

1. What are terminals and non terminal symbols in grammar

Ans:-

Terminals: The terminal symbols are the elementary symbol of the language defined by a formal grammar. A terminal symbol which doesn't appear on the left hand side of any production. A grammar contains a set of terminal symbol such as symbol +, -, *, /, etc. The digits from 0 to 9, The bold and letters such as id.

Non-terminal: The Non-terminal symbol (or syntactic variables) are replaced by groups of terminal symbol according to the production rules. The following are non-terminal. The lower case names such as expression operator, operand, statement, etc. The letter S is the start symbol, The grammar symbol can be terminal or non-terminal. The capital letters near the end of the alphabet such as X, Y, Z etc represent grammar symbol.

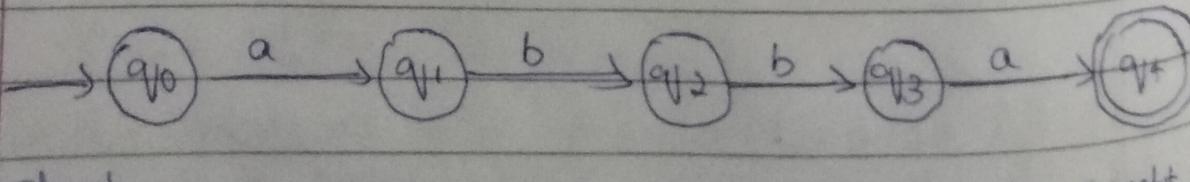
3. Construct a DFA to accept the string abba

Ans:-

Step 1: Identify the minimum string
minimum string = {abba}

Step 2: Identify the alphabets
 $\Sigma = \{a, b\}$

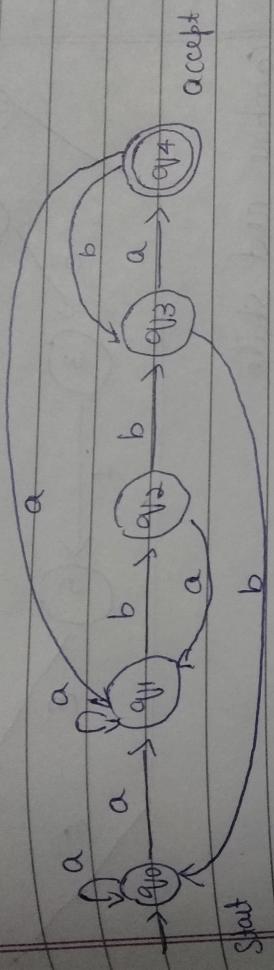
Step 3: Construct a skeleton DFA



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Step 4: Identify the skeleton transition in Step 3

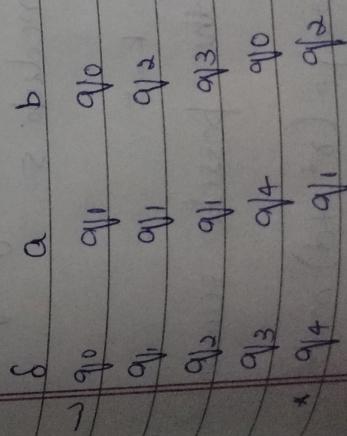
$$\begin{aligned}\delta(q_0, b) &= q_1, \quad \delta(q_1, a) = q_2, \quad \delta(q_2, a) = q_3, \quad \delta(q_3, b) = q_4 \\ \delta(q_4, a) &= q_1, \quad \delta(q_4, b) = q_1\end{aligned}$$



Step 5: Construct the DFA using transition in Step 3 & Step 4

$$\begin{aligned}M &= \{Q, \Sigma, \delta, q_0, F\} \\ Q &= \{q_0, q_1, q_2, q_3, q_4\} \\ \Sigma &= \{a, b\}\end{aligned}$$

