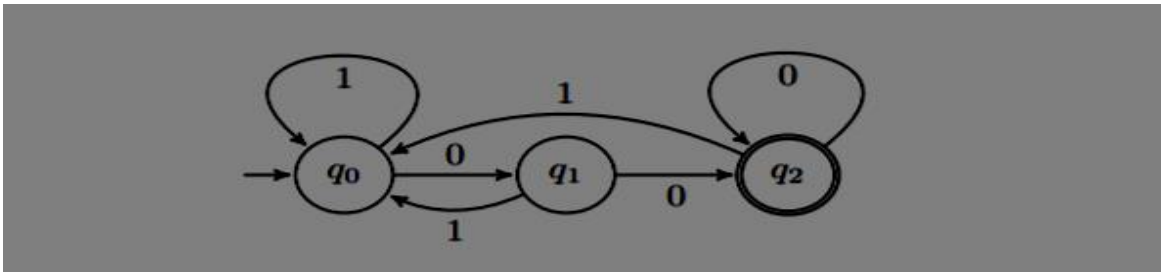
Birla Institute of Technology and Science, Pilani
CS F351, Theory of Computation
Practice Questions on CFG/PDA

Note: You should practice these problems with following perspectives:

a) Design a CFG/PDA for the given language.
b) Decide whether a given language is context free or not.
c) Given a grammar / PDA, deduce its language.
d) Closure properties.

Q1) Give a CFG for: L = {0^i 1^j 0^k | j > i + k} So, 001111100 is in the string.

Q2)Construct a context-free grammar for the following DFA:



Q3)
Give a CFG to generate A = {$a^i b^j c^k$ | i, j, k ≥ 0 and either i = j or j = k}. Is the grammar ambiguous? Why or why not?

Q4)
Give a context-free grammar generating the language L = the complement of the language {$a^n b^n$ | n ≥ 0}.

Q5)
Let T = { 0, 1, (,), ∪, ∗ , ∅, e }. We may think of T as the set of symbols used by regular expressions over the alphabet {0, 1}
a) Your task is to design a CFG G with set of terminals T that generates exactly the regular expressions with alphabet {0, 1}.
b)Using your CFG G, give a derivation and the corresponding parse tree for the string (0 ∪ (10)∗ 1)∗ .

Q6)
Prove that the class of context-free languages is closed under concatenation.

Q7)
Give a context-free grammar that generates postfix expressions using operators +, -, *, and /. Use the terminal "id" to stand for any variable name and "lit" to stand for integer literals. Show a derivation of "id lit id + *" in your grammar.

Q8)

Please convert the following CFG into an equivalent CFG in Chomsky normal form

$A \rightarrow BAB \mid ABA \mid B \mid \epsilon$

$B \rightarrow 00 \mid \epsilon$

Q9)

Let G = (V,T,S,P) be a context-free grammar such that every one of its productions is of the form $A \rightarrow v$, with $|v| = k > 1$. Show that the derivation tree for any $w \in L(G)$ has a height h such that

$$\log_k |w| \leq h \leq \frac{(|w| - 1)}{k - 1}.$$

Q10)

Let L1 be a context-free language and L2 be regular. Show that there exists an algorithm to determine whether or not L1 and L2 have a common element.

Q11)

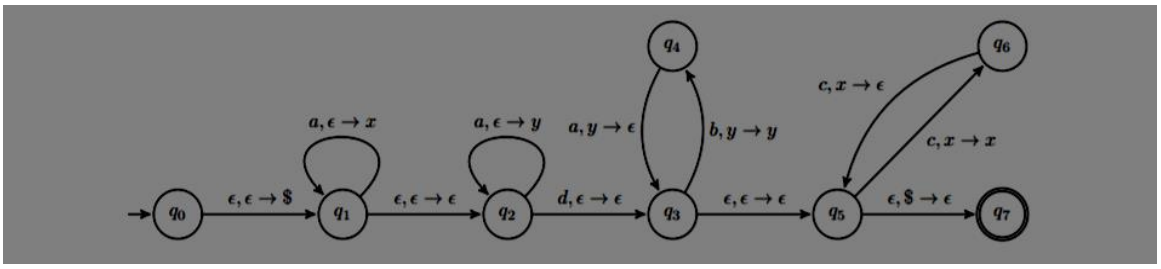
Describe a PDA to accept sets of "balanced" "{" and " }" symbols from a "C" program. That is, the alphabet consists of the two symbols: "{" and "}" Strings are sequences of these symbols such that no prefix of the string has more "}" than "{". For example, the following is legal: {{}{}}, but the following string is not: {}}{.

