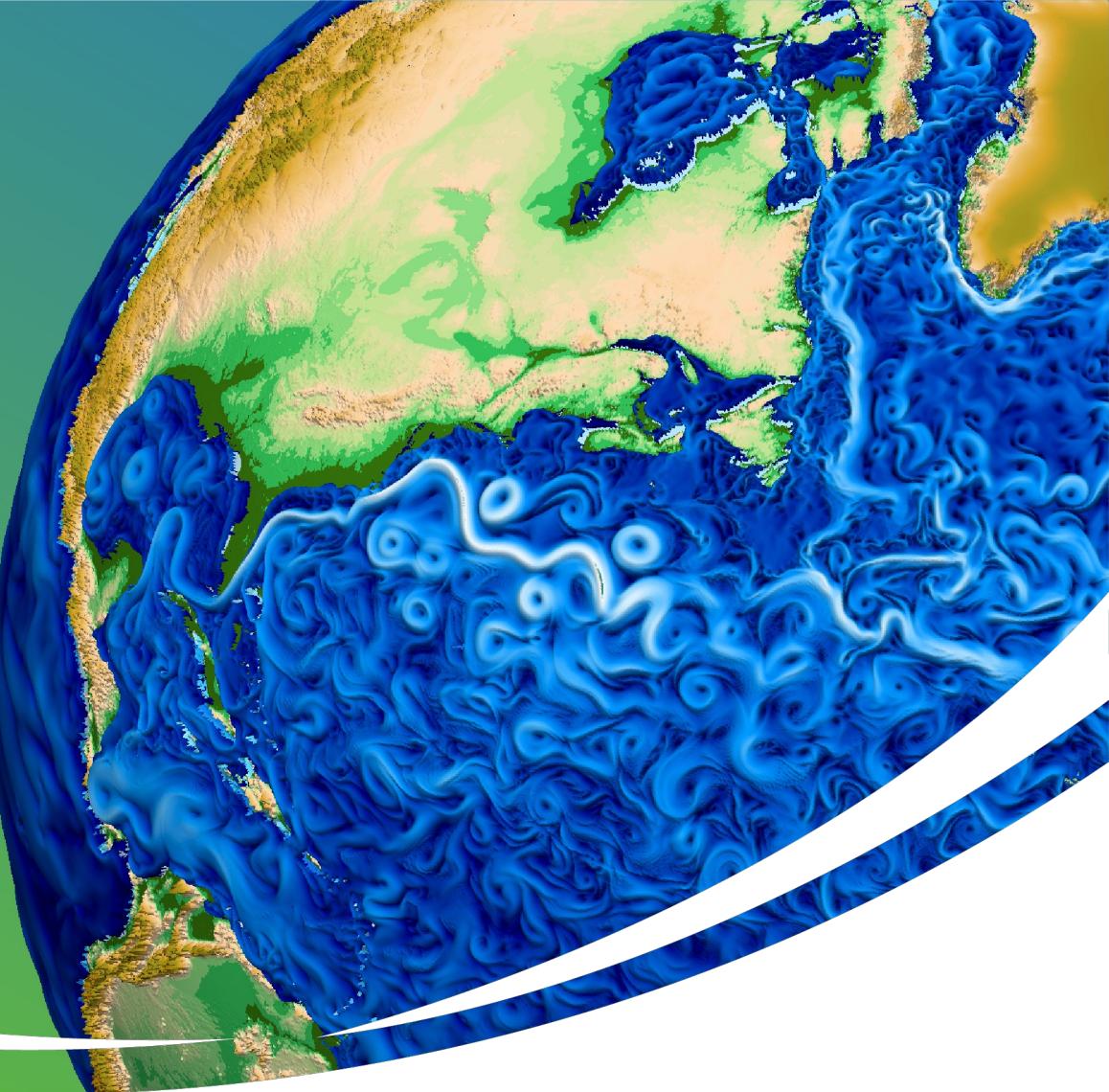


# Reduced-Dimensional Neural Network Surrogate Construction and Calibration of the E3SM Land Model

Khachik Sargsyan (SNL), Daniel Ricciuto (ORNL)



Sandia  
National  
Laboratories

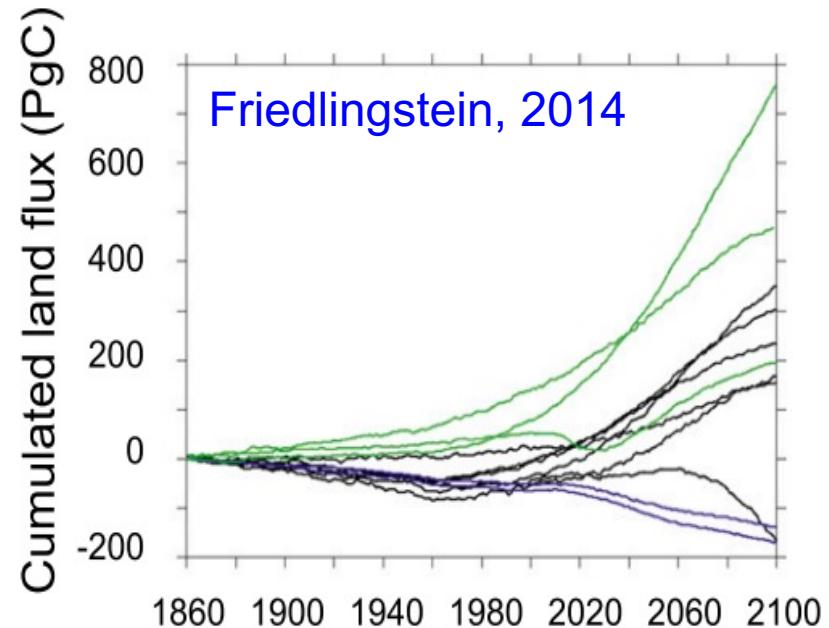


AGU Fall Meeting  
San Francisco  
Dec. 11, 2023

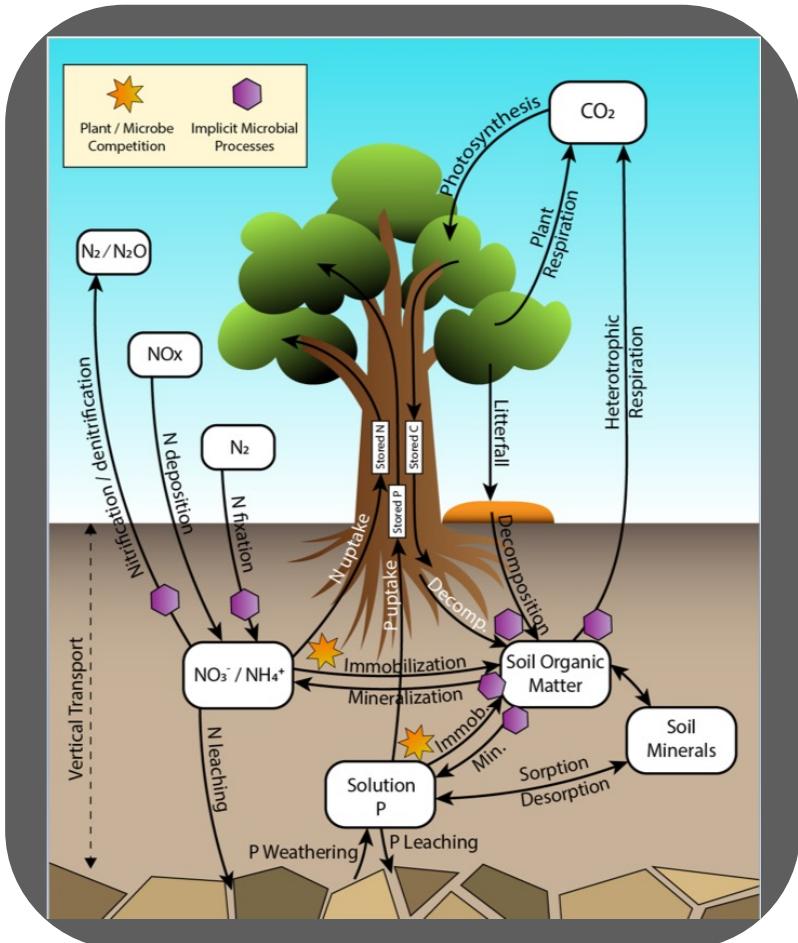


# Motivation and Overview

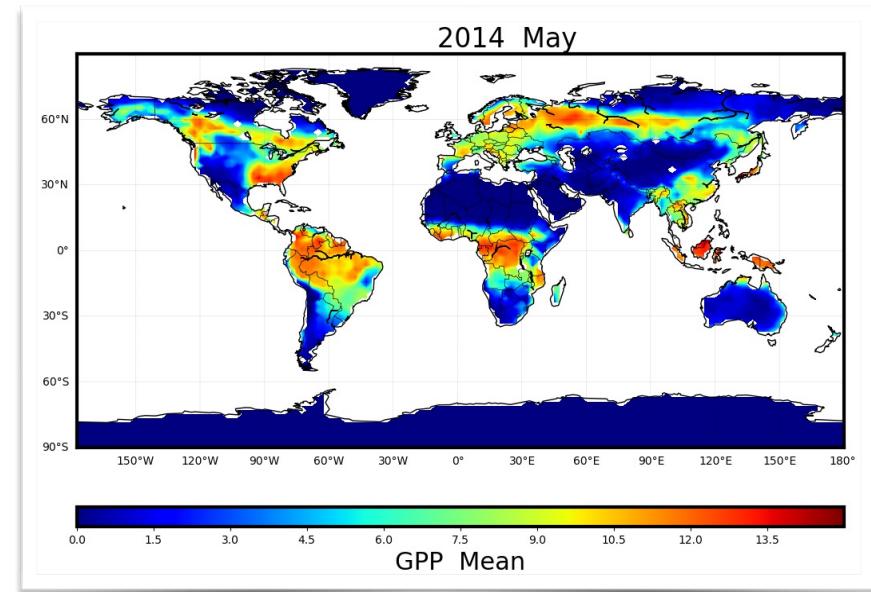
- Need for **model surrogates** for sample-intensive studies, such as ...
  - Global sensitivity analysis (forward UQ)
  - Model calibration (inverse UQ)
- Major **challenges**
  - Expensive model evaluation, small ensembles
  - High dimensional (spatio-temporal) outputs
- Reduced-dimensional, inexpensive surrogate construction via Karhunen-Loève expansions and Neural Networks (KLNN)
- Surrogate enables global sensitivity analysis and Bayesian model calibration



