1.1.19 (c.) $\langle 7, 7, 6, 5, 4, 4, 3, 2 \rangle$

$\rightarrow \langle 6, 5, 4, 3, 3, 2, 1 \rangle \rightarrow \langle 4, 3, 2, 2, 1, 0 \rangle \rightarrow \langle 2, 1, 1, 0, 0 \rangle$

$\rightarrow \langle 0, 0, 0, 0 \rangle \Rightarrow$ Graphic

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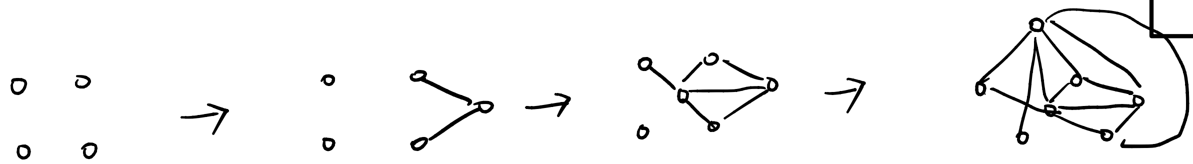
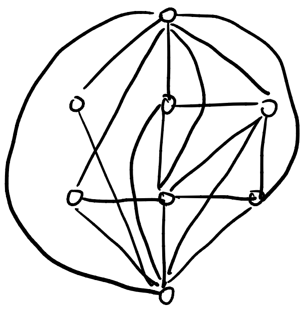


$\rightarrow$



$\xrightarrow{\text{degree}}$

$$\begin{pmatrix} & 7 & \\ 2 & 4 & 5 \\ 3 & 6 & 4 \\ & 7 & \end{pmatrix}$$

1.1.28

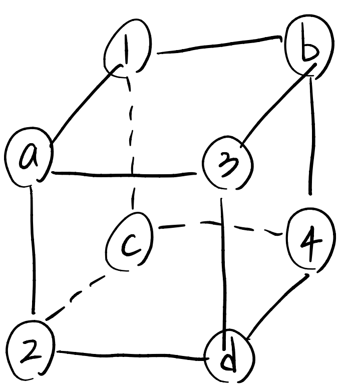
Handshaking : $\Sigma d(v) = 2|E| = 6 \Rightarrow d_1 + d_2 + d_3 = 6$, $d_i \geq 0 \ \forall i$

$d_1 \geq d_2 \geq d_3$, even number of odd degree vertex

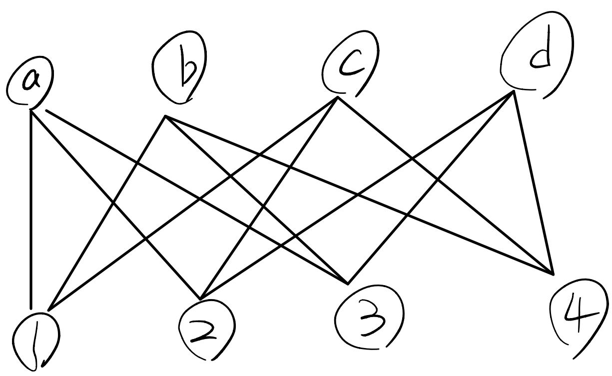
$\Rightarrow \langle 6, 0, 0 \rangle$; $\langle 5, 1, 0 \rangle$; $\langle 4, 2, 0 \rangle$; $\langle 4, 1, 1 \rangle$; $\langle 3, 3, 0 \rangle$

$\langle 3, 2, 1 \rangle$; $\langle 2, 2, 2 \rangle$ $\Rightarrow$ $\boxed{\text{6 degree sequences.}}$ #

