Exercise 1. This is a very simple exercise. It doesn't have any sub-question. Is 57 prime?

Solution 1. Yes.

Exercise 2. This exercise has multiple sub-questions.

2.1 Give an explicit algebraic formula for $\cos \frac{2\pi}{17}$.

2.2 Find a polynomial of which $\cos \frac{\pi}{17}$ is a root.

These identities allowed Gauß to draw a *heptadecagon* (polygon with seventeen sides) from a ruler-and-compass construction.

2.3 Can all polygons be thus obtained?

Solution 2. These questions are related to some of the oldest problems in Mathematics. The problem of building a heptadecagon solely using a ruler and a compass was open for centuries. The first solution is due to Gauß. It was published in 1796 in his famous *Disquisitiones arithmeticae*.

2.1 We have

$$\begin{aligned}\cos\frac{2\pi}{17} =& \frac{1}{16}\left(\sqrt{17}-1+\sqrt{34-2\sqrt{17}}\right)\\ &+\frac{1}{8}\left(\sqrt{17+3\sqrt{17}-\sqrt{34-2\sqrt{17}}-2\sqrt{34+2\sqrt{17}}}\right).\end{aligned}$$

2.2 One of the roots of

$$\begin{aligned}&32768X^{16}-131072X^{14}+212992X^{12}-180224X^{10}\\ &+84480X^8-21504X^6+2688X^4-128X^2+X+1\end{aligned}$$

is $\cos \frac{\pi}{17}$.

2.3 Unfortunately, no, by Wantzel's theorem.