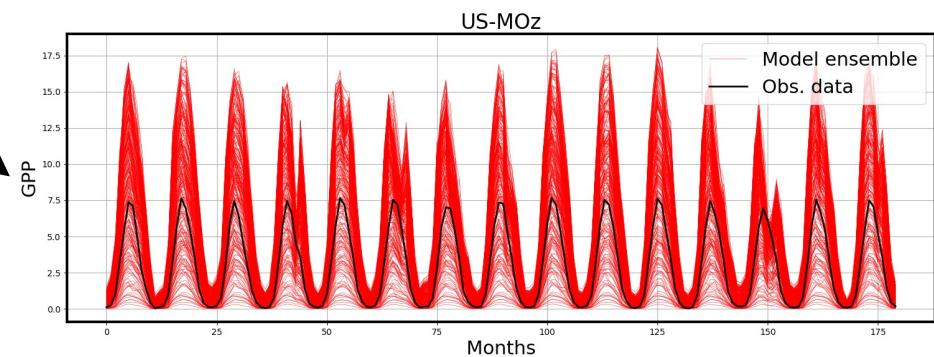
# E3SM Land Model (ELM): focus on carbon and energy cycle



Quantity of Interest:  
Gross primary  
productivity (GPP)...



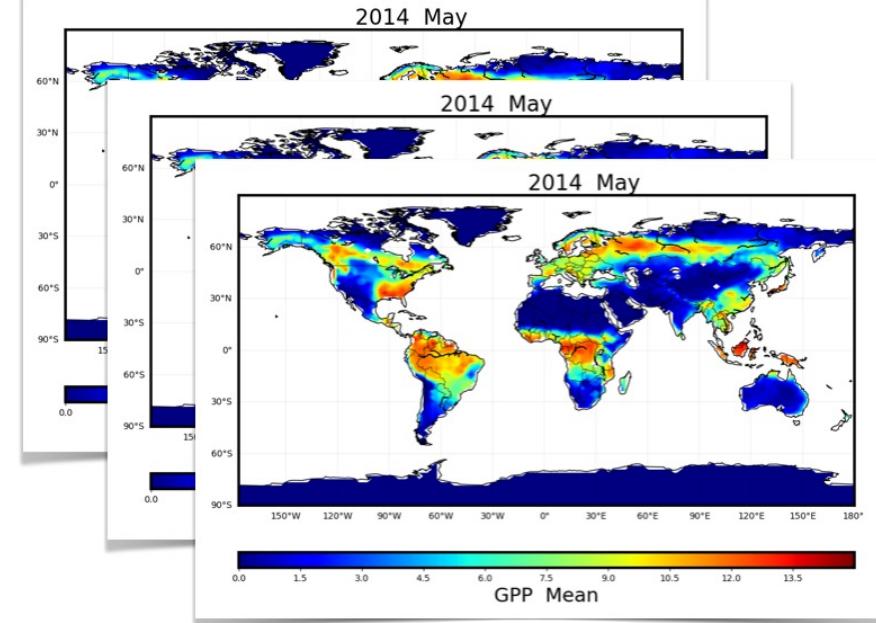
... resolved in space, ...



... and in time.

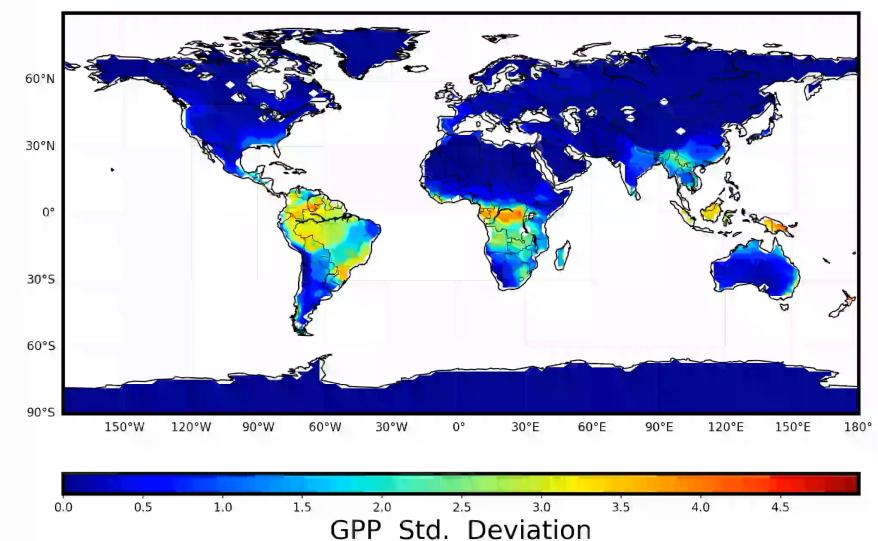
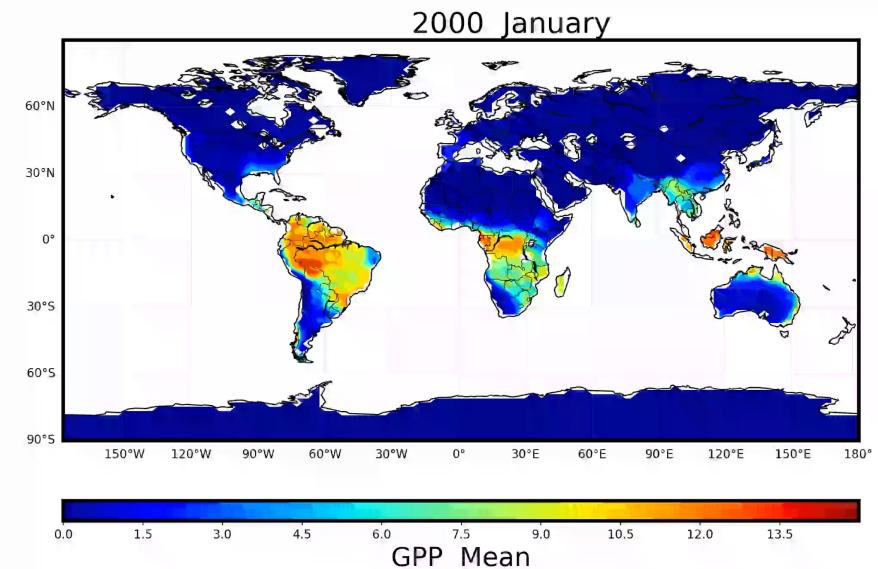
## Model Ensemble (275 samples)

1.9x2.5 resolution, satellite phenology



## Perturbed Parameters

Parameter	Description	Min	Max
flnr	Fraction of leaf in RuBisCO	0	0.25
mbbopt	Stomatal slope (Ball-Berry)	2	13
bbbopt	Stomatal intercept (Ball-Berry)	1000	40000
roota_par	Rooting depth distribution	1	10
vcmaxha	Activation energy for Vcmax	50000	90000
vcmaxse	Entropy for Vcmax	640	700
jmaxha	Activation energy for jmax	50000	90000
dayl_scaling	Day length factor	0	2.5
dleaf	Characteristic leaf dimension	0.01	0.1
xl	Leaf/stem orientation index	-0.6	0.8

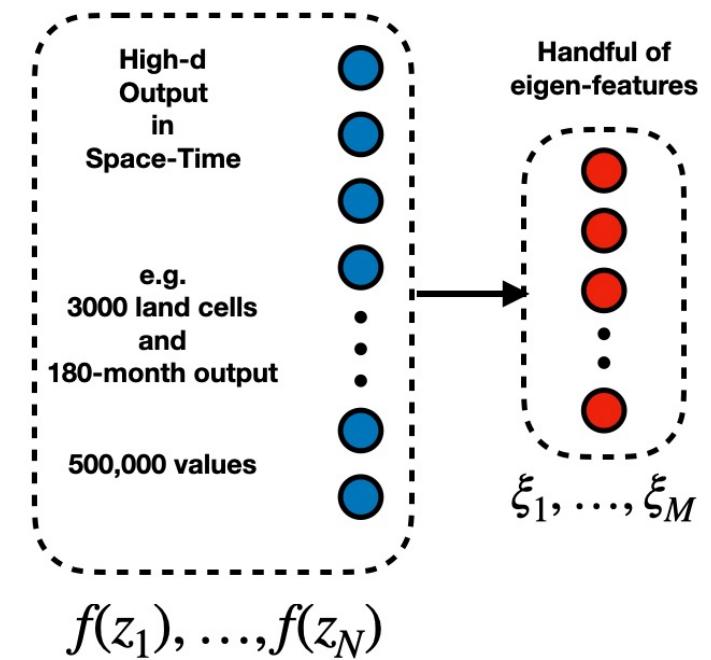


# Dimensionality Reduction via Karhunen-Loève Expansion

$$f(\lambda; z) \approx \bar{f}(z) + \sum_{m=1}^M \xi_m(\lambda) \sqrt{\mu_m} \phi_m(z)$$

Uncertain parameters      "Certain" conditions

- Spatio-temporal model output  $f(\lambda; z)$ , where  $z = (x, y, t)$
- Output field has large dimensionally  $N = N_x \times N_y \times N_t$
- Eigenpairs  $(\mu_m, \phi_m(z))$  are found via eigen-solve
- Analysis reduces to  $M \ll N$  eigenfeatures  $\xi_1, \dots, \xi_M$
- Under the hood: this is essentially an SVD