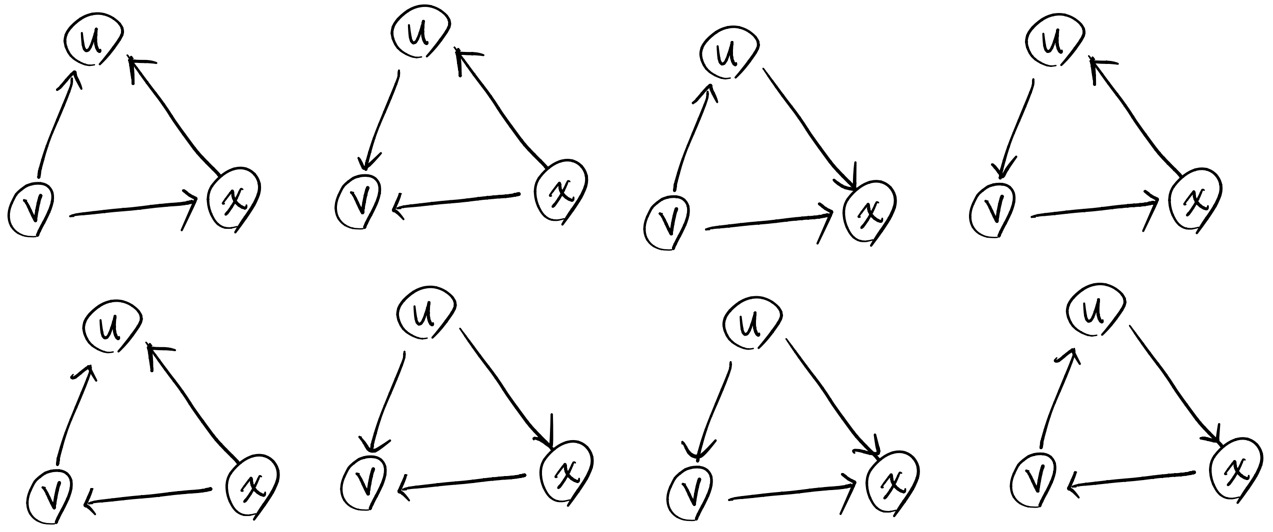
1.2.5.



$\Rightarrow$



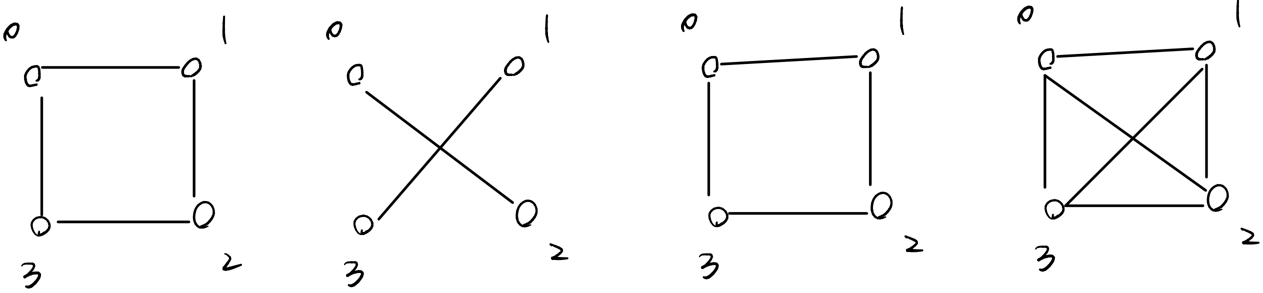
1.2.11. (a.)



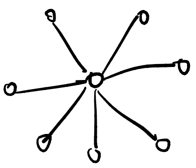
(b.) Every edge has 2 directions to be chosen, and there are $\binom{4}{2} = 6$ edges, so 2^6 different tournaments.

1.2.15. $i+s = j \mod n$; $j+s = i \mod n$

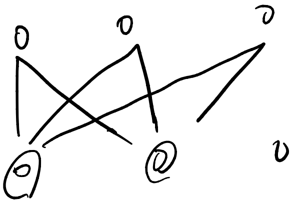
circ(4:1) circ(4:2) circ(4:3) circ(4:1,2)



1.4.16

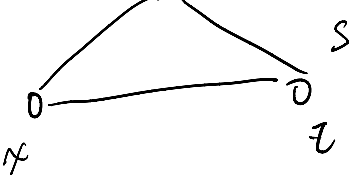


1.4.43



- No length 3 walk because x and y must be in the same partition.
- There are 3 length 2 walk between given x and y.

