Lab Assessment for Module 4

MA-423 : Matrix Computations　　21 November, 2020　　S. Bora

Marks: 14

Important instructions:

- **Put all your work in a single folder and submit a link to download it on MS Teams within the declared deadline.**
- **Submitted folder should contain *only* the programs written for the purpose of the assessment and other supporting programs that are required for those programs to run. If there are any unnecessary programs in the folder, then upto 2 marks may be deducted from the score of every group member.**
- **The outputs for Q3 should be provided in pdf format.**
- **Any kind of cheating or copying from internet will result in heavy penalties for every group member.**

1. Write a program `B = FrancisQRD(A)` that performs a single iteration of Francis's Implicit double shift or degree 2 Shifted QR algorithm on an upper Hessenberg matrix A with generalized Rayleigh Quotient shifting strategy. It should have the following features.

   (i) All arithmetic is entirely real if A is a real matrix. **(3 marks)**

   (ii) It uses reflect.m and applreflect.m codes efficiently (which should be provided). **(4 marks)**

2. Write a program `[B, k] = CallFrancisQR(A, m, tol)` that takes as inputs a properly upper Hessenberg matrix A, an integer `m` (that is either 1 or 2) and a small number `tol` and calls `FrancisQRS(A)` (which uses Rayleigh quotient shifts) if `m = 1` and `FrancisQRD(A)` if `m = 2` to give as outputs a matrix `B` and a positive integer `k` such that `B` is an upper Hessenberg **that is ready to be deflated** and `k` is the number of iterations performed, the decision to stop being based on a criterion that uses `tol`. Additionally the program should also terminate in the very beginning with an error message if the input matrix `A` is not properly Hessenberg. **(3 marks)**

   [**Marking scheme:** 1 mark for the check if `A` is properly Hessenberg + 2 marks for a proper design of the calls to the correct codes and the decision to stop the iterations. In doing so you can assume that deflation with the given shifting strategies is certain.]

3. Convert the following matrices to upper Hessenberg form by using `hess(A)` (if necessary) and run `[B, k] = CallFrancisQR(A, m, tol)`. Report the outputs in <u>format short e</u> for each case with `m = 1` and `m = 2`, and a common value of $\mathtt{tol} = 10^{-10}$. **(4 marks)**

   [**Marking scheme:** 1 mark for each pair of outputs. The `FrancisQRS.m` code must be correct to get credit for its outputs.]

   (i) $A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 9 & 2 \\ 0 & -1 & 2 \end{bmatrix}$ (ii) $A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 3 & 2 \\ -4 & -1 & 2 \end{bmatrix}$.