8 tutorials undetermined coefficients

wtorek, 28 kwietnia 2020 08:39

Example : $y'' + y = x+1$.

I part. (Hom. case)

We are starting with char. eq. for homogeneous part:

$y'' + y = 0 \qquad \lambda^2 + 1 = 0 \Rightarrow \lambda = i$ or $\lambda = -i$.

Let us take $\lambda = i$. Then

$e^{ix} = \cos x + i \sin x$ (*). Taking a linear combination of real and imaginary part of (*) we are getting homogeneous sol.:

$$y_0(x) = C_1 \cos x + C_2 \sin x$$

II part: Let us write a „special" number which is taken from $r(x) = x+1$.

$r(x) = x+1 = (x+1)\cdot e^{0x}$. Hence $\bar{\lambda} = \alpha + i\beta = 0 + i0 = 0$

0 is not a root for char. eq. so

$y_p(x) = Ax + B$ ($r(x)$ is a polynomial of degree of 1)

Now, we need to find A, B.

$y_p'(x) = A \qquad y_p'' = 0$.

Let us put everything into our diff. eq:

$0 + Ax + B = x + 1$

$0 + Ax + B = x + 1.$

$B = 1 \quad A = 1.$

<u>Answ</u>: $y(x) = y_0(x) + y_p(x) = C_1 \cos x + C_2 \sin x + x + 1.$