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Assignment 5

AI1110: Probability and Random Variables Indian Institute of Technology Hyderabad

K S Ananth CS22BTECH11029

11.16.3.7: Question. A fair coin is tossed four times, and a person win Re 1 for each head and lose Rs 1.50 for each tail that turns up. From the sample space calculate how many different amounts of money you can have after four tosses

and the probability of having each of these amounts.

Solution: According to the question:

Variable	Description	Value
n	Number of tosses	4
p	Profit after getting heads	Re 1
q	Loss after getting tails	Rs 1.50
X	Number of heads in n tosses	X

TABLE 0: Given Information

The amount of money the person will have after n tosses is:

$$(X \times 1) - ((n - X) \times 1.50)$$
 (1)

The probability of getting a profit/loss obtained in (1) is:

$$\Pr(X = k) = P_X(k) = {}^{n}C_k(0.5)^k(0.5)^{n-k}$$
 (2)

Now, for 4 tosses as given in the question:

Results	Profit	Loss	Total	Probability
4-H 0-T	4	0	4	1 16
3-H 1-T	3	1.5	1.5	$\frac{1}{4}$
2-H 2-T	2	3	-1	3/8
1-H 3-T	1	4.5	-3.5	$\frac{1}{4}$
0-H 4-T	0	6	-6	1/6

TABLE 0: Amount gained/lost

Therefore, the different amount of money and its probability = $\{4, \frac{1}{16}\}$, $\{1.5, \frac{1}{4}\}$, $\{-1, \frac{3}{8}\}$, $\{-3.5, \frac{1}{4}\}$, $\{-6, \frac{1}{16}\}$.