

Assignment 3

AI1110: Probability and Random Variables
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12.13.5.1: Question. A die is thrown 6 times. If 'getting an odd number' is a success, find the probability of
(i) 5 successes?
(ii) at least 5 successes?
(iii) at most 5 successes?

Answer: (i) $\frac{3}{32}$, (ii) $\frac{7}{64}$, (iii) $\frac{63}{64}$.

Solution: According to the question:

n:	Number of throws	6
p:	Probability of getting an odd number	0.5
q:	Probability of getting an even number	0.5

TABLE 0: Given Information

Let X: Number of times we get odd numbers in 6 throws of a die. Throwing a die and getting an odd or an even is a bernoulli event. So, X has a binomial distribution.

$$\Pr(X = x) = {}^nC_x q^{n-x} p^x \quad (1)$$

$$\Pr(X = x) = {}^nC_x (0.5)^{n-x} (0.5)^x = {}^nC_x (0.5)^{n-x+x} \quad (2)$$

$$= {}^6C_x (0.5)^6 \quad (3)$$

(i) Probability 5 successes.

Putting x=5 in (3)

$$\Pr(X = 5) = {}^6C_5 (0.5)^6 \quad (4)$$

$$= 6 \times \frac{1}{64} = \frac{3}{32} \quad (5)$$

(ii) Probability at least 5 successes.

Using (3):

$$\Pr(X \geq 5) = \Pr(X = 5) + \Pr(X = 6) \quad (6)$$

$$= {}^6C_5 (0.5)^6 + {}^6C_6 (0.5)^6 \quad (7)$$

$$= 6(0.5)^6 + (0.5)^6 = 7(0.5)^6 = \frac{7}{64} \quad (8)$$

(iii) Probability at most 5 successes.
using (3)

$$\Pr(X \leq 5) = 1 - \Pr(X = 6) \quad (9)$$

$$= 1 - {}^6C_6 (0.5)^6 = 1 - (0.5)^6 \quad (10)$$

$$= 1 - \frac{1}{64} = \frac{63}{64} \quad (11)$$