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EXPLORING INCENTIVES FOR "TANKING" IN THE NATIONAL BASKETBALL ASSOCIATION

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1. Motivation

Upon the completion of every National Basketball Association (NBA) season since 1947, the league's teams assemble to select young players from collegiate or international programs. Originally, in an effort to promote parity, the worst-performing teams from the previous season would receive the highest draft picks which would be used to select, presumably, the most promising players.

An obvious downside to this system is that there is some incentive to be among the league's worst teams, and, thus, the word "tanking" was coined to denote intentionally losing to receive a higher draft pick. This design flaw was on full display leading up to the 1983 Draft, which offered the chance to select Virginia's Ralph Sampson, the most highly-touted college prospect in the league's history. To ensure that they would receive the top selection, the Houston Rockets traded the league's reigning Most Valuable Player, Moses Malone, and finished the season with a record of 14 wins and 68 losses after finishing with 46 wins and 36 losses the year before. Rockets fans might not have enjoyed the difficult season, but the team's management determined that adding the top prospect was more important for the franchise than winning that season.

Following this episode, the NBA then instituted a draft lottery in 1985 whereby the worst-performing teams from the previous season would have the highest probability of securing the top selections instead of being guaranteed their draft slots in the reverse order of the standings. However, even with the lottery in place, tanking remains a problem in the NBA to this day (see: "The Process", a four-year tanking strategy recently undertaken by the Philadelphia 76ers). A major reason for this sustained phenomenon is that there is little value in finishing in the "middle of the pack" besides appeasing fans with moderate success, albeit largely inconsequential.

To compound this issue, basketball is a sport where the best player is often able to single-handedly win games. One can clearly see this in the example of LeBron James, considered by most to be the sport's greatest active talent, whose teams have reached the NBA Finals of the playoffs in seven straight seasons despite, at times, having little help from teammates. Additionally, the skill level of the prospects available in any given draft typically follows the form of an exponential decay function; the difference in value of picks 1 and 10 is much greater than that of picks 21 and 30.

Tanking is a symptom of a market failure when considering how the NBA assigns draft positions. However, most of the academic literature in the space has focused on monetary incentives (see Price, Soebbing, et. al, 2010). Meanwhile, we hope to examine performance-based incentives as our objective is to create a system for allocating draft selections whereby there is no incentive to tank and, in fact, each team's optimal strategy is to put forth its greatest effort. We might hypothesize that a system that awards the highest probability of the top selections to the worst-performing team from the cumulative standings of the previous two or three seasons not only improves greatly upon the current system but also maintains the parity that it was designed to create. Unfortunately, it is very difficult to solve (or even simulate) scenarios with more than three periods. Instead, rather than focusing on a specific design that might fix the issue, we explore different reasons that could explain the presence of such a market failure in the two-period case.

We first will look at a series of two-player, two-period games where the players (in this case, NBA teams) choose how much "effort to exert in each period (in this case, NBA seasons) depending on how much they value future success as compared to current success (based on some discount factor). We can account for different scenarios by adjusting budgets for effort (which can be considered the team's skill level) and discount rates. Then, we will move to n-player, two-period games; here, we can both look at working examples and the intuition behind them and then comparative statics with more general conditions. Ultimately, we want to prove that tanking is an issue, understand why it exists, and propose further avenues for research with new methods for exploring potential solutions.

2. General Two-Period Model

In order to simulate tanking incentives, we created a model with two periods, in which players could decide to either tank or not tank in the first period. Each player was assigned a skill level, represented by a budget constraint, and in each period could exert either 0 effort (i.e. tanking) or effort equal to that skill level.

Now we give a formal description of our model:

