

cholesky factorization theorem

first, block multiplication.

$$AB = \begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \end{bmatrix} \begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$$

$$= \begin{bmatrix} A_{11}B_{11} + A_{12}B_{21} & A_{11}B_{12} + A_{12}B_{22} \\ A_{21}B_{11} + A_{22}B_{21} & A_{21}B_{12} + A_{22}B_{22} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 3 \\ 2 & 2 & 4 \end{bmatrix} \begin{bmatrix} 2 & 4 & 1 \\ 1 & 0 & 1 \\ 3 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 \cdot 2 + [2 \ 3] \begin{bmatrix} 1 \\ 3 \end{bmatrix} & 1[4 \ 1] + [2 \ 3] \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix} \\ [0 \ 2] \cdot 2 + \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} & [0 \ 2][4 \ 1] + \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} 13 & 7 & 9 \\ 10 & 3 & 7 \\ 18 & 12 & 12 \end{bmatrix}.$$

theorem 14. if $A_{n \times n}$ is symmetric positive definite, then there exists an upper triangular $R_{n \times n}$ such that $A = R^T R$.

$$A = \begin{bmatrix} a & b^T \\ b & C \end{bmatrix} \Rightarrow$$

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for  $k = 1, 2, \dots, n$ 
  if  $A_{kk} < 0$ , stop, end
   $R_{kk} = \sqrt{A_{kk}}$ 
   $u^T = \frac{1}{R_{kk}} A_{k,k+1:n}$ 
   $R_{k,k+1:n} = u^T$ 
   $A_{k+1:n,k+1:n} = A_{k+1:n,k+1:n} - uu^T$ 
end
    
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then two back-substitutions: $R^T c = b$, $Rx = c$.