

2.16:

PB & J. Suppose 80% of people like peanut butter, 89% like jelly, and 78% like both.

Given that a randomly sampled person likes peanut butter, what's the probability that he also likes jelly?

Answer: 0.975 $P(PB) = 0.80$, $P(J) = 0.89$ and $P(J \text{ and } PB) = 0.78$, so $P(J|PB) = ?$

$$\begin{aligned}
 P(J|PB) &= P(J \text{ and } PB) / P(PB) = 0.78 \\
 &= 0.78/0.80 \\
 &= 0.975
 \end{aligned}$$

2.18:

Health coverage, relative frequencies. The Behavioral Risk Factor Surveillance System (BRFSS) is an annual telephone survey designed to identify risk factors in the adult population and report emerging health trends. The following table displays the distribution of health status of respondents to this survey (excellent, very good, good, fair, poor) conditional on whether or not they have health insurance.

| | | Health Status | | | | | Total |
|-----------------|-------|---------------|-----------|--------|--------|--------|--------|
| | | Excellent | Very good | Good | Fair | Poor | |
| Health Coverage | No | 0.0230 | 0.0364 | 0.0427 | 0.0192 | 0.0050 | 0.1262 |
| | Yes | 0.2099 | 0.3123 | 0.2410 | 0.0817 | 0.0289 | 0.8738 |
| | Total | 0.2329 | 0.3486 | 0.2838 | 0.1009 | 0.0338 | 1.0000 |

(a) Are being in excellent health and having health coverage mutually exclusive?

Answer: No, they are not, a healthy person can have health coverage too as $P(\text{Excellent Health and Have Health Coverage}) = 0.2099$

(b) What is the probability that a randomly chosen individual has excellent health?

Answer: 0.2329

$$P(\text{Excellent Health}) = 0.2329/1 = 0.2329$$

(c) What is the probability that a randomly chosen individual has excellent health given that he has health coverage?

Answer: 0.2402

$$P(\text{Excellent Health} | \text{Health Coverage}) = 0.2099/0.8738 = 0.2402$$

(d) What is the probability that a randomly chosen individual has excellent health given that he doesn't have health coverage?

Answer: 0.1822

$$\begin{aligned}
 P(\text{Excellent Health} | \text{No Health Coverage}) &= P(\text{Excellent Health and No Health Coverage}) / P(\text{No Health Coverage}) \\
 &= 0.0230 / 0.1262 \\
 &= 0.1822
 \end{aligned}$$

(e) Do having excellent health and having health coverage appear to be independent?

Answer: Statistically they are not. However very close, as shown below:

$$\begin{aligned}
 P(\text{Excellent Health and have Health Coverage}) &= 0.2099 \text{ and } P(\text{Have Health Coverage}) = 0.8738 \\
 &= 0.2099/0.8738 = 0.2402 \text{ which is not equal to } P(\text{Excellent Health}) = 0.2329
 \end{aligned}$$

2.20:

Assortative mating. Assortative mating is a nonrandom mating pattern where individuals with similar genotypes and/or phenotypes mate with one another more frequently than what would be expected under a random mating pattern. Researchers studying this topic collected data on eye colors of 204 Scandinavian men and their female partners. The table below summarizes the results. For simplicity, we only include heterosexual relationships in this exercise.

| | | <i>Partner (female)</i> | | | |
|--------------------|-------|-------------------------|-------|-------|-------|
| | | Blue | Brown | Green | Total |
| <i>Self (male)</i> | Blue | 78 | 23 | 13 | 114 |
| | Brown | 19 | 23 | 12 | 54 |
| | Green | 11 | 9 | 16 | 36 |
| | Total | 108 | 55 | 41 | 204 |

(a) What is the probability that a randomly chosen male respondent or his partner has blue eyes?

Answer: 0.7059

$P(\text{Female with Blue Eyes}) = 108/204$ $P(\text{Male with Blue Eye}) = 114/204$ $P(\text{M and F with Blue Eye}) = 78/204$
 $P(\text{Male or Female has Blue Eye}) = P(\text{Female with Blue Eyes}) + P(\text{Male with Blue Eye}) - P(\text{M and F with Blue Eye})$
 $= 108/204 + 114/204 - 78/204 = 0.7059$

(b) What is the probability that a randomly chosen male respondent with blue eyes has a partner with blue eyes?

Answer: 0.6842

$A = \text{Female with Blue Eye}$ $B = \text{Male with Blue Eye}$ $(A \text{ and } B) = \text{Both have Blue Eye}$
 $P(A|B) = P(A \text{ and } B) / P(B) = \frac{78/204}{114/204}$
 $= 78/114 = 0.6842$

(c) What is the probability that a randomly chosen male respondent with brown eyes has a partner with blue eyes?

Answer: 0.3519

$A = \text{Female with Blue Eye}$, $B = \text{Male with Brown Eyes}$, $(A \text{ and } B) = \text{Male with Brown and Female with Blue}$,
 $P(A) = 108/204$, $P(B) = 54/204$, $P(A \text{ and } B) = 19/204$
 $P(A|B) = P(A \text{ and } B) / P(B) = \frac{19/204}{54/204}$
 $= 19/54 = 0.3519$

What about the probability of a randomly chosen male respondent with green eyes having a partner with blue eyes?

Answer: 0.3056

DATA SCIENCE WEEK 2 ASSIGNMENT

A=Female with Blue Eye, B=Male with Green Eyes, (A and B)= Male with Green and Female with Blue ,
 $P(A) = 108/204$, $P(B)=36/204$, $P(A \text{ and } B) = 11/204$

$$P(A|B) = P(A \text{ and } B)/P(B) = \frac{11/204}{36/204}$$

$$= 11/36 = 0.3056$$

(d) Does it appear that the eye colors of male respondents and their partners are independent?
 Explain your reasoning.

Answer: Statistically they are not as shown below:

A= female has Blue Eyes , B = Male has blue eyes A and B= Both have Blue Eyes

$$P(A)= 108/204, P(B)= 114/204 \text{ and } P(A \text{ and } B)=78/204$$

$$P(A|B) = P(A \text{ and } B)/P(B) = 78/204/114/204 = 78/114 = 0.6842$$

$$P(A)=108/204 = 0.5294$$

So, $P(A|B)$ is not equal to $P(A)$, hence the eye colors of male respondent and their partners are independent.

2.26

Twins. About 30% of human twins are identical, and the rest are fraternal. Identical twins are necessarily the same sex { half are males and the other half are females. One-quarter of fraternal twins are both male, one-quarter both female, and one-half are mixes: one male, one female. You have just become a parent of twins and are told they are both girls. Given this information, what is the probability that they are identical?

Answer: 0.4615

Draw a tree diagram for this to have a better understanding of Identical and Fraternal babies(Boy and Girl) as:

| Twins | Identical | | Probability |
|-------|-----------|-----|------------------|
| | GG | 50% | $0.3*0.5= 0.15$ |
| 30% | BB | 50% | $0.3*0.5= 0.15$ |
| | | | |
| | GG | 25% | $0.25*0.7=0.175$ |
| 70% | BB | 25% | $0.25*0.7=0.175$ |
| | BG | 50% | $0.5*0.7=0.35$ |

$$P(\text{Identical}|GG) = P(\text{Identical and GG})/P(GG) = 0.15/(0.15+0.175) = 0.4615$$

