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圖論 HW3

3.1.11 $n=10$, $c(G)=3$

$\Rightarrow$ 3 trees, each have x_1, x_2, x_3 vertices

$\Rightarrow (x_1-1)+(x_2-1)+(x_3-1)=7<9$

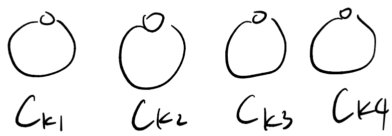
$\Rightarrow$ The number of edge required (9) is greater than the maximum possible #(edges)

3.1.14 A tree with n vertices has $(n-1)$ edges.

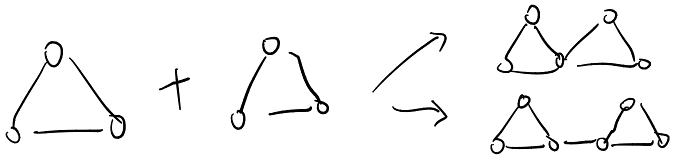
$|E|=\text{even} \longrightarrow |V|=\text{odd}$. $\underline{\sum_{v\in V}\deg(v)} = 2|E| = \underline{\text{even}}$, since $\underline{|V|=\text{odd}}$, at least one vertex has to be even number. (奇數個奇數總和不會是偶數)

3.1.19 If there are 4 edge disjoint cycles with length $k_1 \sim k_4$, then we have:

$|V|=|E|=(k_1+k_2+k_3+k_4)$, and the graph is:



To make them connected, we can either draw an edge between them or choose one vertex from two of them and do the amalgamation:



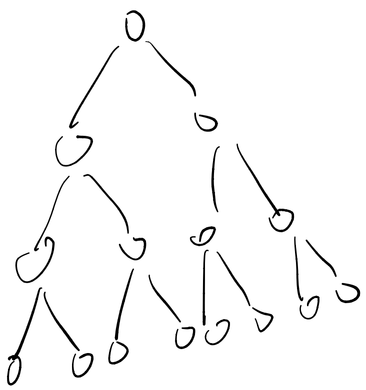
$\Rightarrow |V|=|V|-1$, $|E|=|E|$

$\Rightarrow |V|=|V|$, $|E|=|E|+1$, so either operation increases the difference between $|E|$ and $|V|$ by 1.

Because we have to do 3 operations, the resulting graph must have the relation: $|E|=|V|+3$

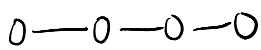
Therefore, we proved that there does not exist a connected n-vertex graph with $(n+2)$ edges that contains 4 edge-disjoint cycles.

3.2.4



Only one type

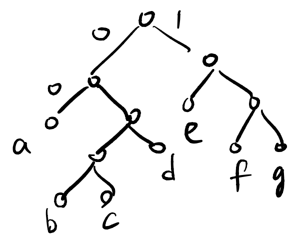
3.2.12



height 3

each with ≤ 3 children

3.5.3 Huffman tree



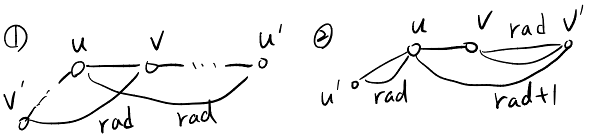
(011)(10)(110)(00)(101)(10)(011)

= defaced #

3.1.29

(→): $Z(T)$ is a single edge, say (u,v). Let u', v' be the farthest point from u, v, then $d(u,u') = d(v,v') = rad(T)$

We know that $Path(u,u')$ and $Path(v,v')$ must pass (u,v), otherwise, $P(u,v') = (u,v) + P(v,v') = rad + 1 > rad$



· In ②, the eccentricity should be $(rad+1)$, a contradiction.

So case ① is the only case, we can see from the figure that $diam(T) = P(u',v') = 2 \times rad - 1$ #

(←): Assume u', v' are the farthest points in G, so $d(u,v) = diam(T)$.