q_0 is the start state
 q_4 is final state



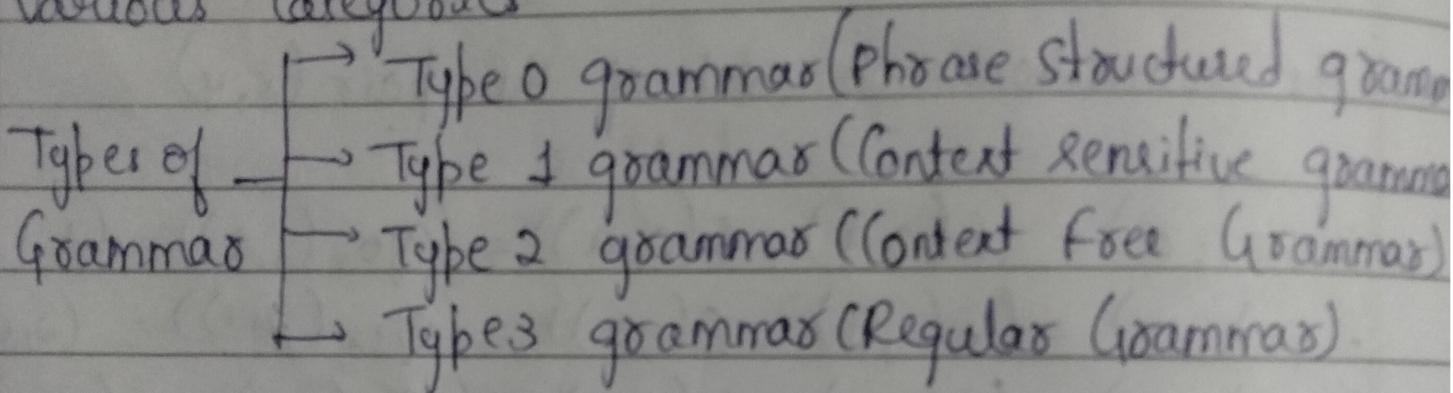
with Epsilon Transition

production

- S is the start symbol

s Explain in detail chomsky hierarchy

Noam Chomsky who is the founder of formal language theory has classified the grammar in various categories



(i) Type 0 Grammar

A grammar $G = (V, T, P, S)$ is said to be type 0 grammar / unrestricted grammar / phrase structure grammar if all the production are of the form $X \rightarrow \beta$ where $\beta \in (VUT)^*$ and $\beta \in (VUT)^*$

The only restriction is that X can not be ϵ i.e., ϵ can not appear on the left hand side of any production family of grammars more

Type 0 language / recursively enumerable language
only turing machine can recognize the language

$$\begin{aligned} S &\rightarrow aAb/\epsilon \\ aA &\rightarrow bAA \\ bA &\rightarrow a \end{aligned}$$

(ii) Type 1 Grammar / Context sensitive Grammar

A grammar $G = (V, T, P, S)$ is said to be type 1 grammar Context sensitive grammar if all the productions are of the form $x \rightarrow \beta$ as in type 0 grammar. There is a restriction on length of β . The length of β i.e., $|\beta| \geq |x|$ & x and $\beta \in (VUT)^*$ i.e., ϵ can not appear on the left hand right hand side of any production. It is ϵ -free grammar

Linear bounded automata (LBA) can be constructed to recognize the language generated from this grammar

$$\begin{aligned} S &\rightarrow aAb \\ aA &\rightarrow bAA \\ bA &\rightarrow aa \end{aligned}$$

(iii) Type 2 Grammar / Context free Grammar

A grammar $G = (V, T, P, S)$ is said to be type 2 grammar or context free grammar if all the productions are of the form $A \rightarrow x$ where $x \in (VUT)^*$ & A is non terminal. The ϵ can appear on the right hand side of any production. Pushdown Automata (PDA) can be constructed to recognize the language generated from this grammar

$$\begin{array}{l} S \rightarrow aB \mid bA \mid \epsilon \\ A \rightarrow aA \mid b \\ B \rightarrow bB \mid a \mid \epsilon \end{array}$$

(iv) Type 3 Grammars / Regular Grammar

The grammar $G = (V, T, P, S)$ is said to type 3 grammar / regular grammar if the grammar is right linear / Left linear. A grammar G is said to be right linear if all the production are of the form

$$A \rightarrow wB$$

$$\text{or } A \rightarrow w$$

where A, B are variable & $w \in T^*$, i.e. T^* i.e., w is string of terminals. A grammar G is said to be left linear

$$A \rightarrow Bw @ A \rightarrow w$$

where $A, B \in V$ & $w \in T^*$

$$S \rightarrow aab \mid bba \mid \epsilon$$

$$A \rightarrow aA \mid b$$

$$B \rightarrow bB \mid a \mid \epsilon$$

is a right linear Grammar

$$S \rightarrow Baa \mid Abb \mid \epsilon$$

$$A \rightarrow Aa \mid b$$

$$B \rightarrow Bb \mid a \mid \epsilon$$

is a left linear grammar

Consider the following grammar

$$S \rightarrow aA$$

$$A \rightarrow ab \mid b$$

$$B \rightarrow Ab \mid a$$

In this grammar each production is either left linear / right linear. But, the grammar is not either left linear / right linear. Such type of grammar is called 1-linear grammar.

