Submission for HW 1

CS 427: Mathematics for Data Science, Autumn 2020-21

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Solution 1

Let $x_1, x_2 \in C$ defined by $C = \{x \mid Ax = b\}$, where $A \in R^{m \times n}$, $b \in R^m$ and $\theta \in R$. Then
$A(\theta x_1 + (1-\theta)x_2) = \theta A x_1 + (1-\theta)Ax_2 = \theta b + (1-\theta)b = b$
$\Rightarrow \theta x_1 + (1-\theta)x_2 \in C$
So, C **is** an affine set.

Solution 2

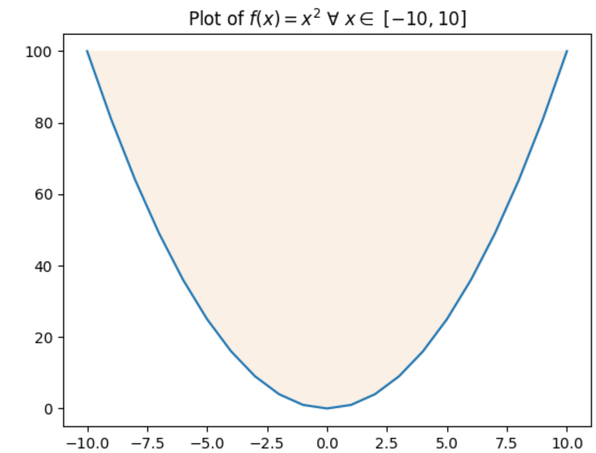
1. **aff** $C = \{x \in R^3 \mid x_3 = 0\}$
2. **conv** $C = C$
3. **int** $C = \phi$
4. **relint** $C = \{x \in R^3 \mid -1 < x_1 < 1, -1 < x_2 < 1, x_3 = 0\}$

Solution 3

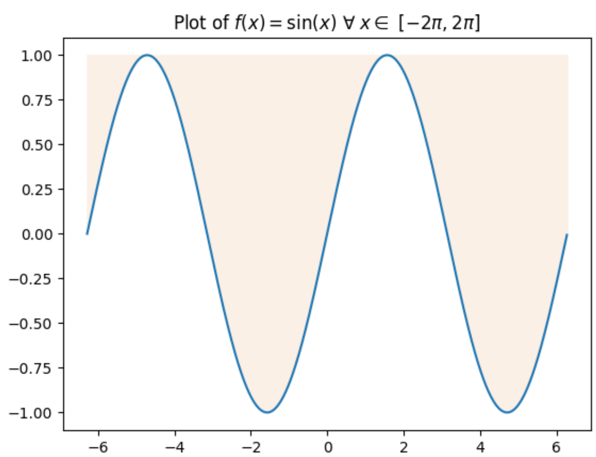
Let $P = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \in R^{2\times 2}$ and $c = \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}$, $x_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $x_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \in R^2$
Note that $c^T x_1 = 1$ and $c^T x_2 = 0$, $x_1^T P x_1 = 0$ and $x_2^T P x_2 = 0$. So, $x_1, x_2 \in C$ where $C = \{x \mid x^T P x \le (c^T x)^2, c^T x \ge 0\}\ \forall\ c, x \in R^2, P \in R^{2\times 2}$
Let $0 \le \theta \le 1$. Consider $\theta x_1 + (1 - \theta x_2) = \begin{pmatrix} 1 \\ 2\theta - 1 \end{pmatrix}$
Note that $c^T(\theta x_1 + (1-\theta)x_2) = \theta \in [0, 1]$
Also note that $(\theta x_1 + (1-\theta)x_2))^T P(\theta x_1 + (1-\theta)x_2) = 4\theta - 4\theta^2$.
However, for $\theta \in \left(0, \frac{4}{5}\right)$, $4\theta - 4\theta^2 > \theta^2$
This means
$(\theta x_1 + (1-\theta)x_2)^T P(\theta x_1 + (1-\theta)x_2) > (c^T(\theta x_1 + (1-\theta)x_2))^2\ \forall\ \theta \in \left(0, \frac{4}{5}\right)$.
$\Rightarrow (\theta x_1 + (1-\theta)x_2) \notin C\ \forall\ \theta \in \left(0, \frac{4}{5}\right)$.
So, C is **not** convex.

