

Dynamic updating about menus

Kim Sarnoff

October 5, 2024

Introduction

- ▶ Inference about ability is a key input into many economic decisions, e.g. hiring.
- ▶ Typically assume evaluators (e.g. employers) act as sophisticated statisticians.
- ▶ But, environment complex:
 - ▶ Information arrives sequentially
 - ▶ About a menu of individuals
 - ▶ With different prior distributions of ability.

Introduction

- ▶ Inference about ability is a key input into many economic decisions, e.g. hiring.
- ▶ Typically assume evaluators (e.g. employers) act as sophisticated statisticians.
- ▶ But, environment complex:
 - ▶ Information arrives sequentially
 - ▶ About a menu of individuals
 - ▶ With different prior distributions of ability.
- ▶ This project: Study dynamic belief formation about individuals evaluated with others.

Outline

Design

Econometric strategy

Aggregate analysis

Sequence effects

Outline

Design

Econometric strategy

Aggregate analysis

Sequence effects

Overview of design

- ▶ Experimental “employers” complete a statistical learning task.
 - ▶ Create groups of experimental “workers” with varying ability.
 - ▶ Worker drawn from a given ability distribution.
 - ▶ Employer receives information on worker ability incrementally and reports beliefs.

Overview of design

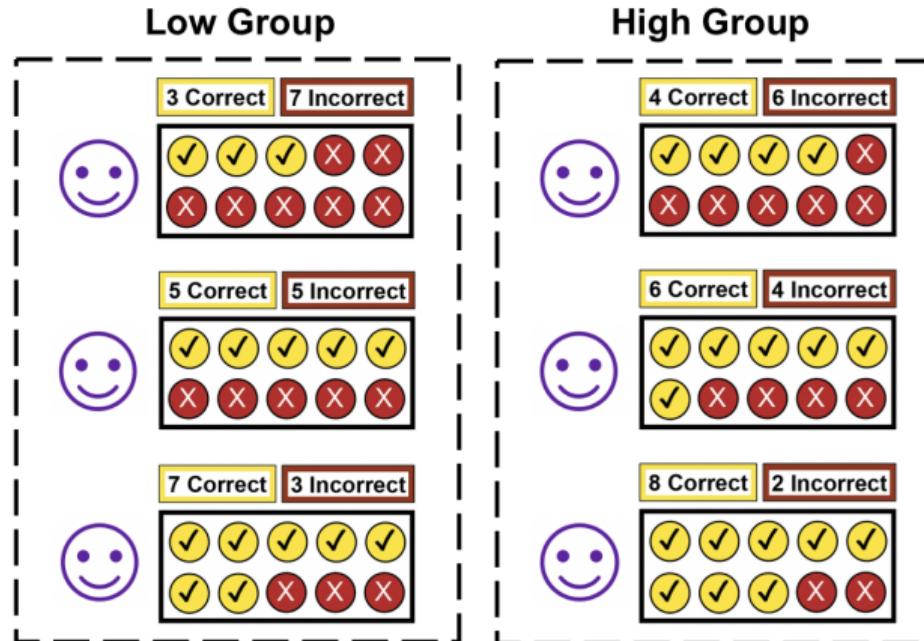
- ▶ Experimental “employers” complete a statistical learning task.
 - ▶ Create groups of experimental “workers” with varying ability.
 - ▶ Worker drawn from a given ability distribution.
 - ▶ Employer receives information on worker ability incrementally and reports beliefs.
- ▶ Vary environment in which learning task done.
 - ▶ Single worker in isolation.
 - ▶ Pairs drawn from same distribution evaluated simultaneously.
 - ▶ Pairs drawn from different distributions evaluated simultaneously.

Workers

- ▶ Hired 160 people ("workers") on Prolific to complete 10 question math quiz.

Workers

- ▶ Hired 160 people (“workers”) on Prolific to complete 10 question math quiz.
- ▶ Create two groups of workers, with different distribution of scores.



Baseline treatment (*Individual*)

- ▶ Employer randomly assigned to low group or high group.

Baseline treatment (*Individual*)

- ▶ Employer randomly assigned to low group or high group.
- ▶ Complete inference task with bookbag and poker chip structure.

Baseline treatment (*Individual*)

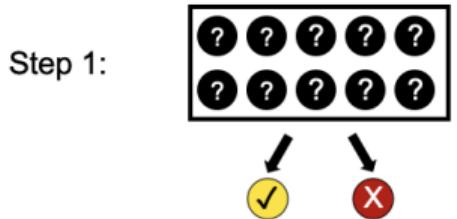
- ▶ Employer randomly assigned to low group or high group.
- ▶ Complete inference task with bookbag and poker chip structure.
 - ▶ Worker randomly selected from group.

Baseline treatment (*Individual*)

- ▶ Employer randomly assigned to low group or high group.
- ▶ Complete inference task with bookbag and poker chip structure.
 - ▶ Worker randomly selected from group.
 - ▶ Three identical rounds of inference about worker's score.

Baseline treatment (*Individual*)

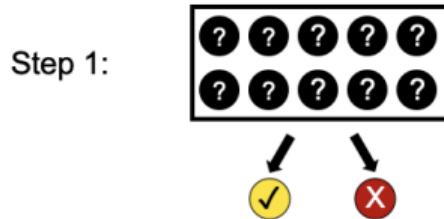
- ▶ Employer randomly assigned to low group or high group.
- ▶ Complete inference task with bookbag and poker chip structure.
 - ▶ Worker randomly selected from group.
 - ▶ Three identical rounds of inference about worker's score.



Step 2: Report full posterior distribution

Baseline treatment (*Individual*)

- ▶ Employer randomly assigned to low group or high group.
- ▶ Complete inference task with bookbag and poker chip structure.
 - ▶ Worker randomly selected from group.
 - ▶ Three identical rounds of inference about worker's score.



Step 2: Report full posterior distribution

- ▶ Complete 8 inference tasks, where worker drawn with replacement.

