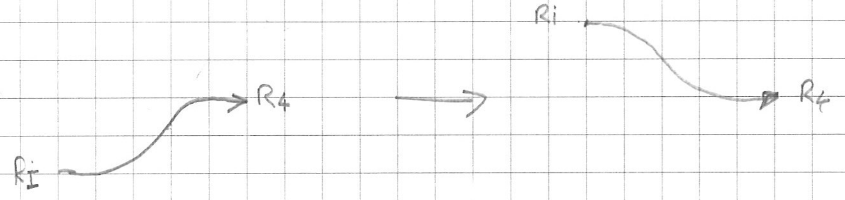
PLEP

① All ramps normalised to be descending, for calculation.



Normalisation is by reflection about final reference, R_4:

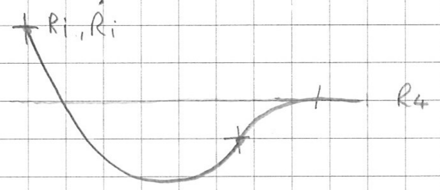
$$R_i = 2R_4 - R_I$$

No exponential section is allowed for ascending ramps.

② Possible forms:

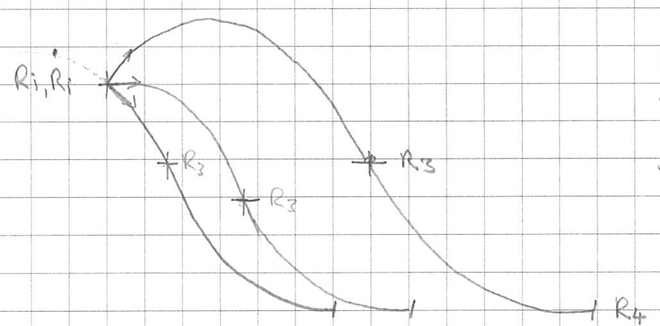
Notes

1. $\overline{P - P}$



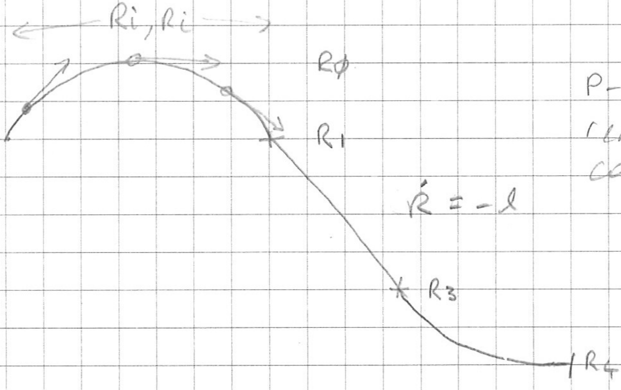
INVERTED $P-P$ is used when $\dot{R}_i$ is too fast to stop without undershooting R_4.

2. $P - P$



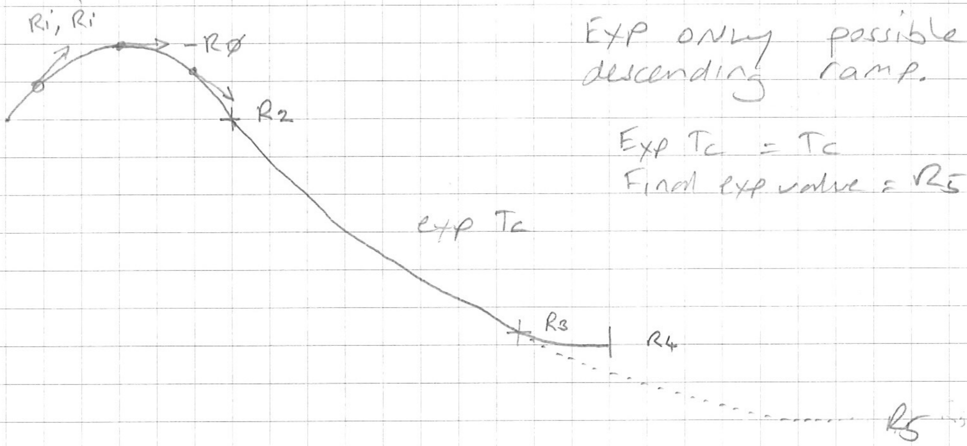
$P-P$ is used when $\dot{R}$ does not exceed max $\dot{R}$ at join between parabolas.