1.4.44



By def. $d(x,z)$ is the shortest path from x to z.

If $d(x,y) + d(y,z) < d(x,z)$, it's a contradiction to "the shortest"

So, either $d(x,y) + d(y,z) = d(x,z)$ (y is on x-z path)

or $d(x,y) + d(y,z) < d(x,z)$ (y is not on x-z path)

1.5.6.

a.) $\langle a, c, e, b, d, a \rangle$

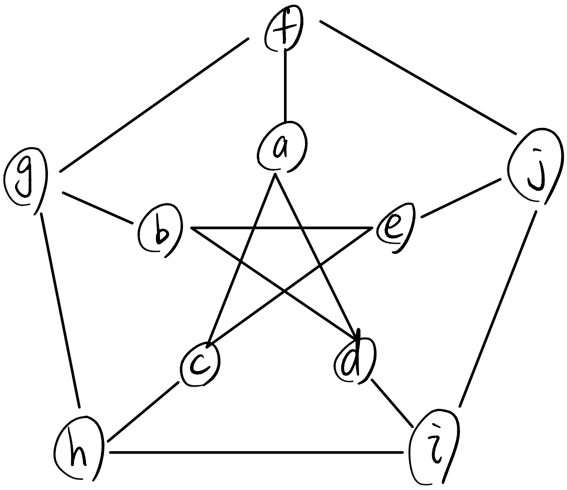
b.) $\langle a, c, e, b, d, i, h, g, f, j \rangle$

c.) 5: $\langle a, c, e, b, d, a \rangle$

6: $\langle a, c, e, j, i, d, a \rangle$

8: $\langle a, c, e, b, d, i, j, f, a \rangle$

9: $\langle a, c, e, b, g, h, i, j, f, a \rangle$



1.5.31 Prove by induction on cycle length.

$m=1$, a closed walk is a cycle by definition

Assume the statement holds for all $m = 2k-1$ closed walk.

Consider a walk $u = \{v_0, e_1, v_1, e_2, \cdots, v_{2k}, e_{2k+1}, v_0\rangle$ of length $m = 2k+1$

If u does not have repeated vertices, it's a cycle by definition.

Say v_i is repeated, then we can split u into 2 closed walk:

$$\begin{cases} u_1 = \langle v_0, e_1, \cdots, v_i, v_j, \cdots, e_{2r+1}, v_0 \rangle \\ u_2 = \langle v_i, e_{i+1}, \cdots, v_i \rangle \end{cases}$$, both of length $m < 2r-1$

Both of them are cycle, by the induction hypothesis, u has a cycle.

By mathematical induction, every closed walk of odd length contains a cycle.

1.5.42.

Consider a longest path $\{v_1 \to v_n\} \in G$, every neighbor pointing to v_1 must be in the path, otherwise $\{v_1 \to v_n\}$ is not the longest.

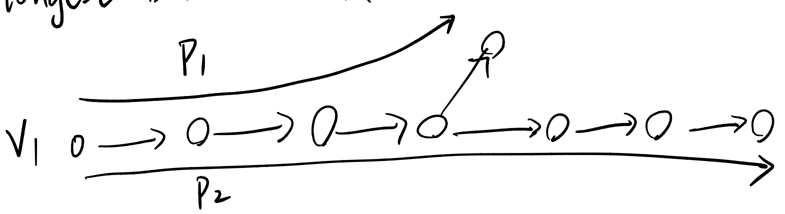
Because every vertex has positive indegree, some other vertices must point to v_1.

While v_n is the "last" vertex in this path (or $\{v_1 \to v_n \to u\}$ is longer)

the vertex that points to v_1 must be in $\{v_1 \to v_n\}$

Therefore, $\{v_1 \to v_k \to v_1\}$, $k \in [2, n]$ forms a directed cycle.

(longest 在此指的是 maximal 而不是 maximum)



$\Rightarrow$ P_1 和 P_2 都適用上面的敘述