COMPUTER AIDED ENGINEERING LAB [ME404]

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TOPICS

- Numerical Integration
- Symbolic Math
- Ordinary differential equation

Numerical Integration

- Trapz
  - trapz stands for **Trapezoidal Rule Integration**.
  - It **approximates the area under a curve** by dividing it into trapezoids and summing their areas.
  - Eg $\int_0^{\pi} \sin(x)\,.\,dx$

```
x = linspace(0,pi,1000);   % points from 0 to pi
y = sin(x);                % function values
Q = trapz(x,y)             % trapezoidal integration
```

- Diff
  - It is used for calculation of derivative.
  - To find **discrete differences** in data (trends, increments).
  - To check **changes across rows/columns** in a matrix.
  - Useful for **slopes, derivatives, changes in sequences**.
  - In numeric array it means Discrete difference.
  - In symbolic math It means derivative

- Eg
  - For numeric data

Differences of numbers

```
x = [2 5 9 15];
dx = diff(x)
```

For numeric data 2D difference

```
A = [1 2 4; 3 5 7; 6 9 12];
drow = diff(A,1,1)   % difference along rows
dcol = diff(A,1,2)   % difference along columns
```

  - In syms
  - Double Derivative of $f(x) = 3x^3 + 4x^2 + 9x + 10$

```
syms x
f = 3*x^3 + 4*x^2 + 9*x + 10;
f1 = diff(f, x)       % first derivative
f2 = diff(f, x, 2)    % second derivative
```

Diff(equation, indept var, order)

diff(f, x, 2)

- Integral2
  - integral2 is used to compute **double integrals** (integration over two variables).
  - It is MATLAB's numeric integrator (not symbolic).
  - General form Q = integral2(fun, xmin, xmax, ymin, ymax)
  - Eg

$$\int_0^1 \int_0^1 (x + y)\,dxdy$$

```
f = @(x,y) x + y;
I = integral2(f, 0, 1, 0, 1)
```

$$\int_{-2}^2 \int_{-2}^2 e^{-(x^2+y^2)}\,dxdy$$

```
f = @(x,y) exp(-(x.^2 + y.^2));
I = integral2(f, -2, 2, -2, 2)
```

SYMBOLIC MATH

- Symbolic Math in MATLAB allows you to represent numbers, variables, and expressions **exactly** (like algebra), instead of approximate floating-point numbers
- It is used for performing calculus operations (differentiation, integration, limits, solving equations) **analytically**, just like you do on paper
- Command use is sym or syms.
- You create symbolic variables (using syms or sym), then apply functions like diff, int, limit, solve, simplify, etc. to manipulate expressions.

- Factorize
  - Used to break a polynomial or expression into its **simplest factors**.
  - Helps in simplifying equations and solving roots easily.
  - Eg – Find factors of $x^2 - 9$.

```
syms x
f = x^2 - 9;
factor(f)
```

- Solve
  - Solves algebraic equations or systems of equations **symbolically**.
  - Returns **exact solutions** instead of numeric approximations.
  - Eg solve $x^2 - 4 = 0$

```
syms x
eqn = x^2 - 4 == 0;
solve(eqn, x)
```

- Expand
  - Expands factored or compact expressions into **full polynomial form**.
  - Useful for simplifying multiplication of polynomials.
  - Eg expand $(x + 2)^2$

```
syms x
f = (x + 2)^2;
expand(f)
```

- Linsolve
  - Solves systems of **linear equations** written in matrix form.
  - Preferred for efficiency when dealing with multiple equations.
  - Eg solve $2x + y = 7$; $x - y = 1$

```
A = [2 1; 1 -1];
B = [7; 1];
X = linsolve(A, B)
```

- Dsolve
  - Solves **ordinary differential equations (ODEs)** symbolically.
  - Gives the **general or particular solution** in exact form.
  - Eg solve $\frac{dy}{dx} = y$

```
syms y(x)
ode = diff(y, x) == y;   % dy/dx = y
dsolve(ode)
```

- Diff
  - Used to calculate **derivatives** of functions symbolically.
  - Can compute higher-order derivatives too.
  - Eg calculate derivative of $x^3 + 2x^2 + 5$

```
syms x
f = x^3 + 2*x^2 + 5;
diff(f, x)
```

- Int
  - Performs **integration** of functions symbolically.
  - Supports both **indefinite** and **definite** integrals.
  - Eg find $\int x^2\, dx$

```
syms x
f = x^2;
int(f, x)
```

Find the following integrals:

a) $\int_0^{\infty} e^{-x} dx$

b) $\int_0^{\infty} e^{-x^2} dx$

c) $\int_0^{\infty} e^{-x^2} x\sin(x) dx$

ORDINARY DIFFERENTIAL EQUATION

- A **differential equation** is an equation that involves a function and its derivatives (rates of change).
- It tells you **how something changes with respect to another variable** (usually time t).
- An **ordinary differential equation (ODE)** means the equation involves **only one independent variable** (like time t).
- They can be **Linear** or **Non-Linear**.