Two workers from same group (*Same*)

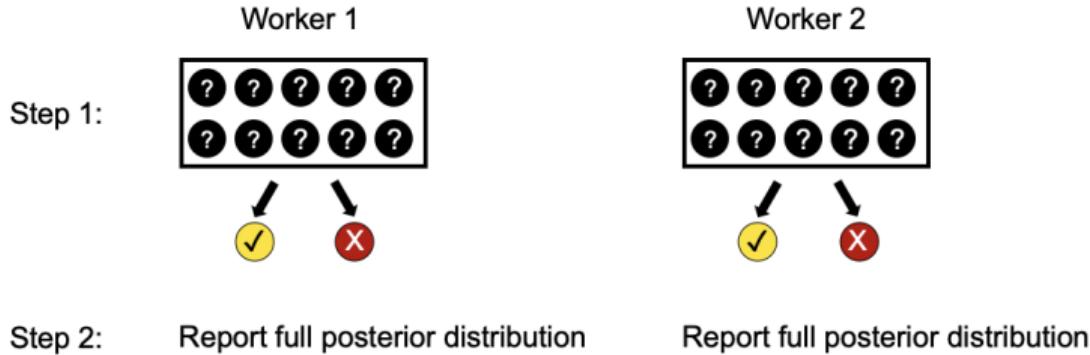
- ▶ Employer randomly assigned to low group or high group.

Two workers from same group (*Same*)

- ▶ Employer randomly assigned to low group or high group.
- ▶ Completes inference task.
 - ▶ **Two workers** randomly selected from group, with replacement.

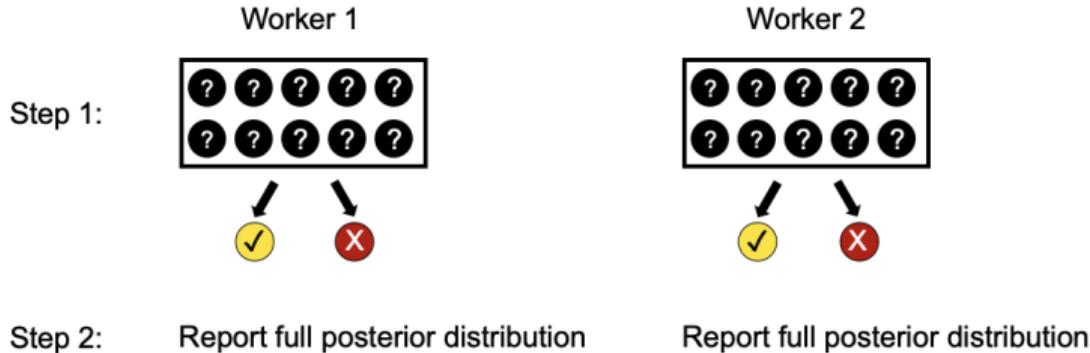
Two workers from same group (*Same*)

- ▶ Employer randomly assigned to low group or high group.
- ▶ Completes inference task.
 - ▶ **Two workers** randomly selected from group, with replacement.
 - ▶ Three identical rounds of inference about **each worker's** score.



Two workers from same group (*Same*)

- ▶ Employer randomly assigned to low group or high group.
- ▶ Completes inference task.
 - ▶ **Two workers** randomly selected from group, with replacement.
 - ▶ Three identical rounds of inference about **each worker's** score.



- ▶ Complete **4** inference tasks, where **two workers** drawn with replacement.

Two workers from different groups (*Diff*)

- ▶ Employer observes **low group and high group.**

Two workers from different groups (*Diff*)

- ▶ Employer observes **low group and high group.**
- ▶ Complete inference task.
 - ▶ **One worker** randomly selected from **each group.**

Two workers from different groups (*Diff*)

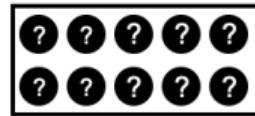
- Employer observes **low group and high group**.
- Complete inference task.
 - **One worker** randomly selected from **each group**.
 - Three identical rounds of inference about **each worker's** score.

Step 1:
Worker from low group



Step 2: Report full posterior distribution

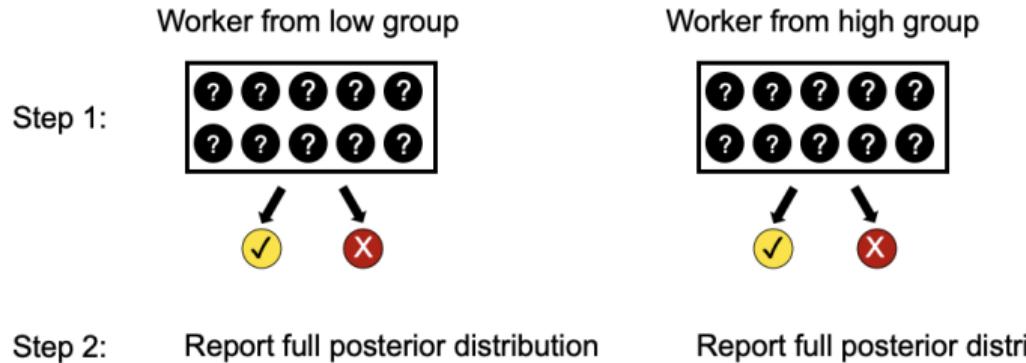
Worker from high group



Report full posterior distribution

Two workers from different groups (*Diff*)

- ▶ Employer observes **low group and high group**.
- ▶ Complete inference task.
 - ▶ **One worker** randomly selected from **each group**.
 - ▶ Three identical rounds of inference about **each worker's** score.



- ▶ Complete **4** inference tasks, where **worker from each group** drawn with replacement.