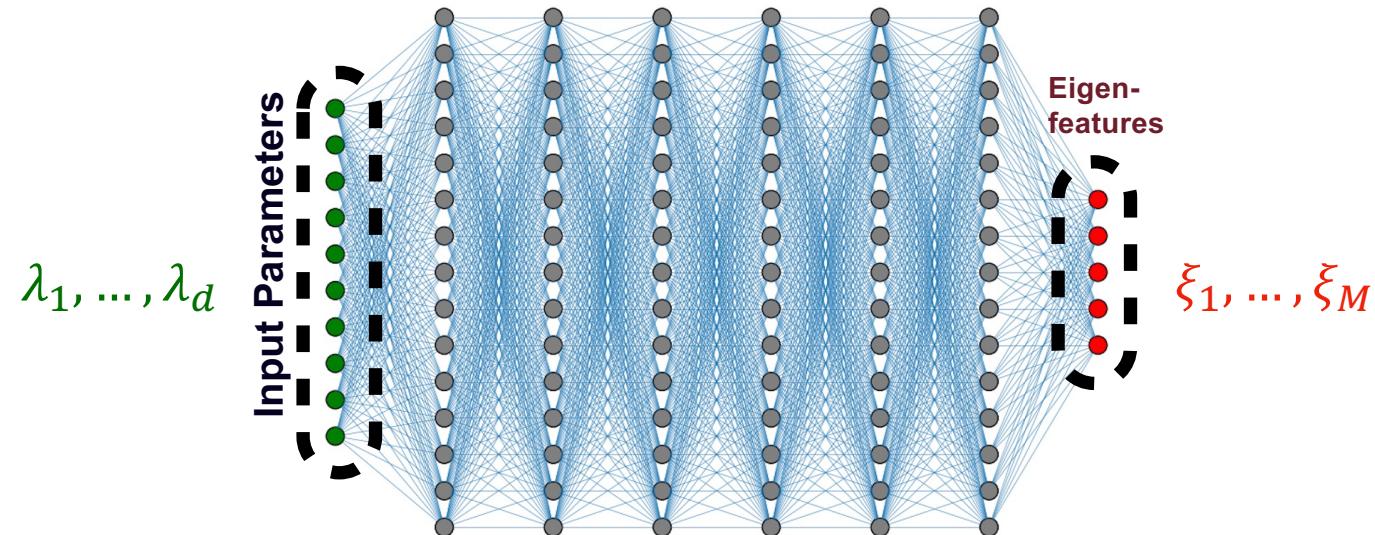




# KL+NN = reduced dimensional spatio-temporal surrogate

The goal is to construct a surrogate with respect to uncertain parameters  $\lambda$ , such that  
 $f(\lambda; z_i) \approx f_s(\lambda; z_i)$  for all conditions  $z_i$ .

Instead of building surrogate for each individual  $z_i$  for  $i = 1, \dots, N$ ,  
we construct neural network (NN) surrogate for  $\xi_1, \dots, \xi_M$  where  $M \ll N$ .



$$f(\lambda; z) \approx \bar{f}(z) + \sum_{m=1}^M \xi_m(\lambda) \sqrt{\mu_m} \phi_m(z)$$

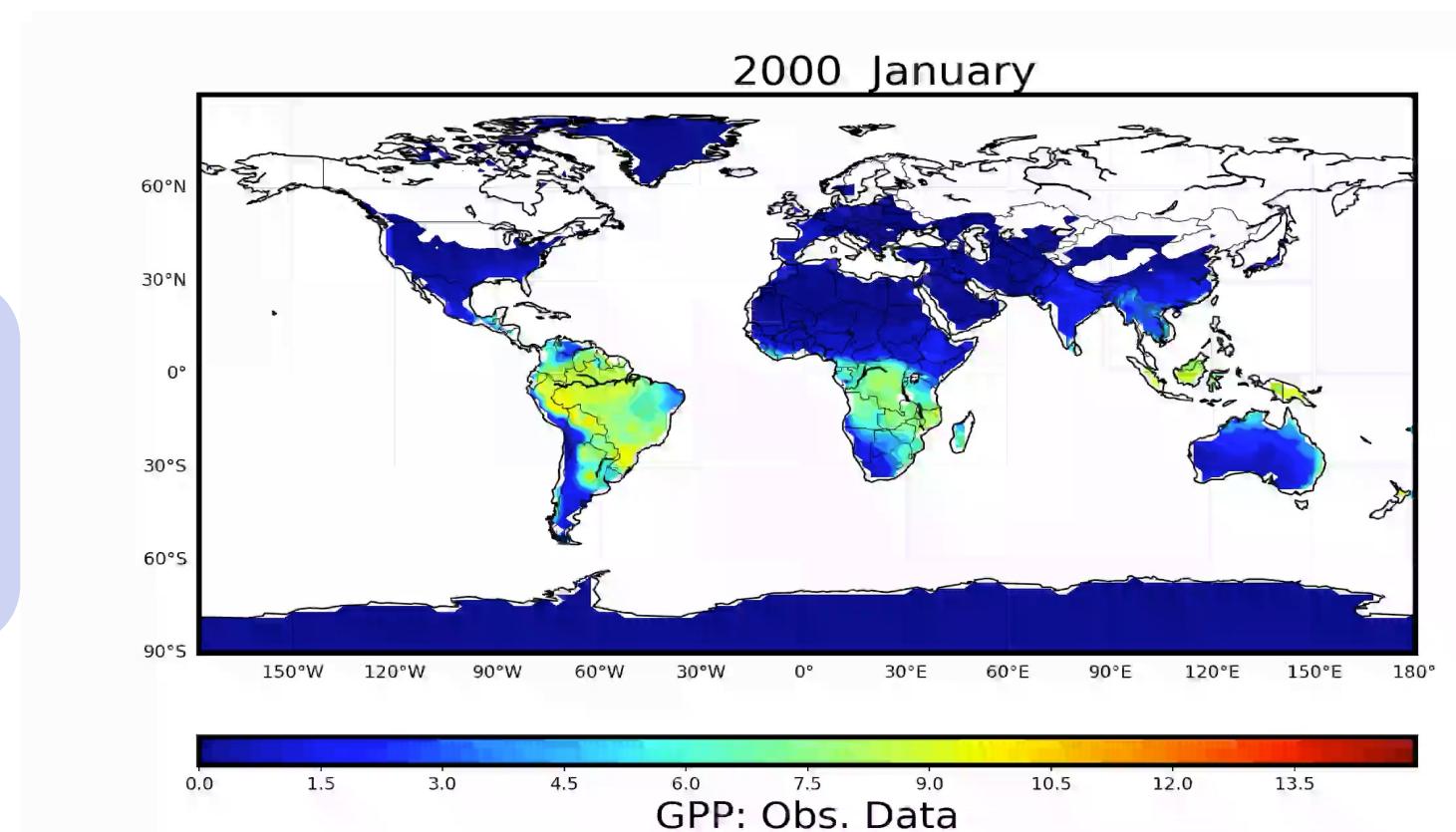
$\uparrow$

$\xi_m^{NN}(\lambda)$

# Reference Data

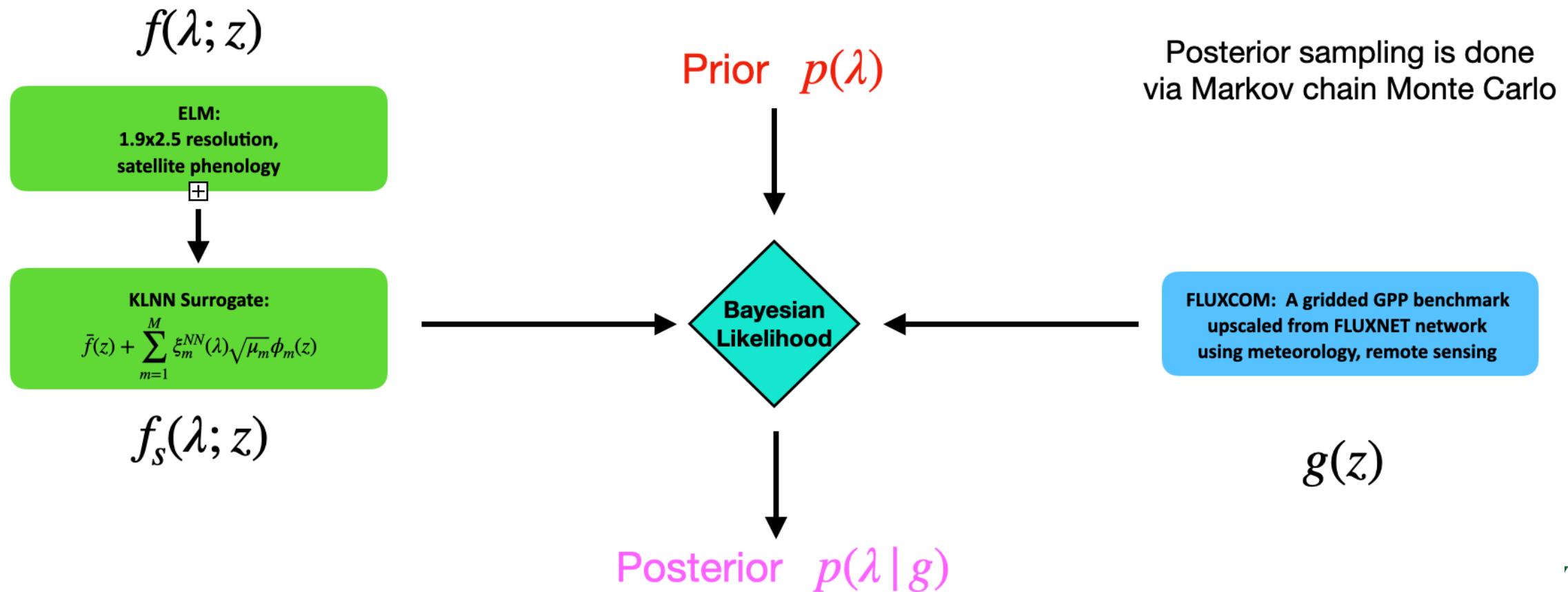
FLUXCOM: A gridded GPP benchmark upscaled from FLUXNET network using meteorology, remote sensing

<https://www.fluxcom.org/>



Bayes' formula

$$p(\lambda | g) \propto p(g | \lambda) p(\lambda)$$



# Bayesian Likelihood is constructed in the reduced space

Bayes' formula

$$p(\lambda|g) \propto p(g|\lambda)p(\lambda)$$

KLNN surrogate:

$$f(\lambda; z) \approx \bar{f}(z) + \sum_{m=1}^M \xi_m^{NN}(\lambda) \sqrt{\mu_m} \phi_m(z)$$

