Computability - Exercise 3

All answers should be proved formally

Due Wednesday, June 11 09:59:59 AM

1. For each of the following languages over the alphabet $\{0, 1\}$, write a regular expression for the language. No need to prove your answer (but make sure you are correct).
   (a) All words of odd length. (hint: is it easier to think of even?)
   (b) All words in which the number of 1's is strictly smaller then 5.
   (c) All words that do not contain neither 00 not 11 as a subwords.

2. Are the following languages regular? If ,in your proof, you choose to describe an automaton or a regular expression, there is no need to prove the correctness of the construction.

   All the languages are over $\Sigma = \{0, 1\}$
   (a) $L = \{1^{2^n} \mid n \geq 0\}$
   (b) $L = \{0^m 1^n \mid 0 \leq m \leq n \leq 1000\}$
   (c) $L = \{w \mid$ The number of 01 substrings in w equals the number of 10 substrings in $w\}$
   (d) $L = \{ww \mid w \in \Sigma^*\}$
   (e) For a fixed natural $n \geq 0$, the language $L_n = \{ww \mid w \in \Sigma^n\}$

3. Define deterministic infinite automaton in the same way that DFA's are defined, with the only difference that the set of states can be infinite, and so is the set of accepting states. What languages are accepted by deterministic infinite automata?

4. In this question (taken from the exam in 2006) we define a new type of automata: universal automata. The definition of universal automaton is very similar to that of a nondeterministic automaton. The only

difference is that a word w is accepted by a universal automaton $\mathcal{A}$ iff *all* the runs of $\mathcal{A}$ on w are accepting (rather then if there exists a run that is accepting).

For example: look at the universal automaton $\mathcal{A}$ in Figure 1:

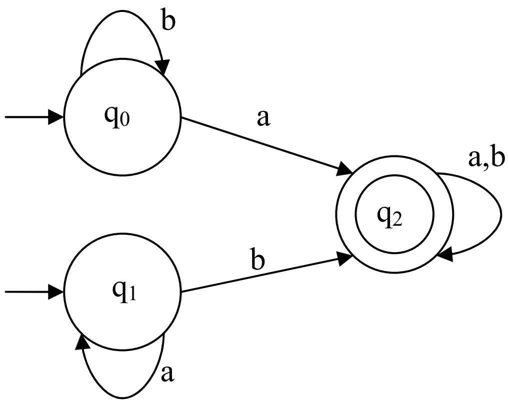

Figure 1: Example: $\mathcal{A}$ a universal automaton

Note that $bbb \notin L(\mathcal{A})$ because $q_0q_0q_0q_0$ is a non-accepting run of $\mathcal{A}$ on bbb.

(a) What is $L(A)$?

(b) Prove or refute: for every language $L \subseteq \Sigma^*$ it holds that L is regular iff L is accepted by some universal automaton

5. (Optional question)
Prove that a DFA with n states accepts an infinite language iff it accepts a word w such that $n \leq |w| \leq 2n$ (where $|w|$ is the length of w).

6. (Optional question)
Show that the language $L = \{a^ib^jc^k \mid i,j,k \geq 0 \text{ and if } i \geq 1 \text{ then } j = k\}$ satisfies all the conditions of the pumping lemma. Prove that L is not regular. Does this fact contradict the pumping lemma?