cholesky factorization theorem

first, block multiplication.

$$AB = \left[\begin{array}{c|cc} x & x & x \\ \hline x & x & x \\ x & x & x \end{array}\right]\left[\begin{array}{c|cc} x & x & x \\ \hline x & x & x \\ x & x & x \end{array}\right] = \left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array}\right]\left[\begin{array}{c|c} B_{11} & B_{12} \\ \hline B_{21} & B_{22} \end{array}\right]$$
$$= \left[\begin{array}{c|c} A_{11}B_{11} + A_{12}B_{21} & A_{11}B_{12} + A_{12}B_{22} \\ \hline A_{21}B_{11} + A_{22}B_{21} & A_{21}B_{12} + A_{22}B_{22} \end{array}\right]$$

$$\left[\begin{array}{c|cc} 1 & 2 & 3 \\ \hline 0 & 1 & 3 \\ 2 & 2 & 4 \end{array}\right]\left[\begin{array}{c|cc} 2 & 4 & 1 \\ \hline 1 & 0 & 1 \\ 3 & 1 & 2 \end{array}\right] = \begin{bmatrix} 1 \cdot 2 + \begin{bmatrix} 2 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \end{bmatrix} & 1\begin{bmatrix} 4 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 3 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 2 \end{bmatrix} 2 + \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \end{bmatrix} & \begin{bmatrix} 0 \\ 2 \end{bmatrix}\begin{bmatrix} 4 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix} \end{bmatrix}$$
$$= \left[\begin{array}{c|cc} 13 & 7 & 9 \\ \hline 10 & 3 & 7 \\ 18 & 12 & 12 \end{array}\right].$$

theorem 14. if A_{nxn} is symmetric positive definite, then there exists an upper triangular R_{nxn} such that $A = R^TR$.

$$A = \left[\begin{array}{c|c} a & b^T \\ \hline b & C \end{array}\right] \Rightarrow$$

$$\begin{array}{l} \textbf{for } k = 1, 2, \ldots, n \\ \quad \textbf{if } A_{kk} < 0, \textbf{ stop}, \textbf{ end} \\ \quad R_{kk} = \sqrt{A_{kk}} \\ \quad u^T = \frac{1}{R_{kk}} A_{k,k+1:n} \\ \quad R_{k,k+1:n} = u^T \\ \quad A_{k+1:n,k+1:n} = A_{k+1:n,k+1:n} - uu^T \\ \textbf{end} \end{array}$$

then two back-substitutions: $R^Tc = b$, $Rx = c$.