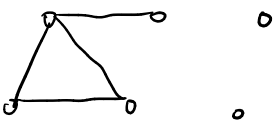
109511028 黃威凱

圖形理論 HW2

2.1.6

①


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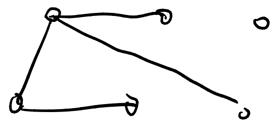

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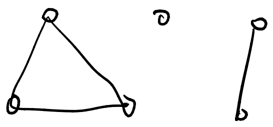

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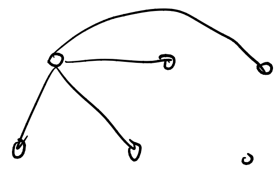

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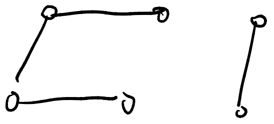

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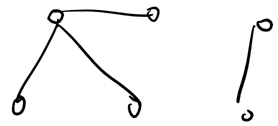

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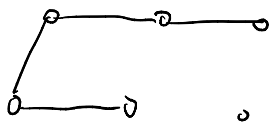

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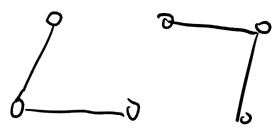
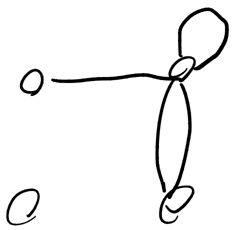


2.1.14

①


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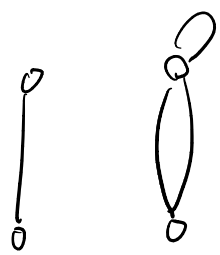

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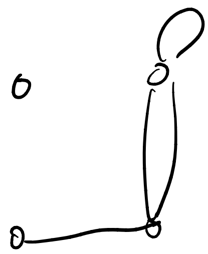

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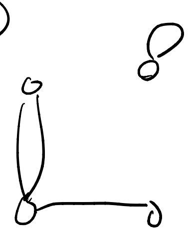

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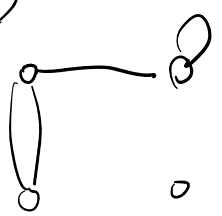

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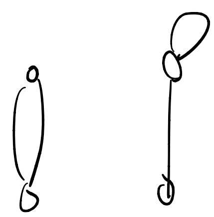


2.2.7 ☆ $(u,v) \xrightarrow{f} (f(u),f(v))$

Symmetry	Vertex Permutation	Edge Permutation
Multi-edge	(u) (v) (w) (x)	(a) (e) (c) (d) (b)
–	(u) (v) (w) (x)	(ae) (c) (d) (b)
–	(u) (v) (w) (x)	(a) (e) (c d) (b)
–	(u) (v) (w) (x)	(ae) (c d) (b)
Reflection through b.	(u v) (w x)	(ac) (ed) (b)
	(u v) (w x)	(ad) (ec) (b)

2.2.11.

circ$(n:S)$: $(i,j) \in E \iff i+s=j \pmod n$ or $j+s=i \pmod n$

vertex-transitive : for every vertex pair $u,v \in V$, $\exists$ automorphism that maps u to v.

pf: The vertex function $i \mapsto i+k \pmod n$ is an automorphism.

For all vertices, we can map i to j if $|i-j| = k \in S$

This creates edge-disjoint cycles, consider $\pi = (\underbrace{1 \ (1+k) \ (1+2k) \cdots}_{lcm(n,k)/k})(\cdots)\cdots(\cdots)$

Vertices in the same cycle are transitive, and every pair of cycles are also transitive.

2.2.25

orbit: $\{x_1, x_2 \cdots\}$, edges(or vertices) in an orbit are transitive.

→ orbit 內元素隨便換都可以找到至少一個 automorphism.

vertex-orbit : $\{1,3,5\}$, $\{0,2,4\}$

edge-orbit : $\{13, 35, 51\}$, $\{01, 23, 45\}$, $\{02, 24, 40\}$

2.3.22

- A clique in G is a maximal subset of mutually adjacent vertices in G.
- clique number : $\omega(G)$ = maximum clique size.
- independence : : $\alpha(G)$ = MIS.
- eccentricity : max distance
- radius : min ecc.

