Euler homogeneous eq.

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Idea:

Eul. eq.: $ax^2y'' + bxy' + cy = 0$

notation: $\frac{dy}{dt} = \dot{y}$

$x = e^t$

$$y'(x) = \frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx} = e^{-t}\cdot\dot{y} = \frac{1}{x}\dot{y}$$

By chain rule:

Statement:

If $y = f(u)$ and $u = g(x)$, then this abbreviated form is written in Leibniz notation as:

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}.$$ [1]

For the second derivative we are getting:

$$\begin{aligned} y''(x) &= \frac{dy'}{dx} = \frac{dy'}{dt}\frac{dt}{dx} = \left[\frac{d}{dt}\left(e^{-t}\dot{y}\right)\right]e^{-t} = \left[\left(-e^{-t}\right)\dot{y} + e^{-t}\ddot{y}\right]e^{-t} \\ &= e^{-2t}\left(-\dot{y} + \ddot{y}\right) = \frac{1}{x^2}\left(\ddot{y} - \dot{y}\right), \end{aligned}$$

(From the 05-05 lecture)

Plugging both derivatives into our eq. we get:

$$a\cancel{x^2}\cdot\underbrace{\frac{1}{\cancel{x^2}}(\ddot{y} - \dot{y})}_{y''} + bx\underbrace{\frac{1}{x}y}_{y'} + c\cdot y = 0$$

As you can see we have second order diff. eq. with constant coefficients.