GMRES

CONSIDER $Ax = b$ AND SERIES

$$\underbrace{\text{SPAN}\{x_0, Ax_0, A^2x_0, \dots, A^kx_0\}} \sim \underbrace{A^0x_0, A^1x_0, A^2x_0, \dots}$$

KRYLOV SPACE
+ APPROX SOLUTION FOUND IN THIS SPACE

LIKE BASIS FUNCTIONS (IE: $1, x, x^2, \dots$) BUT WITH A.

IE, WHAT IS $c_0x_0 + c_1Ax_0 + c_2A^2x_0 + \dots + c_kA^kx_0$ THAT MINIMIZES RESIDUAL?

IE, $r = A(\underbrace{c_0x_0 + c_1Ax_0 + c_2A^2x_0 + \dots + c_kA^kx_0}_{x_k}) - b$

K ITERATES, CHANGING Ax

$x_0 \xrightarrow{Ax} x_k, \quad x_0 \xrightarrow{Ax} x_{k+1}$

($x_0, Ax_0, A^2x_0, \dots, A^mx_0$
THERES A MAXIMUM M
SUCH IF $K_M = K_{M+1}$.) SIDE NOTE

GRAM-SCHMIDT / MOD

```
FOR j = 1:n
  y = A_j
  FOR i = 1:j-1
    r_ij = q_i^T A_j   y IF GS/MOD
    y = y - r_ij q_i
  r_jj = ||y||_2
  q_j = y / r_jj
```

② REORTHOGONALIZED W/ EACH ITERATION K

GMRES

x_0, $r = b - Ax_0$, $q_1 = r/\|r\|_2$

```
FOR k = 1:m , m ≤ n
  y = A q_k
  FOR j in k
    h_jk = q_j^T y
    y = y - h_jk q_j
  h_{k+1,k} = ||y||_2     ~ IF ZERO, SKIP NEXT LINE + END
  q_{k+1} = y / h_{k+1,k}
```

MINIMIZE $\left\| H c_k - \begin{bmatrix} \|r\|_2 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \right\|_2$

$x_k = Q_k c_k + x_0$

$A = QR \Rightarrow QRx = b \Rightarrow Rx = Q^{-1}b$

$AQ_k = Q_{k+1}H_k \Rightarrow H_kc_k = Q_{k+1}^{-1}b = \begin{bmatrix} \|r\|_2 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$ LEAST SQUARES PROB: $H_k c_k = \begin{bmatrix} \|r\|_2 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$

② ORTHOGONALIZAT W/ LEAST SQUARES