Submission for HW 3 (Programming Assignment)

CS 427: Mathematics for Data Science, Autumn 2020-21

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1 Question 1

Plot a 3D graph and a contour map of $f(x, y) = x^2 - y^2 \ \forall \ x, y \in [-5, 5]$

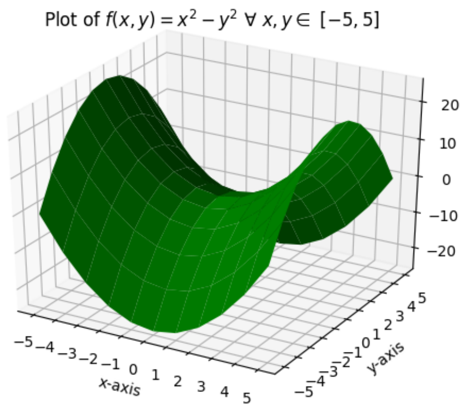

Figure 1: 3D graph

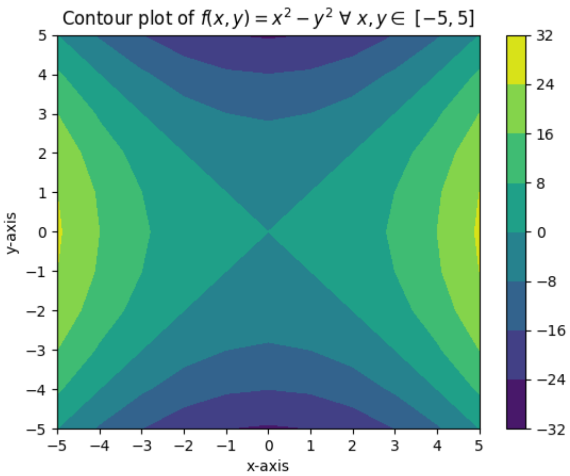

Figure 2: Contour map

2 Question 2

Randomly generate a set of 24 points that belong to the set $\{(x, y) : x, y \in [-5, 5]$. Create a scatter plot and outline the convex hull of the set you just created.

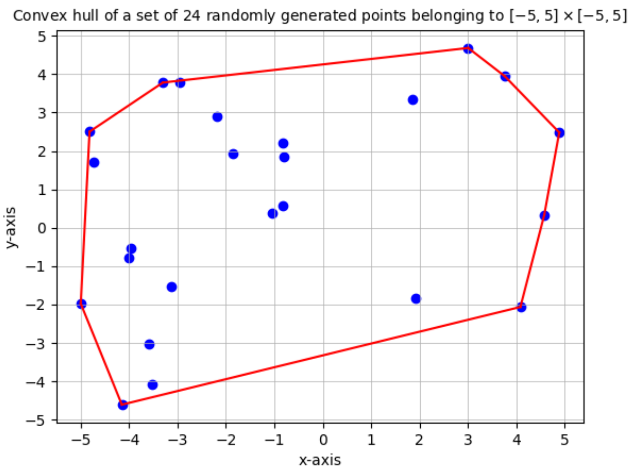

Figure 3: Scatter plot and convex hull

3 Question 3

Check if the function $f(x) = x^T Ax$ for $A \in R^{2\times2}$ where all components of x are integers in $[-10, 10]$, is convex. Find 11 counter examples if it is not.

I take a random vector x where $x_{ij} \in [-10, 10]$ for $i, j = \{1, 2\}$. I then rotate and scale it using the matrix $A := \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$. Here $f(x) = x^T Ax = (x^2 + y^2)\cos\theta + 2xy\sin\theta$. There is a result which states that $f(x) = x^T Ax$ is convex if and only if A is positive semidefinite. In the manner I defined the matrix A, it is symmetric. However, it can be analysed that for some values of θ, f takes negative values. So I provide 11 such values of A for which f is not convex. For better visualisation, I plotted the rotated and scaled vector Ax for some values of A. I also plotted the corresponding surfaces of f.
[1]Here are the counter-examples in terms of the matrix A:
$\begin{pmatrix} 0.27 & 0.96 \\ 0.96 & 0.27 \end{pmatrix} \begin{pmatrix} 0.17 & 0.99 \\ 0.99 & 0.17 \end{pmatrix} \begin{pmatrix} 0.07 & 1.0 \\ 1.0 & 0.07 \end{pmatrix} \begin{pmatrix} -0.03 & 1.0 \\ 1.0 & -0.03 \end{pmatrix} \begin{pmatrix} -0.13 & 0.99 \\ 0.99 & -0.13 \end{pmatrix} \begin{pmatrix} -0.23 & 0.97 \\ 0.97 & -0.23 \end{pmatrix} \begin{pmatrix} -0.32 & 0.95 \\ 0.95 & -0.32 \end{pmatrix}$
$\begin{pmatrix} -0.42 & 0.91 \\ 0.91 & -0.42 \end{pmatrix} \begin{pmatrix} -0.5 & 0.86 \\ 0.86 & -0.5 \end{pmatrix} \begin{pmatrix} -0.59 & 0.81 \\ -0.59 & 0.81 \end{pmatrix} \begin{pmatrix} -0.67 & 0.75 \\ 0.75 & -0.67 \end{pmatrix}$
Clearly as seen in the below plots, f loses its convexity as we vary the value of θ from 0 to π.