~~as x,y,z are regular languages~~

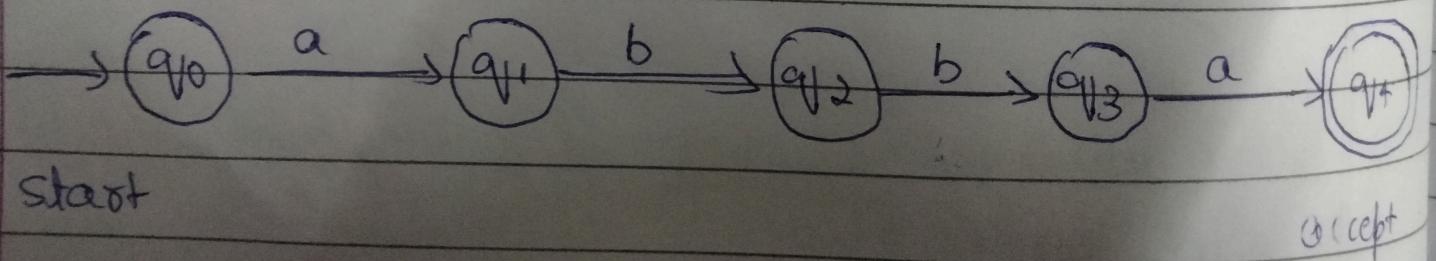
3. Construct a DFA to accept the string abba

Ans:-

Step 1: Identify the minimum string
minimum string = {abba}

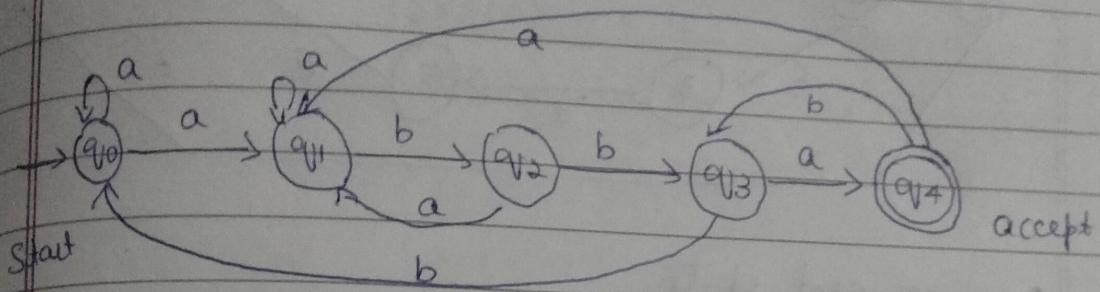
Step 2: Identify the alphabets
 $\Sigma = \{a, b\}$

Step 3: Construct a skeleton DFA



in Step 3 transition not defined

$$\delta(q_0, b) = ?, \quad \delta(q_1, a) = ?, \quad \delta(q_2, a) = ?, \quad \delta(q_3, b) = ?$$
$$\delta(q_4, a) = ?, \quad \delta(q_4, b) = ?$$