Predictions

- ▶ Bayesian benchmark: Calculate posterior distribution of score according to

$$P(score|y) = \frac{P(y|score)P(score)}{\sum_{score} P(y|score)P(score)}$$

Predictions

- ▶ Bayesian benchmark: Calculate posterior distribution of score according to

$$P(score|y) = \frac{P(y|score)P(score)}{\sum_{score} P(y|score)P(score)}$$

- ▶ $P(score|y)$ only depends on prior and signal, not treatment

Signals

	Individual	Same Group		Different Groups	
	Group G	Group G	Group G	Group G	Group G'
Worker 1	0,0,1	Pair 1	0,0,1	0,0,1	0,1,1
Worker 2	0,1,1		0,1,1	0,1,1	1,0,0
Worker 3	0,0,0		0,0,0	1,1,1	1,1,1
Worker 4	1,1,0		1,1,0	0,0,1	0,0,1
Worker 5	0,1,1	Pair 2	0,1,1	0,1,1	0,0,1
Worker 6	1,0,0		1,0,0	1,0,0	0,1,1
Worker 7	1,1,1		1,1,1	1,1,1	0,0,0
Worker 8	0,0,1		0,0,1	0,0,1	1,1,0

Payment

- ▶ *Individual*
 - ▶ 24 rounds overall: 3 rounds each for 8 workers.
 - ▶ Pay for half of the rounds, chosen randomly, with BSR.

Payment

- ▶ *Individual*
 - ▶ 24 rounds overall: 3 rounds each for 8 workers.
 - ▶ Pay for half of the rounds, chosen randomly, with BSR.
- ▶ Pair treatments
 - ▶ 12 rounds overall: 3 rounds each for 4 worker pairs.
 - ▶ Pay for one worker, chosen randomly, each round.

Sample

- ▶ Total sample is 180
 - ▶ 30 participants for each group in individual treatment.
 - ▶ 30 participants for each group in same group treatment.
 - ▶ 60 participants for different groups treatment.
- ▶ Estimation sample is 150
 - ▶ Drop participants who make mistakes in more than half the rounds.

Outline

Design

Econometric strategy

Aggregate analysis

Sequence effects

Econometric strategy

- Evaluator i reports belief about worker w in time t
- For given worker, t goes from 1 to 3

$$\begin{aligned} & \overbrace{\log \left(\frac{P(score_j|signal)_{i,w,t}}{P(score_k|signal)_{i,w,t}} \right)}^{\text{log posterior odds}} = \\ & \beta_C * 1[s_t = \text{correct}] * \overbrace{\log \left(\frac{P(correct|score_j)_{i,w,t}}{P(correct|score_k)_{i,w,t}} \right)}^{\text{LLR if signal in } t \text{ correct}} \\ & + \beta_I * 1[s_t = \text{incorrect}] * \overbrace{\log \left(\frac{P(incorrect|score_j)_{i,w,t}}{P(incorrect|score_k)_{i,w,t}} \right)}^{\text{LLR if signal in } t \text{ incorrect}} \\ & + \delta * \overbrace{\log \left(\frac{P(score_j)_{i,w,t-1}}{P(score_k)_{i,w,t-1}} \right)}^{\text{log prior odds}} + \epsilon_{i,w,t} \end{aligned}$$

Econometric strategy

- ▶ Suppose true model is each i has fixed weights on prior & data:

$$\begin{aligned} \log \left(\frac{P(score_j|signal)_{i,w,t}}{P(score_k|signal)_{i,w,t}} \right) = \\ \beta_{Ci} * 1[s_t = \text{correct}] * \log \left(\frac{P(correct|score_j)_{i,w,t}}{P(correct|score_k)_{i,w,t}} \right) \\ + \beta_{Ii} * 1[s_t = \text{incorrect}] * \log \left(\frac{P(incorrect|score_j)_{i,w,t}}{P(incorrect|score_k)_{i,w,t}} \right) \\ + \delta_i * \log \left(\frac{P(score_j)_{i,w,t-1}}{P(score_k)_{i,w,t-1}} \right) + \epsilon_{i,w,t} \end{aligned}$$

Econometric strategy

- ▶ Suppose true model is each i has fixed weights on prior & data:

$$\begin{aligned} \log \left(\frac{P(score_j|signal)_{i,w,t}}{P(score_k|signal)_{i,w,t}} \right) = \\ \beta_{Ci} * 1[s_t = correct] * \log \left(\frac{P(correct|score_j)_{i,w,t}}{P(correct|score_k)_{i,w,t}} \right) \\ + \beta_{Ii} * 1[s_t = incorrect] * \log \left(\frac{P(incorrect|score_j)_{i,w,t}}{P(incorrect|score_k)_{i,w,t}} \right) \\ + \delta_i * \log \left(\frac{P(score_j)_{i,w,t-1}}{P(score_k)_{i,w,t-1}} \right) + \epsilon_{i,w,t} \end{aligned}$$

- ▶ If run population regression, $\epsilon_{i,w,t}$ contains $\delta_i, \beta_{Ci}, \beta_{Ii} \implies E[\epsilon_{i,w,t} | \mathbf{X}] \neq 0$
- ▶ Instrumenting with Bayesian prior not a solution [▶ Detail](#)

Simulated maximum likelihood

$$y_{it} = \begin{cases} \beta_{Ci}1[s_t = \text{correct}]s_{it} + \beta_{Ii}1[s_t = \text{incorrect}]s_{it} + \epsilon_{it} & \text{if } t = 1 \\ \beta_{Ci}1[s_t = \text{correct}]s_{it} + \beta_{Ii}1[s_t = \text{incorrect}]s_{it} + \delta_i y_{i,t-1} + \epsilon_{it} & \text{if } t > 1 \end{cases}$$

