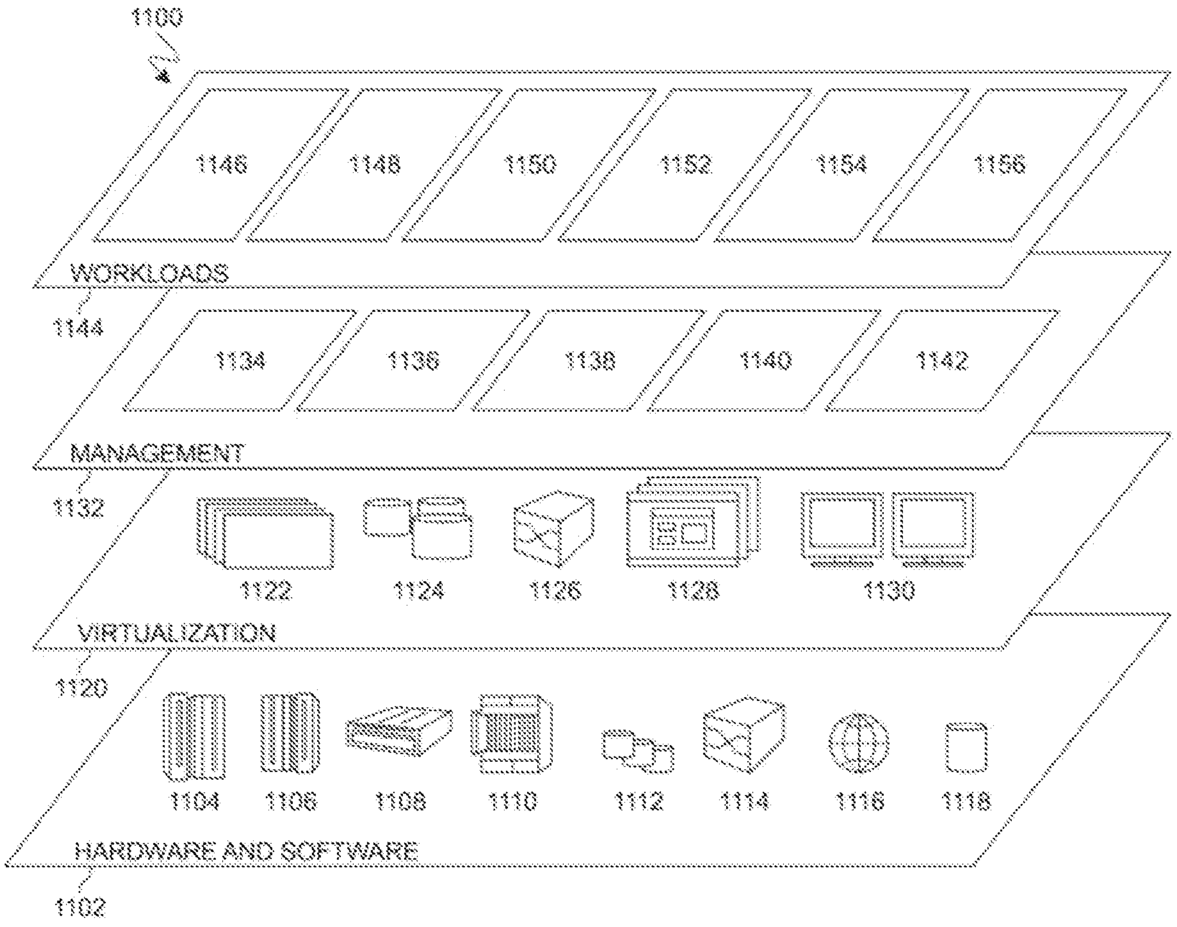

1100
1146
1148
1150
1152
1154
1156
WORKLOADS
1144
1134
1136
1138
1140
1142
MANAGEMENT
1132
1122
1124
1126
1128
1130
VIRTUALIZATION
1120
1104
1106
1108
1110
1112
1114
1116
1118
HARDWARE AND SOFTWARE
1102

FIG. 7

LEARNING ROBUST PREDICTORS USING GAME THEORY

STATEMENT REGARDING PRIOR DISCLOSURES BY THE INVENTOR OR A JOINT INVENTOR

[0001] The following disclosure is submitted under 35 U.S.C. § 102(b)(1)(A): "Invariant Risk Minimization Games," Kartik Ahuja, Karthikeyan Shanmugam, Kush Varshney, and Amit Dhurandhar, Feb. 11, 2020.

BACKGROUND

[0002] The present invention relates generally to the field of computing, and more particularly to machine learning.

[0003] Machine learning is a field of artificial intelligence (AI) which enables computers to learn from, and make decisions, based on data without being explicitly programmed. Generally, a machine learning algorithm may be used to build a model which maps known inputs to unknown outputs in a set of training data. In environments in which the distribution between the training data and testing data are the same and/or similar, current models—based on the standard risk minimization paradigm—may perform well. However, these models, which often rely on learning spurious features, may fail to produce accurate predictions in environments where the distribution between the training data and testing data are different.

SUMMARY

[0004] Embodiments of the present invention disclose a method, computer system, and a computer program product for invariant risk minimization games. The present invention may include defining a plurality of environment-specific classifiers corresponding to a plurality of environments. The present invention may also include constructing an ensemble classifier associated with the plurality of environment-specific classifiers. The present invention may further include initiating a game including a plurality of players corresponding to the plurality of environments. The present invention may also include calculating a nash equilibrium of the initiated game. The present invention may further include determining an ensemble predictor based on the calculated nash equilibrium. The present invention may include deploying the determined ensemble predictor associated with the calculated nash equilibrium to make predictions in a new environment.

BRIEF DESCRIPTION OF THE SEVERAL VIEWS OF THE DRAWINGS

[0005] These and other objects, features and advantages of the present invention will become apparent from the following detailed description of illustrative embodiments thereof, which is to be read in connection with the accompanying drawings. The various features of the drawings are not to scale as the illustrations are for clarity in facilitating one skilled in the art in understanding the invention in conjunction with the detailed description. In the drawings:

[0006] FIG. **1** illustrates a networked computer environment according to at least one embodiment;

[0007] FIG. **2** is a schematic block diagram of a first machine learning environment according to at least one embodiment;

[0008] FIG. **3** is a schematic block diagram of a second machine learning environment according to at least one embodiment;

[0009] FIG. **4** is an operational flowchart illustrating a process for best response training according to at least one embodiment;

[0010] FIG. **5** is a block diagram of internal and external components of computers and servers depicted in FIG. **1** according to at least one embodiment;

[0011] FIG. **6** is a block diagram of an illustrative cloud computing environment including the computer system depicted in FIG. **1**, in accordance with an embodiment of the present disclosure; and

[0012] FIG. **7** is a block diagram of functional layers of the illustrative cloud computing environment of FIG. **6**, in accordance with an embodiment of the present disclosure.

DETAILED DESCRIPTION

[0013] Detailed embodiments of the claimed structures and methods are disclosed herein; however, it can be understood that the disclosed embodiments are merely illustrative of the claimed structures and methods that may be embodied in various forms. This invention may, however, be embodied in many different forms and should not be construed as limited to the exemplary embodiments set forth herein. Rather, these exemplary embodiments are provided so that this disclosure will be thorough and complete and will fully convey the scope of this invention to those skilled in the art. In the description, details of well-known features and techniques may be omitted to avoid unnecessarily obscuring the presented embodiments.

[0014] The present invention may be a system, a method, and/or a computer program product at any possible technical detail level of integration. The computer program product may include a computer readable storage medium (or media) having computer readable program instructions thereon for causing a processor to carry out aspects of the present invention.

[0015] The computer readable storage medium can be a tangible device that can retain and store instructions for use by an instruction execution device. The computer readable storage medium may be, for example, but is not limited to, an electronic storage device, a magnetic storage device, an optical storage device, an electromagnetic storage device, a semiconductor storage device, or any suitable combination of the foregoing. A non-exhaustive list of more specific examples of the computer readable storage medium includes the following: a portable computer diskette, a hard disk, a random access memory (RAM), a read-only memory (ROM), an erasable programmable read-only memory (EPROM or Flash memory), a static random access memory (SRAM), a portable compact disc read-only memory (CD-ROM), a digital versatile disk (DVD), a memory stick, a floppy disk, a mechanically encoded device such as punch-cards or raised structures in a groove having instructions recorded thereon, and any suitable combination of the foregoing. A computer readable storage medium, as used herein, is not to be construed as being transitory signals per se, such as radio waves or other freely propagating electromagnetic waves, electromagnetic waves propagating through a waveguide or other transmission media (e.g., light pulses passing through a fiber-optic cable), or electrical signals transmitted through a wire.

[0016] Computer readable program instructions described herein can be downloaded to respective computing/processing devices from a computer readable storage medium or to an external computer or external storage device via a network, for example, the Internet, a local area network, a wide area network and/or a wireless network. The network may comprise copper transmission cables, optical transmission fibers, wireless transmission, routers, firewalls, switches, gateway computers and/or edge servers. A network adapter card or network interface in each computing/processing device receives computer readable program instructions from the network and forwards the computer readable program instructions for storage in a computer readable storage medium within the respective computing/processing device.

