PMEs for HPH^T through the lower triangular solves, L uses top right quadrant

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L, T, M, and v are temporary variables.

H, x, P, and z are fully computed inputs/constants.

Hats on variables indicate initial inputs to that loop, and are omitted where they're not needed to ensure clarity.

In all cases, partition:

$$H \rightarrow \left(\frac{H_T}{\overline{H_B}}\right)$$

$$x \rightarrow \left(\frac{x_T}{\overline{x_B}}\right)$$

$$z \rightarrow \left(\frac{z_T}{\overline{z_B}}\right)$$

In all of these algorithm, the partitioning lines move from top-left to bottom-right. For operands that don't have (or are partitioned as not having) lefts and rights, the left-to-right movement is irrelevant.

Algorithm 1

$$\left(\begin{array}{c} T_T = H_T P \\ \hline\hline T_B = \hat{T}_B \end{array}\right)$$

$$\left(\begin{array}{c} v_T = z_T - H_T x \\ \hline\hline v_B = \hat{T}_B \end{array}\right)$$

$$\left(\begin{array}{c||c} L_{TL} = T_T H_T^T & * \\ \hline\hline L_{BL} = \hat{L}_{BL} & L_{BR} = \hat{L}_{BR} \end{array}\right)$$

$$\left(\begin{array}{c||c} L_{TL} = \mathrm{CHOL}(\hat{L}_{TL}) & * \\ \hline\hline L_{BL} = \hat{L}_{BL} & L_{BR} = \hat{L}_{BR} \end{array}\right)$$

$$\left(\begin{array}{c} M_T = \mathrm{TRSM}(L_{TL}, T_T) \\ \hline\hline M_B = \hat{M}_B \end{array}\right)$$

$$\left(\begin{array}{c} v_T = \mathrm{TRSV}(L_{TL}, \hat{v}_T) \\ \hline\hline v_B = \hat{v}_B \end{array}\right)$$

Algorithm 2 This is fundamentally a minor variation on algorithm 1 that computes L more greedily without using it more.

$$\left(\begin{array}{c} T_T = H_T P \\ \hline\hline T_B = \hat{T}_B \end{array}\right)$$

$$\left(\begin{array}{c} v_T = z_T - H_T x \\ \hline\hline v_B = \hat{T}_B \end{array}\right)$$

$$\left(\begin{array}{c||c} L_{TL} = T_T H_T^T & * \\ \hline\hline L_{BL}^T = T_T H_B^T & L_{BR} = \hat{L}_{BR} \end{array}\right)$$

$$\left(\begin{array}{c||c} L_{TL} = \mathrm{CHOL}(\hat{L}_{TL}) & * \\ \hline\hline L_{BL} = \hat{L}_{BL} & L_{BR} = \hat{L}_{BR} \end{array}\right)$$

$$\left(\begin{array}{c} M_T = \mathrm{TRSM}(L_{TL}, T_T) \\ \hline\hline M_B = \hat{M}_B \end{array}\right)$$

$$\left(\begin{array}{c} v_T = \mathrm{TRSV}(L_{TL}, \hat{v}_T) \\ \hline\hline v_B = \hat{v}_B \end{array}\right)$$

Algorithm 3 This is algorithm 2, except it takes advantage of the work done on L.

$$\left(\begin{array}{c} T_T = H_T P \\ \hline\hline T_B = \hat{T}_B \end{array}\right)$$

$$\left(\begin{array}{c} v_T = z_T - H_T x \\ \hline\hline v_B = \hat{T}_B \end{array}\right)$$

$$\left(\begin{array}{c||c} L_{TL} = T_T H_T^T & * \\ \hline\hline L_{BL}^T = T_T H_B^T & L_{BR} = \hat{L}_{BR} \end{array}\right)$$

$$\left(\begin{array}{c||c} L_{TL} = \mathrm{CHOL}(\hat{L}_{TL}) & * \\ \hline\hline L_{BL} = \mathrm{TRSM}(L_{TL}, \hat{L}_{BL}) & L_{BR} = \hat{L}_{BR} \end{array}\right)$$

$$\left(\begin{array}{c} M_T = \mathrm{TRSM}(L_{TL}, T_T) \\ \hline\hline M_B = \hat{M}_B \end{array}\right)$$

$$\left(\begin{array}{c} v_T = \mathrm{TRSV}(L_{TL}, \hat{v}_T) \\ \hline\hline v_B = \hat{v}_B \end{array}\right)$$