4.1.15

4.1.15 Prove Proposition 4.1.2.

Proposition 4.1.2. *Let T be the output tree produced by any instance of Tree-Growing in a graph. Then T is an ordered tree with respect to the discovery order of its vertices.* ◊ (Exercises)

4.2.17

4.2.17 Verify that a preorder traversal of a depth-first search tree recreates the discovery order.

4.2.20

4.2.20 Prove Proposition 4.2.3.

Proposition 4.2.3. *When breadth-first search is applied to an undirected graph, the endpoints of each non-tree edge are either at the same level or at consecutive levels.*

Proof: The result uses an argument analogous to the one given in the proof of Proposition 4.2.1. ◊ (Exercises)

Proposition 4.2.1. *Depth-first search trees have no cross-edges.*

Proof: Let T be the output tree produced by a depth-first search, and let e be a non-tree edge whose endpoints x and y satisfy $dfnumber(x) < dfnumber(y)$. At the point when the depth-first search discovered vertex x, edge e became a frontier edge. Since e never became a tree edge, the search had to have discovered vertex y before backtracking to vertex x for the last time. Thus, y is in the subtree rooted at x and, hence, is a descendant of x. ◊

4.3.11

4.3.11 State a sufficient condition to guarantee that a given weighted graph does not have a unique minimum spanning tree. Is the condition also necessary?

4.4.10

4.4.10 Prove Proposition 4.4.4.

Proposition 4.4.4. *Let G be a connected graph. All vertices on a cycle are in the same bridge component of G.* ◊ (Exercises)

4.4.15

4.4.15 Prove Proposition 4.4.13.

Proposition 4.4.13. *If there is no freedom room in the maze, Tarry's algorithm will stop after each tunnel is traversed eactly twice, once in each direction.* ◊ (Exercises)

4.5.20

4.5.20 Prove that a subgraph of a connected graph G is a subgraph of the relative complement of some spanning tree if and only if it contains no edge-cuts of G.

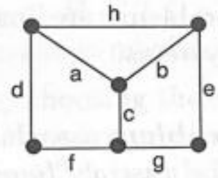
4.5.21

4.5.21 Prove Proposition 4.5.10.

Proposition 4.5.10. *The fundamental edge-cut with respect to an edge e of a spanning tree T consists of e and exactly those edges in the relative complement of T whose fundamental cycles contain e.* ♢ *(Exercises)*

4.6.7

4.6.7 a. Show that the collection $\{\{a,c,d,f\}, \{b,c,e,g\}, \{a,b,h\}\}$ of edge subsets of E_G forms a basis for the cycle space $W_C(G)$ of the graph G shown. b. Find a different basis by choosing some spanning tree and using the associated fundamental system of cycles.



4.6.14

4.6.14 Prove Theorem 4.6.6.

Theorem 4.6.6. *Let T be a spanning tree of a connected graph G. Then the fundamental system of edge-cuts associated with T is a basis for the edge-cut space $W_S(G)$.* ♢ *(Exercises)*

4.6.15

4.6.15 Prove Proposition 4.6.9.

Proposition 4.6.9. *A subgraph H of a graph G is an element of the edge-cut space of G if and only if it has an even number of edges in common with every subgraph in the cycle space of G.* ♢ *(Exercises)*