Project observed data to the KL eigenspace:

$$g(z) \approx \bar{f}(z) + \sum_{m=1}^M \eta_m \sqrt{\mu_m} \phi_m(z)$$

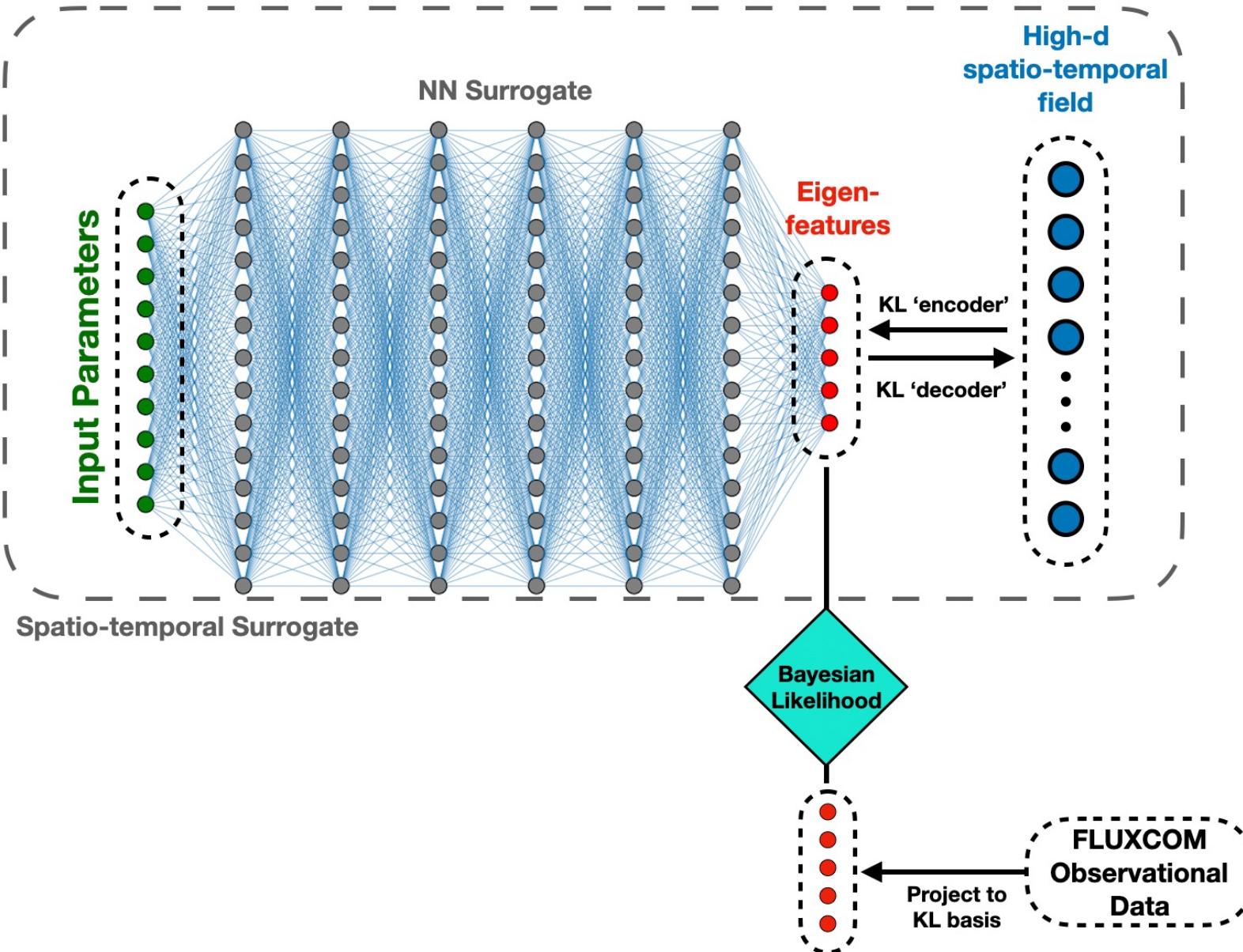
**Pointwise likelihood (naïve) :**

$$L_g(\lambda) \equiv p(g|\lambda) \propto \exp\left(-\sum_{i=1}^N \frac{(g(z_i) - f(\lambda; z_i))^2}{2\sigma_i^2}\right)$$

**Reduced likelihood :**

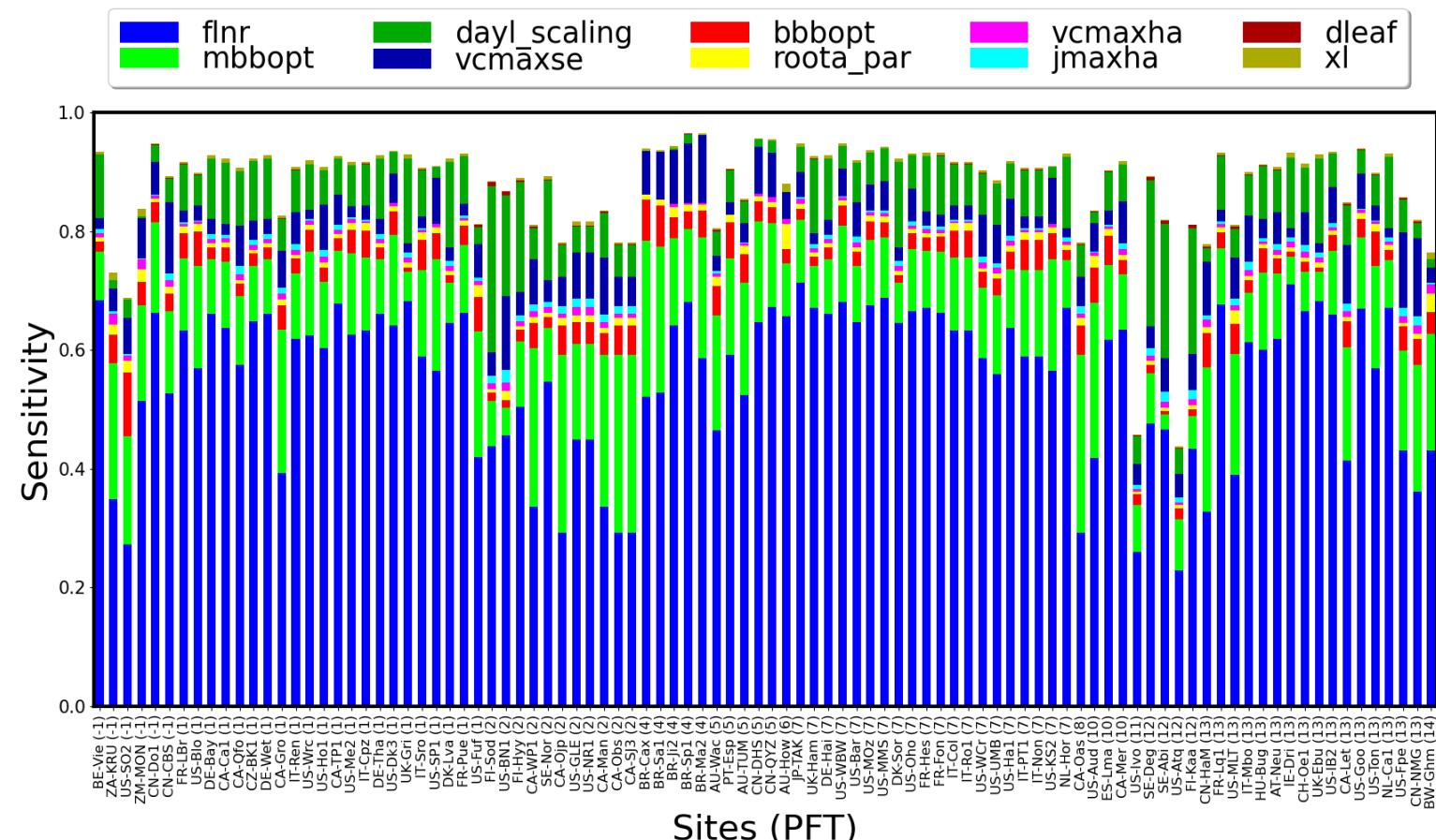
$$L_g(\lambda) \equiv p(g|\lambda) \propto \exp\left(-\sum_{m=1}^M \frac{(\eta_m - \xi_m^{NN}(\lambda))^2}{2\sigma^2}\right)$$

Eigenfeatures  $\xi_m$ 's are uncorrelated, zero-mean, unit variance, hence iid gaussian likelihood is a much better assumption in the reduced space.