[1]Please refer to `plots.py` in my GitHub repository at https://github.com/ksanu1998/MDS_HW_Solutions to view the code used to generate counter-examples and plots in this assignment.

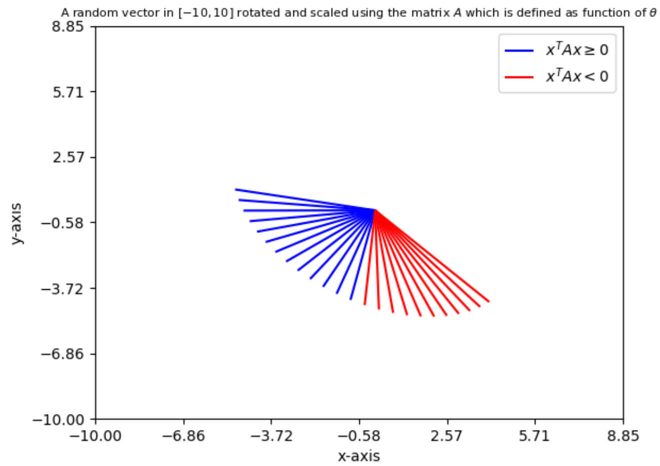

Figure 4: Rotated and scaled vectors Ax for different matrices A

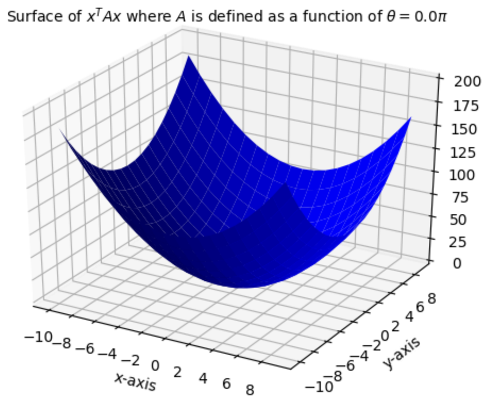

Figure 5: Surface of f for $\theta = 0$

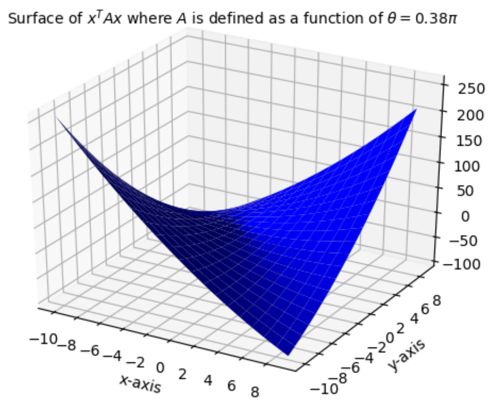

Figure 6: Surface of f for $\theta = 0.38\pi$

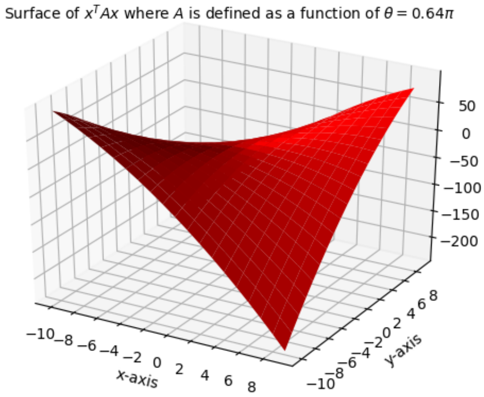

Figure 7: Surface of f for $\theta = 0.64\pi$

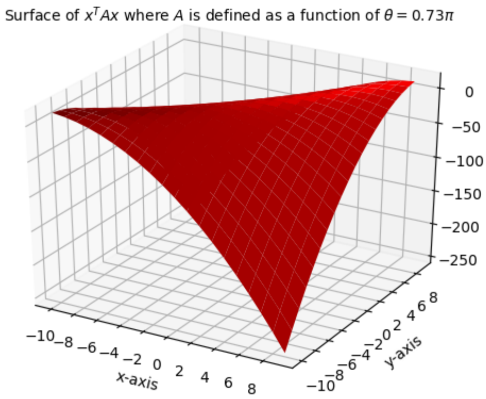

Figure 8: Surface of f for $\theta = 0.73\pi$

Scan this QR code to access the GitHub repository of my homweork solutions at
https://github.com/ksanu1998/MDS_HW_Solutions