3. $P - L - P$



$P-L-P$ is used if $\dot{R}$ reaches max $\dot{R}$ (l) before connecting with P deceleration

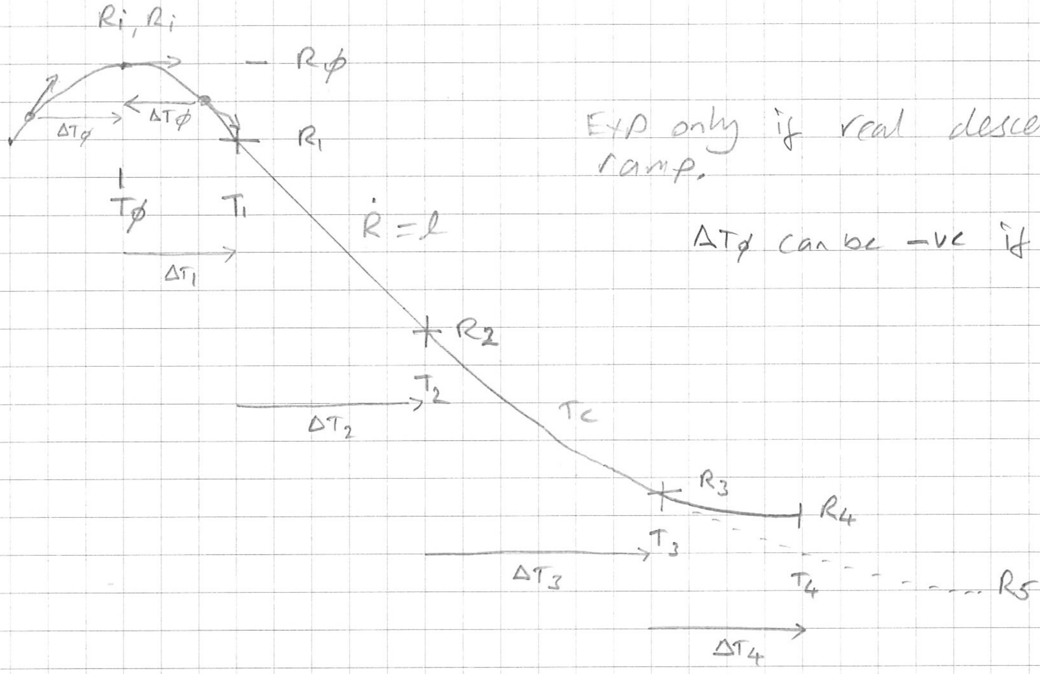
4. P - E - P



Exp only possible for true descending ramp.

Exp $T_c = T_c$
Final exp value = R_5 at $t = \infty$

5. P - L - E - P



Exp only if real descending ramp.

$\Delta T\phi$ can be -ve if $\dot{R}_i < \phi$

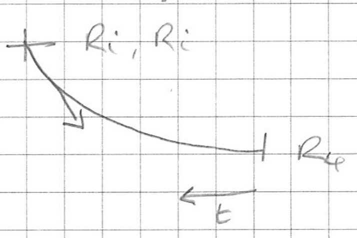
③ PLEP parameters (after normalisation)

R_i	Initial reference
$\dot{R}_i$	Initial rate of change of reference
a	parabolic acceleration / deceleration
l	Max linear rate of change
T_c	Time constant for exp
R_5	Final ref for exp at $t \to \infty$
R_4	Final reference.

(4) Calculated parameters

$R\phi$:	Top of initial parabola
$R1$	End of initial parabola
R_2	End of linear
R_3	End of exp
$T\phi$	Time of top of parabola (may be −ve)
T_1	Time of end of parabola
T_2	Time of end of linear
T_3	Time of end of exp
T_4	Time of end of PCEP

(5) CONDITION EQUATIONS

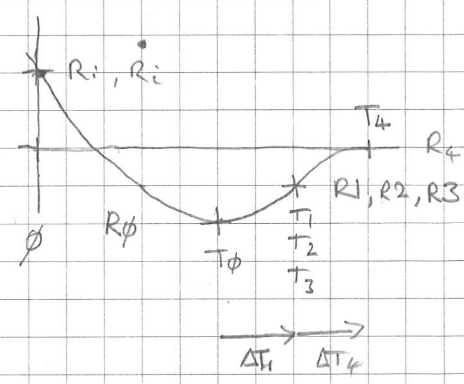


$\overline{P-P}$

$$\left.\begin{aligned} R &= R_4 + \tfrac{1}{2} a t^2 \\ \dot{R} &= at \end{aligned}\right\} R = R_4 + \frac{1}{2}\frac{\dot{R}^2}{a}$$