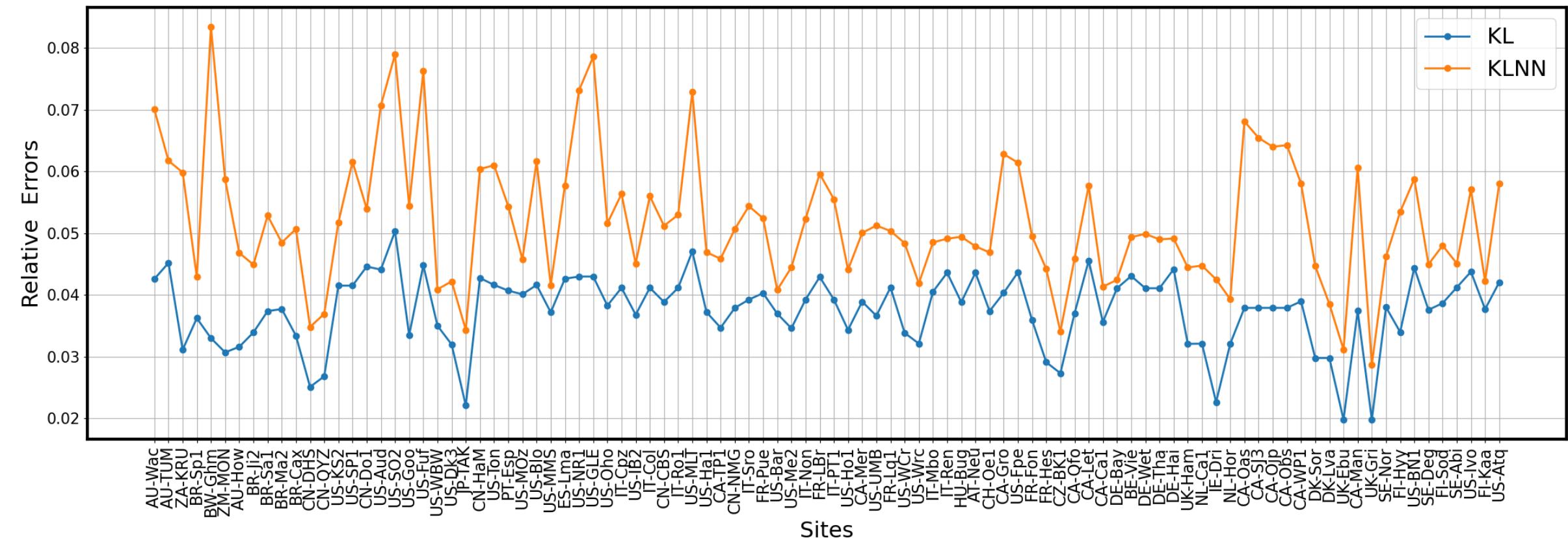


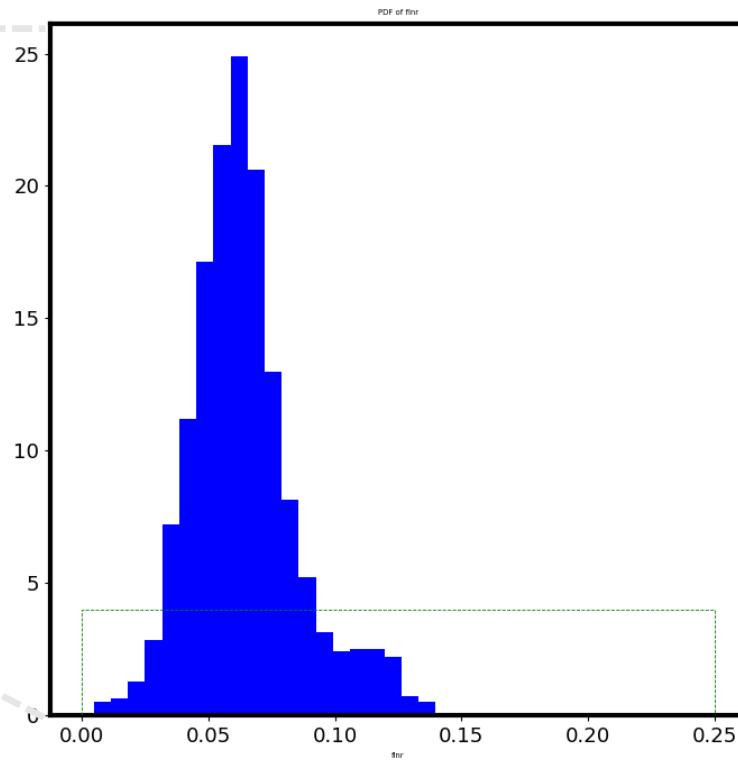
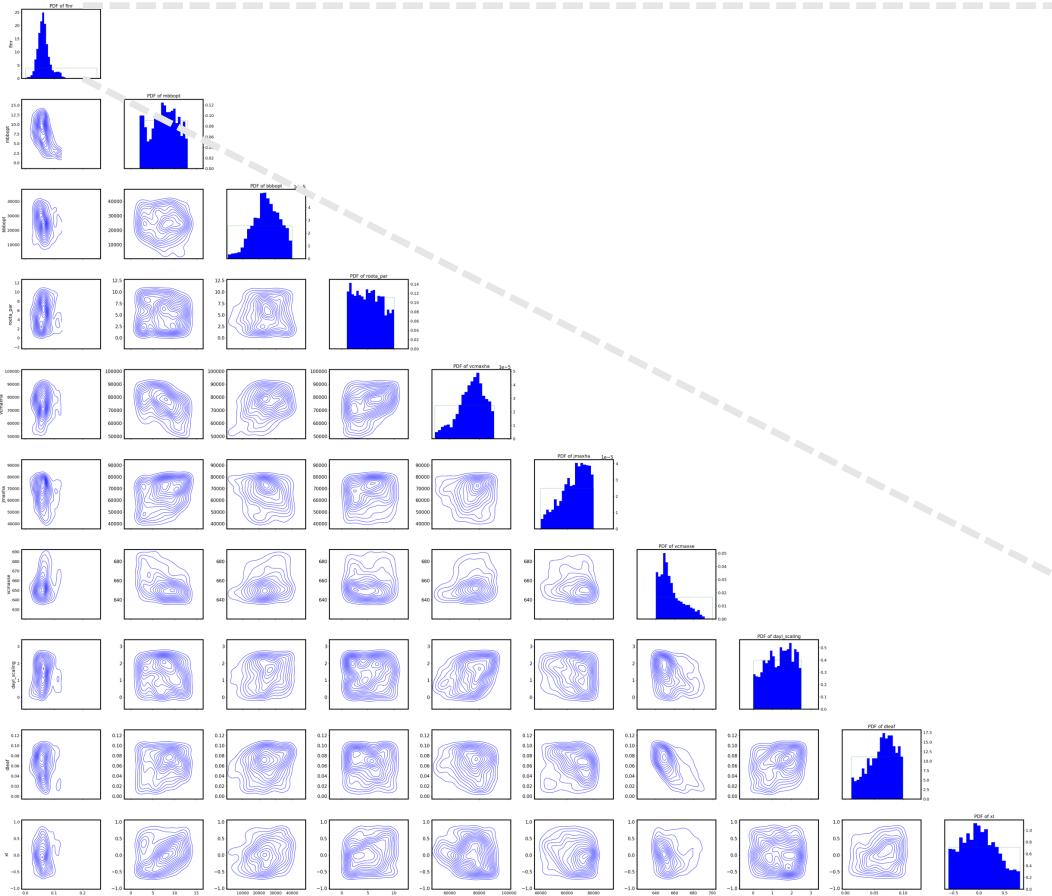
Surrogate-enabled calibration workflow incorporates both forward and inverse UQ tasks

# Sensitivity at 96 FLUXNET sites: RuBisCO leaf fraction as the most impactful parameter



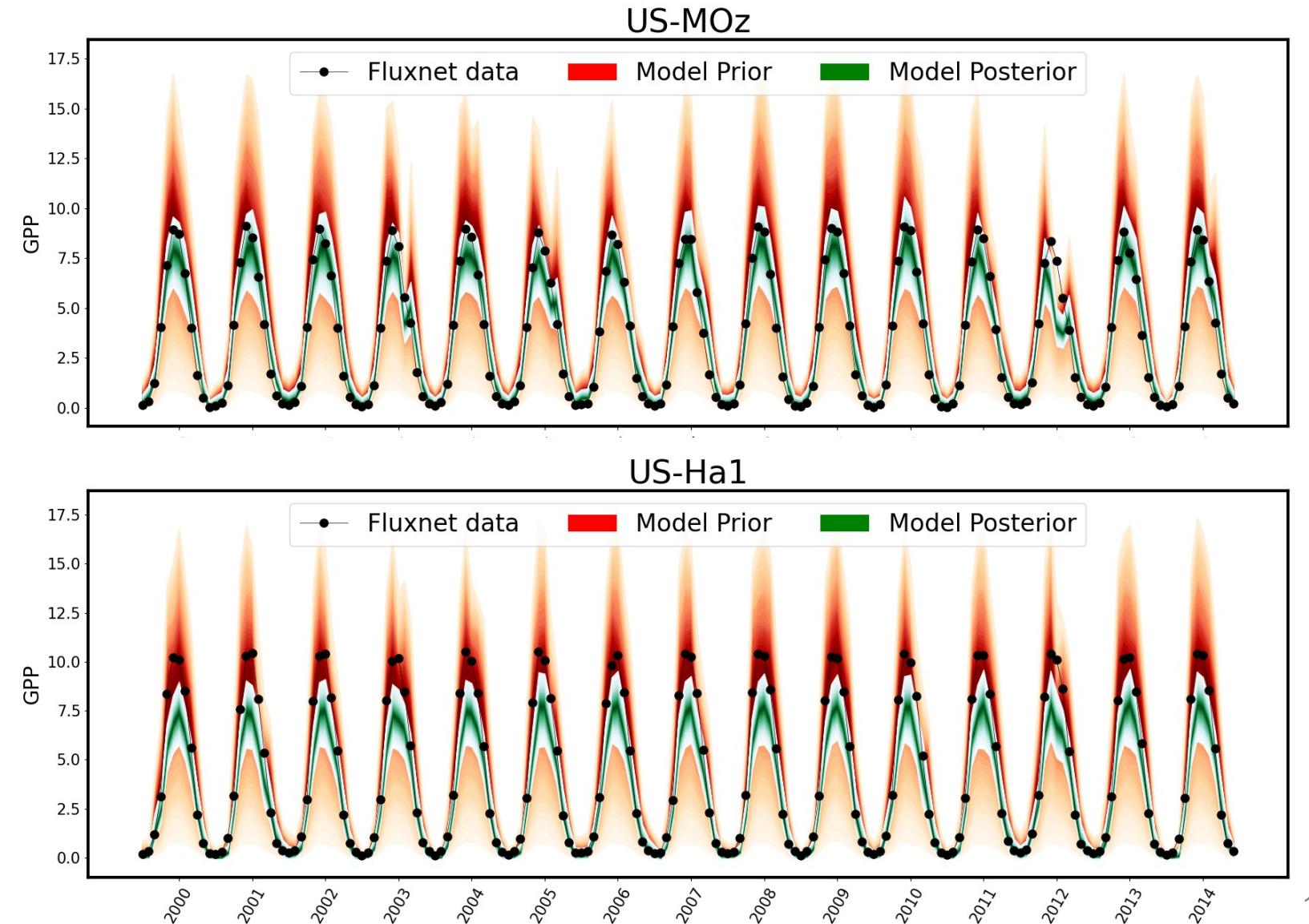
Instead of  $96 \times 180 = 17280$  surrogates, we build  
a single NN surrogate in the reduced, 8-dimensional latent space



