6.1.2

In Exercises 6.1.1 through 6.1.4, determine which graphs in the given graph family are eulerian.

6.1.2 The complete bipartite graph $K_{m,n}$.

6.1.20

6.1.20 Referring to the initial step of Algorithm 6.1.1, why is it always possible to construct a closed trail, regardless of the choice of starting vertex?

6.1.26

6.1.26 Prove or disprove: The line graph of *any* eulerian graph is eulerian.

6.2.4

6.2.4 Find an appropriate extension of the line-graph definition (§1.2) to digraphs, and show that the line graph of the deBruijn digraph $D_{2,3}$ is the deBruijn digraph $D_{2,4}$.

6.2.6

6.2.6 Give an alternative proof of Proposition 6.2.1 using line graphs.

Proposition 6.2.1. *The* $(2,n)$*-deBruijn digraph* $D_{2,n}$ *is eulerian.*

Proof: The deBruijn graph $D_{2,n}$ is strongly connected, since if $a_1a_2\cdots a_{n-1}$ and $b_1b_2\cdots b_{n-1}$ are any two vertices of $D_{2,n}$, then the vertex sequence:

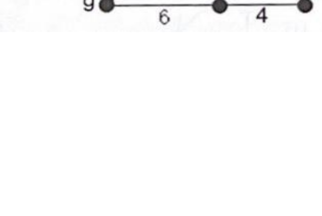
$$a_1a_2\cdots a_{n-1};\quad a_2\cdots a_{n-1}b_1;\quad a_3\cdots a_{n-1}b_1b_2;\quad \ldots;\quad b_1b_2\cdots b_{n-1}$$

defines a directed trail from $a_1a_2\cdots a_{n-1}$ to $b_1b_2\cdots b_{n-1}$. Moreover, for every vertex $b_1b_2\cdots b_{n-1}$ of $D_{2,n}$, the only outgoing arcs from $b_1b_2\cdots b_{n-1}$ are $b_1b_2\cdots b_{n-1}0$ and $b_1b_2\cdots b_{n-1}1$, and the only incoming arcs to $b_1b_2\cdots b_{n-1}$ are $0b_1b_2\cdots b_{n-1}$ and $1b_1b_2\cdots b_{n-1}$. Thus, $indegree(b_1b_2\cdots b_{n-1}) = outdegree(b_1b_2\cdots b_{n-1}) = 2$, which implies, by Theorem 6.1.2, that $D_{2,n}$ is eulerian. ◇

6.2.15

In Exercises 6.2.12 through 6.2.15, apply Algorithm 6.2.2 to find a minimum-weight postman tour for the given weighted graph. Determine whether the solution is unique.

6.2.15



6.3.3

In Exercises 6.3.1 through 6.3.4, determine which graphs in the given graph family are hamiltonian.

6.3.**3** The n-vertex wheel W_n.

6.3.8

In Exercises 6.3.6 through 6.3.10, draw the specified graph or prove that it does not exist.

6.3.**8** An 8-vertex simple graph with more than 8 edges that is hamiltonian but not eulerian.

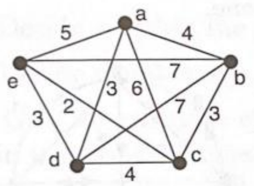
6.3.12

6.3.**12** Prove that a bipartite graph that is hamiltonian must have an even number of vertices.

6.4.6

In Exercises 6.4.6 through 6.4.9, apply a modified version of each of Algorithms 6.4.1 through 6.4.3 to the specified graph to try to find a minimum-weight open hamiltonian path that starts at vertex a. Base your modification on Transformation 1. Indicate the vertex sequence and total edge-weight for each of the three outputs.

6.4.**6**



6.4.10

6.4.**10** Prove or disprove: In the n-dimensional hypercube Q_n (§1.2), the initial and terminal vertices of every open hamiltonian path are adjacent.