Simulated maximum likelihood

$$y_{it} = \begin{cases} \beta_{Ci}1[s_t = \text{correct}]s_{it} + \beta_{li}1[s_t = \text{incorrect}]s_{it} + \epsilon_{it} & \text{if } t = 1 \\ \beta_{Ci}1[s_t = \text{correct}]s_{it} + \beta_{li}1[s_t = \text{incorrect}]s_{it} + \delta_i y_{i,t-1} + \epsilon_{it} & \text{if } t > 1 \end{cases}$$

$$\begin{pmatrix} \beta_{Ci} \\ \beta_{li} \\ \delta_i \end{pmatrix} \sim \text{MVN} \left(\begin{pmatrix} \mu_{\beta_{Ci}} \\ \mu_{\beta_{li}} \\ \mu_{\delta_i} \end{pmatrix}, \begin{pmatrix} \sigma_{\beta_{Ci}}^2 & \sigma_{\beta_{Ci}\beta_{li}} & \sigma_{\beta_{Ci}\delta_i} \\ \sigma_{\beta_{li}}^2 & \sigma_{\beta_{li}}^2 & \sigma_{\beta_{li}\delta_i} \\ \sigma_{\delta_i\beta_{Ci}} & \sigma_{\delta_i\beta_{li}} & \sigma_{\delta_i}^2 \end{pmatrix} \right)$$

$$\epsilon_{it} \sim_{iid} N(0, \sigma^2)$$

Estimation

- ▶ Log likelihood of an observation is:

$$\mathcal{L}_{it} = \ln(\phi[y_{it} - \theta_i x_{it}])$$

where $\theta_i = [\beta_{Ci}, \beta_{Ii}, \delta_i]$, $x_{it} = [s_{it}, y_{i,t-1}(\theta_i)]'$

Estimation

- ▶ Log likelihood of an observation is:

$$\mathcal{L}_{it} = \ln(\phi [y_{it} - \theta_i x_{it}])$$

where $\theta_i = [\beta_{Ci}, \beta_{Ii}, \delta_i]$, $x_{it} = [s_{it}, y_{i,t-1}(\theta_i)]'$

- ▶ Don't observe $\theta_i \implies$ can't do maximum likelihood directly. Need to integrate it out:

$$\mathcal{L}_{it} = \ln \left(\int \phi [y_{it} - \theta_i x_{it}] f(\theta_i) d\theta_i \right)$$

Estimation

- Log likelihood of an observation is:

$$\mathcal{L}_{it} = \ln(\phi [y_{it} - \theta_i x_{it}])$$

where $\theta_i = [\beta_{Ci}, \beta_{Ii}, \delta_i]', x_{it} = [s_{it}, y_{i,t-1}(\theta_i)]'$

- Don't observe $\theta_i \implies$ can't do maximum likelihood directly. Need to integrate it out:

$$\mathcal{L}_{it} = \ln \left(\int \phi [y_{it} - \theta_i x_{it}] f(\theta_i) d\theta_i \right)$$

- Don't know this integral analytically, so simulate it:

$$\mathcal{L}_{it}^{sim} = \ln \left[\frac{1}{S} \sum_s \phi [y_{it} - \theta_{is} x_{it}] \right]$$

Estimation

- Log likelihood of an observation is:

$$\mathcal{L}_{it} = \ln(\phi[y_{it} - \theta_i x_{it}])$$

where $\theta_i = [\beta_{Ci}, \beta_{Ii}, \delta_i]', x_{it} = [s_{it}, y_{i,t-1}(\theta_i)]'$

- Don't observe $\theta_i \implies$ can't do maximum likelihood directly. Need to integrate it out:

$$\mathcal{L}_{it} = \ln \left(\int \phi[y_{it} - \theta_i x_{it}] f(\theta_i) d\theta_i \right)$$

- Don't know this integral analytically, so simulate it:

$$\mathcal{L}_{it}^{sim} = \ln \left[\frac{1}{S} \sum_s \phi[y_{it} - \theta_{is} x_{it}] \right]$$

- Maximize the simulated likelihood: $\sum_i \sum_t \mathcal{L}_{it}^{sim}$

Outline

Design

Econometric strategy

Aggregate analysis

Sequence effects

Baseline model

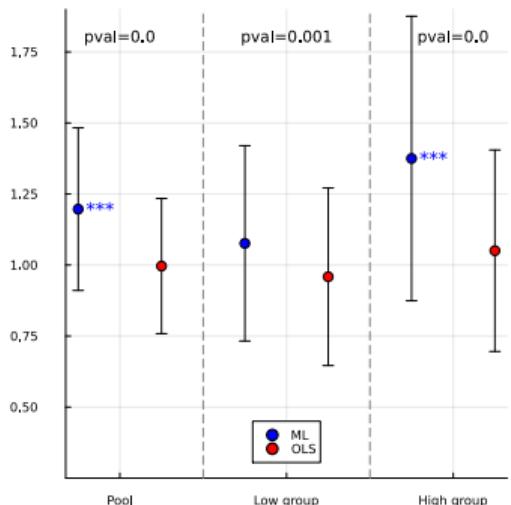
$$y_{it} = \begin{cases} \beta_{Ci}1[s_t = \text{correct}]s_{it} + \beta_{li}1[s_t = \text{incorrect}]s_{it} + \epsilon_{it} & \text{if } t = 1 \\ \beta_{Ci}1[s_t = \text{correct}]s_{it} + \beta_{li}1[s_t = \text{incorrect}]s_{it} + \delta_i y_{i,t-1} + \epsilon_{it} & \text{if } t > 1 \end{cases}$$

- Will show estimate of $\mu_{\beta_{Ci}}, \mu_{\beta_{li}}, \mu_{\delta_i}$

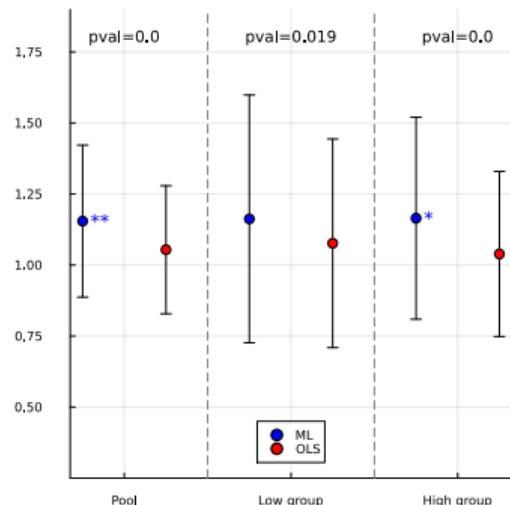
$$\begin{pmatrix} \beta_{Ci} \\ \beta_{li} \\ \delta_i \end{pmatrix} \sim \text{MVN} \left(\begin{pmatrix} \mu_{\beta_{Ci}} \\ \mu_{\beta_{li}} \\ \mu_{\delta_i} \end{pmatrix}, \begin{pmatrix} \sigma_{\beta_{Ci}}^2 & \sigma_{\beta_{Ci}\beta_{li}} & \sigma_{\beta_{Ci}\delta_i} \\ \sigma_{\beta_{li}} & \sigma_{\beta_{li}}^2 & \sigma_{\beta_{li}\delta_i} \\ \sigma_{\delta_i\beta_{Ci}} & \sigma_{\delta_i\beta_{li}} & \sigma_{\delta_i}^2 \end{pmatrix} \right)$$