Limiting case $R_i = R_4 + \dfrac{(-\dot{R}_i)^2}{2a}$ Note $\dot{R}_i$ is −ve, a is +ve

So. $\overline{P-P}$ if $\dot{R}_i < -\sqrt{2a(R_i - R_4)}$



with $T\phi = -\dfrac{\dot{R}_i}{a}$

$$R\phi = R_i - \frac{\dot{R}_i^2}{2a}$$

$$\Delta T_1 = \Delta T_4 = \sqrt{\frac{R_4 - R\phi}{a}}$$

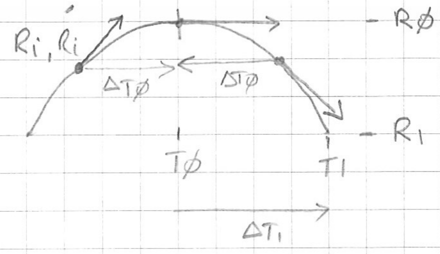
$$R_1 = R_2 = R_3 = \frac{R_4 - R\phi}{2}$$

$$T_1 = T_2 = T_3 = T\phi + \Delta T_1$$

$$T_4 = T_3 + \Delta T_4$$

P - P, P - L - P, P - E - P, P - L - E - P

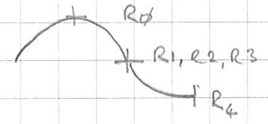
$$T\phi = \frac{\dot{R}_i}{a} \qquad (a \text{ +ve})$$



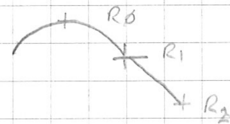
$$R\phi = R_i + \frac{\dot{R}_i^2}{2a}$$

Parabola can join:

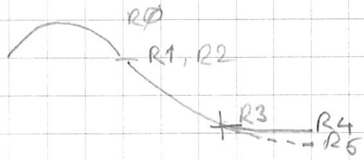
a. Parabola $\Delta T_{1_{PP}} = \sqrt{\frac{(R\phi - R_4)}{a}}$



b. Linear $\Delta T_{1_{PL}} = \frac{\ell}{a}$



c. Exponential $\Delta T_{1_{PE}} = \sqrt{\left(T_c^2 + \frac{2(R\phi - R_5)}{a}\right)} - T_c$



If $(\Delta T_{1_{PP}} < \Delta T_{1_{PL}})$ && $(\Delta T_{1_{PP}} < \Delta T_{1_{PE}})$ $\longrightarrow$ P - P

else if $(\Delta T_{1_{PE}} < \Delta T_{1_{PL}})$ $\longrightarrow$ P - E - P

else $\longrightarrow$ P - L - P or P - L - E - P

Derivation of $\Delta T_{1_{PE}}$:

for Parabola: $\dot{R}_1 = -a\Delta T_1$

for exp: $\dot{R}_1 = \frac{-R_1 + R_5}{T_c}$

$\Big\} \; -aT_c \Delta T_1 = -R_1 + R_5$

where $R_1 = R_o - \frac{a \Delta T_1^2}{2}$

$$\therefore \; aT_c \Delta T_1 - \left(R_o - \frac{a\Delta T_1^2}{2}\right) + R_5 = \phi$$

$$\therefore \; \left(\frac{a}{2}\right)\Delta T_1^2 + (aT_c)\Delta T_1 - (R_o - R_5) = \phi$$

$$\therefore \; \Delta T_1^2 = \frac{-aT_c \pm \sqrt{a^2 T_c^2 + 4\left(\frac{a}{2}\right)(R_o - R_5)}}{2\left(\frac{a}{2}\right)}$$

Taking +ve root: $\Delta T_{1_{PE}} = \sqrt{T_c^2 + \frac{2(R_o - R_5)}{a}} - T_c$

P – P

$$T_0 = \frac{\dot{R}_i}{a} \qquad T_1 = T_2 = T_3 = T_0 + \sqrt{\frac{R_\phi - R_4}{a}} \qquad T_4 = T_3 + \sqrt{\frac{R_\phi - R_4}{a}}$$

$$R_\phi = R_i + \frac{\dot{R}_i^2}{2a} \qquad R_1 = R_2 = R_3 = \frac{R_\phi + R_4}{2}$$