(1) There are n players.
(2) We have two periods.
(3) Each player i has skill $Y_i > 0$.
(4) In each period t, each player i chooses effort $e_i^{(t)}$. We require $e_i^{(t)} \in \{Y_i, 0\}$. In this, we are making the implicit assumption that a Player chooses to either tank the season or not tank, with no middle ground.
(5) Given effort levels in period 1, $(e_1^{(1)}, e_2^{(1)}, \ldots, e_n^{(1)})$, we calculate "smooth order statistics" $r_i^{(1)}$ from the $e_i^{(1)}$. This means that $e_i^{(1)} \mapsto r_i^{(1)}$, where $r_i^{(1)}$ is k if $e_i^{(1)}$ is the k-th largest effort made in the first period, and if teams $e_{i_1}^{(1)}, ..., e_{i_k}^{(1)}$ are tied for the l through $l+k$th place, they get order statistics $r_{i_1}^{(1)} ... r_{i_k}^{(1)}$ with value $l + k/2$.
(6) The orders statistics $r_i^{(1)}$ of the player efforts determine two things.
    (a) The utility $v_i^{(1)}$ of each Player in period 1. For now we just assume a linear relation $v_i^{(1)} = (A-B)r_i^{(1)} + B$ for some values $A > B$. For two players, this means that a player gets value A for being in first place, and value B for being in second place, and the average of A, B if tied.
    (b) The amount of help that each player gets in period 2. We write $h_i := h(r_i^{(1)})$, where the function h is monotonically non-increasing in $r_i^{(1)}$, the player's rank in the previous season. The worse a team performs, the more help it receives.

(7) In the second (and final period), every team will try as hard as possible. Moreover, their skill is assisted by a help factor corresponding to how hard they tanked in the previous. We write $e_i^{(2)} = Y_i + h_i(r_i^{(1)})$.
(8) We calculate smooth order statistics $e_i^{(2)} \mapsto r_i^{(2)}$ in the same way as before, and again assume $v_i^{(2)} = (A - B)r_i^{(2)} + B$.
(9) A player i has total payoff $v_i^{(1)} + \delta_i v_i^{(2)}$, where $\delta_1, \delta_2 < 1$ (*strictly*) are team-wise discounts for utility in the second period.

3. Resolution of the Two Player-Model

Setting the number of teams $n = 2$, our model above defines a coordination game with 2 players. We now investigate the pure-strategy Nash equilibria of this game.

Claim 1. In the case where teams have equivalent skill levels and different discount rates, there exists no pure-strategy Nash equilibrium of the coordination game in two players where at least one player strongly prefers tanking.

Proof. In this case, we have that $Y_1 = Y_2 = 1$ but $\delta_1 < \delta_2$ (without loss of generality). This sets up the following two-period game:

		Player 2: tank	Player 2: not tank
Player 1	tank	$\left(\frac{1+\delta_1}{2}(A+B), \frac{1+\delta_2}{2}(A+B)\right)$	$\left(B + \delta_1 A, A + \delta_2 B\right)$
	not tank	$\left(A + \delta_1 B, B + \delta_2 A\right)$	$\left(\frac{1+\delta_1}{2}(A+B), \frac{1+\delta_2}{2}(A+B)\right)$

It follows that the behavior of the players will be symmetric (varying only on their discount rates). Thus, we examine the pure strategy for Player 1 first. In the case that Player 2 tanks, i.e. $e_2^{(1)} = 0$, Player 1 gets $\frac{1+\delta_1}{2}(A+B)$ if he or she tanks and $A + \delta_1 B$ if he or she does not tank. Because $A > B$, we have that tanking is strictly worse than not tanking regardless of the value of δ_1.

In the case that Player 2 does not tank, i.e. $e_2^{(1)} = 1$, Player 1 gets $B + \delta_1 A$ if he or she tanks and $\frac{1+\delta_1}{2}(A+B)$ if he or she does not tank. Again, because $A > B$, tanking is strictly worse than not tanking. Because the players are symmetric, the same logic applies to Player 2's decision-making. Thus, in this specific case, **we find that (not tank, not tank) is the pure-strategy Nash Equilibrium**.

This leads us to an interesting but expected result: when Players have the same skill level, regardless of their discount rate, they strictly prefer not tanking. This mimics real life because parity often dissuades tanking; Players who can be competitive will try to win, tanking generally occurs when a Player believes tanking will greatly improve their chances of winning in future seasons relative to the present. This is a sound baseline model and result to move forward with. □

Claim 2. Where future discount rates are equivalent, there exists no pure-strategy Nash equilibrium of the coordination game in two players where at least one player strongly prefers tanking.

Proof. We assumed our help function was monotonically non-increasing in the smooth order statistic. In two players, any choice of h becomes defined by three numbers $c_1 \geq c_2 \geq c_3$. We may assume $c_3 = 0$, since the order statistic operation on $e_i^{(2)} = Y_i + h(r_i^{(1)})$ is translation invariant. We have

$$h(r_i^{(1)}) = \begin{cases} c_1 & r_i^{(1)} = 0 \\ c_2 & r_i^{(1)} = 1/2 \\ 0 & r_i^{(1)} = 1 \end{cases}$$

In this game, each player seeks to maximize its utility $v_i^{(1)} + \delta v_i^{(2)}$. There are several cases to consider.

