Assignment 3

AI1110: Probability and Random Variables
Indian Institute of Techonology Hyderabad

K S Ananth
CS22BTECH11029

12.13.5.1: Question. A die is thrown 6 times. If 'getting an odd number' is a success, find the probability of
(i) 5 successes?
(ii) at least 5 successes?
(iii) at most 5 successes?

Answer: (i) $\frac{3}{32}$, (ii) $\frac{7}{64}$, (iii) $\frac{63}{64}$.
Solution:According to the question:

n:	Number of throws	6
p:	Probability of getting an odd number	0.5
q:	Probability of getting an even number	0.5

TABLE 0: Given Information

Let X: Number of times we get odd numbers in 6 throws of a die. Throwing a die and getting an odd ar an even is a bernoulli event. So, X has a binomial distribution.

$$\Pr(X = x) = {}^{n}C_{x} q^{n-x} p^{x} \tag{1}$$

$$\Pr(X = x) = {}^{n}C_{x}(0.5)^{n-x}(0.5)^{x} = {}^{n}C_{x}(0.5)^{n-x+x} \tag{2}$$

$$= {}^{6}C_{x}(0.5)^{6} \tag{3}$$

(i) Probability 5 successes.
Putting x=5 in (3)

$$\Pr(X = 5) = {}^{6}C_{5}(0.5)^{6} \tag{4}$$

$$= 6 \times \frac{1}{64} = \frac{3}{32} \tag{5}$$

(ii) Probability at least 5 successes.
Using (3):

$$F_X(6) - F_X(4) = \Pr(X = 5) + \Pr(X = 6) \tag{6}$$

$$= {}^{6}C_{5}(0.5)^{6} + {}^{6}C_{6}(0.5)^{6} \tag{7}$$

$$= 6(0.5)^{6} + (0.5)^{6} = 7(0.5)^{6} = \frac{7}{64} \tag{8}$$

(iii) Probability at most 5 successes.
using (3)

$$F_X(5) = 1 - \Pr(X = 6) \tag{9}$$

$$= 1 - {}^{6}C_{6}(0.5)^{6} = 1 - (0.5)^{6} \tag{10}$$

$$= 1 - \frac{1}{64} = \frac{63}{64} \tag{11}$$