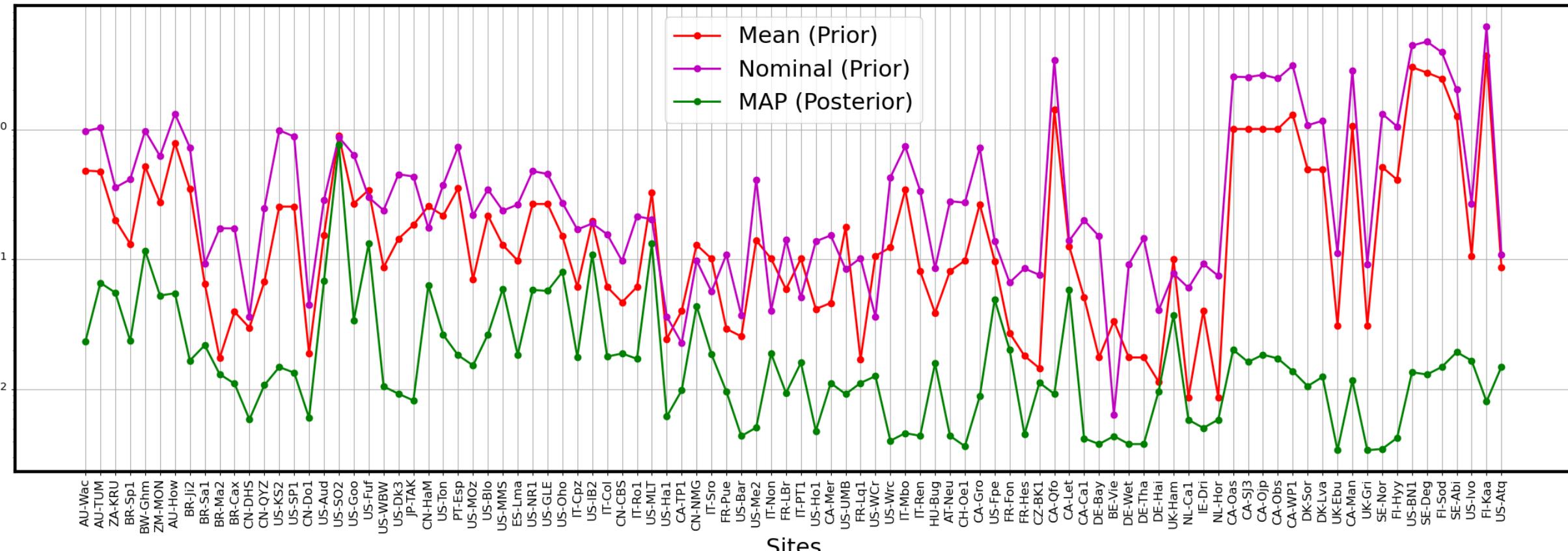


RuBisCO leaf fraction (**flnr**) is the most constrained parameter

Time evolution  
of GPP at select  
FLUXNET sites

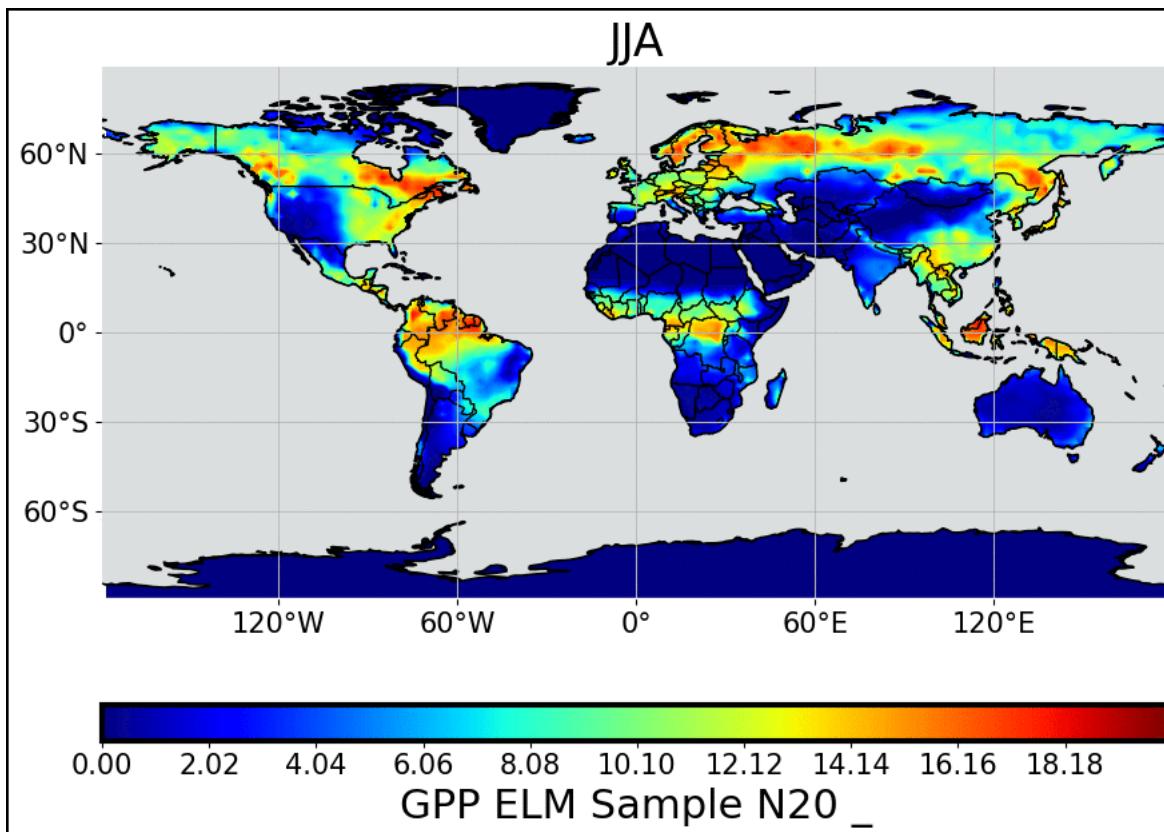


# Calibration brings model prediction closer to reference data

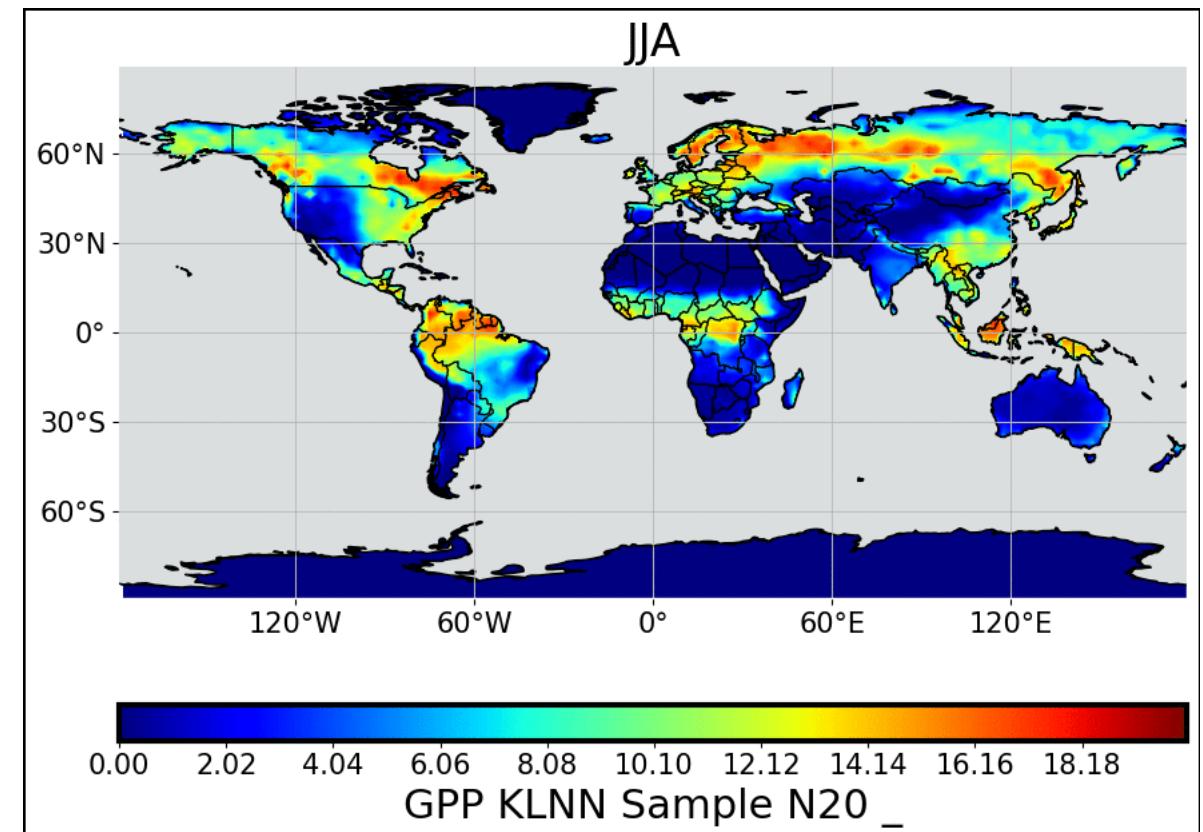


Dimensionality reduction from 4000 cells x 4 seasons = **16000** to **11-dimensional latent space**

