6.1.2 109511028 黃威凱

No. If m or n is odd, the degree of some vertices will not be even.

6.1.20

The graph is guaranteed to have at least one cycle if all vertices are of even degree. A cycle is a closed trail. #

6.1.26

The line graph is constructed by transforming edges into vertices and two vertices are adjacent iff the two edges share an endpoint in the original graph.

All vertices in an Eulerian graph are of even degree, so there are odd number of edges sharing the endpoints of an edge on both sides,

Hence, all edges are adjacent to (odd + odd) = even number of edges in the line graph.

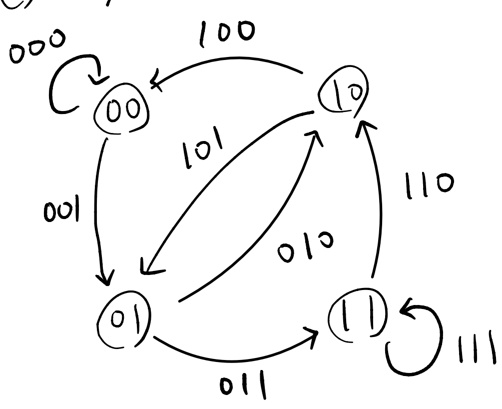
⇒ All vertices in the line graph of an Eulerian graph are of even degree #.

6.2.4.

① Let (u,v) be an arc and u is source, v is tail, then two arc are connected iff two arc shares an endpoint v with v as an arc's source and the other arc's tail. (I.e. $e_1=(s,u)$; $e_2=(u,v)$, then (e_1,e_2) in line graph)

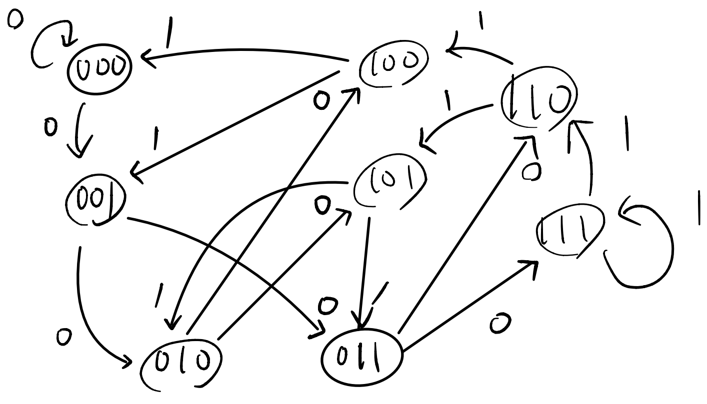
(e.g. 000要連 001 和 000)
001 → 010 和 011)

② $D_{2,3}$



⇒

line graph $(D_{2,4})$



6.2.6.

Following same definition in problem 6.2.4, each arc would have the same indegree = outdegree = 2, because a vertex in a deBrujin digraph has only two outgoing edges $a_1 \sim a_{n-1}0 / a_1 \sim a_{n-1}1$ and 2 incoming edges $\begin{cases} 0a_1 \sim a_{n-1} \\ 1a_1 \sim a_{n-1} \end{cases}$.
Therefore, every vertex in the line graph will have indegree = outdegree = 2.

$\Rightarrow$ The line graph $L(G)$ is eulerian.

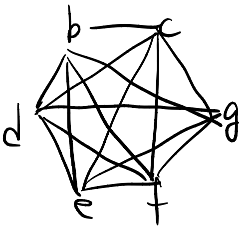
$\Rightarrow$ We can find a trail that visits every edge in $L(G)$.

$\Rightarrow$ Every vertex in $L(G)$ are visited.

$\Rightarrow$ The trail is an eulerian tour in $G \Rightarrow G$ is eulerian.

6.2.15.

① Odd vertices: $\{b, c, d, e, f, g\}$



$$\begin{array}{c|cccccc} & b & c & d & e & f & g \\ \hline b & & 6 & 10 & 10 & 5 & 5 \\ c & & & 7 & 16 & 11 & 11 \\ d & & & & 12 & 7 & 17 \\ e & & & & & 5 & 6 \\ f & & & & & & 8 \\ g & & & & & & \end{array}$$