Individual treatment

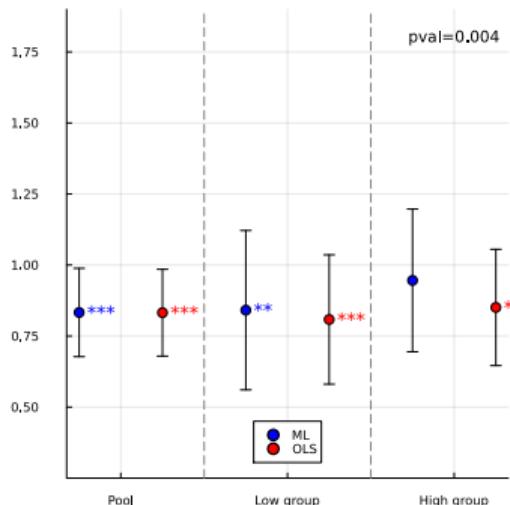
Panel 1: β_c



Panel 2: β_i

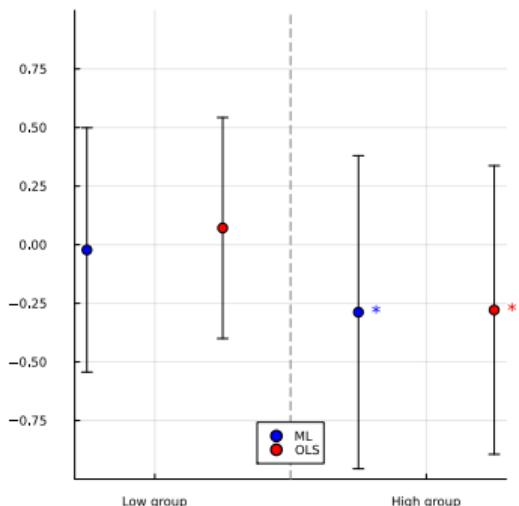


Panel 3: δ

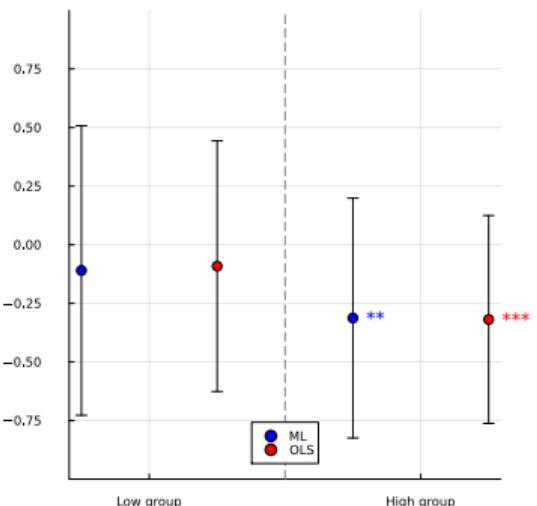


Comparing pair treatments

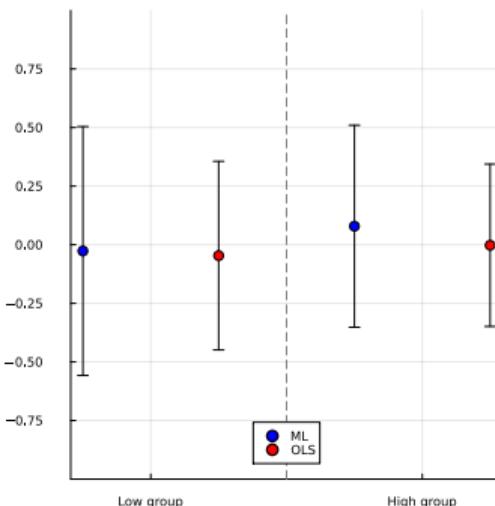
Panel 1: β_c



Panel 2: β_i



Panel 3: δ



Within pair distance in final belief

- ▶ Calculate within pair distance in final belief in *Diff*
- ▶ Compare to within pair distance in simulated pairs from *Same*

	Same Group		Different Groups	
	Group G	Group G	Group G	Group G'
Pair 1	0,0,1		0,0,1	0,1,1
Pair 2	0,1,1		1,0,0	1,0,0
Pair 3	0,0,0		1,1,1	1,1,1
Pair 4	1,1,0		0,0,1	0,0,1

