

Assignment 4

AI1110: Probability and Random Variables
Indian Institute of Technology Hyderabad

K S Ananth
CS22BTECH11029

12.13.5.12: Question. Find the probability of throwing at most 2 sixes in 6 throws of a single die.

Answer: $\frac{35}{18}(\frac{5}{6})^4$.

Solution: According to the question:

n	Number of throws	6
p	Probability of getting 6	$\frac{1}{6}$
q	Probability of getting other than 6	$\frac{5}{6}$

TABLE 0: Given Information

Let X: Number of times we get number in 6 throws of a die. Throwing a die and getting 6 or a number other than 6 is a bernoulli event. So, X has a binomial distribution.

$$\Pr(X = k) = P_X(k) = {}^nC_k q^{n-k} p^k \quad (1)$$

$$P_X(k) = {}^6C_k \left(\frac{5}{6}\right)^{6-k} \left(\frac{1}{6}\right)^k \quad (2)$$

We are supposed to find probability of throwing at most 2 sixes i.e. $\Pr(X \leq 2)$.

$$\Pr(X \leq 2) = F_X(2) \quad (3)$$

$$= P_X(0) + P_X(1) + P_X(2) \quad (4)$$

Using (2) in (4):

$$= {}^6C_0 \left(\frac{5}{6}\right)^6 \left(\frac{1}{6}\right)^0 + {}^6C_1 \left(\frac{5}{6}\right)^5 \left(\frac{1}{6}\right)^1 + {}^6C_2 \left(\frac{5}{6}\right)^4 \left(\frac{1}{6}\right)^2 \quad (5)$$

$$= \left(\frac{5}{6}\right)^6 + \left(\frac{5}{6}\right)^5 + \frac{5}{12} \times \left(\frac{5}{6}\right)^4 \quad (6)$$

$$= \left(\frac{5}{6}\right)^4 \left(\frac{25}{36} + \frac{5}{6} + \frac{5}{12}\right) \quad (7)$$

$$= \left(\frac{5}{6}\right)^4 \left(\frac{70}{36}\right) \quad (8)$$

$$= \frac{35}{18} \left(\frac{5}{6}\right)^4 \quad (9)$$

So, the required probability is $\frac{35}{18}(\frac{5}{6})^4$.