Q12)

Using the Pumping Lemma for CFLs to prove that language $\{0^n 1^m 0^n 1^m \mid n, m \geq 0\}$ is not context-free.

Q13)

Consider the following PDA:



(a) Show that the PDA accepts the word aaadbabacc.

(b) Which language L does the given PDA accept?

Q14)

Create a PDA that recognizes the following context free language: $L = \{a^*wc^k \mid w \in \{a, b\}^*$ and k = the number of as in w}

Q15)
Convert the following CFG G to an equivalent PDA.
$R \rightarrow XRX|S$
$S \rightarrow aTb|bTa$
$T \rightarrow XTX|X|\epsilon$
$X \rightarrow a|b$

Q16)
Prove that the intersection of a CFL and a regular language is always a CFL. Assume you are given a PDA M for the CFL and an NFA A for the regular language, then describe a PDA which accepts the intersection of their languages.

Q17)
Consider the set of all strings a/b, where a and b are positive decimal integers such that a < b. The set of strings then represents all possible decimal fractions. Determine whether or not this is a context-free language.

Q18)
$L = a^i b^j c^m d^n$ where i+j=m+n. what will be the CFG for this?

Q19)
Let L be a given context-free language over the alphabet {a,b}. Construct L1 , L2 as follows.
Let $L1=L- \{ xyx \mid x,y \in \{a,b\}*\}$, and $L2=L \cdot L$

Then which of the above two is/are CFG. Give Proper Reason

Q20)
Consider the following Language L defined as follow
$L = \{ a^n b^m \mid n<=m \text{ and } m>= 481\}$
Is above Language CFL or Not justify your answer

Q21)
$L1 = a*b*c^n d^n$ $L2 = a^n b^n c*d*$
Find $L1 \cap L2$

Q22)
$L = \{ W \# W \mid W \text{ is in } (0+1)* \text{ and } \# \text{ is a special symbol}$
Is L CFG or Not. Justify Your Answer

Q23) Which of the following languages is/are deterministic context-free?

$L1=\{ ww^R \mid w \in \{a,b\}* \text{ and } w^R \text{ is reverse of } w\}$

$L2=\{ ww^R x \mid w, x \in \{0,1\}*\}$

Give Proper Reason

Q24) Check if the following is CFL?

$a^i b^j c^k$ | (if (i==j) then k is even)

Where “If” is IF operator used in C language

Q25) Suppose that L is Context free and R is Regular.

Statement 1: L–R is necessarily Context free
Statement 2: R–L is necessarily Context free

Which of the above statements is/are true? Give Proper Reason

Q26) Consider the following grammars. Names representing terminals have been specified in capital letters.

Grammar G1 :
stmnt→ WHILE (expr) stmnt
stmnt→ OTHER
expr → ID

Grammar G2 :
stmnt→ WHILE (expr) stmnt
stmnt→ OTHER
expr→ expr+expr
expr→ expr*expr
expr→ ID

Then which of the above two is/are CFG. Give Proper Reason

Q27) Design a context free grammar for the language consisting of all strings over {a,b} that are not the form ww for any string w.

Q28) Consider the following context-free grammar over the alphabet Σ = {a, b, c} with S as the start symbol:
S → abScT | abcT
T → bT | b

Give the Language Represented by this CFG

Q29)
Let L = L1∩L2, where L1 and L2 are languages as defined below:
L1 = {$a^m b^m c a^n b^n$ | m, n >= 0 }
L2 = {$a^i b^j c^k$ | i, j, k >= 0 }
Then L is _____

Q30) Consider a CFG with the following productions.

$S \rightarrow AA \mid B$
$A \rightarrow 0A \mid A0 \mid 1$
$B \rightarrow 0B00 \mid 1$
S is the start symbol, A and B are non-terminals and 0 and 1 are the terminals. The language generated by this grammar is