# Assignment 3
**AI1110**: Probability and Random Variables
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**12.13.5.1: Question**. A die is thrown 6 times. If 'getting an odd number' is a success, find the probability of

(i) 5 successes?
(ii) at least 5 successes?
(iii) at most 5 successes?

**Answer: (i)** $\frac{3}{32}$, **(ii)** $\frac{7}{64}$, **(iii)** $\frac{63}{64}$.
**Solution**:According to the question:

| n: | Number of throws | 6 |
|---|---|---|
| p: | Probability of getting an odd number | 0.5 |
| q: | Probability of getting an even number | 0.5 |

TABLE 3: Given Information

Let X: Number of times we get odd numbers in 6 throws of a die. Throwing a die and getting an odd ar an even is a bernoulli event. So, X has a binomial distribution.

$$\Pr(X = x) = {}^nC_x q^{n-x} p^x \qquad (1)$$

$$\Pr(X = x) = {}^nC_x(0.5)^{n-x}(0.5)^x = {}^nC_x(0.5)^{n-x+x} \quad (2)$$

$$= {}^6C_x(0.5)^6 \qquad (3)$$

(i) Probability 5 successes.
Putting x=5 in (3)

$$\Pr(X = 5) = {}^6C_5(0.5)^6 \qquad (4)$$

$$= 6 \times \frac{1}{64} = \frac{3}{32} \qquad (5)$$

(ii) Probability at least 5 successes.
Using (3):

$$\Pr(X \geq 5) = F_X(6) - F_X(4) \qquad (6)$$

$$= \Pr(X = 5) + \Pr(X = 6) \qquad (7)$$

$$= {}^6C_5(0.5)^6 + {}^6C_6(0.5)^6 \qquad (8)$$

$$= 6(0.5)^6 + (0.5)^6 = 7(0.5)^6 = \frac{7}{64} \qquad (9)$$

(iii) Probability at most 5 successes.
using (3)

$$\Pr(X \leq 5) = F_X(5) \qquad (10)$$

$$= 1 - \Pr(X = 6) \qquad (11)$$

$$= 1 - {}^6C_6(0.5)^6 = 1 - (0.5)^6 \qquad (12)$$

$$= 1 - \frac{1}{64} = \frac{63}{64} \qquad (13)$$