[0017] Computer readable program instructions for carrying out operations of the present invention may be assembler instructions, instruction-set-architecture (ISA) instructions, machine instructions, machine dependent instructions, microcode, firmware instructions, state-setting data, configuration data for integrated circuitry, or either source code or object code written in any combination of one or more programming languages, including an object oriented programming language such as Smalltalk, Python, C++, or the like, and procedural programming languages, such as the "C" programming language or similar programming languages. The computer readable program instructions may execute entirely on the user's computer, partly on the user's computer, as a stand-alone software package, partly on the user's computer and partly on a remote computer or entirely on the remote computer or server. In the latter scenario, the remote computer may be connected to the user's computer through any type of network, including a local area network (LAN) or a wide area network (WAN), or the connection may be made to an external computer (for example, through the Internet using an Internet Service Provider). In some embodiments, electronic circuitry including, for example, programmable logic circuitry, field-programmable gate arrays (FPGA), or programmable logic arrays (PLA) may execute the computer readable program instructions by utilizing state information of the computer readable program instructions to personalize the electronic circuitry, in order to perform aspects of the present invention.

[0018] Aspects of the present invention are described herein with reference to flowchart illustrations and/or block diagrams of methods, apparatus (systems), and computer program products according to embodiments of the invention. It will be understood that each block of the flowchart illustrations and/or block diagrams, and combinations of blocks in the flowchart illustrations and/or block diagrams, can be implemented by computer readable program instructions.

[0019] These computer readable program instructions may be provided to a processor of a general purpose computer, special purpose computer, or other programmable data processing apparatus to produce a machine, such that the instructions, which execute via the processor of the computer or other programmable data processing apparatus, create means for implementing the functions/acts specified in the flowchart and/or block diagram block or blocks. These computer readable program instructions may also be stored in a computer readable storage medium that can direct a computer, a programmable data processing apparatus, and/or other devices to function in a particular manner, such that the computer readable storage medium having instructions stored therein comprises an article of manufacture including instructions which implement aspects of the function/act specified in the flowchart and/or block diagram block or blocks.

[0020] The computer readable program instructions may also be loaded onto a computer, other programmable data processing apparatus, or other device to cause a series of operational steps to be performed on the computer, other programmable apparatus or other device to produce a computer implemented process, such that the instructions which execute on the computer, other programmable apparatus, or other device implement the functions/acts specified in the flowchart and/or block diagram block or blocks.

[0021] The flowchart and block diagrams in the Figures illustrate the architecture, functionality, and operation of possible implementations of systems, methods, and computer program products according to various embodiments of the present invention. In this regard, each block in the flowchart or block diagrams may represent a module, segment, or portion of instructions, which comprises one or more executable instructions for implementing the specified logical function(s). In some alternative implementations, the functions noted in the blocks may occur out of the order noted in the Figures. For example, two blocks shown in succession may, in fact, be accomplished as one step, executed concurrently, substantially concurrently, in a partially or wholly temporally overlapping manner, or the blocks may sometimes be executed in the reverse order, depending upon the functionality involved. It will also be noted that each block of the block diagrams and/or flowchart illustration, and combinations of blocks in the block diagrams and/or flowchart illustration, can be implemented by special purpose hardware-based systems that perform the specified functions or acts or carry out combinations of special purpose hardware and computer instructions.

[0022] The following described exemplary embodiments provide a system, method and program product for game-theoretic machine learning algorithms. As such, the present embodiment has the capacity to improve the technical field of machine learning by training on data from many environments and finding invariant predictors which may reduce the effect of spurious features by concentrating models on features that have a causal relationship with the outcome. Accordingly, a computing device may be utilized to train a machine learning model which operates invariantly in a plurality of environments. More specifically, a computing device may define one or more classifiers specific to one or more environments, of a plurality of environments. Then, the computing device may construct one or more ensemble classifiers related to all of the one or more classifiers specific to the one or more environments. Next, the computing device may define a game, the game having as players each of the environments of the plurality of environments. Then, the computing device may determine, using best response dynamics, a Nash equilibrium of the game. Thereafter, the computing device, may determine an ensemble predictor at the Nash equilibrium. The computing device may then deploy the ensemble predictor to make predictions in a new environment including unseen data.

[0023] As described previously, a machine learning algorithm may be used to build a model which maps known inputs to unknown outputs in a set of training data. In environments in which the distribution between the training data and testing data are the same and/or similar, current models—based on the standard risk minimization paradigm—may perform well. However, these models, which often rely on learning spurious features, may fail to produce accurate predictions in environments where the distribution between the training data and testing data are different.

[0024] The annals of machine learning are rife with examples of spurious correlations that fail to hold outside of a specific training (and identically distributed test) distribution. In one such case, a convolutional neural network (CNN) was trained to classify camels from cows. However, the training dataset had one source of bias, i.e., most of the pictures of cows were in green pastures, while most pictures of camels were in deserts. The CNN picked up the spurious correlation, i.e., it associated green pastures with cows and failed to classify pictures of cows on sandy beaches correctly. In another case, a neural network used a brake light indicator to continue applying brakes, which was a spurious correlation in the training data.

[0025] To address the problem of models inheriting spurious correlations, one existing technique tries to exploit the varying degrees of spurious correlation naturally present in data collected from multiple data sources to learn robust predictors. The goal of this existing technique is to find a representation such that an optimal classifier, given the representation, is invariant across training environments. However, this formulation leads to a challenging bi-level optimization problem, which is relaxed by limiting the classifier to a simple linear classifier and learning a representation such that the classifier is "approximately locally optimal" in all the training environments.

[0026] Therefore, it may be advantageous to, among other things, provide a way to generate predictors, based on classifiers that are not restricted to linear classifiers, which may operate invariantly in a large number of environments. It may also be advantageous to generate predictors providing better performance with much lower standard deviation than existing techniques.

[0027] According to at least one embodiment, an ensemble of classifiers may be created with each environment controlling one component of the ensemble. Each environment may use the entire ensemble to make predictions. According to at least one embodiment, all the environments may play a game where each environment's action is to decide its contribution to the ensemble such that the environment minimizes its risk. In disclosed embodiments, the set of predictors that solve the ensemble game may be equal to the set of invariant predictors across the training environments. This result holds for a large class of nonlinear classifiers.

[0028] According to at least one embodiment, a training algorithm may be used to solve the game by implementing best response dynamics. Under this implementation, each environment may periodically take its turn and move its classifier in the direction that minimizes the risk specific to that environment. Empirically, the invariant predictors found using this approach may lead to better or comparable performance with much lower standard deviation than any existing methods on several different datasets. Moreover, as noted previously, the disclosed embodiments do not restrict classifiers to be linear.

[0029] Statistical machine learning has dealt with the distribution shift between the training distribution and test distribution in a number of ways. Conventional approaches may include sample weighting, domain adaptation, and robust optimization. Importance weighting, or more generally sample weighting, may attempt to match training and testing distributions by reweighting samples. It typically assumes that the probability of labels given all covariates does not shift, and in more general cases, requires access to test labels. Domain adaptation tries to find a representation whose distribution is invariant across source and target domains. Domain adaptation is known to have serious limitations even when the distribution of labels shifts marginally across environments. When only training data sources are given, robust optimization techniques find the worst-case loss over all possible convex combinations of the training sources. This assumes that the test distribution is within the convex hull of training distributions, which is not true in many settings.

[0030] Embodiments of the present disclosure may implement various concepts of game theory, as will be further discussed. In one embodiment, a standard normal form game may be implemented. A standard normal form game may be represented by a tuple: $\Gamma=(N, \{S_i\}_{(i\in N)}, \{u_i\}_{(i\in N)})$, where N is a finite set of players. Player $i\in N$ may take actions from a strategy set S_i. The utility of player i may be represented by $u_i:S\rightarrow\mathbb{R}$, where the joint set $S=\Pi_{i\in N}S_i$. The joint strategy of all the players may be given as $s\in S$, the strategy of player i may be represented by s_i and the strategy of the rest of players may be represented by $s_{-i}=(s_{i'})_{i'\neq i}$. If the set of strategies S is finite, then the game Γ may be identified as a finite game. However, if the set of strategies S is uncountably infinite, then the game Γ may be identified as a continuous game. According to one embodiment, a strategy s^* may be a pure strategy Nash equilibrium (NE) if it satisfies:

$$u_i(s_i^*,s_{(-i)}^*)\geq u_i(k,s_{(-i)}^*),\forall k\in S_i,\forall i\in N$$

[0031] Embodiments of the present disclosure may also implement various concepts of invariant risk minimization (IRM). IRM may consider datasets $\{(x_i^e,y_i^e)\}_{i=1}^{n_e}$ from multiple training environments $e\in\varepsilon_{tr}$. The feature value may be represented by $x_i^e\in X$ and the corresponding labels may be represented by $y_i^e\in\mathcal{Y}$, where X may be $X\subseteq\mathbb{R}^n$ and $\mathcal{Y}$ may be $\mathcal{Y}\subseteq\mathbb{R}^k$. Based on the above, a predictor may be represented as $f:X\rightarrow\mathbb{R}^k$.

