

Submission for HW 3 (Programming Assignment)

CS 427: Mathematics for Data Science, Autumn 2020-21

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1 Question 1

Plot a 3D graph and a contour map of $f(x, y) = x^2 - y^2 \forall x, y \in [-5, 5]$

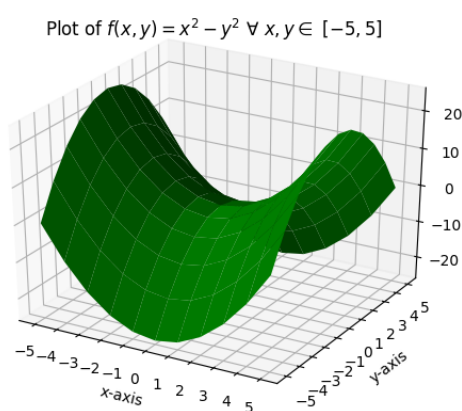


Figure 1: 3D graph

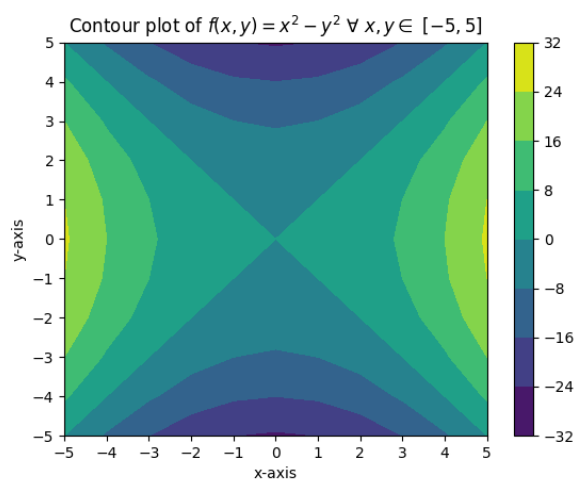


Figure 2: Contour map

2 Question 2

Randomly generate a set of 24 points that belong to the set $\{(x, y) : x, y \in [-5, 5]\}$. Create a scatter plot and outline the convex hull of the set you just created. External libraries are allowed to be used.

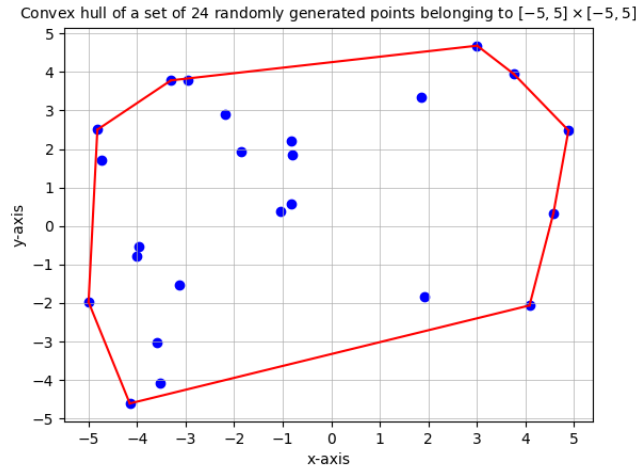


Figure 3: Scatter plot and convex hull

3 Question 3

Check if the function $f(x) = x^T A x$ for $A \in R^{2 \times 2}$ where all components of x are integers in $[-10, 10]$, is convex. Find 11 counter examples if it is not.

I take a random vector x where $x_{ij} \in [-10, 10]$ for $i, j = \{1, 2\}$. I then rotate and scale it using the matrix $A := \begin{pmatrix} \cos \theta & \sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$. Then $f(x) = x^T A x = (x^2 + y^2) \cos \theta + 2xy \sin \theta$ gives the dot product of the new vector with the original vector. There is a result which states that $f(x) = x^T A x$ is convex if and only if A is positive semidefinite. By the way I defined the matrix A , it is symmetric. However, it can be analysed that for some values of θ , f takes negative values. So I provide 11 such values of A for which f is not convex. For better visualisation, I plot the rotated and scaled vector for every value of A . Also, I plot the corresponding surfaces of f .

Here are the counter-examples in terms of matrix A :

$\begin{pmatrix} 0.27 & 0.96 \\ 0.96 & 0.27 \end{pmatrix}$
 $\begin{pmatrix} 0.17 & 0.99 \\ 0.99 & 0.17 \end{pmatrix}$
 $\begin{pmatrix} 0.07 & 1.0 \\ 1.0 & 0.07 \end{pmatrix}$
 $\begin{pmatrix} -0.03 & 1.0 \\ 1.0 & -0.03 \end{pmatrix}$
 $\begin{pmatrix} -0.13 & 0.99 \\ 0.99 & -0.13 \end{pmatrix}$
 $\begin{pmatrix} -0.23 & 0.97 \\ 0.97 & -0.23 \end{pmatrix}$
 $\begin{pmatrix} -0.32 & 0.95 \\ 0.95 & -0.32 \end{pmatrix}$
 $\begin{pmatrix} -0.42 & 0.91 \\ 0.91 & -0.42 \end{pmatrix}$
 $\begin{pmatrix} -0.5 & 0.86 \\ 0.86 & -0.5 \end{pmatrix}$
 $\begin{pmatrix} -0.59 & 0.81 \\ -0.59 & 0.81 \end{pmatrix}$
 $\begin{pmatrix} -0.67 & 0.75 \\ 0.75 & -0.67 \end{pmatrix}$
Clearly as seen in the below plots, f loses its convexity as we vary the value of θ from 0 to π

