Reply Letter for GRSL-00116-2025

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Paper Number: GRSL-00116-2025
Title: Efficient and Accurate Full-Waveform Inversion with Total Variation Constraint
Date of Evaluation: 14-Jan-2025.
Date for Revision: 28-May-2024.

We would like to take this opportunity to thank Dr. Xiuping Jia for kindly handling our manuscript and the reviewers for their carefully reading of our manuscript and giving useful comments. Furthermore, we also appreciate the recommendation to resubmit to the Journal of Selected Topics in Applied Earth Observations and Remote Sensing. We have carefully studied all the comments given by the associate editor and the reviewers and have made revisions, which we hope will meet with your approval. Please kindly see the following pages that contain our detailed responses to each comment raised by the associate editor and each reviewer. We have also uploaded a PDF file of the revised manuscript, where we changed the text color to blue in revised parts.

All numberings (of sections, equations, references, etc.) in the original comments given by the reviewers refer to the previous manuscript, and those in the following replies refer to the revised manuscript. We have highlighted significant changes in the revised manuscript and the replies in blue.

We are grateful for your help and comments, which enabled us to significantly improve the quality of our paper. We hope that our response is sufficient to make our manuscript suitable for publication in the IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing.

Response to the Associate Editor

Associate Editor Comment — (There are no comments.)

Reply: Thank you very much for handling our paper carefully. Following the instruction, we have provided an item-by-item response to the reviewer's comments. Although the details are described in the answers to each reviewer's comment, we would like to summarize the main revisions in brief:

- We have reorganized the paragraphs in the Introduction for improved readability. (For the reply to Comment 1.1)
- We have improved the description of the proposed method. (For the replies to Comments 1.5, 2.1, 2.3, and 3.1)
- We have added references and comparative methods. (For the replies to Comments 1.2, 1.3, 1.4, 2.6, 3.2, and 3.5)
- We have conducted additional experiments with additional noise conditions. (For the replies to Comments 1.4, 2.5, and 3.4)
- We have expanded the experiments by incorporating an additional dataset. (For the reply to Comment 1.5)
- We have conducted further analysis of the proposed method. (For the replies to Comments 2.3 and 2.7)

Response to Reviewers

Reviewer 1

Reviewer General Comment — This paper presents an implementation of TV regularization for solving the FWI problem using the classical reduced-space formulation with an L2-norm misfit function. The author introduces the primal-dual splitting method with proximal operators to eliminate the inner loop required for nonlinear TV mapping. While the proposed algorithm may be useful for certain inverse problems, I find it unsuitable for FWI due to the following reasons:

Reply: Thank you for your careful reading and fruitful comments. (UNC)

Reviewer Comment 1.1 — The first sentence states: "This paper proposes a computationally efficient algorithm to address the Full-Waveform Inversion (FWI) problem with a Total Variation (TV) constraint ..." This claim is misleading. While the proposed method may be beneficial for linear inverse problems, where gradient calculations are inexpensive and the computational cost is dominated by TV regularization, this is not the case for FWI. In FWI, the leading-order cost is dominated by gradient computation, which primarily stems from solving the PDEs. Consequently, the overhead of implementing TV regularization, even with an inner loop, is negligible. In other words, the inner loop for TV regularization is not a computational bottleneck in FWI.

Reply: AAA

Reviewer Comment 1.2 — The proposed method is based on the reduced-space approach, which is known to be highly sensitive to cycle-skipping. Over the past decade, extensive research has demonstrated the advantages of extended-source FWI using Alternating Direction Method of Multipliers (ADMM), which enables TV regularization to be applied efficiently without inner loops. The paper does not adequately compare or contrast the proposed approach with these state-of-the-art methods.

Reply: AAA

Reviewer Comment 1.3 — The numerical example provided is not representative of realistic FWI case. The problem is formulated on a small subset of a benchmark model, placing the inverse problem in a linear regime. Furthermore, as observed in Figure 5, the method requires 5000 iterations, which is highly impractical and significantly deviates from the efficiency of modern FWI algorithms.

Reply: AAA

Reviewer Comment 1.4 — Given these limitations, I believe the proposed method for implementing TV regularization would be better suited for a different class of inverse problems rather than FWI. I recommend the author reconsider the application domain and provide a more thorough comparison with existing state-of-the-art methods.

Reply: AAA

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Reviewer 2

Reviewer General Comment — I appreciate the opportunity to review the manuscript titled "Efficient and Accurate Full-Waveform Inversion with Total Variation Constraint." While I find the core idea interesting and relevant to the field, the manuscript is currently far from being acceptable for publication and requires significant revisions. Below are my key concerns:

Reply: BBB

Reviewer Comment 2.1 — The introduction lacks a clear differentiation between the proposed method and existing TV-regularized FWI approaches, particularly those based on splitting techniques. The authors should explicitly state how their method improves upon prior work (or its connections), such as ADMM-based, split Bregman, FISTA, and proximal Newton approaches.