[0032] According to one embodiment, one goal of IRM may be to use these multiple datasets to construct a predictor f that performs well across many unseen environments $\mathcal{E}_{all}$, where $\mathcal{E}_{all}\supseteq\varepsilon_{tr}$. In one embodiment, the risk achieved by f in environment e may be defined as $R^e(f)=\mathbb{E}_{X^e,Y^e}[\ell(f(X^e),Y^e)]$ where ℓ may represent the loss when $f(X)$ is the predicted value and Y is the corresponding label. To assume that f maps to real values is not restrictive; for instance, in a k-class classification problem, the output of the function f may include the score for each class, which may then be converted into a hard label by selecting the class with the highest score.

[0033] According to one embodiment, a data representation $\Phi:X\rightarrow\mathcal{Z}\subseteq\mathbb{R}^d$ may elicit an invariant predictor $w\cdot\Phi$ across environments $e\in\varepsilon$ if there is a classifier $w:\mathcal{Z}\rightarrow\mathbb{R}^k$ that achieves the minimum risk for all the environments:

$$w \in \arg\min_{\overline{w} \in \mathcal{H}_w} R^e(\overline{w} \circ \Phi),$$

$\forall e \in \varepsilon$. The set of all the mappings Φ is given as $\mathcal{H}_\Phi$ and the set of all the classifiers is given as $\mathcal{H}_w$. IRM may be phrased as the following constrained optimization problem (1):

$$\min_{\Phi \in \mathcal{H}_\Phi, w \in \mathcal{H}_w} \sum_{e \in \varepsilon_{tr}} R^e(w \circ \Phi)$$

$$\text{s.t. } w \in \arg\min_{\overline{w} \in \mathcal{H}_w} R^e(\overline{w} \circ \Phi),$$

$$\forall e \in \varepsilon_{tr}$$

[0034] If a predictor w·Φ satisfies the constraints above, then it may be an invariant predictor across all of the training environments ε_{tr}. However, optimization problem (1) may be quite challenging to solve. Accordingly, embodiments of the present disclosure introduces an alternate characterization for solving optimization problem (1) based on game theory.

[0035] According to one embodiment of the present disclosure, each environment may control its own classifier $w^e \in \mathcal{H}_w$. In one embodiment, an ensemble may be implemented to construct an overall classifier $w^{av}: \mathcal{Z} \rightarrow \mathbb{R}^k$ defined as

$$w^{av} = \frac{1}{|\varepsilon_{tr}|} \sum_{q=1}^{|\varepsilon_{tr}|} w^q,$$

where for each $z \in \mathcal{Z}$,

$$w^{av}(z) = \frac{1}{|\varepsilon_{tr}|} \sum_{q=1}^{|\varepsilon_{tr}|} w^q(z).$$

(The av stands for average.) In one example of binary classification with two environments $\{e_1, e_2\}$, $w^e = [w_1^e, w_2^e]$ may represent the classifier of environment e, where each component may include the score for each class. In the above example, a component j of the ensemble classifier w^{av} may be defined as

$$w_j^{av} = \frac{w_j^{e1} + w_j^{e2}}{2}.$$

These scores may then be input into a softmax function. In one embodiment, the final probability assigned to class j for an input z may be represented by

$$\frac{e^{w_j^{av}(z)}}{e^{w_1^{av}(z)} + e^{w_2^{av}(z)}}.$$

[0036] According to one embodiment, all of the environments may be required to use the ensemble classifier w^{av}, resulting in the following new optimization problem (2):

$$\min_{\Phi \in \mathcal{H}_\Phi, w^{av} \in \mathcal{H}_w} \sum_{e \in \varepsilon_{tr}} R^e(w^{av} \circ \Phi)$$

$$\text{s.t. } w^e \in \arg\min_{\overline{w^e} \in \mathcal{H}_w} R^e\left(\frac{1}{|\varepsilon_{tr}|}\left[\overline{w^e} + \sum_{q \neq e} w^q\right] \circ \Phi\right),$$

$$\forall e \in \varepsilon_{tr}$$

[0037] According to one embodiment, optimization problem (2) may be equivalently restated as the following formulation (3):

$$\min_{\Phi \in \mathcal{H}_\Phi, w^{av}} \sum_{e \in \varepsilon_{tr}} R^e(w^{av} \circ \Phi)$$

$$\text{s.t. } R^e\left(\frac{1}{|\varepsilon_{tr}|}\left[w^e + \sum_{q \neq e} w^q\right] \circ \Phi\right) \leq R^e\left(\frac{1}{|\varepsilon_{tr}|}\left[\overline{w^e} + \sum_{q \neq e} w^q\right] \circ \Phi\right) \forall\, \overline{w^e} \in \mathcal{H}_w,$$

$$\forall e \in \varepsilon_{tr}$$

[0038] It is contemplated that the above formulation (3) may be advantageous for multiple reasons. For example, using the ensemble classifier w^{av} may automatically enforce that the same classifier is used across all of the environments. Further, each environment may be free to select the classifier w^e from the entire set of classifier $\mathcal{H}_w$, unlike in optimization problem (1), where all of the environments are required to choose the same classifier. Additionally, the constraints in formulation (3) are equivalent to the set of a pure Nash equilibrium strategy of a game that will be further detailed below.

[0039] According to one embodiment, the disclosed game may be played between $|\varepsilon_{tr}|$ players (e.g., set of training environments), with each player corresponding to an environment e. In one embodiment, the set of actions available to each environment e may be defined by $w^e \in \mathcal{H}_w$ (e.g., a respective environment-specific classifier of a set of classifiers). At the start of a game, a data representation Φ may be selected from the set of all data representations $\mathcal{H}_\Phi$ and may be observed by all the environments. In one embodiment, the utility function for an environment e may be defined as $u_e[w^e, w^{-e}, \Phi] = -R^e(w^{av}, \Phi)$ where $w^{-e} = \{w^q\}_{q \neq e}$ may include the set of choices of all environments but environment e. This game may be referred to as an Ensemble Invariant Risk Minimization (EIRM) and expressed as the following tuple:

$$\Gamma^{EIRM} = (\varepsilon_{tr}, \mathcal{H}_\Phi, \{\mathcal{H}_w\}_{q=1}^{|\varepsilon_{tr}|}, \{u_e\}_{e \in \varepsilon_{tr}}).$$

[0040] According to one embodiment, a pure Nash equilibrium strategy may be represented as a tuple $(\Phi, \{w^q\}_{q=1}^{|\varepsilon_{tr}|})$. It is contemplated that each pure Nash equilibrium may depend on the representation Φ. Accordingly, representation Φ may be include as a part of the tuple representing the pure Nash equilibrium. In one embodiment, the set of pure Nash equilibria may be defined as S^{EIRM}. Further, a set of all the ensemble predictors constructed from the Nash equilibria may be defined as:

$$\hat{S}^{EIRM} = \left\{\left[\frac{1}{|\varepsilon_{tr}|} \sum_{q=1}^{|\varepsilon_{tr}|} w^q\right] \circ \Phi \,\middle|\, \Phi, \{w^q\}_{q=1}^{|\varepsilon_t|} \in S^{EIRM}\right\}$$

[0041] According to one embodiment, members of the set of pure Nash equilibria S^{EIRM} may be equivalently expressed as the solutions to the following formulation (4):

$$u_e[w^e, w^{-e}, \Phi] \geq u_e[\overline{w^e}, w^{-e}, \Phi], \forall w^e \in \mathcal{H}_w, \forall e \in \varepsilon_{tr}.$$

[0042] According to one embodiment, if $u_e[w^e, w^{-e}, \Phi]$ is replaced with $-R^e(w^{av}, \Phi)$ in the above formulation (4), the inequalities of formulation (3) may be obtained.

[0043] According to at least one embodiment, various strategies may be used to compute the pure Nash equilibrium of the game Γ^{EIRM}. In one embodiment, a best response dynamics strategy may be implemented given its intuitive and natural structure.

[0044] In a first best response dynamics approach (FIG. **2**), a fixed representation Φ may be used. In one embodiment, the representation Φ may be fixed to be identity mapping. In another embodiment, the representation Φ may be selected as some other mapping such as an approximation of a map for Gaussian kernel. Once the representation Φ is fixed, the environments may play according to best response dynamics as follows: Each environment may take its turn (in a periodic manner with each environment going once) and minimize its respective objective. This process may be repeated until a certain criterion is achieved, e.g., maximum number of epochs or desired value of training accuracy. The above approach may not provide an opportunity to optimize representation Φ. In one embodiment, the upper-level optimization objective in formulation (3) may be used as a way to guide search for representation Φ.

