

Assignment 3

AI1110: Probability and Random Variables
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12.13.5.1: Question. A die is thrown 6 times. If 'getting an odd number' is a success, find the probability of

- (i) 5 successes?
- (ii) at least 5 successes?
- (iii) at most 5 successes?

Answer: (i) $\frac{3}{32}$, (ii) $\frac{7}{64}$, (iii) $\frac{63}{64}$.

Solution: According to the question:

n	Number of throws	6
p	Probability of getting an odd number	0.5
q	Probability of getting an even number	0.5

TABLE 3: Given Information

Let X: Number of times we get odd numbers in 6 throws of a die. Throwing a die and getting an odd or an even is a bernoulli event. So, X has a binomial distribution.

$$\Pr(X = k) = P_X(k) = {}^nC_k q^{n-k} p^k \quad (1)$$

$$P_X(k) = {}^nC_k (0.5)^{n-k} (0.5)^k = {}^nC_k (0.5)^{n-k+k} \quad (2)$$

$$= {}^6C_k (0.5)^6 \quad (3)$$

- (i) Probability 5 successes.

Putting x=5 in (3)

$$P_X(5) = {}^6C_5 (0.5)^6 \quad (4)$$

$$= 6 \times \frac{1}{64} = \frac{3}{32} \quad (5)$$

- (ii) Probability at least 5 successes.

Using (3):

$$\Pr(X \geq 5) = F_X(6) - F_X(4) \quad (6)$$

$$= P_X(5) + P_X(6) \quad (7)$$

$$= {}^6C_5 (0.5)^6 + {}^6C_6 (0.5)^6 \quad (8)$$

$$= 6(0.5)^6 + (0.5)^6 = 7(0.5)^6 = \frac{7}{64} \quad (9)$$

- (iii) Probability at most 5 successes.
using (3)

$$\Pr(X \leq 5) = F_X(5) \quad (10)$$

$$= 1 - P_X(6) \quad (11)$$

$$= 1 - {}^6C_6 (0.5)^6 = 1 - (0.5)^6 \quad (12)$$

$$= 1 - \frac{1}{64} = \frac{63}{64} \quad (13)$$