Step 5: Construct the DFA using transition in
step 3 & step 4

$$M = \{Q, \Sigma, \delta, q_0, F\}$$

$$Q = \{q_0, q_1, q_2, q_3, q_4\}$$

$$\Sigma = \{a, b\}$$

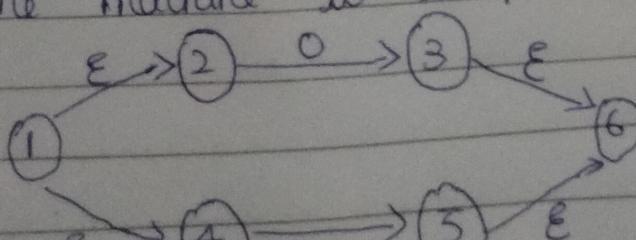
q_0 is the start state

q_4 is final state

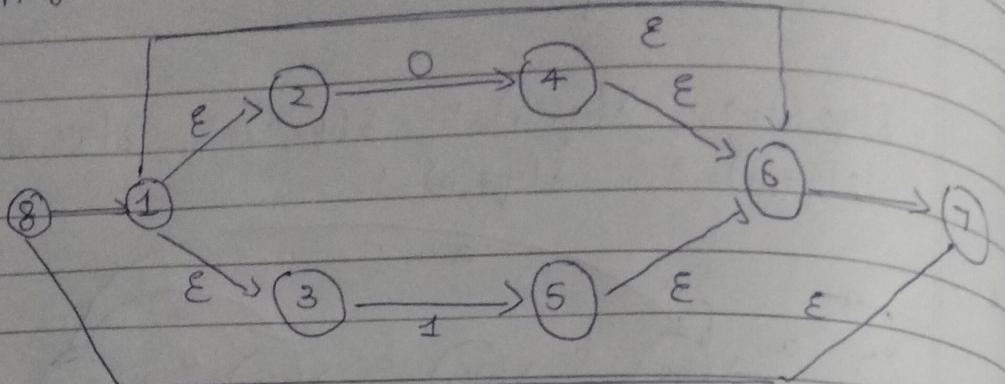
δ	a	b
q_0	q_1	q_0
q_1	q_1	q_2
q_2	q_1	q_3
q_3	q_4	q_0
q_4	q_1	q_2

4. Construct a NFA with Epsilon Transition
 $(0+1)^*$ $1(0+1)$

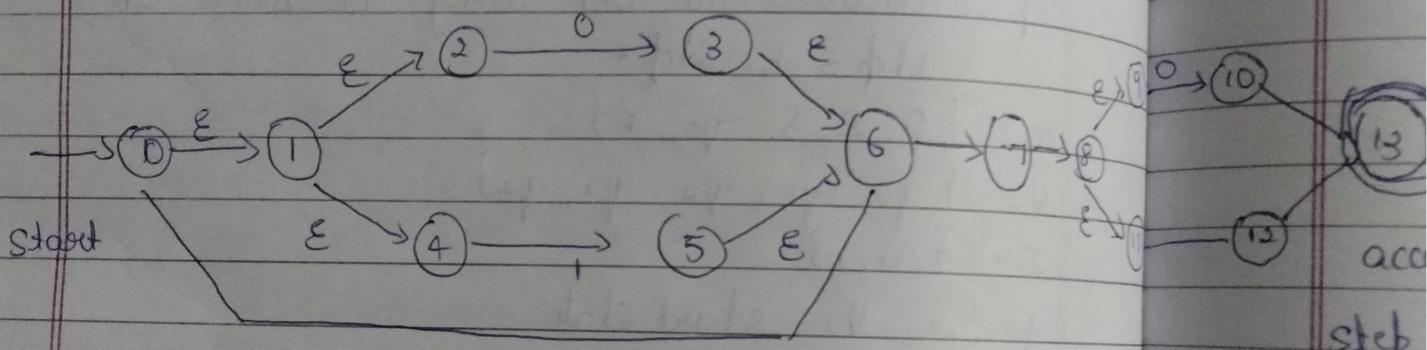
Step 1. The machine to accept '1'



Apply kleene star



Combine next state



5. Construct a DFA to accept the string of 0's & 1's representing zero modulo 5

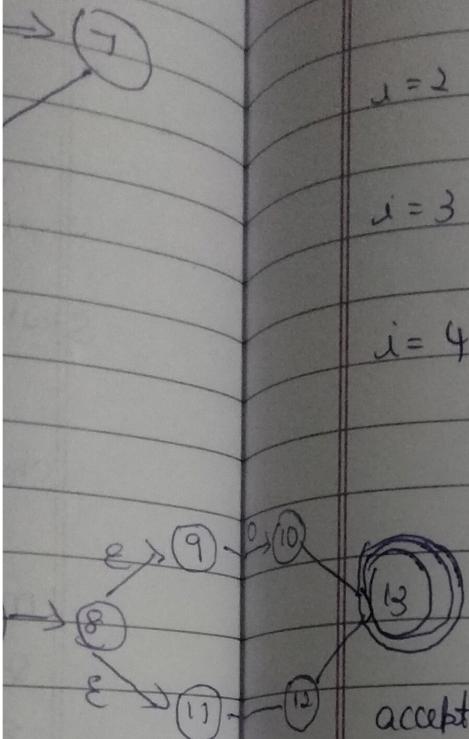
Step 1: $r=2$, $d=\{0,1\}$, $k=5$

After dividing k , the possible remainders

Step 2: $i=0, 1, 2, 3, 4$

Step 3: $\delta(q_i, a) = q_j$ where $j = (\delta + i + d) \bmod k$
with $r=2$ & $k=5$

