

Assignment 5

AI1110: Probability and Random Variables
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11.16.3.7: Question. A fair coin is tossed four times, and a person win Re 1 for each head and lose Rs 1.50 for each tail that turns up. From the sample space calculate how many different amounts of money you can have after four tosses and the probability of having each of these amounts.

Solution: According to the question:

Number of tosses	4
Profit after getting heads	Re 1
Loss after getting tails	Rs 1.50

TABLE 0: Given Information

Let X : Number of times we get heads when tossed for n times.
The amount of money the person will have after n tosses is:

$$(X \times 1) - ((n - X) \times 1.50) \quad (1)$$

The probability of getting a profit/loss obtained in (1) is:

$$\Pr(X = k) = P_X(k) = {}^nC_k(0.5)^k(0.5)^{n-k} \quad (2)$$

Now, for 4 tosses as given in the question:

Results	Profit	Loss	Total
4-H 0-T	4	0	4
3-H 1-T	3	1.5	1.5
2-H 2-T	2	3	-1
1-H 3-T	1	4.5	-3.5
0-H 4-T	0	6	-6

TABLE 0: Amount gained/lost

Therefore, the different amount of money = {4, 1.50, -1, -3.5, -6}.

Using (2) we find the probability of the different possibilities.

$$\Pr(X = 4) = P_X(4) = {}^4C_4(0.5)^4(0.5)^{4-4} = \frac{1}{16} \quad (3)$$

$$\Pr(X = 3) = P_X(3) = {}^4C_3(0.5)^3(0.5)^{4-3} = \frac{1}{4} \quad (4)$$

$$\Pr(X = 2) = P_X(2) = {}^4C_2(0.5)^2(0.5)^{4-2} = \frac{3}{8} \quad (5)$$

$$\Pr(X = 1) = P_X(1) = {}^4C_1(0.5)^1(0.5)^{4-1} = \frac{1}{4} \quad (6)$$

$$\Pr(X = 0) = P_X(0) = {}^4C_0(0.5)^0(0.5)^{4-0} = \frac{1}{16} \quad (7)$$