[0045] In a second best response dynamics approach (FIG. **3**), the representation Φ may be updated periodically by the representation learner using the objective in formulation (4). According to one embodiment, between two updates of the representation Φ, the environments may play according to best response dynamics as described above. An exemplary algorithm of a best response training process is detailed in Table (1) below:

Algorithm: Best Response Training

```
Input: Data for each environment and combined data
Initialize: Randomly initialize $\{w_{cur}{}^e\}_{e=1}^{|\mathcal{E}_{tr}|}$ and $\Phi_{cur}$
from $\mathcal{H}_w$ and $\mathcal{H}_\Phi$ respectively
while iter ≤ $iter_{max}$ do
  if Fixed − Phi then
    $\Phi_{cur} = I$
  end if
  if Variable − Phi then
    $\Phi_{cur} = SGD[\Sigma_e R^e(w_{cur}{}^{av} \circ \Phi_{cur})]$, SGD[.]: step
    update using stochastic gradient descent
  end if
  for p ∈ {1, . . K} do
    for e ∈ {1, . . . , $|\mathcal{E}_{tr}|$} do
      $w_{cur}{}^e = SGD[R^e(w_{cur}{}^{av} \circ \Phi_{cur})]$

        $w_{cur}^{av} = \frac{1}{|\mathcal{E}_{tr}|}\sum_e w_{cur}^e$

    end for
  iter = iter + 1
  end for
end while
```

[0046] Referring to FIG. **1**, an exemplary networked computer environment **100** in accordance with one embodiment is depicted. The networked computer environment **100** may include a computer **102** with a processor **104** and a data storage device **106** that is enabled to run a software program **108** and a invariant risk minimization (IRM) game program **110***a*. The networked computer environment **100** may also include a server **112** that is enabled to run a IRM game program **110***b* that may interact with a database **114** and a communication network **116**. The networked computer environment **100** may include a plurality of computers **102** and servers **112**, only one of which is shown. The communication network **116** may include various types of communication networks, such as a wide area network (WAN), local area network (LAN), a telecommunication network, a wireless network, a public switched network and/or a satellite network. It should be appreciated that FIG. **1** provides only an illustration of one implementation and does not imply any limitations with regard to the environments in which different embodiments may be implemented. Many modifications to the depicted environments may be made based on design and implementation requirements.

[0047] The client computer **102** may communicate with the server computer **112** via the communications network **116**. The communications network **116** may include connections, such as wire, wireless communication links, or fiber optic cables. As will be discussed with reference to FIG. **5**, server computer **112** may include internal components **902***a* and external components **904***a*, respectively, and client computer **102** may include internal components **902***b* and external components **904***b*, respectively. Server computer **112** may also operate in a cloud computing service model, such as Software as a Service (SaaS), Platform as a Service (PaaS), or Infrastructure as a Service (IaaS). Server **112** may also be located in a cloud computing deployment model, such as a private cloud, community cloud, public cloud, or hybrid cloud. Client computer **102** may be, for example, a mobile device, a telephone, a personal digital assistant, a netbook, a laptop computer, a tablet computer, a desktop computer, or any type of computing devices capable of running a program, accessing a network, and accessing a database **114**. According to various implementations of the present embodiment, the IRM game program **110***a*, **110***b* may interact with a database **114** that may be embedded in various storage devices, such as, but not limited to a computer/mobile device **102**, a networked server **112**, or a cloud storage service.

[0048] According to the present embodiment, a user using a client computer **102** or a server computer **112** may use the IRM game program **110***a*, **110***b* (respectively) to generate one or more predictors which may provide out of distribution generalization and operate invariantly in previously unseen environments. The IRM game system and method are explained in more detail below with respect to FIGS. **2** to **4**.

[0049] Referring now to FIG. **2**, a schematic block diagram of a first machine learning environment **200** implementing the IRM game program **110***a*, **110***b* according to at least one embodiment is depicted. According to one embodiment, first machine learning environment **200** may include one or more components (e.g., client computer **102**; server computer **112**; communication network **116**) of the computer environment **100** discussed above with reference to FIG. **1**.

[0050] According to one embodiment, environment **200** may include one or more computer systems **202** running the IRM game program **110***a*, **110***b*. According to one embodiment, the computer systems **202** running the IRM game program **110***a*, **110***b* may include specialized computing devices for machine learning. In various embodiments, the computer systems **202** associated with the IRM game program **110***a*, **110***b* may include a workstation, a personal computing device, a laptop computer, a desktop computer, a thin-client terminal, a tablet computer, a smart telephone, a smart watch or other smart wearable, or other electronic devices.

[0051] According to one embodiment, the IRM game program **110***a*, **110***b* may be stored in a tangible storage device of computer system **202** and executed using a processor associated with computer system **202**. In one embodiment, the IRM game program **110***a*, **110***b* may include a single computer program or multiple program modules or sets of instructions being executed by the processor of the associated computer system **202**. The IRM game program **110***a*, **110***b* may include routines, objects, components, units, logic, data structures, and actions that may perform particular tasks or implement particular abstract data types. The IRM game program **110***a*, **110***b* may be practiced in distributed cloud computing environments where tasks may be performed by remote processing devices which may be linked through the communication network (e.g., communication network **116**). In one embodiment, the IRM game program **110***a*, **110***b* may include program instructions that may be collectively stored on one or more computer-readable storage media.

[0052] As described previously, the standard risk minimization paradigm of machine learning may be brittle when operating in environments where test data distributions are often different from the training data distributions. Training data in a single environment may include a source of bias which may lead a machine learning model to learn spurious correlations instead of the causative features. Training on data from many environments and finding invariant predictors may reduce the effect of spurious features by concentrating models on features that have a causal relationship with the outcome.

[0053] According to one embodiment, the IRM game program **110***a*, **110***b* may address the problem of models inheriting spurious correlations by exploiting the varying degrees of spurious correlations naturally present in data collected from multiple data sources to learn a robust predictor using a game-theoretic perspective. In machine learning environment **200**, IRM game program **110***a*, **110***b* may implement a fixed IRM (F-IRM) game **204** to provide fixed representation based best response training, as will be further detailed below.

[0054] According to one embodiment, IRM game program **110***a*, **110***b* may implement one or more environments (may also be referred to as training environments) associated with corresponding datasets (may also be referred to as training datasets). As illustrated in FIG. **2**, IRM game program **110***a*, **110***b* may define a first environment **206** (e.g., environment **1**) including a first dataset **208** (e.g., dataset **1**) and a second environment **210** (e.g., environment **2**) including a second dataset **212** (e.g., dataset **2**). In various embodiments, IRM game program **110***a*, **110***b* may define any number of additional environments **214** (e.g., environment $|\varepsilon_{tr}|$, where $|\varepsilon_{tr}|$ may refer to the total number of training environments) and a corresponding number of additional datasets **216** (e.g., dataset $|\varepsilon_{tr}|$). The various environments (e.g., first environment **206**, second environment **210**) may operate on a same computer system (e.g., computer system **202**) or may be implemented using different computer systems linked through the communication network.

[0055] In one embodiment, the respective datasets (e.g., first dataset **208**, second dataset **212**) may be received by the corresponding environments (e.g., first environment **206**, second environment **210**) from various locations (e.g., environment **1** may receive dataset **1** from a first hospital location; environment **2** may receive dataset **2** from a second hospital location). The data across the respective datasets may be different such that each environment (e.g., first environment **206**, second environment **210**) may include an arbitrary distribution of data. By implementing the above-described set of training environments receiving the corresponding datasets (having arbitrary distribution of data), the IRM game program **110***a*, **110***b* may construct a predictor, such as, for example an ensemble predictor **218**, which may perform well (e.g., invariantly) across many new (e.g., unseen) environments, where the new environments may represent a superset of the set of training environments.

[0056] According to one embodiment, a predictor (e.g., $w\cdot\Phi$), such as, for example the ensemble predictor **218** may be composed of a classier (w) and a data representation or representation (Φ). In one embodiment, a classifier may refer to an algorithm that maps an input data to an output label (e.g., indicating a specific class or category) and the data representation may refer to a transformation of the raw input data which may enable the classifier to predict the output label based on learning features of the input data.

[0057] According to one embodiment, the ensemble predictor **218** may be generated as an output for solving the invariant risk minimization (IRM) game (e.g., ensemble invariant risk minimization game) initiated by the IRM game program **110***a*, **110***b*. Generally, the IRM game may be played between the set of environments (e.g., first environment **206**, second environment **210**), where each player may correspond to a respective environment (e.g., first player=first environment **206**).

[0058] According to one embodiment, the ensemble predictor **218** may be composed of a classier (w) and the representation (Φ), as previously described. At the initialization of the game, the IRM game program **110***a*, **110***b* may define the representation or representation learner **220** (e.g., Φ), which may be observed by all the of the environments taking part in the IRM game. In one embodiment, the IRM game program **110***a*, **110***b* may include two approaches to the IRM game. The first approach, illustrated in FIG. **2**, may be referred to as a fixed representation based best response training (FIRM game **204**) and a second approach, which will be described with reference to FIG. **3**, may be referred to as a variable representation based best response training (variable IRM (V-IRM) game **302**).

[0059] When playing the F-IRM game **204**, as in FIG. **2**, the IRM game program **110***a*, **110***b* may fix the representation **220** to an identify function (e.g., Φ=1). As a result of the representation **220** being fixed, the ensemble predictor **218** generated from F-IRM game **204** may be defined by the ensemble classifier **226**, as will be further detailed below.

[0060] Continuing with the FIRM game **204**, the IRM game program **110***a*, **110***b* may endow each environment (e.g., first environment **206**, second environment **210**) with its own environment-specific classifier, which may be selected from a set of all classifiers (e.g., linear and non-linear classifiers). For example, the first environment **206** may select (or be assigned) a first environment-specific classifier **222** (e.g., w_k^1) and the second environment **210** may select (or be assigned) a second environment-specific classifier **224** (e.g., w_k^2). The subscript k may refer to a version of the environment-specific classifier at a particular round of round-robin updates, as will be further detailed below.