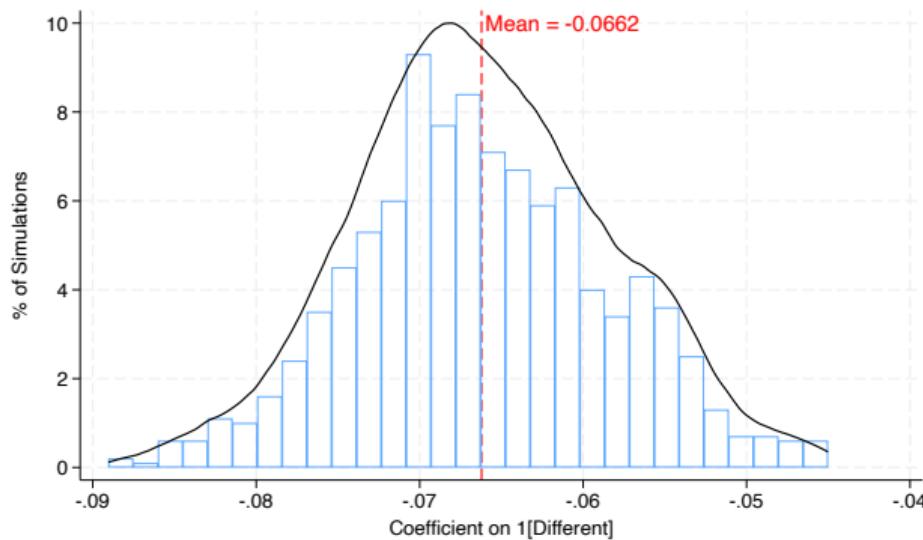
	Group G'
Pair 1	0,0,1
Pair 2	0,1,1
Pair 3	0,0,0
Pair 4	1,1,0

Partner from different group leads to compression within pair.

- Perform 1000 simulations, estimating

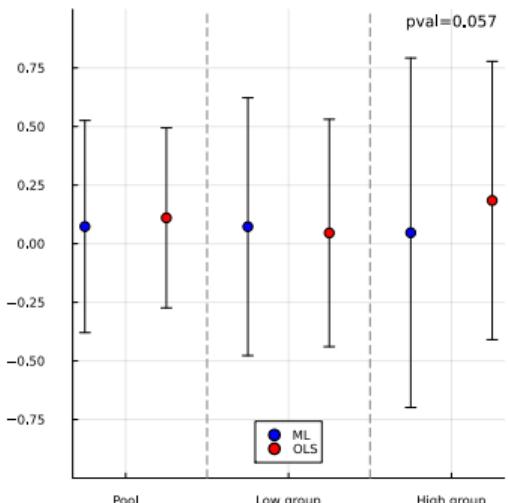
$$\text{Distance within pair} = \beta_0 + \beta_1 \text{[Different]} + \epsilon$$

SE clustered at individual level, where this is simulated individual in *Same*.

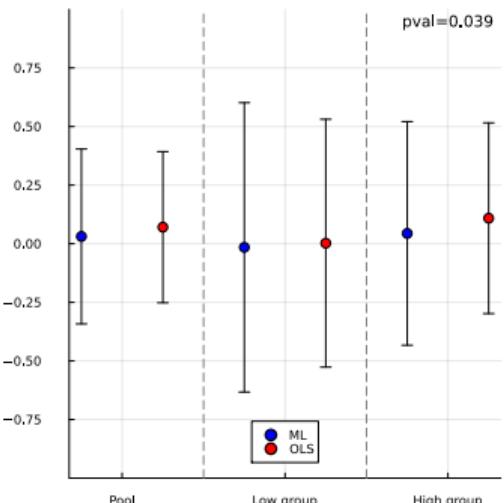


Simultaneous evaluation alone has no effect.

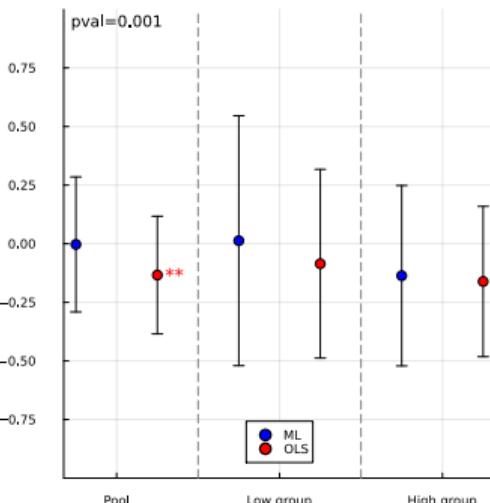
Panel 1: β_c



Panel 2: β_i



Panel 3: δ



Outline

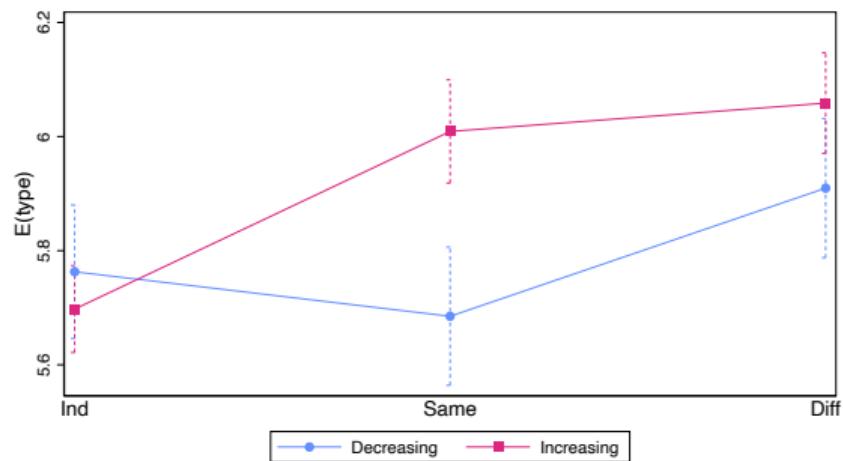
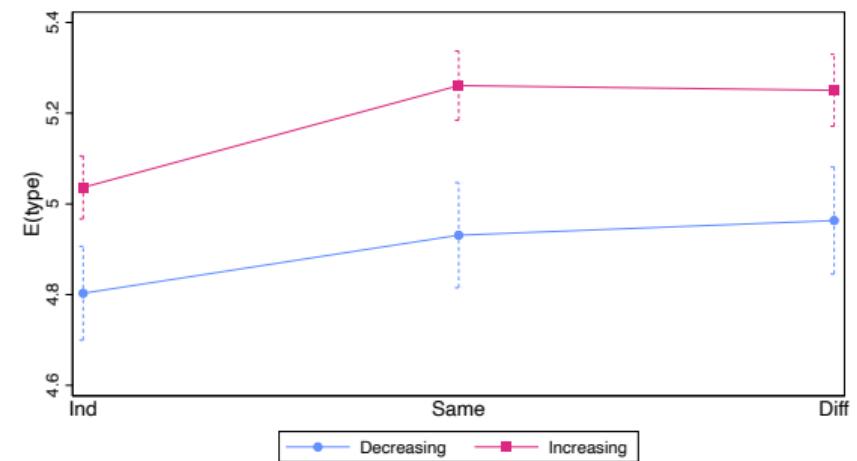
Design

Econometric strategy

Aggregate analysis

Sequence effects

Beliefs generally higher after increasing sequences.

