109511028 黃威凱

4.1.15

Order Tree: A rooted tree where the children of each vertex is assigned an order.

1. Let the start vertex be root.

2. Choose a vertex by tree growing instance, define its order by its discovery order.

3. Tree-growing only selects vertices not in the current tree, at the end, a tree is grown (no cycle).

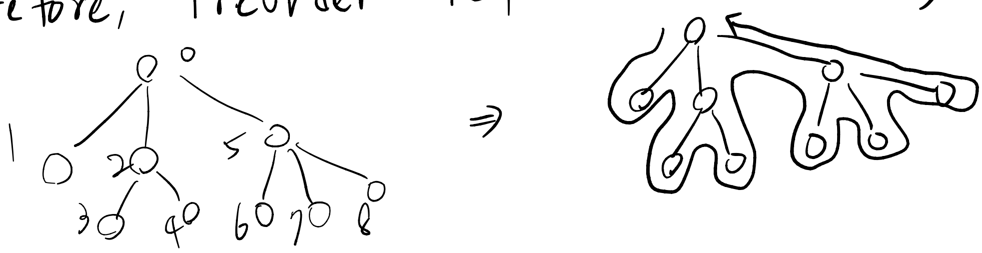
4. Every vertex' children are assigned an order. #

4.2.17.

Preorder visit a node before all of its children, so if we start from the root of the DFS tree, the discovery order is 0 and we are fine.

Then preorder visit its children with the least discovery number in DFS tree, and backtracks when the node has no more unvisited children.

Therefore, Preorder reproduces the discovery order of DFS.

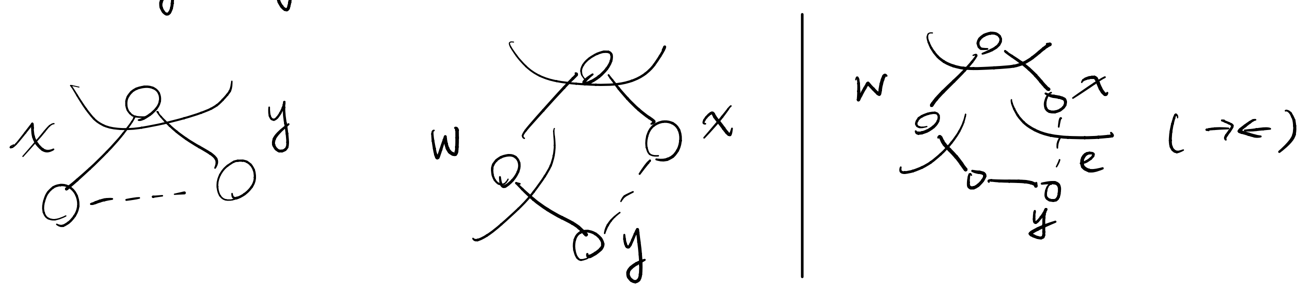


4.2.20

For all non-tree edge $e = (x, y)$, we assume x and y are either at the same level or at consecutive levels.

W.L.O.G, let x be the vertex at higher level (先被加入 Tree).

We can see that y must be added to T either by x's parent or siblings' edge, otherwise y should be added to T by e.

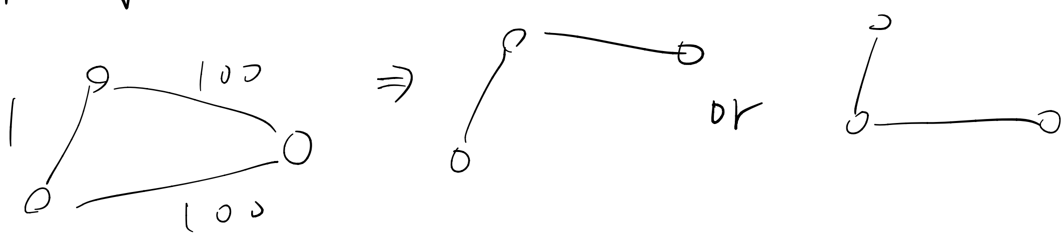


4.3.11

All edges have the same weight we can add x to T by any edge

→ no unique spanning tree.

(←) Not true.



4.4.10

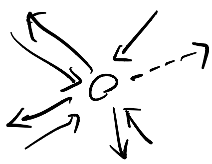
Let x, y be any 2 vertices on the same cycle. Every x-y path must use only cycle edge, and no cycle edges are cut edge, they must be on the same bridge component.

4.4.15

G is connected and we go through a tunnel unmarked if it exists, so every tunnel is visited at least once.

If a tunnel is visited, either it's marked ① (OUT, X) ② (IN, OUT) or ③ (OUT, OUT).

① Because we only go through tunnel that is unmarked or marked IN, we won't go through the tunnel again if it's already (OUT, OUT), that's a tunnel traversed exactly twice.

② For a (IN, OUT) door, we can only go through it from IN again by the description of algorithm, and because <u>all other doors are marked OUT at the moment</u>, each tunnel associated with the room must have been visited twice. ⇒  (一定是来回才會走回原本房間).

③ We won't go through the door marked OUT, so the tunnel (OUT, X) can only be visited one more time from X side, making it (OUT, OUT)

⇒ Every tunnel is traversed exactly twice, once in each direction.

4.5.21

Let $S_e = \{e, e_1' \sim e_k'\}$ be the fund. edge-cut w.r.t. e of a spanning tree T consists of e, and edges $e_1' \sim e_k'$ are in T's relative complement.