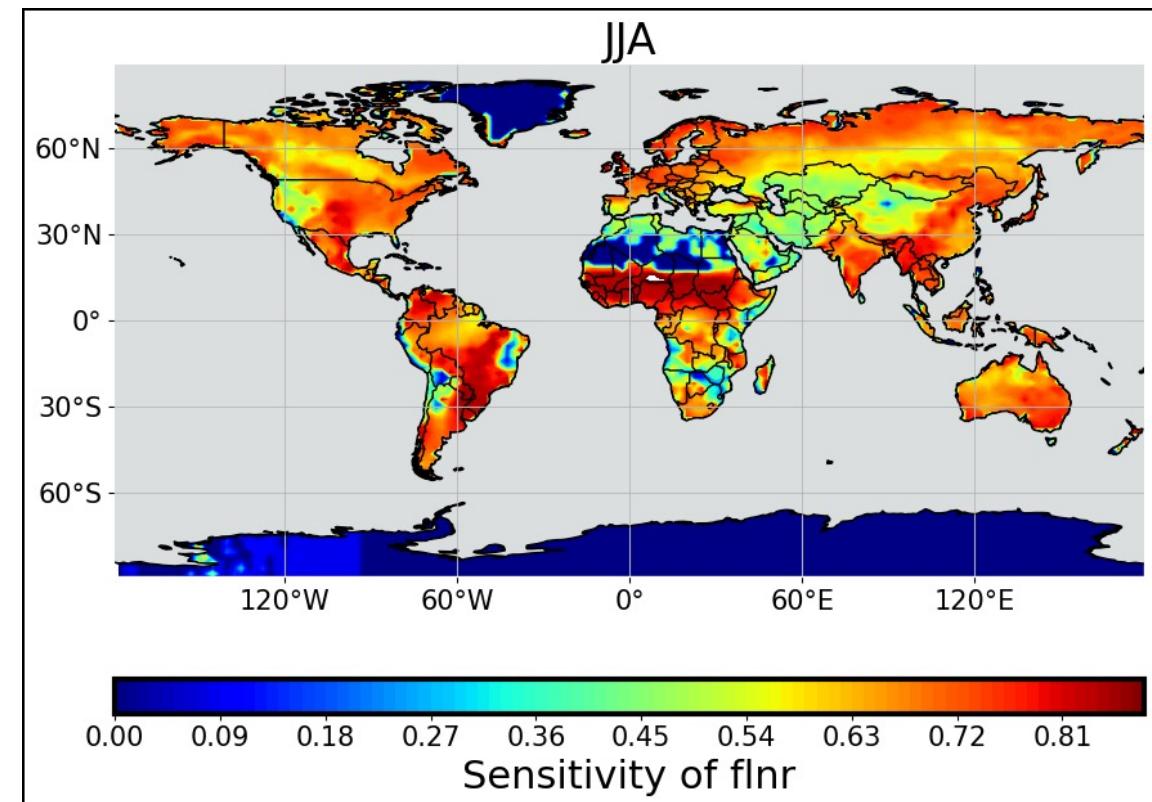
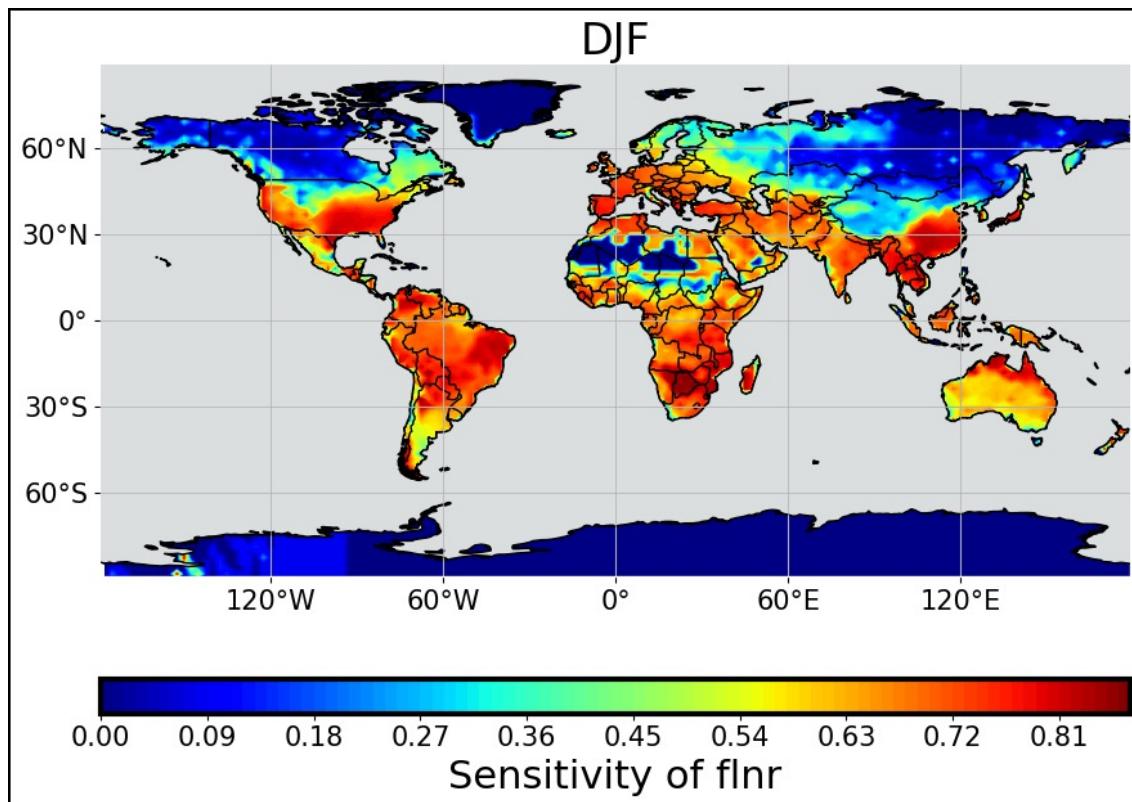
## ELM Model Samples



## KLNN Surrogate Samples

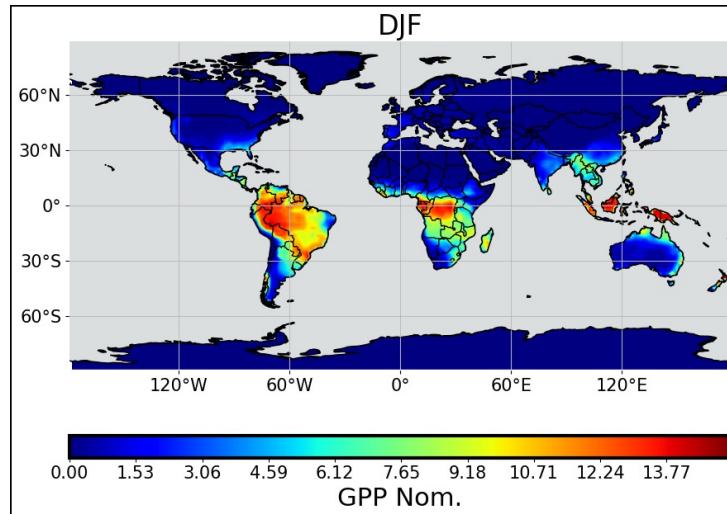


# flnr sensitivity across the globe

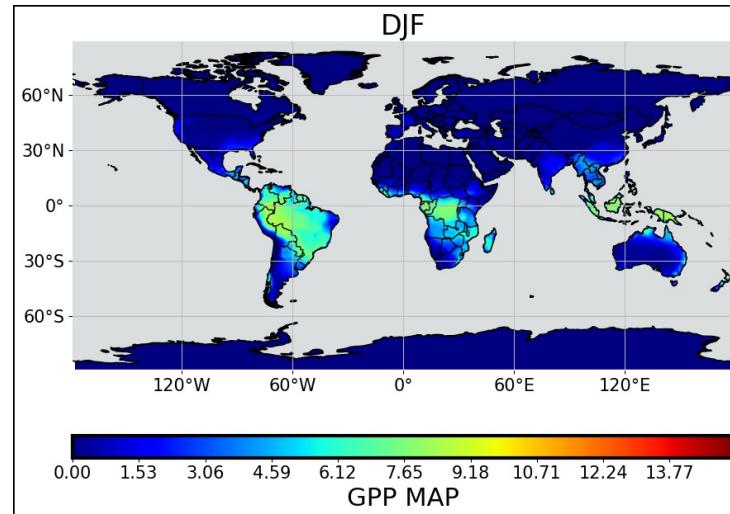




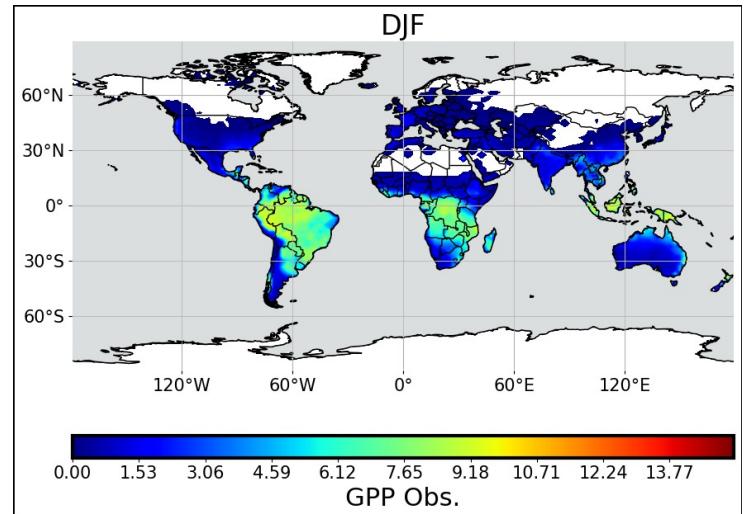
Nominal parameter (prior)



