Assignment 5
AI1110: Probability and Random Variables
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11.16.3.7: Question. A fair coin is tossed four times, and a person win Re 1 for each head and lose Rs 1.50 for each tail that turns up.
From the sample space calculate how many different amounts of money you can have after four tosses and the probability of having each of these amounts.

Solution:According to the question:

Variable	Description	Value
n	Number of tosses	4
A	Amount gained/lost	A
p	Profit when it is heads	Re 1
q	Loss when it is tails	Rs 1.5
X	Number of heads in n tosses	X

TABLE 0: Given Information

The amount of money the person will have after n tosses is:

$$A = (X \times 1) - ((n - X) \times 1.50) \tag{1}$$

$$A = (2.5X) - (1.5n) \tag{2}$$

The probability of getting a profit/loss obtained in (**??**) is:

$$\Pr(X = k) = P_X(k) = {}^nC_k(0.5)^k(0.5)^{n-k} = {}^nC_k(0.5)^n \tag{3}$$

Let $F_X(k)$ denote the cumulative distribution function of X:

$$F_X(k) = \Pr(X \le k) = \sum_{i=0}^{i=k} {}^nC_i \left(\frac{1}{2}\right)^n \tag{4}$$

Let $F_A(k)$ denote the cumulative distribution function of A:

$$F_A(k) = \Pr(A \le k) \tag{5}$$
$$= \Pr(2.5X - 1.5n \le k) \tag{6}$$
$$= \Pr\left(X \le \frac{k + 1.5n}{2.5}\right) \tag{7}$$
$$= F_X\left(\frac{k + 1.5n}{2.5}\right) \tag{8}$$

By (**??**)

$$= \sum_{i=0}^{i=\lfloor \frac{k+1.5n}{2.5} \rfloor} {}^nC_i \left(\frac{1}{2}\right)^n \tag{9}$$

$$p_A(k) = \begin{cases} {}^nC_{\frac{k+1.5n}{2.5}} \left(\frac{1}{2}\right)^n, & \frac{k+1.5n}{2.5} \in I \text{ and } 0 \le \frac{k+1.5n}{2.5} \le n \\ 0, & \text{otherwise} \end{cases}$$
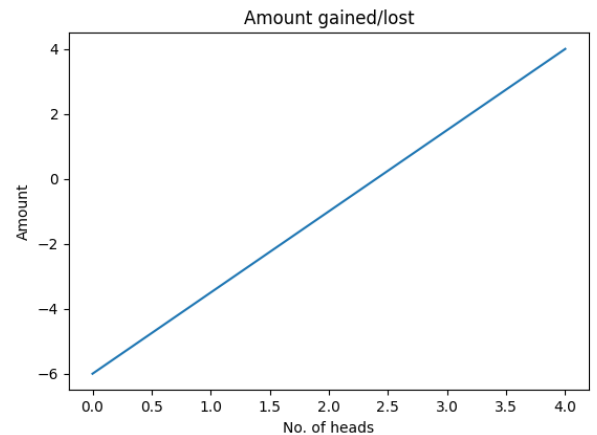

Fig. 0: Plot of amount gained/lost

Now, for 4 tosses as given in the question the different amount of money and its probability = $\{4, \frac{1}{16}\}, \{1.5, \frac{1}{4}\}, \{-1, \frac{3}{8}\}, \{-3.5, \frac{1}{4}\}, \{-6, \frac{1}{16}\}$.