Mathematics for Data Science: CS427 **(Due: 15/10/20 05:00 AM IST)**

Homework 3-HW3

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Problem 1

Prove that a function $f : \mathbb{R}^n \to \mathbb{R}$ is convex if and only if its hessian is positive semi-definite

Problem 2

When is the epigraph of a function a halfspace? When is the epigraph of a function a convex cone? When is the epigraph of a function a polyhedron?

Problem 3

A function $f : \mathbb{R}^n \to \mathbb{R}^n$ is monotone if $\forall x, y \in \text{dom}(f)$

$$(f(x) - f(y))^T (x - y) \geq 0$$

Show that gradient ∇f of f is monotone. Is every monotone mapping a gradient of some convex function?

Problem 4

Suppose $f : \mathbb{R}^n \to \mathbb{R}$ is convex, $g : \mathbb{R}^n \to \mathbb{R}$ is concave, $\text{dom}\, f = \text{dom}\, g = \mathbb{R}^n$, and for all $x, g(x) \leq f(x)$. Show that there exists an affine function h such that for all x, $g(x) \leq h(x) \leq f(x)$. In other words, if a concave function g is an under-estimator of a convex function f, then we can fit an affine function between f and g.

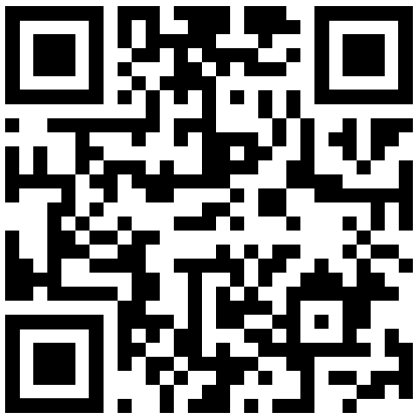
Programming Assignment

Using any programming language of your choice (note that all axis labels and contour legends should be clearly marked)

1. Plot a 3-D graph and a contour map of $f(x, y) = x^2 - y^2 \; \forall x, y \in [-5, 5]$.
2. Randomly generate a set of 24 points that belong to the set $\{(x, y) : \forall x, y \in [-5, -5]\}$. Create a scatter plot and outline the convex hull of the set you just created. External libraries are allowed to be used.
3. Check if the function $f(x) = x^T Ax$ for $A \in \mathbb{R}^{2\times 2}$ where all components of x are integers in $[-10, 10]$, is convex. Find 11 counter examples if it is not.
   ONLY list the values of A and x that you claim are counter examples and also provide the code snippet you used to find them.

Submission Link

Hints to solve the problems will be given after the submission date. Further queries are encouraged to be asked during class.



`https://forms.gle/pMbbBfYarn9Fu4iR9`
Submission format: `HW3_<Roll Number>.pdf`
Please ensure correct naming format. Incorrect formats will be ignored