(1) If $Y_1^{(2)} > Y_2^{(2)} + c_1$, we calculate the following payoff matrix.

		Player 2: tank	Player 2: not tank
Player 1	tank	$\left(\frac{A+B}{2} + \delta A, \frac{A+B}{2} + \delta B\right)$	$\left(B + \delta A, A + \delta B\right)$
	not tank	$\left(A(1+\delta), B(1+\delta)\right)$	$\left(A(1+\delta), B(1+\delta)\right)$

There is one Nash equilibrium at (not tank, not tank). Not tanking is strongly preferred for player 1, and tanking is weakly preferred for player 2.

(2) If $Y_1 = Y_2 + c_1$, then

		Player 2: tank	Player 2: not tank
Player 1	tank	$\left(\frac{A+B}{2} + \delta A, \frac{A+B}{2} + \delta B\right)$	$\left(B + \delta A, A + \delta B\right)$
	not tank	$\left(A + \delta \cdot \frac{A+B}{2}, B + \delta \cdot \frac{A+B}{2}\right)$	$\left(A + \frac{A+B}{2} \cdot \delta, B + \delta \cdot \frac{A+B}{2}\right)$

As before, player 1 strongly prefers not-tanking, and Player 2 weakly prefers not taking, although they are effectively indifferent.

(3) If $Y_2 < Y_1 < Y_2 + c_1$, then

		Player 2	
		tank	not tank
Player 1	tank	$\left(\frac{A+B}{2} + \delta A, \frac{A+B}{2} + \delta B\right)$	$\left(B + \delta A, A + \delta B\right)$
	not tank	$\left(A + \delta \cdot B, B + \delta \cdot A\right)$	$\left(A + \delta B, B + \delta A\right)$

Now, if Player 1 tanks, Player 2 strongly prefers not to tank, and (tank, not tank) is not an equilibrium.
If Player 1 does not tank, the second Player is indifferent. The only Nash equilibrium in this case will be (not tank, not tank).

(4) We solve the case $Y_2 = Y_1$ in claim 1.

$\square$

4. Simulations for the n-Player Model

Because it became intractable to solve for Nash equilibria in the n-dimensional game theoretically, we wrote a program to solve for Nash equilibria in a general $n-$player coordination game.

Simulation of our game necessarily requires defining values for our free variables n, $\{Y_i\}$, δ, A, B and h. In many of the cases below, we also introduce a new parameter α that scales our help function linearly. This is so that we can examine how tanking incentives change as the reward for tanking increases. Code for our solver is available at (`https://github.com/ksagit/EC2099-game-scripts`).

We used the simulator to calculate the equilibria of several worked examples that we hope will provide some intuition for how tanking is incentivized in our model, in the case $n = 5$, which could a good spread of mediocre teams and good teams.

4.1. **Working Examples and Intuition.**

Here, we simulate 5-Player games using the brute-force Nash Equilibrium solver. This allows us to show specific examples of parameter spaces that give us some intuition as to how and when tanking incentives occur.

Example 1: We are able to show a working example of how the discount rate affects agents' willingness to tank. If we choose $\delta = .8$ with budgets $Y_i = \{1, 2, 3, 4, 8\}$ for the five teams respectively and $\alpha = 3$ (meaning the help function has the form $3(n - r_i)$ where r_i is the order statistic, the form of which we discuss in greater detail in 4.2), we see that only Player 4 *only* tanks (with $Y_4 = 4$). However, if we raise $\delta = .9$, meaning we increase the value of future values, Player 3 and Player 4 *both* tank, which makes sense: if you discount future winning less, then you should be more willing to tank.

Example 2: Another example we investigate displays the "catching up" phenomenon in tanking. We can look at two budget sets, $\{1, 2, 5, 6, 15\}$ and $\{1, 2, 5, 6, 10\}$, with the discount rate $\delta = 0.9$ and the linear help function with $\alpha = 2$. In the first case, Player 5 (with budget $Y_5 = 15$) will not tank because it prefers to win in period 1 and then receive whatever utility it can get in period 2. Player 4 (with budget $Y_4 = 6$) will also not tank because it does not have the ability to "catch up" even with the added help. On the other hand, in the second case, Player 5's behavior does not change, but Player 4 now chooses to tank because the added help of finishing in last will put its budget in period 2 above that of Player 5, allowing it to win.