remainder	d	$(2*i+d) \% 5$	$\delta(q_i^j, d) = q_{j+1}^i$
$j=0$	0	$(2*0+0) \% 5 = 0$	$\delta(q_0^0, 0) = q_0^1$
	1	$(2*0+1) \% 5 = 1$	$\delta(q_0^1, 1) = q_1^1$
$j=1$	0	$(2*1+0) \% 5 = 2$	$\delta(q_1^0, 0) = q_2^1$
	1	$(2*1+1) \% 5 = 3$	$\delta(q_1^1, 1) = q_2^2$
$j=2$	0	$(2*2+0) \% 5 = 4$	$\delta(q_2^0, 0) = q_3^1$
	1	$(2*2+1) \% 5 = 0$	$\delta(q_2^1, 1) = q_3^0$
$j=3$	0	$(2*3+0) \% 5 = 1$	$\delta(q_3^0, 0) = q_4^1$
	1	$(2*3+1) \% 5 = 2$	$\delta(q_3^1, 1) = q_4^2$
$j=4$	0	$(2*4+0) \% 5 = 3$	$\delta(q_4^0, 0) = q_4^3$
	1	$(2*4+1) \% 5 = 4$	$\delta(q_4^1, 1) = q_4^4$



Step 4 : $M_2 = (Q, \Sigma, \delta, q_0^0, F)$

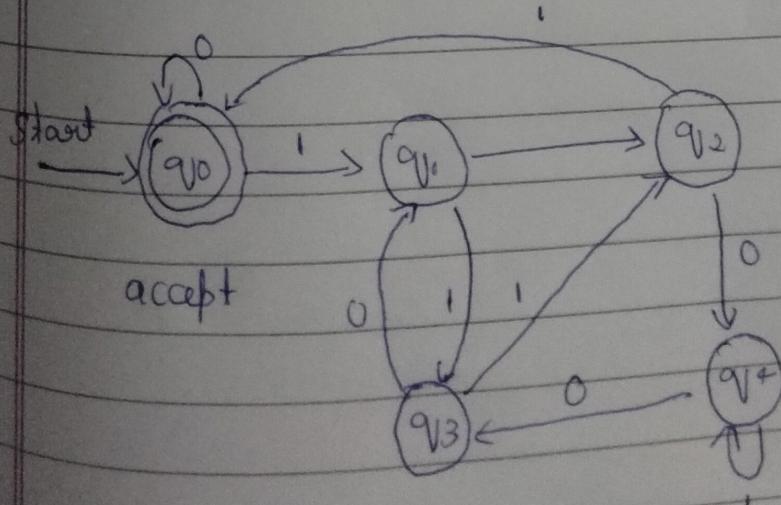
$$Q = \{q_0^0, q_1^1, q_2^2, q_3^3, q_4^4\}$$

$$\Sigma = \{0, 1\}$$

q_0^0 is the start state

$$F = \{q_0^0\}$$

δ = Transition state



→ Transition table

δ	0	1
q_0	q_0	q_1
q_1	q_2	q_3
q_2	q_4	q_0
q_3	q_1	q_2
q_4	q_3	q_4

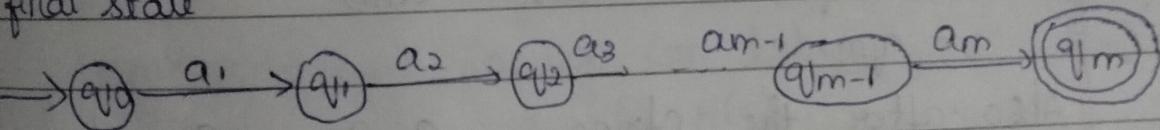
Theorem: Let $M = (Q, \Sigma, \delta, q_0, F)$ be a FA & has n number of states. Let L be the regular language accepted by M . Let for every string x in L , there exists a constant n such that $|x| \geq n$. If string x can be broken into three substrings $u, v & w$ such that $x = uvw$

$$v \neq \epsilon \text{ i.e., } |v| \geq 1 \quad |uvw| \leq n$$

then $uv^i w$ is in L for $i \geq 0$

Proof: Let $M = (Q, \Sigma, \delta, q_0, F)$ be an FA & let L is the language accepted by DFA & is regular. Let $x = a_1 a_2 a_3 \dots a_m$ where $m \geq n$ & each a_i is in Σ . Here, n represent the state of DFA