Let $C_i = \{e_i', \text{tree edge}\}$, be the fund. cycle w.r.t. e_i', $i = 1 \sim k$.

Because $|\text{minimal edge-cut} \cap \text{cycle}|$ is even (# edges $= 2k$)

And the only non-tree edge in common between C_i and S_e is e_i'

There must be a tree edge in common between C_i and S_e and it must be e.

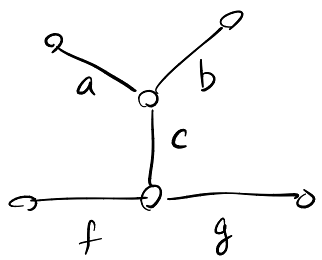
② For other cycles $C_i' = \{\text{tree edge}, e_{k+1}' \sim e_N'\}$, because no non-tree edges are in S_e and hence e should not be in tree edge of C_i' $\Rightarrow$ $|C_i' \cap S_e| = \phi$.

$\Rightarrow$ S_e consists of edge e and <u>exactly</u> the edges in the relative complement of T whose fund. cycles contain e.

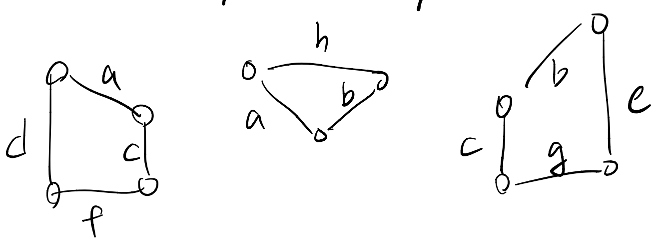
4.6.7.

Theorem 4.6.5. *Let T be a spanning tree of a connected graph G. Then the fundamental system of cycles associated with T is a basis for the cycle space $W_C(G)$.*

(a) Spanning Tree

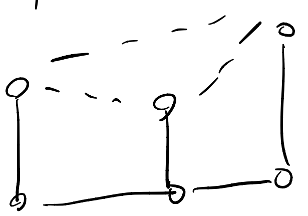


Fundamental system of cycles :



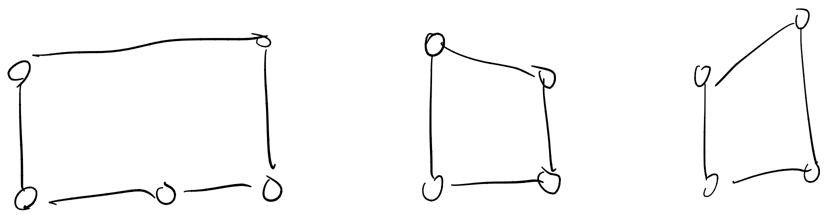
$\Rightarrow \{\{a,c,d,f\}, \{b,c,e,g\}, \{a,b,h\}\}$ is a basis for $W_C(G)$.

(b) ST :



Cycles $= \{\{d,f,g,e,h\}, \{a,c,f,d\}, \{c,g,b,e\}\}$

$\Rightarrow$



4.6.14.

Each fund. EC (edge-cut) associated with T contains one tree edge that is not part of any other fund. EC associated with T.

$\rightarrow$ Any two of the ECs are linearly independent.

Suppose H is any element of $W_S(G)$. Let $e_1 \sim e_r$ be the tree edges of H

Let S_i be the fund. EC w.r.t. edge e_i. $i = 1 \sim r$.

Since each e_i appears only in S_i and e_i is the only tree edge in S_i

$B = H \oplus S_1 \oplus S_2 \cdots \oplus S_r$ contains no tree edges.

Thus B is a subgraph of the relative complement of some tree.

But B is an element of $W_S(G)$, so B must be the null graph.

4.6.15.

($\rightarrow$): Each subgraph in the EC space is a union of edge-disjoint minimal EC. And each subgraph in the cycle space is a union of edge-disjoint cycles. Necessity follows from

Proposition 4.5.8. *A cycle and a minimal edge-cut of a connected graph have an even number of edges in common.*

($\leftarrow$): W.L.O.G, assume G is connected. Otherwise consider each connected component.

Suppose H has an even number of edges in common with each subgraph in the cycle space of G. Let T be a spanning tree of G. Let $e_1 \sim e_r$ be the tree edges of H, and consider $S = S_1 \oplus S_2 \oplus \cdots \oplus S_r$, where S_i is the fund. EC associated with T w.r.t. edge e.

We know that $H \oplus S$ has no tree edges because e_i is the only tree edge in S_i. ($e_1 \sim e_r$ 都只出現 2次, 1次 H, 1次 S_i).

$\rightarrow$ $H \oplus S$ can only consists of edges in the relative complement of T.

Suppose b is an edge in $H \oplus S$, if C is the fund. cycle associated with b, then b is the only common edge in $H \oplus S$ and C.

(C 只有 1 個 non-tree edge)

But $S \in W_S(G)$, (by Theorem) S must have even number of edges in common with $C \in W_C(G)$, as does H. (by assumption)

$\rightarrow$ $H \oplus S$ = even + even − 2 $\text{edge}_{\text{common}}$ = even number of common edges.

In particular, $H \oplus S$ and C must have an even number of edges in common for all $C \in W_C(G)$.

$\rightarrow$ This contradiction implies $H \oplus S$ must be ϕ.

$\rightarrow$ $H \oplus S = \phi \rightarrow H = S \in W_S(G)$.