

Let  $A_1, A_2, \dots, A_n$  be airports, and let  $A_0$  denote the air.

Let  $B_1, \dots, B_m$  represent airplanes, and  $C_1, \dots, C_p$  represent passenger transportation needs.

## 1 Encoding airplane flights

Let the binary variable  $P_{t_1, A_j, B_k}$  encode in 0-1 values (1-true, 0-false) whether airplane  $B_k$  is at state  $A_j$  at time  $t_1$ . More precisely, if  $j=0$ , then the airplane is in the air; if  $j \in \{1, \dots, n\}$ , it is at airport  $A_j$ .

The condition  $\sum_j^{\{0, \dots, n\}} P_{t_1, A_j, B_k} = 1$  for each time point  $t_1 \in \{0, \dots\}$  and airplane number  $k \in \{1, \dots, m\}$  ensures that each airplane is in only one state at each time.

Let  $d(a_1, a_2)$  represent the time required for a flight from airport  $A_{a_1}$  to  $A_{a_2}$ , where  $a_1, a_2 \in \{1, \dots, n\}, a_1 \neq a_2$  (for  $a_1 = a_2$ , we assume 0). We have noted that the distance (time in minutes) between airports does not need to be the same in both directions.

The condition  $P_{t_1, A_i, B_k} \cdot P_{t_2, A_j, B_k} = 0$  for  $|t_1 - t_2| < d(i, j)$ , where  $t_1, t_2 \in \{0, \dots\}$  and  $k \in \{1, \dots, m\}$  and  $i, j \in \{1, \dots, n\}, i \neq j$ , ensures that the time between the airplane's presence at two airports is greater than or equal to the minimum possible travel time.

## 2 Encoding passengers

Let  $L_{C_r, t_1, A_j, B_k}$  be a binary variable that encodes by 1 the situation where passenger  $C_r$ , where  $r \in \{1, \dots, p\}$  is at time  $t_1 \in \{0, \dots\}$  in state  $A_j, j \in \{0, 1, \dots, n\}$  and in airplane  $B_k$ , where  $k \in \{1, \dots, m\}$ .

Let  $A_{x_r}$  denote the destination airport of the  $r$ -th passenger. Additionally, we add the airplane  $B_{m+1}$ , which "stands" at every airport  $A_j$ , meaning  $P_{t_1, A_j, B_{m+1}} = 1$  for  $j \in \{1, \dots, n\}$  and  $P_{t_1, A_0, B_{m+1}} = 0$  for every  $t_1 \in \{0, \dots\}$ . Furthermore, we assume  $L_{C_r, t_1, A_0, B_{m+1}} = 0$  for each  $C_r$  and  $t_1$ .

We impose the condition  $\sum_{j,k} L_{C_r, t_1, A_j, B_k} = 1$  for each  $t_1, C_r$ .

We maximize the sum of the following expressions:

(1)  $\sum_k \sum_j^{\{0, 1, \dots, n\}} L_{C_r, t_1, A_j, B_k} P_{t_1, A_j, B_k}$  - the passenger is where the airplane is.

(2)  $\sum_k \sum_j^{\{0\} \cup \{1, \dots, n\}} L_{C_r, t_1, A_0, B_k} L_{C_r, t_1+1, A_j, B_k}$  - if the passenger is in the air in a given airplane, then in the next moment, they are also in the same airplane.

(3)  $\sum_{u,k} \sum_j^{\{1, \dots, n\} \setminus \{x_r\}} L_{C_r, t_1, A_j, B_u} L_{C_r, t_1+1, A_j, B_k}$  - the passenger at a given airport can change airplanes, unless they are at the destination airport.

(4)  $\sum_u \sum_j^{\{1, \dots, n\} \setminus \{x_r\}} L_{C_r, t_1, A_j, B_u} L_{C_r, t_1+1, A_0, B_u}$  - the passenger follows the departing airplane, unless they have already arrived at their destination.

(5)  $\sum_{u,k} L_{C_r, t_1, A_{x_r}, B_u} L_{C_r, t_1+1, A_{x_r}, B_k}$  - the passenger remains at the destination airport.

Regarding the above constraints:

(1) is always equal to 1 for each  $C_r, t_1$

(2)+(3)+(4)+(5) is always equal to 1 for each  $C_r, t_1$