[0061] According to one embodiment, the IRM game program **110***a*, **110***b* may construct an average or ensemble classifier **226** (e.g., w_k^{av}) based on the environment-specific classifiers (e.g., first environment-specific classifier **222**, second environment-specific classifier **222**) associated with the multiple environments (e.g., first environment **206**, second environment **210**). In at least one embodiment, the environment-specific classifiers (e.g., first environment-specific classifier **222**, second environment-specific classifier **222**) may be provided as components of the ensemble classifier **226**, such that the ensemble classifier **226** may be calculated as an aggregation or average of the individual components (e.g., environment-specific classifiers).

[0062] According to one embodiment, constructing the ensemble classifier **226** may involve averaging the logits produced by each environment-specific classifier. According to one embodiment, a classifier may map a point x (e.g., input) to a set of C reals (e.g., referred to as logits), where the set of C may represent the number of classes in a classification problem. In one embodiment of this classification problem, a softmax operation may be applied to the set of C reals to get a number (represented by C) of probabilities which sum to 1. Each probability may indicate a confidence score associated with a respective class. In one embodiment, calculating the average classifier works as follows: each environment-specific classifier (e.g., w^1, w^2) may include a function which may take x and produce C real numbers—vector of C reals (logits). The average may be represented by

$$\frac{w(x)^1 + w(x)^2 \ldots + w(x)^n}{n}$$

and produce another real vector of length C (averaged logit). In one embodiment, a softmax operation may be applied over this averaged logit to arrive at a soft prediction (e.g., probability scores). The soft predictions may then be plugged into a loss function such as, for example, cross-entropy.

[0063] According to one embodiment, the IRM games (e.g., F-IRM game **204**, V-IRM game **302**) implemented by the IRM game program **110***a*, **110***b* may utilize a best response dynamics strategy to compute a Nash equilibrium, where the Nash equilibrium may be equivalent to the ensemble predictor **218**. In field of game theory, best response dynamics may refer to a strategy or set of strategies which may produce the most favorable outcome for each player, given the other players' strategies. The Nash equilibrium may refer to the point in the game where each player has selected the best response to the other players' strategies (e.g., based on best response dynamics).

[0064] While playing the game, the IRM game program **110***a*, **110***b* may enable each environment to use the ensemble classifier **226** to make predictions associated with their respective datasets. However, the only set of actions available to each environment may include controlling their respective environment-specific classifier component of the ensemble classifier **226**. Under best response dynamics, the environments (e.g., first environment **206**, second environment **210**) may play the game such that each environment may decide its rest response (e.g., modification to its environment-specific classifier) to minimize the risk or loss specific to its own environment.

[0065] According to one embodiment, the IRM game program **110***a*, **110***b* may calculate a performance of each environment using a respective loss or risk function. For example, the first environment **206** may include a first loss function

$$228\left(\text{e.g., } R^1\left(\frac{1}{|\varepsilon_{tr}|}\left[\overline{w^1} + \sum\nolimits_{q\neq 1} w^q\right]\circ\Phi\right)\right)$$

and the second environment **210** may include a second loss function

$$230\left(\text{e.g., } R^2\left(\frac{1}{|\varepsilon_{tr}|}\left[\overline{w^2} + \sum\nolimits_{q\neq 2} w^q\right]\circ\Phi\right)\right).$$

Each loss function may include a component representing the ensemble or average classifier and a component representing the representation, where the ensemble classifier may further include a component representing respective environment-specific classifiers. In F-IRM game **204**, the loss functions may only be impacted by the ensemble classifier component since the representation may be fixed to the identity function. For example, in the first loss function

$$228\left(\text{e.g., } R^1\left(\frac{1}{|\varepsilon_{tr}|}\left[\overline{w^1} + \sum\nolimits_{q\neq 1} w^q\right]\circ\Phi\right)\right),$$

the ensemble classifier function

$$\left(\text{e.g., }\left(\frac{1}{|\varepsilon_{tr}|}\left[\overline{w^1} + \sum\nolimits_{q\neq 1} w^q\right]\right)\right),$$

may include a component w^1 representing the respective environment-specific classifier **222**. According to one embodiment, each environment may improve its performance (e.g., increase its utility) by modifying its respective environment-specific classifier to minimize its respective environment loss function.

[0066] According to one embodiment, the steps of the game may proceed using a round-robin schedule of updates. In each round of the game, every environment (e.g., environment **1**, environment **2** . . . environment $|\varepsilon_{tr}|$) may take its turn (in a periodic manner with each environment going once) and minimize its respective loss or risk function and update the ensemble classifier **226**.

[0067] For example, in a first round (e.g., k=1) of the round-robin updates, at event **1** (**232**), the ensemble classifier **226** may be trained in the first environment **206** using the corresponding dataset **208**. According to one embodiment, the first environment **206** may receive feedback via the first loss function

$$228\left(\text{e.g., } R^1\left(\frac{1}{|\varepsilon_{tr}|}\left[\overline{w^1}+\sum\nolimits_{q\neq 1} w^q\right]\circ\Phi\right)\right)$$

and may modify the weights of the first environment-specific classifier **222** to minimize the first loss function **228**. In one embodiment, each environment may implement stochastic learning with backpropagation. More specifically, each environment may calculate a stochastic gradient descent (SGD) of the respective loss function and modify the respective environment-specific classifier to minimize the respective loss function associated with each environment.

[0068] According to one embodiment, the ensemble classifier **226** may be updated after each environment-specific classifier is updated (e.g., may be referred to as sequential updates). For example, following the modifications to the first environment-specific classifier **222**, the ensemble classifier **226** may be updated accordingly. According to another embodiment, the ensemble classifier **226** may be updated following modifications to all of the environment-specific classifiers. In this embodiment, all of the environment-specific classifier may be modified at the same time (e.g., may be referred to as parallel updates). The ensemble classifier **226** may be updated after the parallel updates to all of the environment-specific classifiers. For example, following one round of modifications to the first and second environment-specific classifiers **222**, **224** (in parallel), the ensemble classifier **226** may be updated accordingly.

[0069] Continuing with the first round (e.g., k=1) of the round-robin updates, the second environment **210** may then take its turn at event **2** (**234**). In one embodiment, the second environment **210** may proceed in a manner similar to the process described at event **1** (**232**) with reference to the first environment **206**. The second environment **210** may modify its respective environment-specific classifier **224** to minimize its respective second loss function **230**. In this example, the ensemble classifier **226** may then be updated based on the modifications to the second environment-specific classifier **224** (e.g., sequential updates). According to one embodiment, the first round of the round-robin updates may proceed to the last environment in the set of environments (e.g., Environment $|\varepsilon_{tr}|$) at event $|\varepsilon_{tr}|$ (**236**) in a similar manner described above with reference to first and second environments **206**, **210**.

[0070] This cycle of round-robin updates may iterate until a maximum number of iterations (max-iterations) is reached. In one embodiment, the IRM game program **110***a*, **110***b* may define the max-iterations based on criterion such as, for example, maximum number of epochs (e.g., passes through the entire dataset) or desired value of training accuracy. In at least one embodiment, the max-iterations may be associated with the number of cycles necessary to compute the Nash equilibrium of the F-IRM game **204**.

[0071] According to one embodiment, the ensemble predictor **218** may be determined at the Nash equilibrium of the FIRM game **204**. In some embodiments, the Nash equilibrium of the F-IRM game **204** may generate a set of ensemble predictors **218**. According to one embodiment, the IRM game program **110***a*, **110***b* may fix the representation **220** to an identity function, a previously described. As such, in various embodiments of the F-IRM game **204**, the ensemble predictor **218** may be equivalent to the ensemble classifier **226**. Further, it is contemplated that any ensemble predictor **218** obtained from solving the Nash equilibrium of the F-IRM game **204** over the training environments (e.g., environment **1**, environment **2** . . . environment may perform as an invariant predictor across all unseen or new environments when deployed by a computing device.

[0072] Referring now to FIG. **3**, a schematic block diagram of a second machine learning environment **300** implementing the IRM game program **110***a*, **110***b* according to at least one embodiment is depicted. According to one embodiment, second machine learning environment **300** may include one or more components (e.g., client computer **102**; server computer **112**; communication network **116**) of the computer environment **100** discussed above with reference to FIG. **1**. In at least one embodiment, second machine learning environment **300** may include other components which may be similar to first machine learning environment **200**. As such, similar components may share the same reference numbers between FIG. **2** and FIG. **3**.

[0073] According to one embodiment, machine learning environment **300** may be implemented using one or more computer systems **202** running the IRM game program **110***a*, **110***b*. Further, machine learning environment **300** may include training environments (e.g., environment **1** (first environment **206**), environment **2** (second environment **210**) . . . environment $|\varepsilon_{tr}|$ (additional environments **214**)). In one embodiment, the IRM game program **110***a*, **110***b* may define a respective environment-specific classifier for each corresponding environment. For example, the first environment **206** may select (or be assigned) a first environment-specific classifier **222** (e.g., w_k^1) and the second environment **210** may select (or be assigned) a second environment-specific classifier **224** (e.g., w_k^2). According to one embodiment, the various environment-specific classifiers may be aggregated to construct the ensemble classifier **226** and the ensemble classifier **226** may be used to determine the ensemble predictor **218** after solving the IRM game initiated by the IRM game program **110***a*, **110***b*.

