Slides (Module 2) on Matrix Inverse with Examples

Module 2
(Inverse of a Matrix)

Inverse of a Matrix

- Inverse of a $n \times n$ square matrix A is a matrix A' or A^{-1} such that

$$AA' = I = A'A$$

or

$$AA^{-1} = I = A^{-1}A$$

where I is $n \times n$ identity matrix.

If the inverse matrix A' exists, then A is invertible.

A^{-1} → More popular notation

For example,

$A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$ and $A' = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$ So, A' is an inverse of the matrix A.

$$AA' = \begin{bmatrix} 6-5 & -10+10 \\ 3-3 & -5+6 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_{2\times 2}$$

$$A'A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_{2\times 2}$$

Another example,

$A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ Is it invertible?

Let A' exist and $A' = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$AA' = \begin{bmatrix} a+2c & b+2d \\ 2a+4c & 2b+4d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

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Comparing component-wise,

$$a + 2c = 1$$
$$b + 2d = 0$$
$$2a + 4c = 0$$
$$2b + 4d = 1$$

Is it possible that the 1st and the 3rd equation satisfy together?

<u>Theorem</u>: If A is invertible, A^{-1} is unique.

Assume that there are two inverses A', A''

So, $AA' = A'A = I = AA'' = A''A$

Due to Associativity of Matrix Multiplication

$$A' = A'I = A'(AA'') = (A'A)A'' = IA'' = A''$$

Be careful ! $A^{-1} \neq \frac{1}{A}$

<u>Theorem</u>:
If A is invertible, the system of linear equations $A\boldsymbol{x} = \boldsymbol{b}$ has the unique solution $\boldsymbol{x} = A^{-1}\boldsymbol{b}$ for any $b \in \mathbb{R}^n$

<u>Theorem</u>:
If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then A is invertible if $(ad - bc) \neq 0$.
In that case, we can write $A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

If $ad - bc = 0$, A is not invertible.

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This is actually the determinant of A

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