Let w be the only center of T, so $\max(d(w,u), d(w,v)) = rad(T)$.

Since $diam(T) = 2rad(T) - 1$, $d(u,v) = d(u,w) + d(w,v) = 2rad(T) - 1$,

Therefore, one of the distances is $rad(T) - 1$, assume $d(u,w)$.

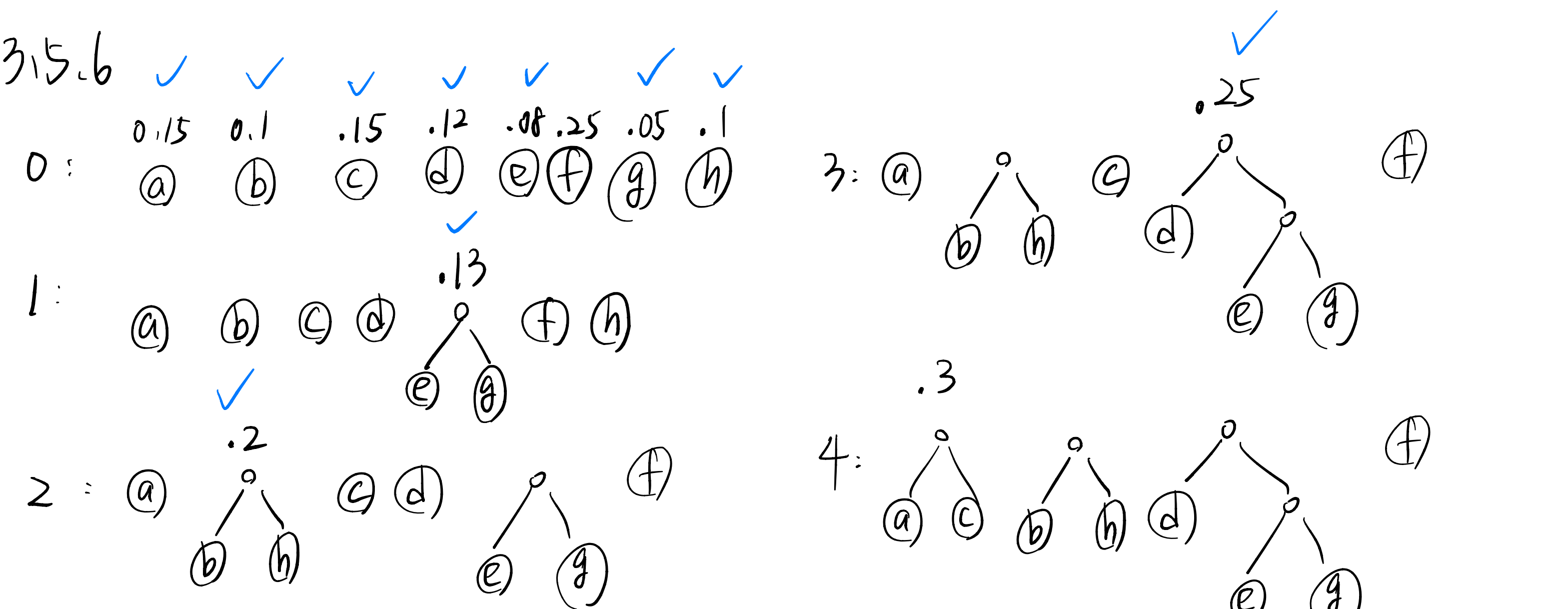
However, assume w' is the first vertex on (w,v) path $(w - w' - \cdots - v)$,

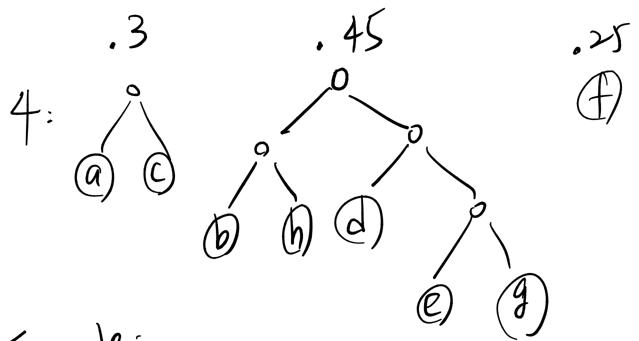
then we have $d(w',u) = d(w,u) + 1 = rad(T)$, and $d(w',v) = rad(T) - 1$.