[0074] As described previously, the IRM game program **110***a*, **110***b* may include two approaches to the IRM game. The first approach, referred to as a fixed representation based best response training (F-IRM game **204**), was described with reference to FIG. **2**. According to at least one embodiment, the second approach, referred to as a variable representation based best response training (V-IRM game **302**), may be implemented by the second machine learning environment **300**, as will be detailed further with reference to FIG. **3**.

[0075] According to one embodiment, in the V-IRM game **302**, the IRM game program **110***a*, **110***b* may define a representation learner **304** (may also be referred to as a representation) which may be variable, unlike the fixed representation **220** in F-IRM game **204**. In at least one embodiment, the representation learner **304** may be updated periodically (e.g., to modify the representation) to minimize a total loss function **306** (e.g., $\Sigma_e R^e(w^{av}\cdot\Phi)$) associated with the representation **304**.

[0076] Another difference in the training process between the F-IRM game **204** and the V-IRM game **302** may be found in the handling of the various training datasets. In V-IRM game **302**, the representation learner **304** may first receive all datasets **308** before the respective datasets are fed into the corresponding environments. In one embodiment, a goal of the representation learner **304** may be to transform the data (e.g., in all datasets **308**) such that the ensemble classifier **226** may learn the causal features to generate a robust predictor (e.g., ensemble predictor **218**). The transformed datasets may then be transmitted to the corresponding environments. For example, a first dataset **310** (e.g., Dataset **1**) may include a first set of transformed data for first environment **206** and a second dataset **312** (e.g., Dataset **2**) may include a second set of transformed data for second environment **210**. The first and second datasets **310**, **312** may initially be included in the all datasets **308** received by the representation learner **304**. Following the transformation of the data in the all datasets **308**, the first and second datasets **310**, **312** may be fed to the corresponding environments.

[0077] According to one embodiment, the V-IRM game **302** implemented by the IRM game program **110***a*, **110***b* may also utilize the best response dynamics strategy to compute a Nash equilibrium, where the Nash equilibrium may be equivalent to the ensemble predictor **218**. In V-IRM game **302**, the representation learner **304** may be updated periodically (e.g., once per round k of round-robin updates) and each environment may play the game according to best response dynamics (as described with reference to FIG. **2**, e.g., take its turn and update the environment-specific classifier to minimize loss function) between two updates (e.g., sequential updates) of the representation learner **304**.

[0078] For example, in a first round (e.g., k=1) of the round-robin updates, at event **1** (**314**), the first environment **206** (including data transformed by the representation learner **304**) may minimize the respective loss function **228** by modifying the respective environment-specific classifier **222**, as described previously with reference to FIG. **2**. The ensemble classifier **226** may then be updated based on the modifications to the first environment-specific classifier **222**. According to one embodiment, the ensemble classifier **226** may be updated based on sequential updates (e.g., following each environment-specific classifier) or parallel updates (e.g., following parallel updates to all of the environment-specific classifiers), as described previously with reference to FIG. **2**.

[0079] Continuing with the first round (e.g., k=1) of the round-robin updates, the second environment **210** (including data transformed by the representation learner **304**) may then take its turn at event **2** (**316**) and proceed in a manner similar to the process described at event **1** (**314**) with reference to the first environment **206**. The second environment **210** may modify its respective environment-specific classifier **224** to minimize its respective second loss function **230**. The ensemble classifier **226** may then be updated based on the modifications to the second environment-specific classifier **224**. According to one embodiment, the first round of the round-robin updates may proceed to the last environment (e.g., environment $|\varepsilon_{tr}|$) at event $|\varepsilon_{tr}|$ (**318**) in a similar manner as described above with reference to the first and second environments **206**, **210**. Once all the environments have taken a turn in the first round (e.g., k=1) of the round-robin updates, at event $|\varepsilon_{tr}|+1$ (**320**), the representation learner **304** may modify the representation to minimize the total loss function **306** (e.g., using SGD with backpropagation). In one embodiment, the IRM game program **110***a*, **110***b* may calculate the total loss function **306** by aggregating the respective environment-specific loss functions (e.g., environment-specific classifier **222**, environment-specific classifier **224**). Between the first round (e.g., k=1) update and a second round (e.g., k=2) of the representation learner **304**, each environment may take its turn to play the game according to the best response dynamics strategy described above.

[0080] According to one embodiment, the cycle of round-robin updates in the V-IRM game **302**) may iterate until the max-iterations is reached. In at least one embodiment, the max-iterations may be associated with the number of cycles necessary to compute the Nash equilibrium of the V-IRM game **302**.

[0081] According to one embodiment, the ensemble predictor **218** may be determined at the Nash equilibrium of the V-IRM game **302**. In some embodiments, the Nash equilibrium of the V-IRM game **302** may generate a set of ensemble predictors **218**. According to one embodiment, the ensemble predictor **218** in the V-IRM game **302** may be different from the ensemble classifier **226** in that the ensemble predictor **218** may be composed of the ensemble classifier **226** and the representation updated by the representation learner **304** to minimize the total loss function **306**.

[0082] Further, it is contemplated that any ensemble predictor **218** obtained from solving the Nash equilibrium of the V-IRM game **302** over the training environments (e.g., environment **1**, environment **2** . . . environment may perform as an invariant predictor across all unseen or new environments when deployed by a computing device.

[0083] Referring now to FIG. **4**, an operational flowchart illustrating an exemplary best response training process **400** used by the IRM game program **110***a*, **110***b* according to at least one embodiment is depicted. The best response training process **400** may be described with reference to FIGS. **2** and **3**.

[0084] At **402**, a plurality of environment-specific classifiers corresponding to a plurality of environments are defined. According to one embodiment, the IRM game program **110***a*, **110***b* may implement multiple environments (e.g., training environments), as described with reference to FIGS. **2** and **3**. According to one embodiment, the IRM game program **110***a*, **110***b* may endow each environment with its own environment-specific classifier, which may be selected from a set of all classifiers (e.g., selected from linear and non-linear classifiers).

[0085] Then at **404**, a representation corresponding to the plurality of environments is defined. According to one embodiment, the type of representation implemented by the IRM game program **110***a*, **110***b* may dictate a respective approach to the best response training process **400**. If the representation is fixed to an identity function, process **400** may proceed as a fixed representation based best response training process in which the IRM game program **110***a*, **110***b* may implement the FIRM game, as described previously with reference to FIG. **2**. However, if the representation indicates a variable representation which may be updated by a representation learner, process **400** may proceed as a variable representation based best response training process in which the IRM game program **110***a*, **110***b* may implement the V-IRM game, as described previously with reference to FIG. **3**.

[0086] Then at **406**, an ensemble classifier associated with the plurality of environment-specific classifiers is constructed. The IRM game program **110***a*, **110***b* may implement an average or ensemble classifier to learn features which have a causal relationship with the outcome from a wide distribution of training data. According to one embodiment, the IRM game program **110***a*, **110***b* may construct the ensemble classifier based on the environment-specific classifiers associated with the multiple environments. In at least one embodiment, the environment-specific classifiers may be provided as components of the ensemble classifier, such that the ensemble classifier may be calculated as an aggregation or average of the individual components (e.g., environment-specific classifiers), as described with reference to FIG. **2**.

[0087] Then at **408**, a game is initiated including a plurality of players corresponding to the plurality of environments. According to one embodiment, the IRM game program **110***a*, **110***b* may initiate a F-IRM game, as described with reference to FIG. **2** or a V-IRM game, as described with reference to FIG. **3**, depending on the representation learner defined by the IRM game program **110***a*, **110***b*. Notwithstanding the type of game initiated by the IRM game program **110***a*, **110***b*, each environment (e.g., training environment), may represent a corresponding player in the game, as described previously with reference to FIGS. **2** and **3**.

[0088] Then at **410**, a Nash equilibrium of the initiated game is calculated. According to one embodiment, the IRM games (e.g., F-IRM game, V-IRM game) implemented by the IRM game program **110***a*, **110***b* may utilize a best response dynamics strategy to compute a Nash equilibrium. As described with reference to FIG. **2**, in the field of game theory, best response dynamics may refer to a strategy or set of strategies which may produce the most favorable outcome for each player (e.g., each environment), given the other players' strategies (e.g., other environments' strategies). The Nash equilibrium may refer to the point in the game where each player has selected the best response to the other players' strategies (e.g., based on best response dynamics).

[0089] According to one embodiment, the IRM game program **110***a*, **110***b* may enable each environment to use the ensemble classifier to make predictions associated with their respective datasets. However, the only set of actions available to each environment may include controlling their respective environment-specific classifier component of the ensemble classifier.

[0090] According to one embodiment, in the F-IRM game, (e.g., representation fixed to identity function), each environment may take its turn and update its respective environment-specific classifier to improve the utility of the respective environment-specific classifier (e.g., minimize loss function specific to its own environment). This cycle may continue until max-iterations of the F-IRM game resulting in the Nash Equilibrium.

[0091] According to one embodiment, in the V-IRM game (e.g., representation variable), the representation learner defined by the IRM game program **110***a*, **110***b* may periodically update the representation to minimize its total risk (e.g., minimize total loss function). Between two updates of the representation by the representation learner, each environment may take its turn and update its respective environment-specific classifier to improve the utility of the respective environment-specific classifier. This cycle may continue until max-iterations of the V-IRM game resulting in the Nash Equilibrium.