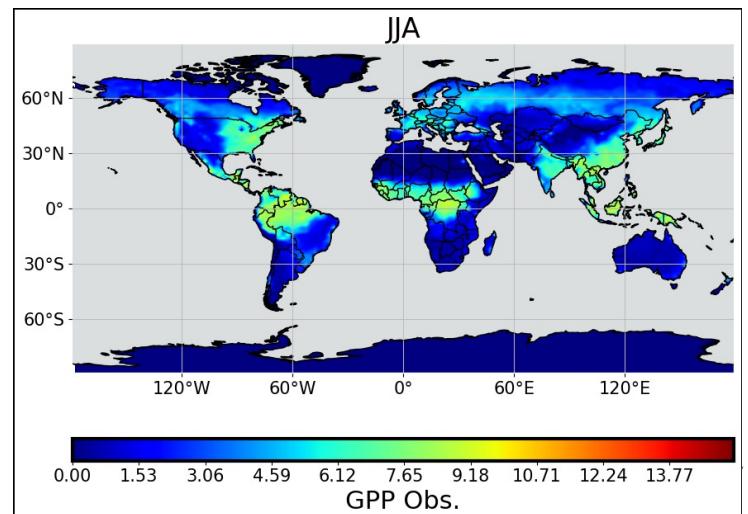
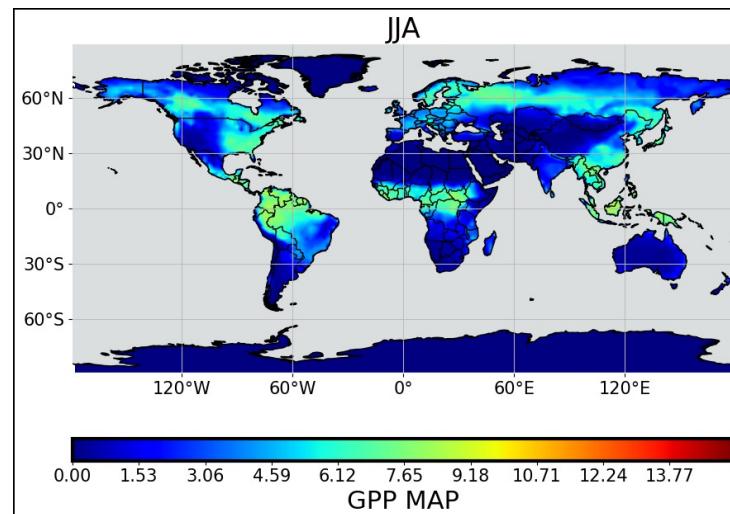
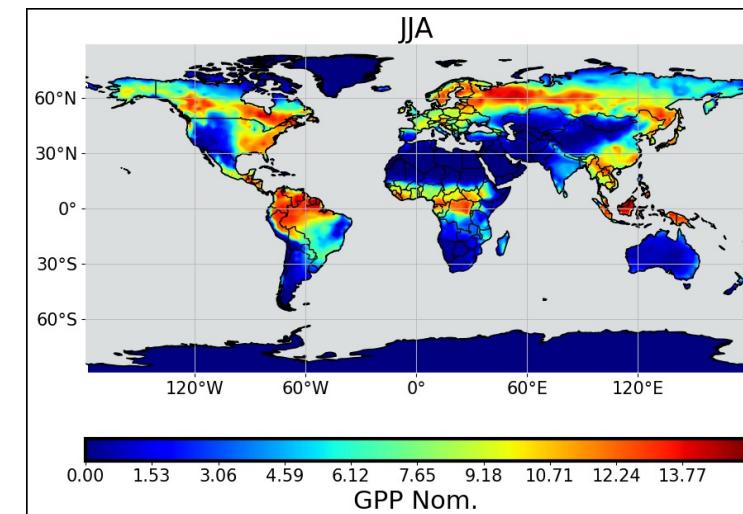
Max a posteriori (MAP)



Reference data



Winter

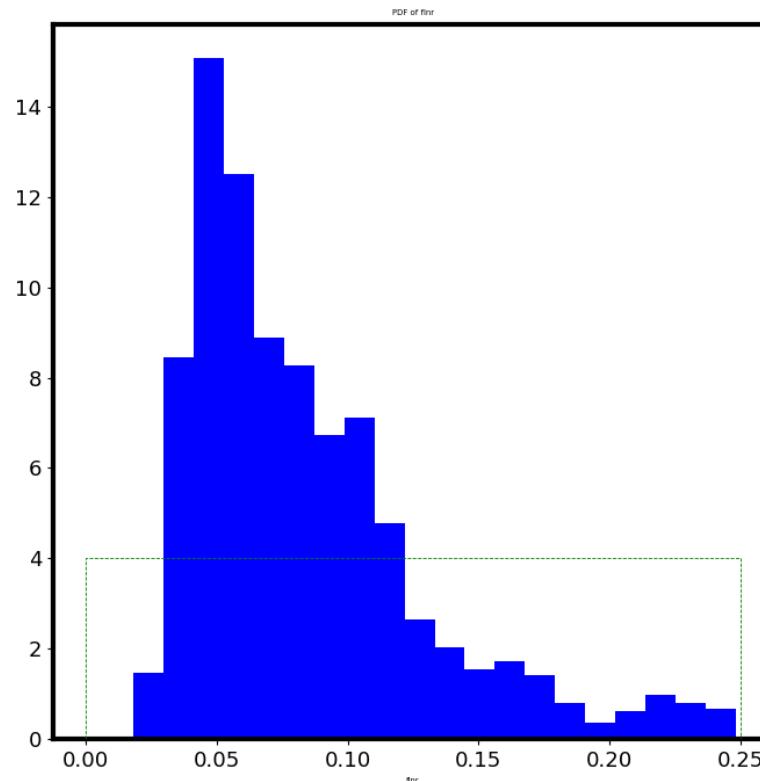


Summer

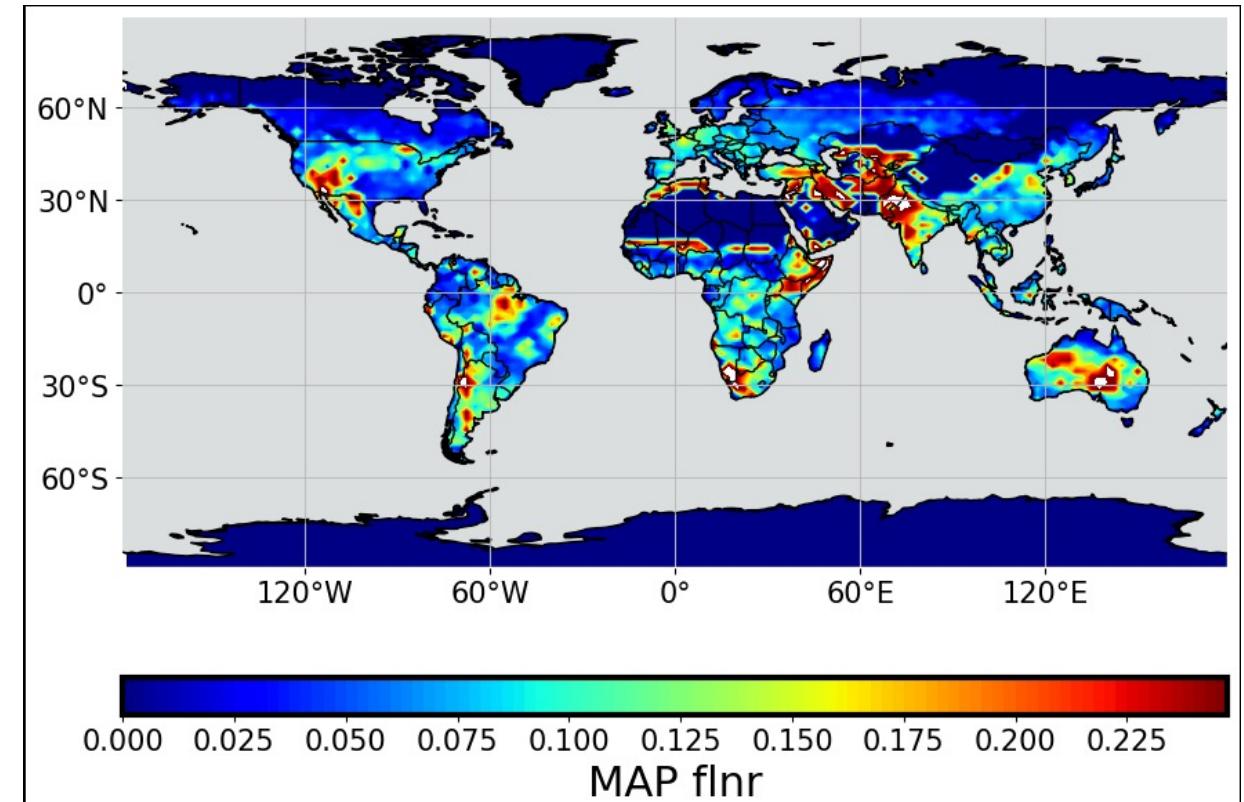
# Two calibration regimes

Ongoing work: PFT-dependent reparameterization  
to improve model's ability to match reference data.

Fixed global flnr parameter



Local flnr parameter





# Summary

---

- Karhunen-Loève (KL) decomposition reduces the spatio-temporal output dimensionality, taking advantage of correlations over space and time.
- Neural network (NN) surrogate in the reduced eigenspace leads to a spatio-temporal **KLNN** surrogate that is a small fraction of ELM cost.
- KLNN surrogate enables sampling based global sensitivity analysis and Bayesian calibration performed in the eigenspace.
- Several orders of magnitude reduction of output dimensionality, and of the simulation cost with ~5% accuracy impact.
- Ongoing work: PFT-dependent reparameterization to improve model's ability to match reference data.



# Additional Material

# Bayesian Likelihood in the reduced space

KLNN surrogate:

$$f(\lambda; z) \approx \bar{f}(z) + \sum_{m=1}^M \xi_m^{NN}(\lambda) \sqrt{\mu_m} \phi_m(z)$$

Project observed data to the KL eigenspace:

$$g(z) \approx \bar{f}(z) + \sum_{m=1}^M \eta_m \sqrt{\mu_m} \phi_m(z)$$

**Pointwise likelihood (old) :**

$$L_g(\lambda) \equiv p(g|\lambda) \propto \exp\left(-\sum_{i=1}^N \frac{(g(z_i) - f(\lambda; z_i))^2}{2\sigma_i^2}\right)$$

**Data model (old) :**

$$g(z_i) = f(\lambda; z_i) + \sigma_i \epsilon_i$$

i.i.d. Normal

$$L_g(\lambda) \equiv p(g|\lambda) \propto \exp\left(-\sum_{m=1}^M \frac{(\eta_m - \xi_m^{NN}(\lambda))^2}{2\sigma^2}\right)$$

**Data model (new) :**

$$\eta_m = \xi_m^{NN}(\lambda) + \tilde{\sigma} \epsilon_m$$

$\downarrow$

MVN (physics-based)

$$g(z_i) = f(\lambda; z_i) + \sum_{m=1}^M \tilde{\epsilon}_m \sqrt{\mu_m} \phi_m(z_i)$$



## FLUXNET Sites