Notice that $ecc(w') = rad(T)$, so w' is also a center. (→←)

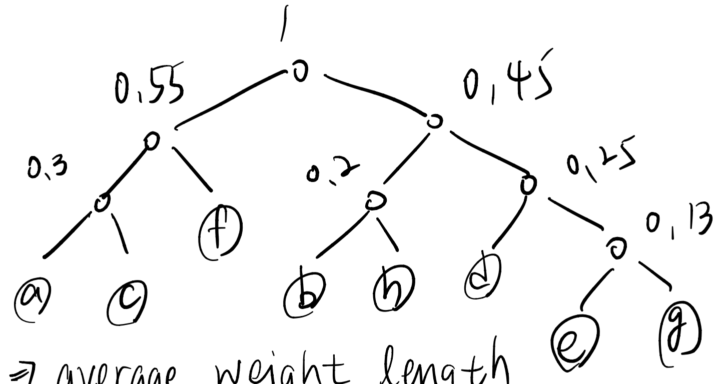
⇒ $diam(T) = 2rad(T) - 1 \rightarrow Z(T)$ must be an edge #

3.5.6





⇒



Encode:

defaced ⇒ (110)(1110)(01)(000)(001)-(1110)(110)

baggage ⇒ (100)(000)(1111)(1111)(000)(1111)(1110) #

⇒ average weight length $= 1 + 0.55 + 0.45 + 0.3 + 0.2 + 0.25 + 0.13$

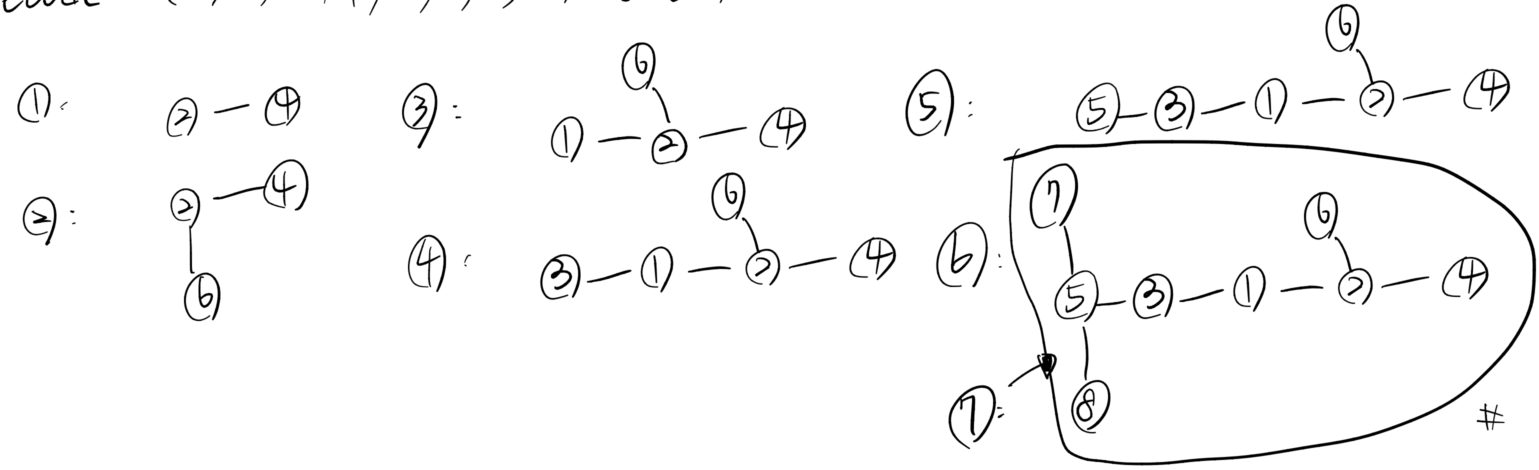
$= 2.88$ #

3.7.5

Encode: (2, 2, 1, 3, 5, 5) → 5 — 8

4 3 5 1 7 2 6 7   1 2 3 4 5 6

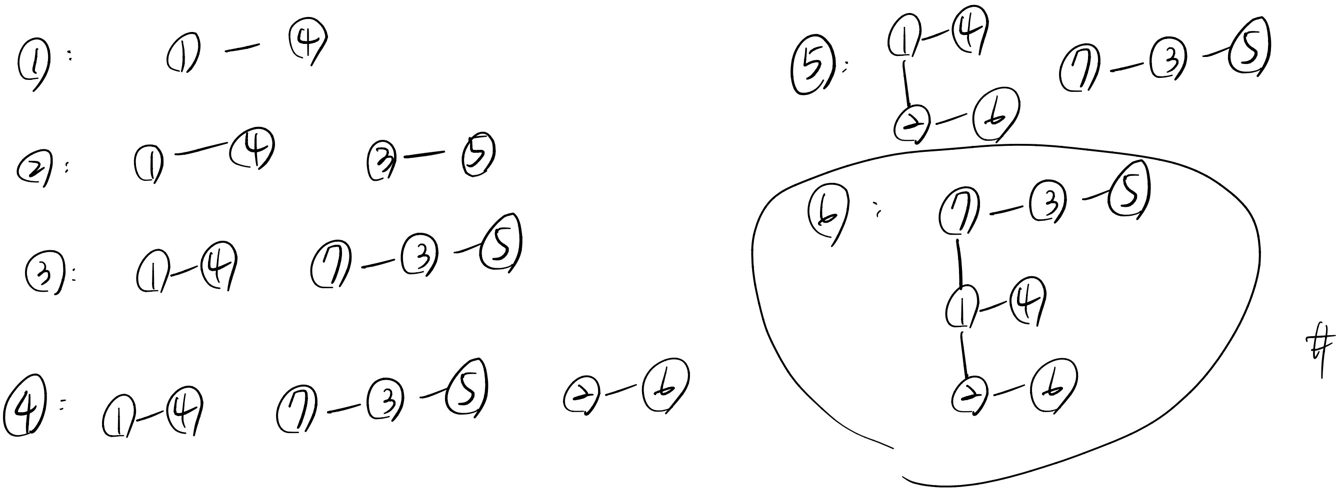
Decode: (1, 2, 3, 4, 5, 6, 7, 8) ; (2, 2, 1, 3, 5, 5)



3.7.9 $n-2=5 \Rightarrow n=7$

L: 1, 2, 3, 4, 5, 6, 7 ; P = 1, 3, 7, 2, 1

6 5 3 1 2 4 6   1 2 3 4 5



3.7.15.

Prüfer Encoding creates a unique (n-2) sequence for each n-vertices labeled tree, so $f_e: T_n \to P_{n-2}$ is a one-to-one function.

To pro

For any $P_{n-2} = (p_1, p_2, \dots, p_{n-2})$, we follow the decoding process, say b is the min. number not in P_{n-2}, then $P'_{n-2} = (p_2, p_3, \dots, p_{n-2}, b)$. Each time we put different numbers into P_{n-2}, so eventually, $P''_{n-2} = (b_1, b_2, \dots, b_{n-2})$, $L = (b_{n-1}, b_n)$, and we add the edge (b_n, b_{n-1}) to the tree constructed before.

Hence, we have shown that every Prüfer Sequence can be used to reconstruct a tree. Which means $f_e: T_n \to P_{n-2}$ is also an onto function, and its inverse function is $f_d: P_{n-2} \to T_n$.