Computability - Exercise 2

All answers should be proved formally

Due Wednesday, June 4, 10:00 AM

1. For $n > 0$, let L_n be the following language over the alphabet $\Sigma = \{a, b\}$.

   $$L_n = \{w : \text{ the } n\text{-th letter of } w \text{ counting from the } \textbf{end} \text{ is } a\}.$$

   (a) Construct the nondeterministic automaton that recognizes L_n and has $O(n)$ states.

   (b) Prove that any deterministic automaton that recognizes L_n has to have at least 2^n states.

2. For a language $L \subseteq \Sigma^*$, let

   $$Pref(L) = \{x : \text{ there exists } y \text{ such that } xy \in L\},$$

   and

   $$Suff(L) = \{x : \text{ there exists } y \text{ such that } yx \in L\}.$$

   Show that if L is regular then so are $Pref(L)$ and $Suff(L)$.

3. For a word $w = w_1 w_2 \cdots w_n$, the reverse of w, denoted w^R is the word w written in reverse order, i.e., $w_n \cdots w_2 w_1$. For a language $L \subseteq \Sigma^*$, let $L^R = \{w^R | w \in L\}$.
   Show that if L is regular then so is L^R.

4. Consider the following finite language (over $\Sigma = \{0, 1\}$):

   $$L_n = \{ww \mid w \in \{0, 1\}^n\}.$$

   (a) Prove that every *nondeterministic* finite automaton for L_n must contain at least 2^n states.

(b) Show a nondeterministic finite automaton for $\overline{L_n}$ with $O(n)$ states. Where $\overline{L_n}$ is the complement of L_n.

5. Draw an equivalent deterministic finite automaton for the automaton in Figure 1.

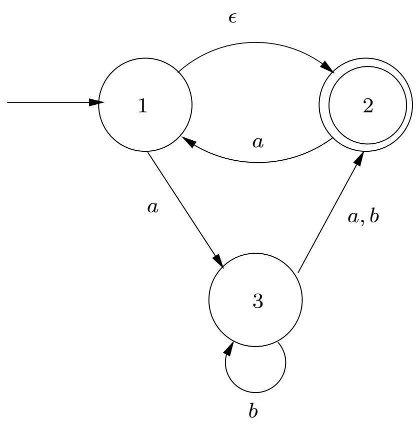

Figure 1: Determinize me!

If you use the determinization construction taught in class (with or without omitting unreachable states) there is no need to provide a proof.

6. (optional)

For a language L (over Σ), define the language $L_{\frac{1}{2}}$ (over Σ) as follows: $L_{\frac{1}{2}} = \{w : \exists y \text{ such that } |w| = |y| \text{ and } wy \in L\}$

Prove that if L is regular then so is $L_{\frac{1}{2}}$.