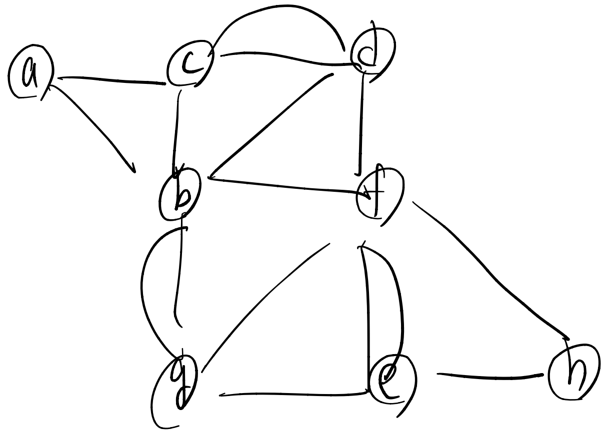
②

$(c,d) = 7$

$(e,f) = 5$

$(b,g) = 5$

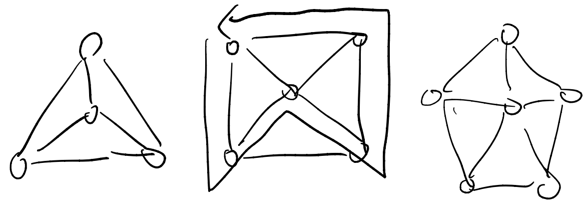
③



$\langle a, b, g, f, e, h, f, e, g, b, f, d, b, c, d, c, a \rangle$

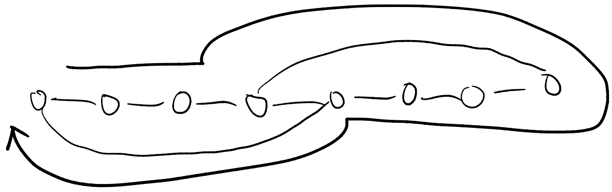
6.3.3.

All n-vertex wheels are Hamiltonian.



(Remove one edge from cycle, go to center and go back to cycle)

6.3.8.



$\Rightarrow$ Hamiltonian, $|V| = 8$, $|E| = 9$

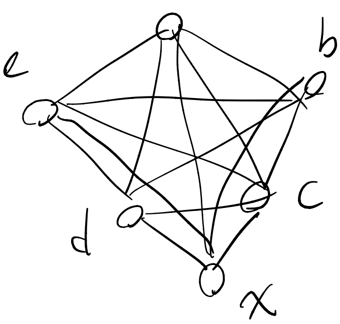
Not Eulerian.

6.3.12

A bipartite graph can only have even length cycle.

Hamiltonian is a cycle, so a bipartite graph that is Hamiltonian must be a single even length cycle $\Rightarrow |V| = \text{even}$. #

6.4.6.

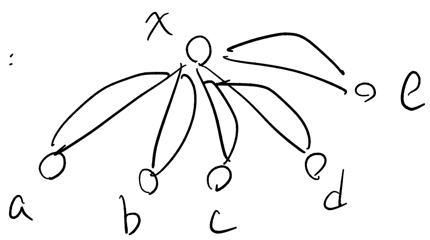


$\Rightarrow K_6$

① NNA: $\langle x, a, d, e, c, b, x \rangle$

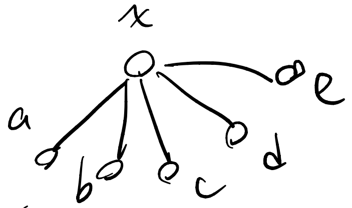
$\xrightarrow{\text{remove } x} \langle a, d, e, c, b \rangle \Rightarrow 3+3+2+3 = \underline{\underline{11}}$ ①

② Double tree:



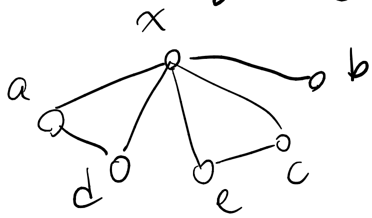
$\xrightarrow{\text{follow Eulerian}} \langle a, b, c, d, e \rangle \Rightarrow 4+3+4+3 = \underline{\underline{14}}$ ②

③ Matching:

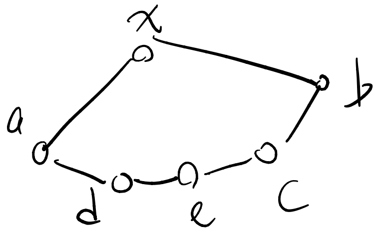


$\Rightarrow$ Min. weight perfect matching $= \{ (a,d), (c,e), (b,x) \} = 2+3+0 = 5$

$\xrightarrow{T^* + M^*}$

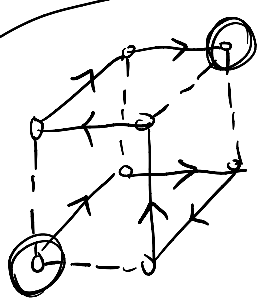


$\xrightarrow{\text{follow Eu.}}$



$\xrightarrow{\text{remove } x} \langle a, d, e, c, b \rangle = \underline{\underline{11}}$ # ③

6.4.10. $\longrightarrow$ disprove



not adjacent

#