[0092] Then at **412**, an ensemble predictor is determined based on the calculated Nash equilibrium of the initiated game. According to one embodiment, one or more ensemble predictors may be generated as an output for solving the FIRM or V-IRM game initiated by the IRM game program **110***a*, **110***b*. Since each game implements the best response dynamics strategy to compute a Nash equilibrium, solving the game for the Nash equilibrium may result in determining the one or more ensemble predictors.

[0093] According to one embodiment, the ensemble predictor may be composed of the ensemble classifier and the representation. In a F-IRM game, where the representation may be fixed to an identity function, the ensemble predictor may be equivalent to the ensemble classifier. However, in a V-IRM game, where the representation may be variable, the ensemble predictor may be different from the ensemble classifier and may depend on the final representation generated by the representation learner.

[0094] Thereafter at **414**, the determined ensemble predictor is deployed to make predictions in a new environment. By implementing the above-described best response training process **400**, the IRM game program **110***a*, **110***b* may construct a robust ensemble predictor which may perform well (e.g., invariantly) across many new (e.g., unseen) environments, where the new environments may represent a superset of the set of training environments, as described previously with reference to FIGS. **2** and **3**.

[0095] The IRM game program **110***a*, **110***b* may improve the functionality of a computer because the IRM game program **110***a*, **110***b* may enable the computer to build robust predictors which may perform successfully in new environments where the distribution of the data is unknown. The IRM game program **110***a*, **110***b* may enable operationalizing machine learning in domains such as, computer vision and natural language processing. For example, the IRM game program **110***a*, **110***b* may enable training a self-driving car to apply its car brakes based on features beyond just detecting the brake lights of the car in front. As such, if a situation requires applying the brakes, a self-driving car model trained using the IRM game program **110***a*, **110***b* may apply the car brakes, even if the brake lights of the car in front is not visible.

[0096] It may be appreciated that FIGS. **2** to **4** provide only an illustration of one embodiment and do not imply any limitations with regard to how different embodiments may be implemented. Many modifications to the depicted embodiment(s) may be made based on design and implementation requirements.

[0097] FIG. **5** is a block diagram **900** of internal and external components of computers depicted in FIG. **1** in accordance with an illustrative embodiment of the present invention. It should be appreciated that FIG. **5** provides only an illustration of one implementation and does not imply any limitations with regard to the environments in which different embodiments may be implemented. Many modifications to the depicted environments may be made based on design and implementation requirements.

[0098] Data processing system **902**, **904** is representative of any electronic device capable of executing machine-readable program instructions. Data processing system **902**, **904** may be representative of a smart phone, a computer system, PDA, or other electronic devices. Examples of computing systems, environments, and/or configurations that may represented by data processing system **902**, **904** include, but are not limited to, personal computer systems, server computer systems, thin clients, thick clients, hand-held or laptop devices, multiprocessor systems, microprocessor-based systems, network PCs, minicomputer systems, and distributed cloud computing environments that include any of the above systems or devices.

[0099] User client computer **102** and network server **112** may include respective sets of internal components **902** *a, b* and external components **904** a, b illustrated in FIG. **5**. Each of the sets of internal components **902** *a, b* includes one or more processors **906**, one or more computer-readable RAMs **908** and one or more computer-readable ROMs **910** on one or more buses **912**, and one or more operating systems **914** and one or more computer-readable tangible storage devices **916**. The one or more operating systems **914**, the software program **108**, and the IRM game program **110***a* in client computer **102**, and the IRM game program **110***b* in network server **112**, may be stored on one or more computer-readable tangible storage devices **916** for execution by one or more processors **906** via one or more RAMs **908** (which typically include cache memory). In the embodiment illustrated in FIG. **5**, each of the computer-readable tangible storage devices **916** is a magnetic disk storage device of an internal hard drive. Alternatively, each of the computer-readable tangible storage devices **916** is a semiconductor storage device such as ROM **910**, EPROM, flash memory or any other computer-readable tangible storage device that can store a computer program and digital information.

[0100] Each set of internal components **902** *a, b* also includes a R/W drive or interface **918** to read from and write to one or more portable computer-readable tangible storage devices **920** such as a CD-ROM, DVD, memory stick, magnetic tape, magnetic disk, optical disk or semiconductor storage device. A software program, such as the software program **108** and the IRM game program **110***a* and **110***b* can be stored on one or more of the respective portable computer-readable tangible storage devices **920**, read via the respective R/W drive or interface **918** and loaded into the respective hard drive **916**.

[0101] Each set of internal components **902** *a, b* may also include network adapters (or switch port cards) or interfaces **922** such as a TCP/IP adapter cards, wireless wi-fi interface cards, or 3G or 4G wireless interface cards or other wired or wireless communication links. The software program **108** and the IRM game program **110***a* in client computer **102** and the IRM game program **110***b* in network server computer **112** can be downloaded from an external computer (e.g., server) via a network (for example, the Internet, a local area network or other, wide area network) and respective network adapters or interfaces **922**. From the network adapters (or switch port adaptors) or interfaces **922**, the software program **108** and the IRM game program **110***a* in client computer **102** and the IRM game program **110***b* in network server computer **112** are loaded into the respective hard drive **916**. The network may comprise copper wires, optical fibers, wireless transmission, routers, firewalls, switches, gateway computers and/or edge servers.

[0102] Each of the sets of external components **904** a, b can include a computer display monitor **924**, a keyboard **926**, and a computer mouse **928**. External components **904** a, b can also include touch screens, virtual keyboards, touch pads, pointing devices, and other human interface devices. Each of the sets of internal components **902** *a, b* also includes device drivers **930** to interface to computer display monitor **924**, keyboard **926** and computer mouse **928**. The device drivers **930**, R/W drive or interface **918** and network adapter or interface **922** comprise hardware and software (stored in storage device **916** and/or ROM **910**).

[0103] It is understood in advance that although this disclosure includes a detailed description on cloud computing, implementation of the teachings recited herein are not limited to a cloud computing environment. Rather, embodiments of the present invention are capable of being implemented in conjunction with any other type of computing environment now known or later developed.

[0104] Cloud computing is a model of service delivery for enabling convenient, on-demand network access to a shared pool of configurable computing resources (e.g. networks, network bandwidth, servers, processing, memory, storage, applications, virtual machines, and services) that can be rapidly provisioned and released with minimal management effort or interaction with a provider of the service. This cloud model may include at least five characteristics, at least three service models, and at least four deployment models.

[0105] Characteristics are as follows:

[0106] On-demand self-service: a cloud consumer can unilaterally provision computing capabilities, such as server time and network storage, as needed automatically without requiring human interaction with the service's provider.

[0107] Broad network access: capabilities are available over a network and accessed through standard mechanisms that promote use by heterogeneous thin or thick client platforms (e.g., mobile phones, laptops, and PDAs).

[0108] Resource pooling: the provider's computing resources are pooled to serve multiple consumers using a multi-tenant model, with different physical and virtual resources dynamically assigned and reassigned according to demand. There is a sense of location independence in that the consumer generally has no control or knowledge over the exact location of the provided resources but may be able to specify location at a higher level of abstraction (e.g., country, state, or datacenter).

[0109] Rapid elasticity: capabilities can be rapidly and elastically provisioned, in some cases automatically, to quickly scale out and rapidly released to quickly scale in. To the consumer, the capabilities available for provisioning often appear to be unlimited and can be purchased in any quantity at any time.

[0110] Measured service: cloud systems automatically control and optimize resource use by leveraging a metering capability at some level of abstraction appropriate to the type of service (e.g., storage, processing, bandwidth, and active user accounts). Resource usage can be monitored, controlled, and reported providing transparency for both the provider and consumer of the utilized service.

[0111] Service Models are as follows:

[0112] Software as a Service (SaaS): the capability provided to the consumer is to use the provider's applications running on a cloud infrastructure. The applications are accessible from various client devices through a thin client interface such as a web browser (e.g., web-based e-mail). The consumer does not manage or control the underlying cloud infrastructure including network, servers, operating systems, storage, or even individual application capabilities, with the possible exception of limited user-specific application configuration settings.

[0113] Platform as a Service (PaaS): the capability provided to the consumer is to deploy onto the cloud infrastructure consumer-created or acquired applications created using programming languages and tools supported by the provider. The consumer does not manage or control the underlying cloud infrastructure including networks, servers, operating systems, or storage, but has control over the deployed applications and possibly application hosting environment configurations.

[0114] Infrastructure as a Service (IaaS): the capability provided to the consumer is to provision processing, storage, networks, and other fundamental computing resources where the consumer is able to deploy and run arbitrary software, which can include operating systems and applications. The consumer does not manage or control the underlying cloud infrastructure but has control over operating systems, storage, deployed applications, and possibly limited control of select networking components (e.g., host firewalls).

[0115] Deployment Models are as follows:

[0116] Private cloud: the cloud infrastructure is operated solely for an organization. It may be managed by the organization or a third party and may exist on-premises or off-premises.