Example 3: Finally, we look at a scenario we like to call the "LeBron James" effect, which is when the value of the help function is so large that tanking becomes an extremely enticing prospect. In real life, this would translate to having a once-in-a-lifetime player like LeBron James at the top of the prospect class; we mimic this using an exponential help function, $h(r_i^{(1)}) = \alpha \cdot e^{-r_i}$ where $r_i^{(1)}$ is the order statistic. If we have such an h, then regardless of the budgets, if we have α large enough, we see one of two results: no Nash equilibrium, or a singular Nash equilibrium with no tanking.

This is intuitive because if the value of the prospect is so high, then everyone, in theory, would like to tank. However, that is not a Nash equilibrium because if you believe everyone else is tanking, then you will not tank so you can win the first period. That said, it is not possible for just one team to tank, because then all teams who are not first will tank instead which removes the possibility of a Nash equilibrium (creating the "rock-paper-scissors" effect).

4.2. **Comparative Statics.** Let us fix a simple help function

$$h(r_i^{(1)}) = \alpha \cdot (n - r_i^{(1)})$$

That is, a player's help in the second period is proportional positive difference between the integer value of their order statistic and the total number of players. (For example, the worst-finishing player in the previous season receives $\alpha \cdot (n-1)$ help if there are no ties).

It is natural to ask how the Nash equilibria of an n-player game change as we increase our help parameter α. In particular, we define an equilibrium of the n-player coordination game already defined to be **strongly tanked** if at least one player, conditional on the strategies of the other players, strongly prefers his current strategy of tanking.

To provide an approximate answer to this question, we used simulations. First, we computed Nash equilibria for 100 games with the help function above and $\delta = .9$, where the team budgets Y_i were drawn independently from a χ_1^2 distribution. In figure 1, we plotted the (Laplace-smoothed) average of the fraction of stable equilibria in each game that were strongly tanked.

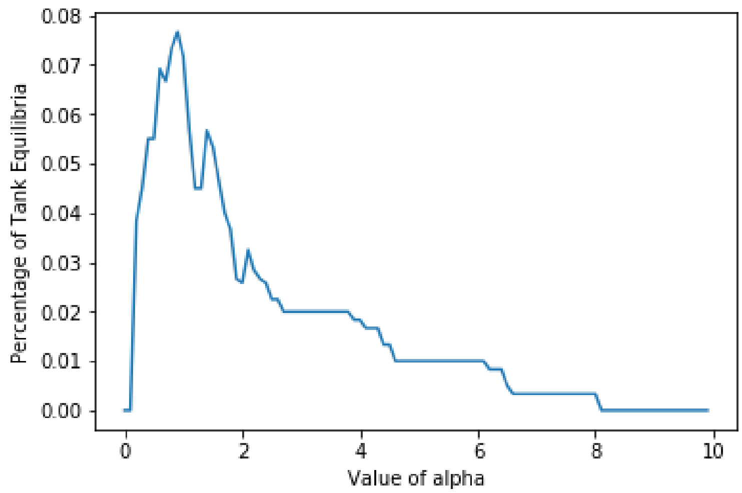

FIGURE 1. Averaged over 100 simulations with $n = 5$

We note a sharp increase in the proportion of tanking equilibria initially, but as the value of α increased we then saw a reduction in the number of equilibria with tanking being strongly preferred for at least one Player. This seems to follow from our intuition above, as the initial increase in tanking incentive causes Players to take advantage of the "catch up" effect and overtake the teams in front of them for period 2. However, as α becomes too large, the incentive for tanking is so strong that all teams consider doing so, which removes the possibility of a Nash equilibrium involving tanking (even the first team will tank).

The next thing we considered was how the proportion of strongly tanked equilibria increased as a function of δ.