P – E – P

$$T_0 = \frac{\dot{R}_i}{a} \qquad T_1 = T_2 = T_0 + \Delta T_{1_{PE}}$$

$$R_0 = R_i + \frac{\dot{R}_i^2}{2a} \qquad R_1 = R_2 = R_\phi - \frac{a\,\Delta T_{1_{PE}}^2}{2}$$

$$\Delta T_4 = T_c - \sqrt{T_c^2 - \frac{2(R_4 - R_5)}{a}}$$

$$\Delta T_3 = -T_c \ln\left\{\frac{R_4 - R_5 + \frac{a\,\Delta T_4^2}{2}}{R_2 - R_5}\right\}$$

$$R_3 = R_4 + \frac{a\,\Delta T_4^2}{2}$$

NOTE: TOTAL TIME FOR PEP IF $\dot{R}_i = \phi$

$$T_{PEP} = \Delta T_{1_{PE}} + \Delta T_3 + \Delta T_4$$

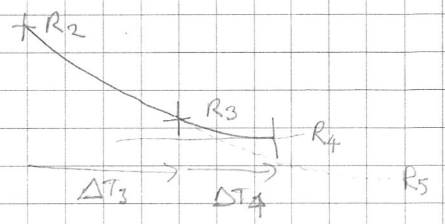
$$= \sqrt{T_c^2 + \frac{2(R_\phi - R_5)}{a}} \cancel{- T_c}$$

$$- T_c \ln\left\{\frac{R_4 - R_5 + \frac{a}{2}\left(T_c - \sqrt{T_c^2 - \frac{2(R_4 - R_5)}{a}}\right)}{R_2 - R_5}\right\}$$

$$\cancel{+ T_c} - \sqrt{T_c^2 - \frac{2(R_4 - R_5)}{a}}$$

$$T_{PEP} = \left(T_c^2 + \frac{2(R_\phi - R_5)}{a}\right)^{1/2} - \left(T_c^2 - \frac{2(R_4 - R_5)}{a}\right)^{1/2} - T_c \ln\left\{\frac{R_4 - R_5 + \frac{aT_c}{2} - \left(\frac{(aT_c)^2}{4} - \frac{a(R_4 - R_5)}{2}\right)^{1/2}}{R_2 - R_5}\right\}$$

Derivation of ΔT_4:



exp: $R = (R_2 - R_5)\, e^{-\frac{t}{T_c}} + R_5$

$$\dot{R} = \frac{-R + R_5}{T_c}$$

par: $R = R_4 + \frac{1}{2} a t^2$

$$\dot{R} = -at$$

So at join: $-R + R_5 = -a T_c \Delta T_4$

$$-R_4 - \frac{1}{2} a \Delta T_4^2 + R_5 = -a T_c \Delta T_4$$

$$\therefore \quad \left(\frac{a}{2}\right) \Delta T_4^2 - a T_c \, \Delta T_4 + (R_4 - R_5) = \phi$$

$$\Delta T_4 = \frac{a T_c \pm \sqrt{a^2 T_c^2 - 4 \frac{a}{2} (R_4 - R_5)}}{2\left(\frac{a}{2}\right)}$$

Take minimum root:

$$\therefore \quad \Delta T_4 = T_c - \sqrt{T_c^2 - \frac{2(R_4 - R_5)}{a}}$$

root is only real if $T_c^2 > \frac{2}{a}(R_4 - R_5)$

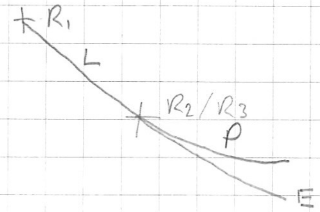
so $R_5 \geqslant R_4 - \frac{a T_c^2}{2}$

Derivation of ΔT_3:

At join: $R_3 = (R_2 - R_5) e^{-\frac{\Delta T_3}{T_c}} + R_5 = R_4 + \frac{1}{2} a \Delta T_4^2$

$$\therefore \quad \Delta T_3 = -T_c . \ln \left\{ \frac{R_4 - R_5 + \frac{a \Delta T_4^2}{2}}{R_2 - R_5} \right\}$$

P - L - P or P - L - E - P



$\dot{R} = -L$ for Linear descent.

exp: $\dot{R} = -L = \frac{-R_2 + R_5}{T_c} \Rightarrow R_2 = R_5 + L T_c$

Par: $R_3 = R_4 + \frac{L^2}{2a}$

P - L - P if $R_3 > R_2$ else P - L - E - P

if $R_4 + \frac{L^2}{2a} > R_5 + L T_c$

P - L - P: $R_2 = R_3 = R_4 + L^2/2a$

$$\Delta T_2 = \frac{R_1 - R_2}{L}$$

$$\Delta T_4 = \frac{L}{a}$$

P - L - E - P $\quad R_2 = R_5 + L T_c$

ΔT_4, ΔT_3, R_3 as for P - E - P.