[0117] Community cloud: the cloud infrastructure is shared by several organizations and supports a specific community that has shared concerns (e.g., mission, security requirements, policy, and compliance considerations). It may be managed by the organizations or a third party and may exist on-premises or off-premises.

[0118] Public cloud: the cloud infrastructure is made available to the general public or a large industry group and is owned by an organization selling cloud services.

[0119] Hybrid cloud: the cloud infrastructure is a composition of two or more clouds (private, community, or public) that remain unique entities but are bound together by standardized or proprietary technology that enables data and application portability (e.g., cloud bursting for load-balancing between clouds).

[0120] A cloud computing environment is service oriented with a focus on statelessness, low coupling, modularity, and semantic interoperability. At the heart of cloud computing is an infrastructure comprising a network of interconnected nodes.

[0121] Referring now to FIG. **6**, illustrative cloud computing environment **1000** is depicted. As shown, cloud computing environment **1000** comprises one or more cloud computing nodes **100** with which local computing devices used by cloud consumers, such as, for example, personal digital assistant (PDA) or cellular telephone **1000**A, desktop computer **1000**B, laptop computer **1000**C, and/or automobile computer system **1000**N may communicate. Nodes **100** may communicate with one another. They may be grouped (not shown) physically or virtually, in one or more networks, such as Private, Community, Public, or Hybrid clouds as described hereinabove, or a combination thereof. This allows cloud computing environment **1000** to offer infrastructure, platforms and/or software as services for which a cloud consumer does not need to maintain resources on a local computing device. It is understood that the types of computing devices **1000**A-N shown in FIG. **6** are intended to be illustrative only and that computing nodes **100** and cloud computing environment **1000** can communicate with any type of computerized device over any type of network and/or network addressable connection (e.g., using a web browser).

[0122] Referring now to FIG. **7**, a set of functional abstraction layers **1100** provided by cloud computing environment **1000** is shown. It should be understood in advance that the components, layers, and functions shown in FIG. **7** are intended to be illustrative only and embodiments of the invention are not limited thereto. As depicted, the following layers and corresponding functions are provided:

[0123] Hardware and software layer **1102** includes hardware and software components. Examples of hardware components include: mainframes **1104**; RISC (Reduced Instruction Set Computer) architecture based servers **1106**; servers **1108**; blade servers **1110**; storage devices **1112**; and networks and networking components **1114**. In some embodiments, software components include network application server software **1116** and database software **1118**.

[0124] Virtualization layer **1120** provides an abstraction layer from which the following examples of virtual entities may be provided: virtual servers **1122**; virtual storage **1124**; virtual networks **1126**, including virtual private networks; virtual applications and operating systems **1128**; and virtual clients **1130**.

[0125] In one example, management layer **1132** may provide the functions described below. Resource provisioning **1134** provides dynamic procurement of computing resources and other resources that are utilized to perform tasks within the cloud computing environment. Metering and Pricing **1136** provide cost tracking as resources are utilized within the cloud computing environment, and billing or invoicing for consumption of these resources. In one example, these resources may comprise application software licenses. Security provides identity verification for cloud consumers and tasks, as well as protection for data and other resources. User portal **1138** provides access to the cloud computing environment for consumers and system administrators. Service level management **1140** provides cloud computing resource allocation and management such that required service levels are met. Service Level Agreement (SLA) planning and fulfillment **1142** provide pre-arrangement for, and procurement of, cloud computing resources for which a future requirement is anticipated in accordance with an SLA.

[0126] Workloads layer **1144** provides examples of functionality for which the cloud computing environment may be utilized. Examples of workloads and functions which may be provided from this layer include: mapping and navigation **1146**; software development and lifecycle management **1148**; virtual classroom education delivery **1150**; data analytics processing **1152**; transaction processing **1154**; and machine learning **1156**. A IRM game program **110***a*, **110***b* provides a way to build robust predictors which may perform successfully in new environments where the distribution of the data is unknown.

[0127] The descriptions of the various embodiments of the present invention have been presented for purposes of illustration, but are not intended to be exhaustive or limited to the embodiments disclosed. Many modifications and variations will be apparent to those of ordinary skill in the art without departing from the scope of the described embodiments. The terminology used herein was chosen to best explain the principles of the embodiments, the practical application or technical improvement over technologies found in the marketplace, or to enable others of ordinary skill in the art to understand the embodiments disclosed herein.

What is claimed is:

1. A computer-implemented method comprising:

defining, by a computing device, a plurality of environment-specific classifiers corresponding to a plurality of environments;

constructing, by the computing device, an ensemble classifier associated with the plurality of environment-specific classifiers;

initiating, by the computing device, a game including a plurality of players corresponding to the plurality of environments;

calculating, by the computing device, a nash equilibrium of the initiated game;

determining, by the computing device, an ensemble predictor based on the calculated nash equilibrium of the initiated game; and

deploying, by the computing device, the determined ensemble predictor associated with the calculated nash equilibrium to make predictions in a new environment.

2. The method of claim **1**, wherein the calculated nash equilibrium of the initiated game includes a best response dynamics strategy.

3. The method of claim **1**, further comprising:

receiving a corresponding dataset for each environment of the plurality of environments, wherein the received corresponding dataset for each environment includes an arbitrary distribution of data.

4. The method of claim **1**, further comprising:

defining a representation corresponding to the plurality of environments; and

fixing the defined representation to an identity function.

5. The method of claim **1**, further comprising:

defining a representation corresponding to the plurality of environments; and

updating the defined representation periodically by a representation learner to minimize a total loss function associated with the defined representation.

6. The method of claim **1**, further comprising:

training the constructed ensemble classifier by each environment of the plurality of environments;

modifying a respective environment-specific classifier by a corresponding environment of the plurality of environments to minimize a respective loss function specific to the corresponding environment; and

updating the trained constructed ensemble classifier based on the modified respective environment-specific classifier.

7. The method of claim **1**, further comprising:

modifying the plurality of environment-specific classifier in parallel; and

updating the constructed ensemble classifier responsive to the modified plurality of environment-specific classifiers in parallel.

8. The method of claim **1**, wherein the determined ensemble predictor includes the constructed ensemble classifier updated based on minimizing a respective loss function associated with each environment of the plurality of environments.

9. The method of claim **8**, wherein the determined ensemble predictor further comprises a representation updated to minimize a total loss function associated with the representation.

10. The method of claim **5**, further comprising:

modifying the plurality of environment-specific classifiers by the plurality of environments, respectively, to minimize a respective loss function associated with each environment of the plurality of environments between two updates of the defined representation.

11. A computer system for invariant risk minimization games, comprising:

one or more processors, one or more computer-readable memories, one or more computer-readable tangible storage media, and program instructions stored on at least one of the one or more computer-readable tangible storage media for execution by at least one of the one or more processors via at least one of the one or more memories, wherein the computer system is capable of performing a method comprising:

defining a plurality of environment-specific classifiers corresponding to a plurality of environments;

constructing an ensemble classifier associated with the plurality of environment-specific classifiers;

initiating a game including a plurality of players corresponding to the plurality of environments;

calculating a nash equilibrium of the initiated game;

determining an ensemble predictor based on the calculated nash equilibrium of the initiated game; and

deploying the determined ensemble predictor associated with the calculated nash equilibrium to make predictions in a new environment.

12. The computer system of claim **11**, wherein the calculated nash equilibrium of the initiated game includes a best response dynamics strategy.

13. The computer system of claim **11**, further comprising:

receiving a corresponding dataset for each environment of the plurality of environments, wherein the received corresponding dataset for each environment includes an arbitrary distribution of data.

14. The computer system of claim **11**, further comprising:

defining a representation corresponding to the plurality of environments; and

fixing the defined representation to an identity function.

15. The computer system of claim **11**, further comprising:

defining a representation corresponding to the plurality of environments; and

updating the defined representation periodically by a representation learner to minimize a total loss function associated with the defined representation.

16. The computer system of claim **11**, further comprising:

training the constructed ensemble classifier by each environment of the plurality of environments;

modifying a respective environment-specific classifier by a corresponding environment of the plurality of environments to minimize a respective loss function specific to the corresponding environment; and

updating the trained constructed ensemble classifier based on the modified respective environment-specific classifier.

17. The computer system of claim **11**, further comprising:

modifying the plurality of environment-specific classifier in parallel; and

updating the constructed ensemble classifier responsive to the modified plurality of environment-specific classifiers in parallel.

18. The computer system of claim **11**, wherein the determined ensemble predictor includes the constructed ensemble classifier updated based on minimizing a respective loss function associated with each environment of the plurality of environments.

19. The computer system of claim **11**, wherein the determined ensemble predictor further comprises a representation updated to minimize a total loss function associated with the representation.

20. A computer program product for invariant risk minimization games, comprising:

one or more computer-readable storage media and program instructions collectively stored on the one or more computer-readable storage media, the program instructions executable by a processor to cause the processor to perform a method comprising:

defining a plurality of environment-specific classifiers corresponding to a plurality of environments;

constructing an ensemble classifier associated with the plurality of environment-specific classifiers;

initiating a game including a plurality of players corresponding to the plurality of environments;

calculating a nash equilibrium of the initiated game;

determining an ensemble predictor based on the calculated nash equilibrium of the initiated game; and

deploying the determined ensemble predictor associated with the calculated nash equilibrium to make predictions in a new environment.

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