

A color set is a finite set, the elements of which are called colors.

A 1-Adinkra with color set C is (V, E, χ, Δ, g) where

- V is a finite set of vertices
- $E \subset V \times V$ is a set of edges
- $\chi : E \rightarrow C$ is a map called the coloring
- $\Delta : E \rightarrow \{1, -1\}$ is a map called the dashing
- $g : V \rightarrow \mathbb{Z}$ is a map called the grading

These are required to satisfy the following:

- If $(v, w) \in E$, then $(w, v) \in E$. Furthermore, $\chi(v, w) = \chi(w, v)$ and $\Delta(v, w) = \Delta(w, v)$.
- For every $v \in V$ and $c \in C$, there exist exactly one $w \in V$ so that $(v, w) \in E$ and $\chi(v, w) = c$.
- If $c_1, c_2 \in C$ with $c_1 \neq c_2$, and $v \in V$, then there exist w, x , and $y \in V$ so that $(v, w), (w, x), (x, y)$, and $(y, v) \in E$, and $\chi(v, w) = \chi(x, y) = c_1$ and $\chi(w, x) = \chi(y, v) = c_2$ and $\Delta(v, w)\Delta(w, x)\Delta(x, y)\Delta(y, v) = -1$.
- If $(v, w) \in E$, then $|g(v) - g(w)| = 1$.

A 2-Adinkra with disjoint color sets C_1 and C_2 is $(V, E, \chi, \Delta, g_L, g_R)$ where

- V is a finite set of vertices
- $E \subset V \times V$ is a set of edges
- $\chi : E \rightarrow C$ is a map called the coloring
- $\Delta : E \rightarrow \{1, -1\}$ is a map called the dashing
- $g_L : V \rightarrow \mathbb{Z}$ and $g_R : V \rightarrow \mathbb{Z}$ are maps called the left grading and right grading.

These are required to satisfy:

- The first requirement for a 1-Adinkra still holds.
- The second and third requirements for a 1-Adinkra still hold with $C = C_1 \cup C_2$.
- The fourth requirement is replaced by: if $(v, w) \in E$ and $\chi(v, w) \in C_1$, then $|g_L(v) - g_L(w)| = 1$ and $g_R(v) = g_R(w)$. If $(v, w) \in E$ and $\chi(v, w) \in C_2$, then $|g_R(v) - g_R(w)| = 1$ and $g_L(v) = g_L(w)$.

Let C_1 and C_2 be disjoint color sets. Let $A_1 = (V_1, E_1, \chi_1, \Delta_1, g_1)$ be a 1-Adinkra with color set C_1 ; and let $A_2 = (V_2, E_2, \chi_2, \Delta_2, g_2)$ be a 1-Adinkra with color set C_2 . We can define the product of these Adinkras as the following 2-Adinkra with color sets (C_1, C_2) .

$$A_1 \times A_2 = (V, E, \chi, \Delta, g_L, g_R)$$

where

$$\begin{aligned} V &= V_1 \times V_2 \\ E &= E_1 \cup E_2 \text{ where} \\ E_1 &= \{((v_1, w), (v_2, w)) \mid (v_1, v_2) \in E_1, \text{ and } w \in V_2\} \\ E_2 &= \{((v, w_1), (v, w_2)) \mid v \in V, \text{ and } (w_1, w_2) \in E_2\} \\ \chi((v_1, w), (v_2, w)) &= c_1(v_1, v_2) \text{ for all } ((v_1, w), (v_2, w)) \in E_1 \\ \chi((v, w_1), (v, w_2)) &= c_2(w_1, w_2) \text{ for all } ((v, w_1), (v, w_2)) \in E_2 \\ g_L(v, w) &= g_1(v) \\ g_R(v, w) &= g_2(w) \\ \Delta((v_1, w), (v_2, w)) &= \Delta_1(v_1, v_2) \\ \Delta((v, w_1), (v, w_2)) &= (-1)^{g_1(v)} \Delta_2(w_1, w_2) \end{aligned}$$

Let $A_1 = (V_1, E_1, \chi_1, \Delta_1, g_{L1}, g_{R1})$ and $A_2 = (V_2, E_2, \chi_2, \Delta_2, g_{L2}, g_{R2})$ be 2-Adinkras with the same color set C . A homomorphism from A_1 to A_2 is a map

$$\phi : V_1 \rightarrow V_2$$

satisfying the following:

- If $(v, w) \in E_1$, then $\phi(v, w) \in E_2$ and $\chi_1(v, w) = \chi_2(\phi(v, w))$.
- If $v \in V_1$ then $g_{1L}(v) = g_{2L}(\phi(v))$.
- If $v \in V_1$ then $g_{1R}(v) = g_{2R}(\phi(v))$.

Note that there is no condition on the dashings Δ_1 and Δ_2 .