

INITIAL VALUE PROBLEM (IVP): $y' = f(t, y)$
 $y(a) = y_a$
 $t \in [a, b]$

EXAMPLE 01: $y' = t y + t^3 = f(t, y)$
 $y(0) = y_0 = 1$
 $t \in [0, 1]$

EULERS: $w_0 = y_0 = 1$

$$w_{i+1} = w_i + h \cdot f(t_i, w_i) \xrightarrow{\text{SWAP}} w_i + h \cdot (t_i w_i + t_i^3) \xrightarrow{\text{SIMPLIFY}} w_i + h(t_i w_i + t_i^3)$$

EXPLICIT TRAPEZOIDAL:

$$w_0 = y_0 = 1$$

$$\begin{aligned} w_{i+1} &= w_i + \frac{h}{2} [f(t_i, w_i) + f(t_i + h, w_i + h f(t_i, w_i))] \\ &= w_i + \frac{h}{2} [t_i w_i + t_i^3 + (t_i + h)(w_i + h(t_i w_i + t_i^3)) + (t_i + h)^3] \\ &= w_i + \dots \xrightarrow{\text{SWAP}} \dots \xrightarrow{\text{SIMPLIFY}} \end{aligned}$$