COMPUTER AIDED ENGINEERING LAB [ME404]

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Topics :

- Solving Linear Equations
- Solving Non-Linear Equations
- Curve Fitting
- Interpolation

Linear Equations

- MATLAB provides two direct ways to solve systems of linear algebraic equations. The most efficient way is to employ the backslash, or "left-division," operator as in

$$x = A\backslash b$$

- The second is to use matrix inversion:

$$x = inv(A) * b$$

- The matrix inverse solution is less efficient than using the backslash.
- E.g. Find the solution of following set of linear algebraic equations

$$x + 5y = 10;\ 3x - 8y = 5$$

- Solution: a=[1 5;3 -8];

  b=[10 5]';

  c=a\b

  Ans: $x = 4.5652\,,\ \ y = 1.0870$

Method 3 – Using Linsolve

$$x + 5y = 10;\ 3x - 8y = 5$$

Solution:

a=[1 5;3 -8];
b=[10 5]';
C = linsolve(a,b)

Ans: $x = 4.5652\,,\ \ y = 1.0870$

Non-Linear Equations

- A nonlinear equation is one where at least one term is not a constant times a variable to the power of 1.
- Examples : $x^2 + y^2 = 1$

  $e^x - 3x = 0$

  $\sin(x) + x = 0$
- Single-variable nonlinear equations

  Form: $f(x) = 0$
- E.g. Solve the non-linear equation $x^3 - 5x + 3 = 0$ *at* $x=1$
- Solution: $f = @(x)\ x^3 - 5x + 3;$

  $x = 1;$

  root = $fzero(f, x)$

  Ans: root=0.6566

- Multi-variable nonlinear equations

  Form: $f1(x1, x2, \ldots, x_n) = 0$

  $f2(x1, x2, \ldots, x_n) = 0$

- E.g. Solve the non-linear equation ${x_1}^2 + {x_2}^2 - 4 = 0;\ e^{x_1} + x_2 - 1 = 0$ at $x = [1,1]$
- Solution: $f = @(x)\ [\ x_1^2 + x_2^2 - 4; e^{x_1} + x_2 - 1];$

  $x = [1, 1];$

  sol= $fsolve(f, x)$

  Ans: $x_1 = -1.8163$, $x_2 = 0.8374$

- E.g. Solve the non-linear equation using symbolic approach $x^2 * y + x * y^2$
- Solution:

```
syms x y
expr = x^2 * y + x * y^2
f = factor(expr)

Ans:[ x * y,  x + y ]
```

Curve Fitting

- Curve fitting is the process of finding a curve (function) that best fits a set of data points.
- If the curve passes through all points → **Interpolation**
- If the curve approximates the trend → **Regression**
- **Types of Curve Fitting**
  - **Linear fit** → $y = ax + b$
  - **Polynomial fit** → $y = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0$
  - **Nonlinear fit** → $y = ae^{bx}, y = \text{asin}(bx)$
  - **Custom models** → Any user-defined equation.

• MATLAB Functions for Curve Fitting

Function	Use
polyfit(x,y,n)	Polynomial fitting, returns coefficients
polyval(p,x)	Evaluates the polynomial at given points
nlinfit	Nonlinear regression
interp1	Interpolation for 1D data

- E.g. Fit the 2^{nd} degree curve on given data x=0 to 5 with step size 0.5 and y=[1.0 1.6 2.5 3.5 5.0 7.4 9.0 12.0 14.5 18.0 22.0]
- Solution: $x = 0{:}\,0.5{:}\,5;$

$$y = [1.0\ 1.6\ 2.5\ 3.5\ 5.0\ 7.4\ 9.0\ 12.0\ 14.5\ 18.0\ 22.0];$$

$$p = polyfit(x, y, 2); \quad \%\ 2nd - degree\ polynomial$$

$$\mathrm{y_{fit}} = polyval(p, x);$$

$$plot(x, y, 'ro', x, y_{fit}, 'b-');$$

$$legend('Data', 'Fitted\ curve');$$

Interpolation

- Interpolation is the process of estimating values between known data points.
- It constructs a function that passes exactly through the given data points.
- Unlike curve fitting (regression), interpolation does not smooth or approximate — it
- matches the data exactly.
- MATLAB Functions for Interpolation
  - interp1(x, y, xq, method) — 1D interpolation
  - interp2 — 2D interpolation
  - interp3 — 3D interpolation
  - griddata — scattered data interpolation

- E.g. Interpolate the given data x=0 to 10 with step size 2 and $y = [0\ 1\ 4\ 9\ 16\ 25]$ and query points are xq=0 to 10 with step size 0.5
- Solution: $x = 0:2:10;$ % Known x

  $y = [0\ 1\ 4\ 9\ 16\ 25];$ % Known y

  $xq = 0:0.5:10;$ % Query points

```
y_linear = interp1(x, y, xq, 'linear’);
y_spline = interp1(x, y, xq, 'spline’);
plot(x, y, 'ro', xq, y_linear, 'b-', xq, y_spline, 'g--’);
legend('Data points', 'Linear', 'Spline');
```

Exercises

1. Find the solution of following set of linear algebraic equations

$$2x + y + 5z + w = 9$$
$$x + y - 3z - w = -5$$
$$3x + 6y - 2z + w = 8$$
$$2x + 2y + 2z - 3w = 3$$

2. A shopkeeper sells 3 pens, 2 pencils, 4 erasers for ₹30 and 2 pens, 3 pencils, 3 erasers for ₹26 and 4 pens, 1 pencil, 2 erasers for ₹28. Find cost of each system of linear algebraic equations.
3. Solve the equations: $e^x - 3x = 0$ at $x = 1$,

   $e^x - 2x - 3 = 0$ at $x = 1$,
4. Solve the system of non-linear equation $x^2 + y^2 = 25\,; x.y = 12$
5. Fit a straight line using polyfit and plot both the data and the fitted line.

   $x = [1,2,3,4,5]\,, y = [2.2,2.8,3.6,4.5,5.1]$
6. Fit a 2nd-degree polynomial using polyfit and find y for $x = 4.5$.

   $x = 0:1:6. v = [1.1.8.3.3.4.5.6.2.8.1.10.5]$

7. Find the value at $x = 5$ using linear interpolation.

   $x = [0,2,4,6,8,10], y = [0,4,8,14,16,20]$

8. Estimate y for $x = 2.5$ using spline interpolation and plot the result.

   $x = 0{:}\,2{:}\,12, y = [1,3,7,13,21,31,43]$