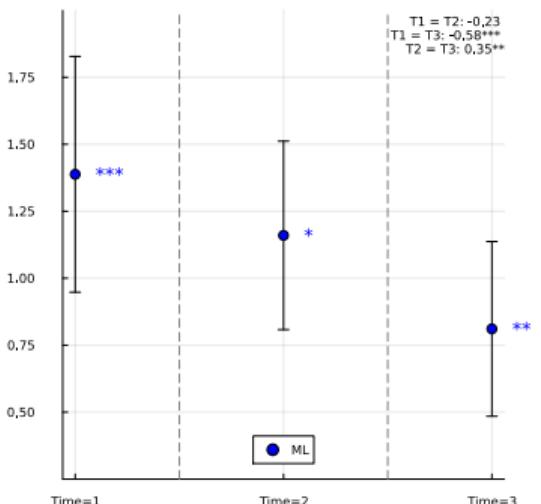


Model with time effects

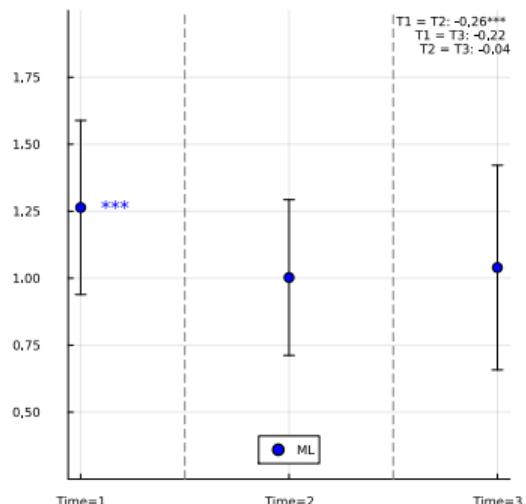
$$y_{it} = \begin{cases} [\beta_{Ci} + \beta_{Ct}]1[s_t = \text{correct}]s_{it} + [\beta_{li} + \beta_{lt}]1[s_t = \text{incorrect}]s_{it} + \epsilon_{it} & \text{if } t = 1 \\ [\beta_{Ci} + \beta_{Ct}]1[s_t = \text{correct}]s_{it} + [\beta_{li} + \beta_{lt}]1[s_t = \text{incorrect}]s_{it} + [\delta_i + \delta_t]y_{i,t-1} + \epsilon_{it} & \text{if } t > 1 \end{cases}$$

Time specific effects in *Individual*

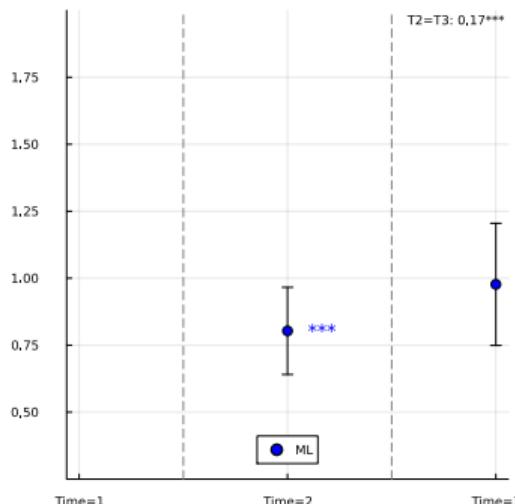
Panel 1: β_c



Panel 2: β_i



Panel 3: δ



Conclusion

- ▶ Study sequential updating, varying whether workers viewed in tandem or alone.
- ▶ Updating is time dependent.
- ▶ When pair comes from different groups, there is compression.
- ▶ Increasing sequences, for the most part, increase beliefs.

Appendix

Interface

Figures

Final belief: simplexes

Effect of signal order

Endogeneity

Other

Individual: Worker draw



3 Correct | 7 Incorrect

✓	✓	✓	✗	✗
✗	✗	✗	✗	✗

5 Correct | 5 Incorrect

✓	✓	✓	✓	✓
✗	✗	✗	✗	✗

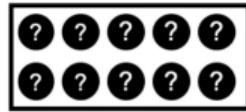
7 Correct | 3 Incorrect

✓	✓	✓	✓	✓	✓
✓	✓	✗	✗	✗	✗

Choose a worker

Observe signal

Worker #1 😊



Choose a ball ↴

Individual: Posterior

Worker #1 😊

Draw 1 Draw 2 Draw 3

Past Draws:

Evaluation (review instructions)
You have **100%** left to distribute.

What's the % chance Worker #1 is this worker?

3 Correct **7 Incorrect**

What's the % chance Worker #1 is this worker?

5 Correct **5 Incorrect**

What's the % chance Worker #1 is this worker?

7 Correct **3 Incorrect**

Signal draw for pairs

The image displays two identical mobile application interfaces side-by-side. Each interface features a top section with a white background containing a black outline of a hand pointing upwards. Below this is a dashed rectangular frame containing three rows of data, each row consisting of a purple smiley face icon, a performance summary box, and a grid of 10 small circles.

