AIMMS Campus
Robust optimization problems

1 Minimum cost network flow

Consider a graph depicted in Figure 1. The numbers on the edges stand for the nominal cost of transporting a unit of resource along the edge, and a possible (additive) perturbation to this cost, respectively. That means, the unit cost per edge can become higher by the amount indicated by the second number.

Formulate the problem of minimizing the total cost of transporting two units of resource from the bottom-left to the top-right node in this graph, under the 'nominal' cost of the edges. How would you test the performance of the solution found assuming that the edge costs can be perturbed?

2 Production planning

Production planner needs to decide on the product mix to be manufactured in a factory throughout the upcoming month. The goal is to maximize the combined products (we assume that everything that gets produced, can also be sold).

A unit of each product requires a certain amount of time (in hours) spent on various processing machines. The per-product data on this is given in Table 1. The number of available machines of each type is as follows: It is assumed that throughout the month there are 24 working days of 8 working hours each. Formulate the problem of maximizing the total profit subject to constraints on the resource availability. You do not need to consider any scheduling issues.

Table 1. Product data for the production planning problem.

Product	Grinding	Vertical drilling	Horizontal drilling	Boring	Planing	Profit
0	0.5	0.1	0.2	0.05	-	7.0
1	0.7	0.2	-	0.03	-	6.0
2	-	-	0.8	-	0.01	8.0
3	-	0.3	-	0.07	-	4.0
4	0.3	-	-	0.10	0.05	8.0
5	0.2	0.6	-	-	-	9.0
6	0.5	-	0.6	0.08	0.05	3.0

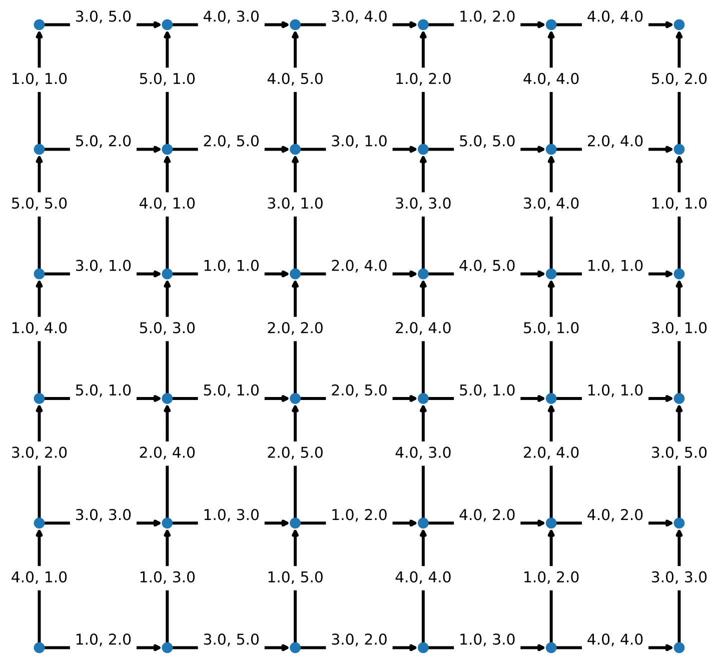

Figure 1. Network for the minimum cost flow case.

Next, assuming that the processing times of the products on machines are not known exactly, how would you test the robustness of a given production plan?

3 Inventory planning (multi-stage)

The company BIM (Best International Machines) produces two types of microchips, logic chips (1gr silicon, 1gr plastic, 4gr copper) and memory chips (1gr germanium, 1gr plastic, 2gr copper).

BIM hired Caroline to manage the acquisition and the inventory of these raw materials. She conducted a data analysis which lead to the following prediction of monthly demands for her chips:

chip type	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
logic chip	88	125	260	217	238	286	248	238	265	293	259	244
memory chip	47	62	81	65	95	118	86	89	82	82	84	66

As you recall, BIM has the following stock at the beginning of the year:

copper	silicon	germanium	plastic
480	1000	1500	1750

The company would like to have at least the following stock at the end of the year:

copper	silicon	germanium	plastic
200	500	500	1000

Each material can be acquired at each month, but the unit prices of each material type vary as follows:

material	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
copper	1	1	1	2	2	3	3	2	2	1	1	2
silicon	4	3	3	3	5	5	6	5	4	3	3	5
germanium	5	5	5	3	3	3	3	2	3	4	5	6
plastic	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1

The inventory is limited by a capacity of a total of 9000 units per month, regardless of the composition of products in stock. The holding costs of the inventory are 0.05 per unit per month, regardless of the material type. Due to budget constraints, Caroline cannot spend more than 5000 per month on new material acquisition.

Note that Caroline aims at minimizing the acquisition and holding costs of the materials while meeting the required quantities for production. The production is made to order, meaning that no inventory of chips is kept.

Formulate the problem Caroline faces as a linear programming problem. How would you test the performance of the solution to this problem in face of uncertainty about the chip demand?

4 Robust counterparts exercise 1

Consider the following constraint:

$$(2 + z_1)x_1 + (3 - z_1 + z_2)x_2 + (4 - z_2)x_3 \leq 4, \quad \forall\, z \in Z = \{(z_1, z_2) : |z_i| \leq 1,\ i = 1, 2\}.$$

5 Robust counterparts exercise 2

Consider the following constraint:

$$(2 + z_1)x_1 + (3 - z_1 + z_2)x_2 + (4 - z_2)x_3 \leq 4, \quad \forall\, z \in Z = \{(z_1, z_2) :\ |z_1| + |z_2| \leq 1\}.$$

6 Robust counterparts exercise 3

Consider the following constraint:

$$\sum_{i=1}^{n} z_i x_i \leq 4, \quad \forall\, z \in Z = \{z : |z_i| \leq 1,\ i = 1, \ldots, n,\ \sum_{i=1}^{n} |z_i| \leq \Gamma\}.$$

7 Robust counterparts exercise 4

Consider the following constraint:

$$\sum_{i=1}^{n} z_i x_i \leq 4, \quad \forall\, z \in Z = \{z : \sqrt{\sum_{i=1}^{n} z_i^2} \leq \Gamma\}.$$