Reply: AAA

Reviewer Comment 2.2 — Some statements in the introduction are misleading. For instance, TV regularization is well-suited for piecewise constant models, not piecewise smooth ones, as claimed in the manuscript. The authors should correct such inaccuracies.

Reply: AAA

Reviewer Comment 2.3 — The theoretical formulation requires further clarification. Certain concepts are presented without adequate justification.

Reply: AAA

Reviewer Comment 2.4 — The tuning of the regularization parameter in the proposed method remains unclear. The authors should discuss how their approach overcomes known challenges related to parameter selection in TV-regularized FWI.

Reply: AAA

Reviewer Comment 2.5 — The design of numerical tests does not convincingly demonstrate the advantages of the proposed method. The authors should test their method on more challenging models.

Reply: AAA

Reviewer Comment 2.6 — The model size used in the experiments is quite small. The authors should justify their choice and, test their method on larger models. Also, the data should contain large offset data that are critical for FWI.

Reply: AAA

Reviewer Comment 2.7 — A key missing element is the residual curve for both standard FWI and TV-FWI. This would provide insights into the convergence behavior of the proposed method.

Reply: AAA

Reviewer Comment 2.8 — Given that FWI typically relies on L-BFGS for velocity updates rather than simple gradient descent, the authors should discuss whether their PDS method can be adapted for L-BFGS.

Reply: AAA

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Reviewer 3

Reviewer General Comment — The manuscript presents a novel algorithm for full-waveform inversion (FWI) that incorporates a Total Variation (TV) constraint into the inversion process via a primal-dual splitting (PDS) method. The goal is to improve both the computational efficiency and the accuracy of subsurface reconstructions. Overall, the paper addresses an important challenge in seismic imaging, and the proposed method is of potential interest to the geophysical community. However, several aspects require further clarification and refinement before publication. I therefore recommend major revisions.

Reply: BBB

Reviewer Comment 3.1 — While the approach is innovative, the manuscript would benefit from a more explicit comparison with existing TV-constrained FWI methods (e.g., references [21–24]). It is not entirely clear how the proposed formulation and algorithm differ from these earlier approaches beyond the elimination of inner loops. A discussion emphasizing the advantages—both theoretical and practical—would help contextualize your contribution.

Reply: AAA

Reviewer Comment 3.2 — The selection of the TV bound parameter α (ranging from 100 to 700) and the step sizes (γ_1 and γ_2) is not sufficiently justified. Given that FWI is highly sensitive to such hyperparameters, it would be beneficial to include a discussion on how these parameters could be selected in practice, possibly through data-driven approaches or based on prior geological knowledge.

Reply: AAA

Reviewer Comment 3.3 — Although the algorithm appears efficient, more details on its convergence behavior are needed. For example, how many iterations does the method typically require to converge relative to standard FWI? Is there any guarantee (theoretical or empirical) on the stability of the inversion, particularly when noisy data are used?

Reply: AAA

Reviewer Comment 3.4 — The use of a box constraint to keep velocity values within [l, u] is common; however, further discussion of the physical implications of the TV constraint would enhance the paper. For instance, how does enforcing TV relate to known geological structures or stratigraphic features? Can you provide examples or discuss cases where the TV constraint may excessively smooth features that are important for interpretation?

Reply: AAA

Reviewer Comment 3.5 — Although the manuscript claims improved computational efficiency, it lacks quantitative runtime comparisons. Including a table or a discussion of the actual computational times relative to a conventional FWI implementation would significantly strengthen this claim.

Reply: AAA

Reviewer Comment 3.6 — The figures depicting velocity models (Figs. 3 and 4) are central to evaluating the performance. To improve these figures, please provide individual color bars for each figure and discuss in more detail the "wave-like artifacts" observed in standard FWI and how the TV constraint mitigates these artifacts. Additionally, please provide a more detailed discussion of the "point artifacts" near the surface, which appear similar to source-receiver artifacts; note that the number of these artifacts (four in Fig. 3a) is much smaller than the number of sources (20) or receivers (101). Finally, please change the labeling in Fig. 4 from (e–h) to (a–d), as Figs. 3 and 4 are two separate figures, not one combined figure.

Reply: AAA

Reviewer Comment 3.7 — The conclusion could benefit from a more critical discussion of the limitations of the proposed approach. For example, under what conditions might the TV constraint be too restrictive? What future work is planned to extend or validate the method further?

Reply: AAA

Reviewer Comment 3.8 — The definitions and descriptions of the variables should be improved. For example, the operator D appears in equation (2) without any definition; please clarify its meaning.

Reply: AAA

Reviewer Comment 3.9 — Since the authors plan to share their source code, additional documentation—including replacing Japanese with English—is necessary. I attempted to understand the code but found it difficult due to insufficient guidance.

Reply: AAA