Homework 5

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1 贝叶斯决策

(1)

$$g_i(x) = -\frac{1}{2}(x-\mu_i)^T\Sigma_i^{-1}(x-\mu_i) - \frac{d}{2}\ln(2\pi) - \frac{1}{2}\ln|\Sigma_i| + \ln p(\omega_i)$$

两类分类判别边界满足 $g_1(x) = g_2(x)$

$g_1(x) - g_2(x) = (\mu_1^T\Sigma_1^{-1} - \mu_2^T\Sigma_2^{-1})x + (-\frac{1}{2}\mu_1^T\Sigma_1^{-1}\mu_1 + \frac{1}{2}\mu_2^T\Sigma_2^{-1}\mu_2)$

$= ([1 \quad 2] - [3 \quad 1])x + \frac{1}{2}(-[1 \quad 2]\begin{bmatrix}1\\2\end{bmatrix} + [3 \quad 1]\begin{bmatrix}3\\1\end{bmatrix})$

$= -2x_1 + x_2 + \frac{5}{2}$

决策边界为 $-2x_1 + x_2 + \frac{5}{2} = 0$

(2) 决策边界会平移。

(3)

$$\Sigma_1^{-1} = \begin{bmatrix}1 & 0\\ -1/2 & -1/2\end{bmatrix}$$

$$\begin{aligned}
&g_1(x) - g_2(x) = (-\frac{1}{2}x^T\Sigma_1^{-1}x + \frac{1}{2}x^T\Sigma_2^{-1}x) + (\mu_1^T\Sigma_1^{-1} - \mu_2^T\Sigma_2^{-1})x \\
&+ (-\frac{1}{2}\mu_1^T\Sigma_1^{-1}\mu_1 + \frac{1}{2}\mu_2^T\Sigma_2^{-1}\mu_2 - \frac{1}{2}\ln|\Sigma_1| + \frac{1}{2}\ln|\Sigma_2|) \\
&= \frac{1}{2}(-[x_1 \quad x_2]\begin{bmatrix}1 & 0\\ -1/2 & -1/2\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} + [x_1 \quad x_2]I\begin{bmatrix}x_1\\x_2\end{bmatrix}) \\
&+ ([0 \quad 0]\begin{bmatrix}1 & 0\\ -1/2 & -1/2\end{bmatrix} - [1 \quad 2]\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix})\begin{bmatrix}x_1\\x_2\end{bmatrix} \\
&+ \frac{1}{2}(-[0 \quad 0]\begin{bmatrix}1 & 0\\ -1/2 & -1/2\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix} + [1 \quad 2]\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix} - \ln\begin{vmatrix}1 & 0\\ 1 & 2\end{vmatrix} + \ln\begin{vmatrix}1 & 0\\ 0 & 1\end{vmatrix}) \\
&= \frac{3}{4}x_2^2 + \frac{1}{4}x_1x_2 - x_1 - 2x_2 + \frac{5}{2}
\end{aligned}$$

决策边界为 $\frac{3}{4}x_2^2 + \frac{1}{4}x_1x_2 - x_1 - 2x_2 + \frac{5}{2} = 0$

2 隐含马尔可夫模型

(1)

$$\alpha_1 = \begin{bmatrix}0.2\\0.4\\0.4\end{bmatrix} * \begin{bmatrix}0.5\\0.4\\0.3\end{bmatrix} = \begin{bmatrix}0.1\\0.16\\0.12\end{bmatrix}$$

$$\begin{bmatrix}0.1\\0.16\\0.12\end{bmatrix} * \begin{bmatrix}0.5 & 0.2 & 0.3\\0.3 & 0.5 & 0.2\\0.2 & 0.3 & 0.5\end{bmatrix} = \begin{bmatrix}0.05 & 0.02 & 0.03\\0.048 & 0.08 & 0.032\\0.024 & 0.036 & 0.06\end{bmatrix}$$
$$\alpha_2 = \begin{bmatrix}0.122\\0.136\\0.122\end{bmatrix} * \begin{bmatrix}0.5\\0.6\\0.7\end{bmatrix} = \begin{bmatrix}0.061\\0.0816\\0.0854\end{bmatrix}$$

$$\begin{bmatrix}0.061\\0.0816\\0.0854\end{bmatrix} * \begin{bmatrix}0.5 & 0.2 & 0.3\\0.3 & 0.5 & 0.2\\0.2 & 0.3 & 0.5\end{bmatrix} = \begin{bmatrix}0.0305 & 0.0122 & 0.0183\\0.02448 & 0.0408 & 0.01632\\0.01708 & 0.02562 & 0.0427\end{bmatrix}$$
$$\alpha_3 = \begin{bmatrix}0.07206\\0.07862\\0.07732\end{bmatrix} * \begin{bmatrix}0.5\\0.4\\0.3\end{bmatrix} = \begin{bmatrix}0.03603\\0.031448\\0.023196\end{bmatrix}$$

$$P(O|\lambda) = 0.03603 + 0.031448 + 0.023196 = 0.090674$$

(2)

$$\delta_1 = \begin{bmatrix}0.2\\0.4\\0.4\end{bmatrix} * \begin{bmatrix}0.5\\0.4\\0.3\end{bmatrix} = \begin{bmatrix}0.1\\0.16\\0.12\end{bmatrix},\ \phi_1 = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\begin{bmatrix}0.1\\0.16\\0.12\end{bmatrix} * \begin{bmatrix}0.5 & 0.2 & 0.3\\0.3 & 0.5 & 0.2\\0.2 & 0.3 & 0.5\end{bmatrix} = \begin{bmatrix}0.05 & 0.02 & 0.03\\0.048 & 0.08 & 0.032\\0.024 & 0.036 & 0.06\end{bmatrix}$$
$$\delta_2 = \begin{bmatrix}0.05\\0.08\\0.06\end{bmatrix} * \begin{bmatrix}0.5\\0.6\\0.7\end{bmatrix} = * \begin{bmatrix}0.025\\0.048\\0.042\end{bmatrix},\ \phi_2 = \begin{bmatrix}1\\2\\3\end{bmatrix}$$

$$\begin{bmatrix}0.025\\0.048\\0.042\end{bmatrix} * \begin{bmatrix}0.5 & 0.2 & 0.3\\0.3 & 0.5 & 0.2\\0.2 & 0.3 & 0.5\end{bmatrix} = \begin{bmatrix}0.0125 & 0.005 & 0.0075\\0.0144 & 0.024 & 0.0096\\0.0084 & 0.0126 & 0.021\end{bmatrix}$$
$$\delta_3 = \begin{bmatrix}0.0144\\0.024\\0.021\end{bmatrix} * \begin{bmatrix}0.5\\0.4\\0.3\end{bmatrix} = \begin{bmatrix}0.0072\\0.0096\\0.0063\end{bmatrix},\ \phi_3 = \begin{bmatrix}2\\2\\3\end{bmatrix}$$

$q_3 = S_2,\ q_2 = S_2,\ q_1 = S_2$，最可能的隐含序列状态为 $(S_2S_2S_2)$