(a.) $\{w,z\}$, $\{v,w,y\}$, $\{u,v,y\}$, $\{u,x,y\}$

(b.) $\omega(G) = 3$

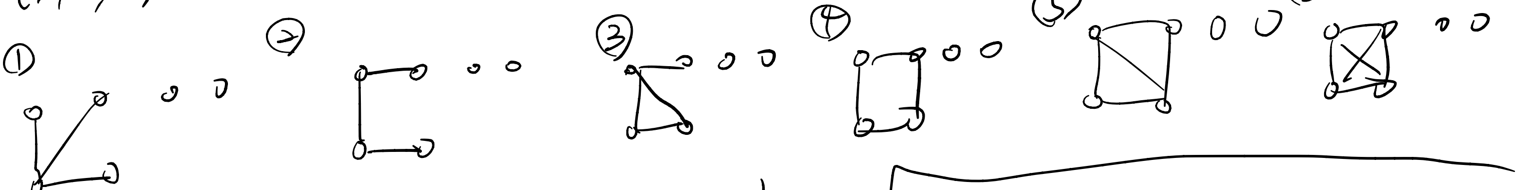
(c.) $\{u,w\}$, $\{x,v,z\}$, $\{y,z\}$
$\{u,z\}$ $\{x,w\}$
$\{x,z\}$

(d.) $\alpha(G) = 3$

(e.) $\{w,v,y\}$ (ecc = rad = 2)

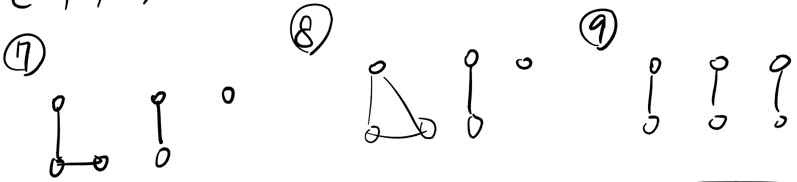
2.3.32 $x_1' + x_2' + x_3' = 3$

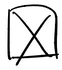
(3,0,0):

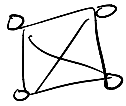


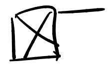
(2,1,0):

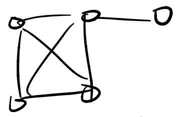
(1,1,1):



① D = 

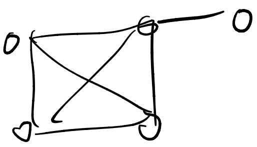
⇒ G(D): 

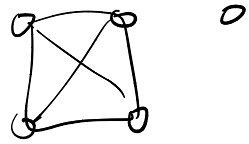
② D = 

⇒ G(D): 

2.3.40

Counterexample:

G: 

H: 

There are only 2 possible D that could have V(H).

Neither can satisfy H = G(D).

#

2.3.41

- induced graph on vertex subset: U ⇒ G(U): 包含所有U的頂點以及兩端點都∈U的edge。
- ~ edge ~: D ⇒ G(D): 所有edge∈D，以及所有edge有碰到的點。

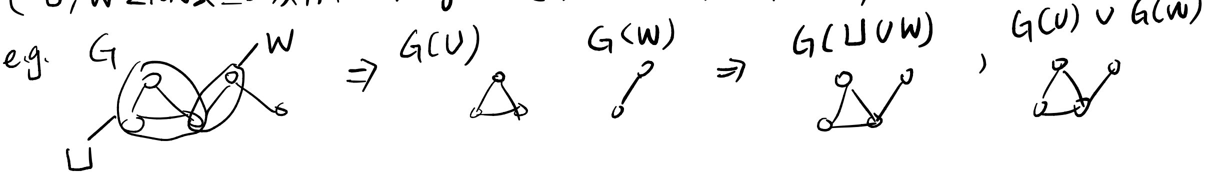
⇒ Let $x \in U$, $y \in W$, if $(x,y) \in E(G)$, then $(x,y) \in E(G(U \cup W))$

But $(x,y) \notin E(G(U) \cup G(W))$ because (x,y) is not in either $E(G(U))$ or $E(G(W))$