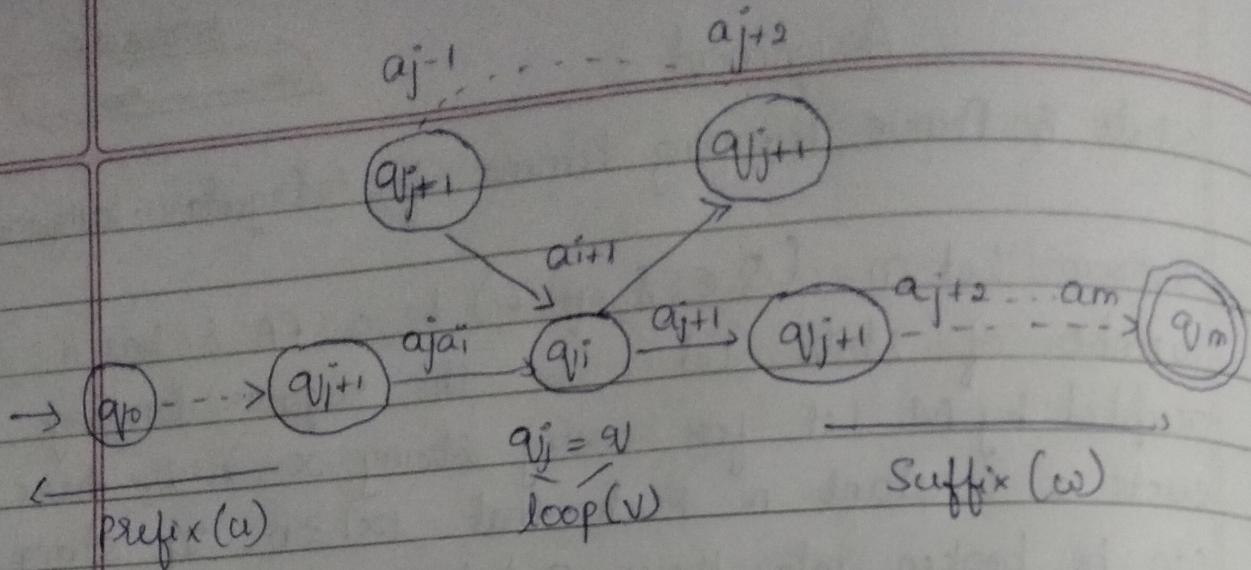
Since we have m input symbol, naturally we should have $m+1$ states in the sequence $q_0, q_1, q_2, \dots, q_m$, where q_0 will be start state & q_m will be the final state



Since $|x| \geq n$, by the pigeon hole principle it is not possible to have distinct transitions. One of the states can have a loop. Let the string x is into 3 substrings

The first group is string prefix a_1, a_2, \dots, a_i i.e., $a_1 a_2 \dots a_i$ the second group is the loop string $a_{i+1} a_{i+2} \dots a_j$ i.e., $v = a_{i+1} a_{i+2} \dots a_{j-1} a_j$

The third group is the loop string suffix from $a_{j+1} a_{j+2} \dots a_m$ i.e., $w = a_{j+1} a_{j+2} \dots a_m$



But when $i=1$ the string uvw is accepted by DFA when $i=2$, the string uvw is accepted by DFA. So, if $i > 0$, the machine goes from q_0 to q_j on I/P string u , circles from q_j to q_j based on the value of i then goes to accepting state on I/P string w , if the string n is split into sub string uvw , then for all $i \geq 0$

$$uviw \in L$$

This can be expressed using transitions

$$\begin{aligned} & \delta(q_0, a_1 a_2 \dots a_{i-1} a_i a_{j+1} a_{j+2} \dots a_m) \\ &= \delta(\delta(q_0, a_1 a_2 \dots a_{i-1} a_i), a_{j+1} a_{j+2} \dots a_m) \\ &= \delta(q_j, a_{j+1} a_{j+2} \dots a_m) \\ &= \delta(q_k, a_{k+1} a_{k+2} \dots a_m) \\ &= q_m \end{aligned}$$

Also after the string $a_1 a_2 \dots a_i$ the machine will be in state q_{i-1} . Since q_i and q_{i-1} are same $a_{i+1} a_{i+2} \dots a_j$ any no. of time & the machine will stay in q_i only finally, if the I/P string is $a_{j+1} a_{j+2} \dots a_m$ The machine enters into final state q_m