E.g. $\frac{dy}{dt} = -2y \longrightarrow$ Linear

$\frac{dy}{dt} = ry(1 - \frac{y}{K}) \longrightarrow$ Non-Linear

- ode23
  - A **basic solver** (low order).
  - Good if you only need **rough accuracy** or your equation has **jumps/discontinuities**.
  - Runs faster for simple problems, but not very precise.
  - E.g. $\frac{dy}{dt} = -2y$ ➡

```
f = @(t,y) -2*y;     % y' = -2y
[t,y] = ode23(f,[0 5],1);
plot(t,y)
title('ode23: simple decay')
```

- ode45
  - MATLAB’s **default solver**.
  - Balances **accuracy** and **speed**.
  - Works well for **most smooth problems** that are not stiff.
  - Usually the first solver you should try.
  - E.g. $\frac{dy}{dt} = y^2 - t, y(0) = 1$

```
f = @(t,y) y^2 - t;
y0 = 1;
tspan = [0 1];
[t, y] = ode45(f, tspan, y0);
plot(t, y, 'LineWidth', 2)
xlabel('t’) ; ylabel('y(t)’)
title('Simple Nonlinear ODE solved with ode45’) ; grid on
```

- ode15s
  - A solver made for **stiff problems** (when some parts of the solution change very fast, others very slow).
  - ode45 will get stuck (take tiny steps and run forever) on stiff problems, but ode15s handles them easily.
  - Uses more math inside (implicit method), but is much faster and stable on stiff equations.

```
f = @(t,y) -1000*y + 3000 - 2000*exp(-t);
y0 = 0;
tspan = [0 1];
[t, y] = ode15s(f, tspan, y0);
plot(t, y, 'LineWidth', 2)
xlabel('t')
ylabel('y(t)')
title('Stiff ODE solved with ode15s')
grid on
```

- Solve an ODE using ode45 and ode15s function. Consider the following equation $\frac{dy}{dt} = 2y + \frac{1}{t}$ with initial values: y(0.1) = 2.4. Consider 0.1 ≤ t ≤ 6. Obtain the solution using ode45 and ode15s functions of MATLAB. Plot the solution.

EXERCISE

1. Solve a system of ODE using ode45 function. Consider the following equation with initial values: y1(0) = 1, y2(0) = −1 and y3(0) = 0. Consider 0 ≤ t ≤ π/2. Obtain the solution using ode45 functions of MATLAB. Plot the solution.

$$\frac{dy_1}{dt} = 2y_1 + y_2 + 5y_3 + e^{-2t}$$

$$\frac{dy_2}{dt} = -3y_1 - 2y_2 - 8y_3 + 2e^{-2t}$$

$$\frac{dy_3}{dt} = 3y_1 + 3y_2 + 2y_3 + \cos(3t)$$

2. The motion of a damped spring-mass system is described by the following ordinary differential equation:

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = 0$$

where x =displacement from equilibrium position (m), t = time (s), m =20-kg mass, and c = the damping coefficient (N · s/m). The damping coefficient c takes on three values of 5 (under damped), 40 (critically damped) and 200 (over- damped). The spring constant k =20 N/m. The initial velocity is zero, and the initial displacement x = 1 m. Solve this equation over the time period 0≤t ≤15 s. Plot the displacement versus time for each of the three values of the damping coefficient on the same plot.

3. Define the following function symbolically $f\ (x)=(x2-4x)\ (x2-4x+1)-20.$

a) Expand $f\ (x)$ using expand.

b) Factorize $f(x)$ using the symbolic function factorize.

4. For a simply supported beam with load w, the deflection equation is given Solve for $y(x)$ symbolically.

$$EI\frac{d^4y}{dx^4}=w$$

5. For displacement $x(t)=e^{-0.1t}\,cos(2t)$ find velocity and acceleration using symbolic math.

6. Solve an ODE using $ode\ 23$ function. Consider the following equation with initial values: $\mathrm{T}(0)\ =200℃.$ Consider $0\ \leq\ t\ \leq 200$. Take $T_\infty=25℃, k=0.05$. Obtain the solution using $ode23$ function of MATLAB. Plot the solution

7. Solve an ODE using $ode45$ function. Consider the following equation with initial values: $y(0) = 0.5$. Consider $0 \leq t \leq 25$. Take $C = 0.517$ and $\tau = 3$. Obtain the solution using $ode45$ function of MATLAB. Plot the solution.

$$\frac{dy}{dt} = \frac{(c-y)}{\tau}$$

8. Solve the above question using $ode23, ode15$s and compare all results.

9. Given some tabulated values of the velocity of an object. Obtain an estimate of the distance travelled in the interval [0, 3]. Distance=$\int_0^3 V\, dt$

Time (s)	0	1	2	3
Velocity (m/s)	0	10	12	14