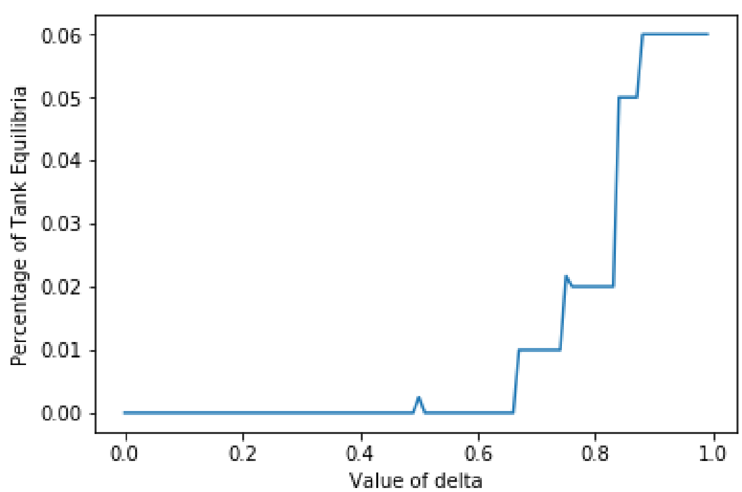

FIGURE 2. Averaged over 100 simulations with n=5

This result is intuitive - as teams increase their valuation of their future performance, they are more likely to tank to improve it.

In addition, our theoretical result in the first section is confirmed - we ran 1,000 simulations of two-player games with randomly drawn skills Y_i and varying help functions and there were no strongly preferred tanking equilibria.

5. Conclusion

Both through looking at two-player games and exploring the comparative statics of n-player games, it is not patently obvious why tanking exists and what system might be implemented to solve it. However, we have been able to show a number of results that bolster our understanding of this market design flaw.

We have shown through theoretical modeling that tanking does not exist in the two-player game, regardless of budgets and discount rates. On a similar note, we showed through simulation that strongly-preferred tanking does not exist with equal budgets in the n-player game. Through illustrative examples, we were able to explore the intuition behind certain phenomenons within the (somewhat arbitrarily chosen) five-player game. Such effects include: a higher discount rate's increasing the incidence of tanking; the "catching up" effect, whereby a team will tank only if it helps it win in the next period; and the presence of a once-in-a-lifetime prospect in that year's draft.

We then looked at a number of comparative statics assessing how the proportion of Nash equilibria that included at least one "tanking" team (a team that strongly prefers tanking) changed as other factors in the model changed.

6. Future work

There are 2 directions in which we are planning on continuing work in this area:

6.1. **Theory:** We have still not been able to test some ideas about improving the NBA because many models we tried to come up with were intractable. Two ideas we want to further explore are:

- We would like to describe the mixed Nash equilibria. It would also be nice to be able to explain comparative statics in mixed Nash equilibria, and be able to explain how changes in the pay-out function might change the structure of the Nash equilibria. There was no nice theoretical result about the set of pure Nash equilibria, but we expect more interesting results about mixed equilibria.
- We were unable to evaluate some policy changes we initially thought might reduce tanking. This happened mostly because we were unable to easily solve analytically for Nash equilibria in the infinite period game or m period game for general m. We would like to find a model that can easily generalizes to multiple periods and which admits easily solvable Nash equilibria.

6.2. **Experiments:** We are interested in extending our experiments to be able to solve for multiple period Nash equilibria for small number of teams (with the potential for help to be decayed over time so we better represent the length of careers). This would be an interesting extension because it would better help us understand year-to-year decision-making by teams (Players) and whether long-term tanking strategies, similar to what the Philadelphia 76ers have done, can exist in Nash equilibria.

We could also change the utility function. Using order statistics is convenient for tractability, but more realistic would be a function that favors the top teams and slopes up steeply to the top n teams, where the top n teams are championship contenders. Similarly, we could change the help function because in many sports, such as basketball, the top first few draft picks yield much higher average returns than the ones after.

To this end, we would use a logistic function $\frac{c}{1+\exp(-k(x-n_0))}$ centered at $n-n_0$, where n_0 are the number of teams that are considered championship contenders, x is the order statistic, and c and k are scaling factors. Moreover, we would use a help function of the form krx^{-n}, where x is the order statistic of the team, k' is another scaling factor, and r is the range of the budgets of the teams in question. We were able to run this experimentally and found that very frequently this model yields strongly tanked equilibria with the team in the $n_0 - 1$ having a strong preference for tanking.

Another avenue worth exploring would be using a machine learning model to simulate an n-team, n-period model. We have found it difficult to explore intuition or find results using Nash equilibria because they generally reduce the available solution space for any game; however, using machine learning with given strategy parameters could give us a better sense of how teams would actually perform over multiple periods.

Ultimately, these extensions, both theoretical and experimental, would allow us to explore the efficacy of possible solutions, which was difficult to do given the complexity of the game/model laid out in this paper. Possible solutions include linking teams' records together, using multi-year cumulative standings to determine the order, or even removing the incentives to tank altogether by squeezing the draft lottery probabilities together.