Solution 4

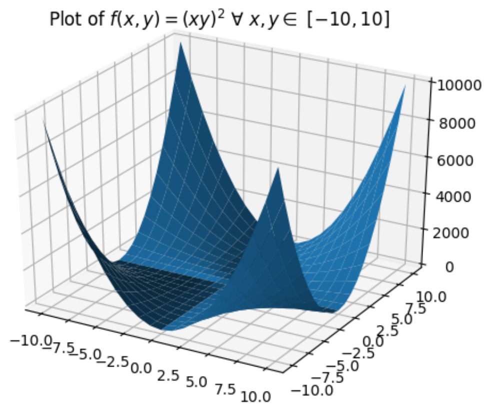
1. Plot of $f(x) = x^2 \ \forall x \in [-10, 10]$



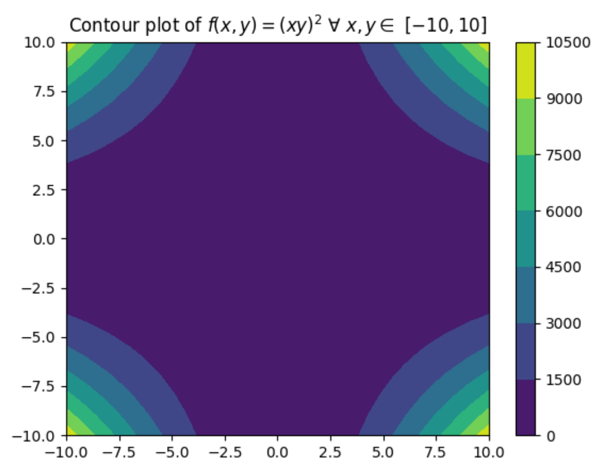
2. Plot of $f(x) = \sin(x) \ \forall x \in [-2\pi, 2\pi]$



3. Plot of $f(x, y) = (xy)^2 \ \forall x, y \in [-10, 10]$



4. Contour plot of $f(x, y) = (xy)^2 \ \forall x, y \in [-10, 10]$



```python
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits import mplot3d

def x2():
    x0 = np.array(range(-10, 11))
    y0 = eval('x0**2')
    plt.fill_between(x0, y0, 100, color='linen')
    plt.plot(x0, y0)
    plt.title(r'Plot of $f(x) = x^{2}\ \forall\ x \in\ [-10,10]$')
    plt.savefig('x2.png')
    plt.close()

def sinx():
    x1 = np.array(np.arange(-2*np.pi, 2*np.pi, 0.01))
    y1 = np.sin(x1)
    plt.fill_between(x1, y1, 1, color='linen')
    plt.plot(x1, y1)
    plt.title(r'Plot of $f(x) = \sin(x)\ \forall\ x \in\ [-2\pi,2\pi]$')
    plt.savefig('sinx.png')
    plt.close()

def xy2():
    x2 = np.array(range(-10, 11))
    y2 = np.array(range(-10, 11))
    X2, Y2 = np.meshgrid(x2, y2)
    Z2 = eval('(X2*Y2)**2')
    fig = plt.figure()
    ax = plt.axes(projection="3d")
    ax.plot_surface(X2,Y2,Z2)
    ax.set_title(r'Plot of $f(x,y) = (xy)^{2}\ \forall\ x,y \in\ [-10,10]$')
    plt.savefig('xy2.png')
    plt.close()

def xy2contour():
    x3 = np.array(range(-10, 11))
    y3 = np.array(range(-10, 11))
    X3, Y3 = np.meshgrid(x3, y3)
    Z3 = eval('(X3*Y3)**2')
    fig3,ax3=plt.subplots(1,1)
    cp = ax3.contourf(X3,Y3,Z3)
    fig3.colorbar(cp)
    ax3.set_title(r'Contour plot of $f(x,y) = (xy)^{2}\ \forall\ x,y \in\ [-10,10]$')
    plt.savefig('xy2contour.png')
    plt.close()

x2()
sinx()
xy2()
xy2contour()
```

Python code to generate the plots in **Solution 4**

Scan this QR code to access the GitHub repository of my homweork solutions at https://github.com/ksanu1998/MDS_HW_Solutions