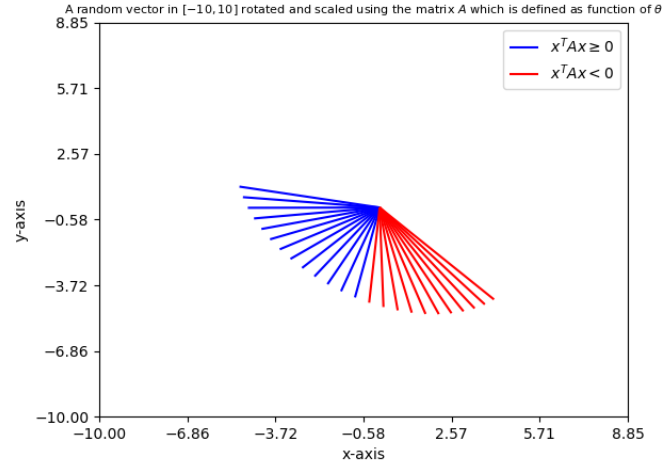


Figure 4: Rotated and scaled vectors (Ax) for different matrices A

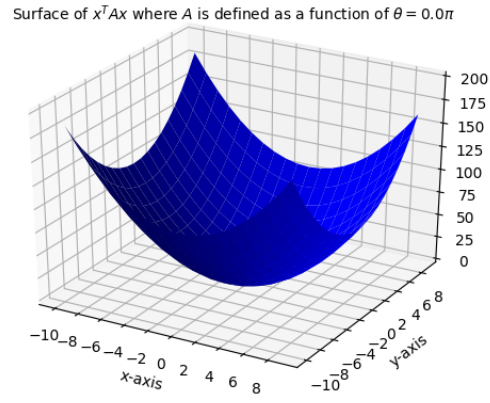


Figure 5: Surface of f for $\theta = 0$

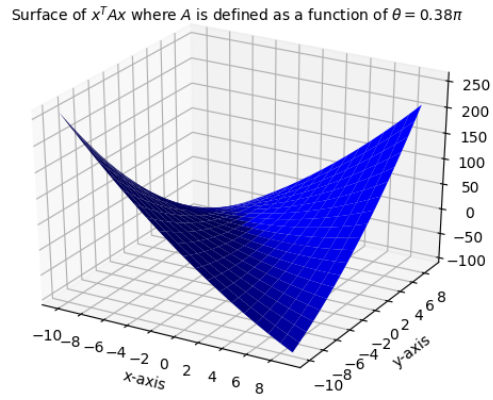


Figure 6: Surface of f for $\theta = 0.38\pi$

Surface of $x^T A x$ where A is defined as a function of $\theta = 0.64\pi$

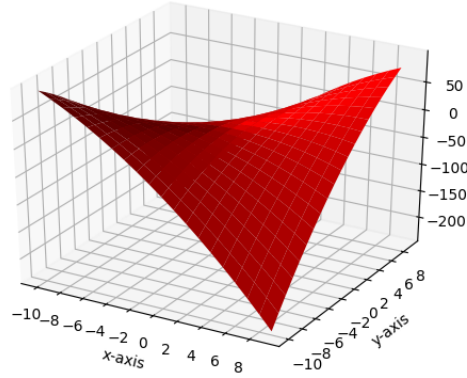


Figure 7: Surface of f for $\theta = 0.64\pi$

Surface of $x^T A x$ where A is defined as a function of $\theta = 0.73\pi$

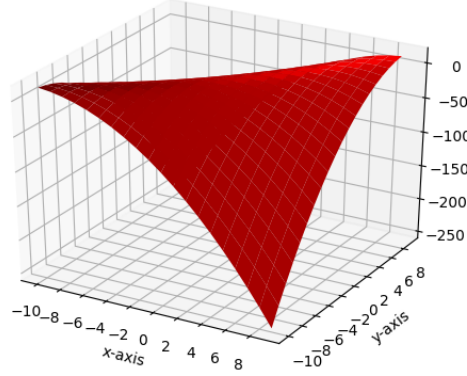
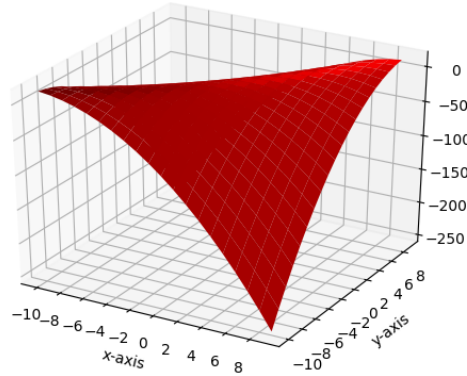


Figure 8: Surface of f for $\theta = 0.73\pi$

Surface of $x^T A x$ where A is defined as a function of $\theta = 0.73\pi$



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https://github.com/ksanu1998/MDS_HW_Solutions