4rd tutorials

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Exact diff. eq.

1. Solve the differential equations:

   a) $(x+2y)y' + x + y = 0$;

   b) $(x-2y)y' - x + y = 0$;

   c) $(x-2y)y' + x^2 + y = 0$;

   d) $2y(x-1)y' + 3x^2 + y^2 = 0$.

Sol: a) $\underbrace{(x+2y)}_{M(x,y)}dy + \underbrace{(x+y)}_{N(x,y)}dx = 0 \qquad (*)$

Test of exactness: Th. Poincaré ...

$$\boxed{\frac{\partial M(x,y)}{\partial x} = \frac{\partial N(x,y)}{\partial y}}$$

$\frac{\partial M(x,y)}{\partial x} = 1 \qquad \frac{\partial M}{\partial y} = 1$

From Poincaré theorem potential function s.t.

$$\frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial y}dy = 0 \qquad (**)$$

Now, comparing left-hand sides of $(*)$, $(**)$ we get:

$F(x,y) = \int M\,dx + h(y)$, for some function h (Observe that $\frac{\partial}{\partial x}\left(\int M dx + h(y)\right) = M$)

We have $F(x,y) = \frac{x^2}{2} + xy + h(y)$.

One more time, comparing left-hand sides of $(*)$, $(**)$ we get:

$\frac{\partial}{\partial y}\left(\frac{x^2}{2} + xy + h(y)\right) = x + y \equiv \cancel{x} + h'(y) = \cancel{x} + y \equiv h'(y) = y \equiv h(y) = \frac{y^2}{2} + c_1$

Answ: $F(x,y) = C \equiv \frac{x^2}{2} + xy + \frac{y^2}{2} + c_1 = C \equiv \frac{x^2}{2} + xy + \frac{y^2}{2} = D,\ D \in \mathbb{R}$