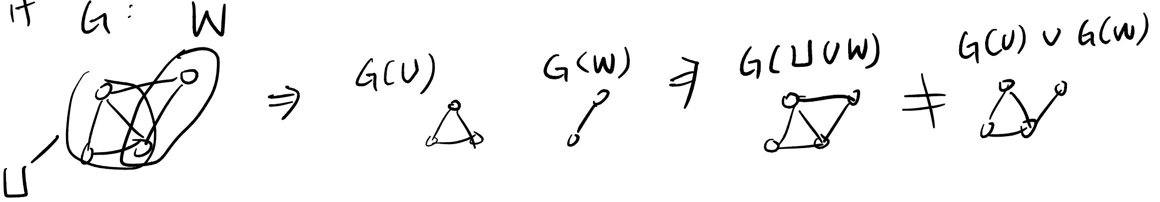
⇒ $G(U) \cup G(W) = G(U \cup W)$ if $(x,y) \notin E(G)$ $\forall x \in U, y \in W, x \neq y$

(U, W之間頂點沒有任何edge相連，但可以用共同的點)

e.g. G W U ⇒ G(U) G(W) ⇒ G(U∪W), G(U) ∪ G(W)

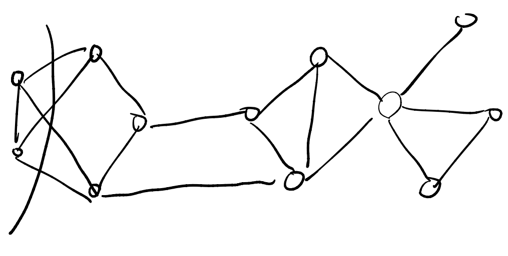


but if G: W U ⇒ G(U) G(W) ⇒ G(U∪W) ≠ G(U) ∪ G(W)



2.4.19

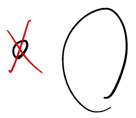
Graph:



⇒ largest minimal edge-cut = 4.

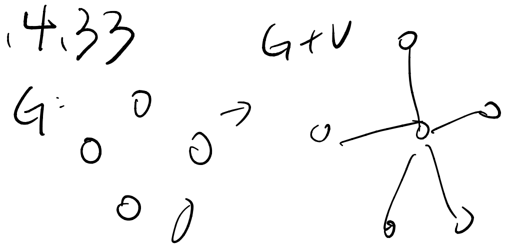
2.4.27 If G is not connected, at most one card will be connected.

E.g.



Therefore, if at least 2 cards are connected, G must be connected.

2.4.33



→ 5 cut edges.

2.4.39 (考慮 x, y 在 $G-u$ 的(不)同 Component, 2種 case)

If x, y are in the same component of $G-v$, then for any z in other components, since $(x,z), (y,z) \in E(\bar{G})$, v is not a cut vertex for any x, y in $\bar{G}$

If x, y = different ~, $(x,y) \in E\bar{G}$. Therefore v is not cut vertex in $\bar{G}$.

⇒ x, y are any 2 vtx in G, so done.

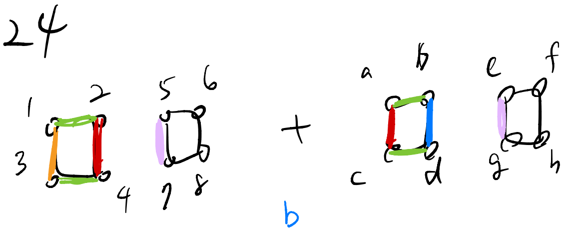
⇒ if v is cut vertex in G, then it's not in $\bar{G}$.

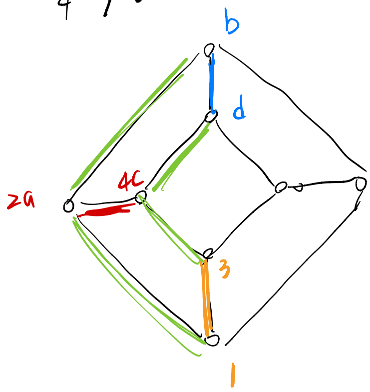
2.5.11

∵ $\deg(c) = 4$ and no vertices in the left graph have degree $=4$

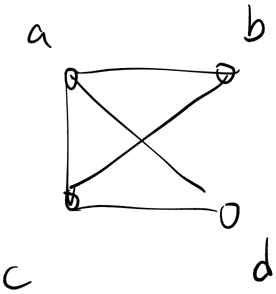
∴ not isomorphic

2.7.24





2.6.7



① $\langle a, b, c, d \rangle$

② $\langle a, b, a, d \rangle$

③ $\langle a, c, a, d \rangle$

④ $\langle a, d, a, d \rangle$

⑤ $\langle a, d, c, d \rangle$