Left Screen Data:

- Row 1:** 3 Correct | 7 Incorrect
Grid: ✓✓✓✗✗✗✗✗✗
- Row 2:** 5 Correct | 5 Incorrect
Grid: ✓✓✓✓✓✗✗✗✗✗
- Row 3:** 7 Correct | 3 Incorrect
Grid: ✓✓✓✓✓✓✓✗✗✗

Bottom Center: Choose a worker

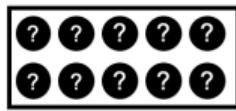
Right Screen Data:

- Row 1:** 3 Correct | 7 Incorrect
Grid: ✓✓✓✗✗✗✗✗✗
- Row 2:** 5 Correct | 5 Incorrect
Grid: ✓✓✓✓✓✗✗✗✗✗
- Row 3:** 7 Correct | 3 Incorrect
Grid: ✓✓✓✓✓✓✓✗✗✗

Bottom Center: Choose a worker

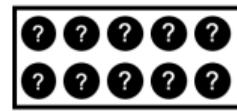
Posterior report for pairs

L Worker #1 😊



Choose a ball

R Worker #1 😊



Choose a ball

Appendix

Interface

Figures

Final belief: simplexes

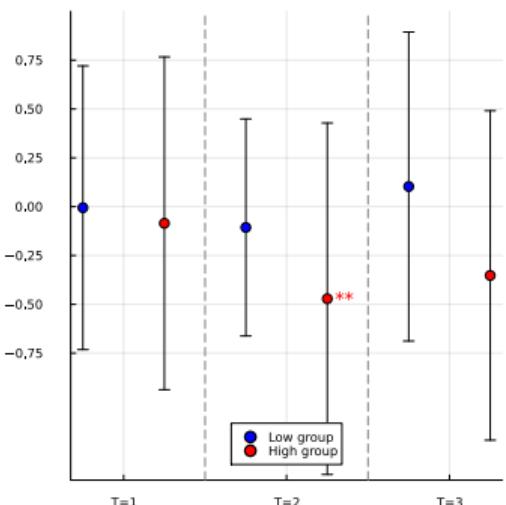
Effect of signal order

Endogeneity

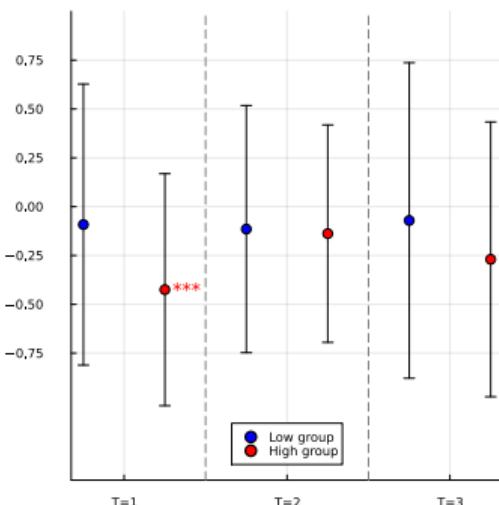
Other

Effect of partner from different group, by time

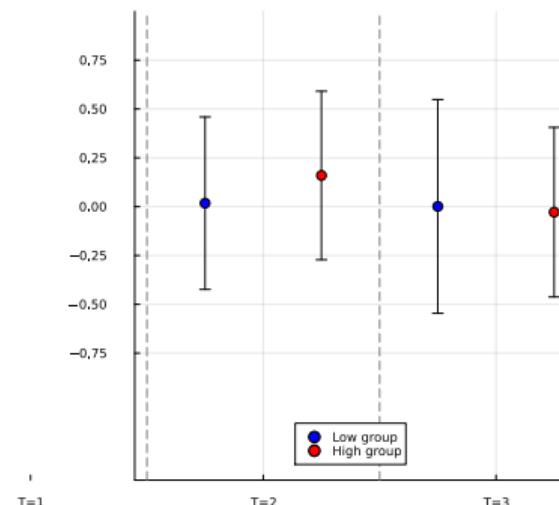
Panel 1: β_c



Panel 2: β_i

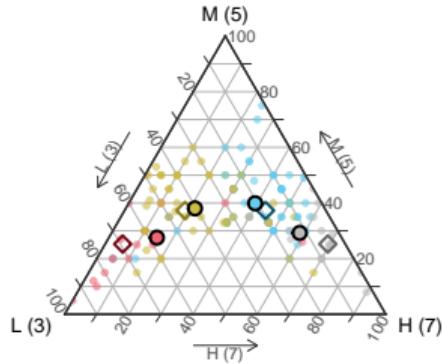


Panel 3: δ

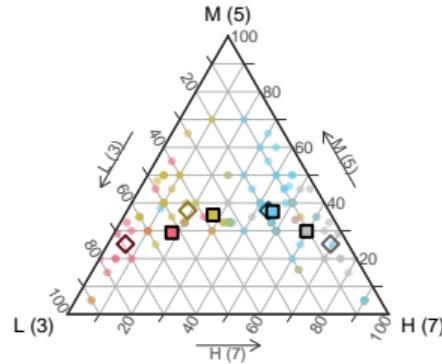


Scatter plot of final beliefs by treatment

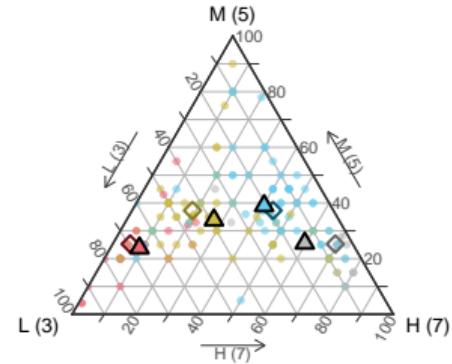
Medium ability (3, 5, 7), Individual



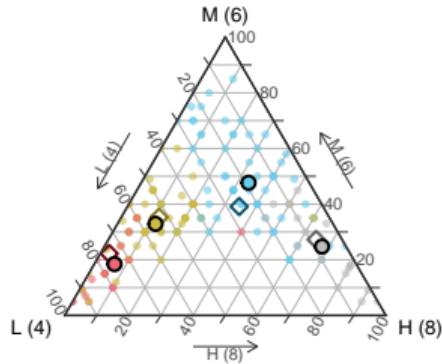
Medium ability (3, 5, 7), Same



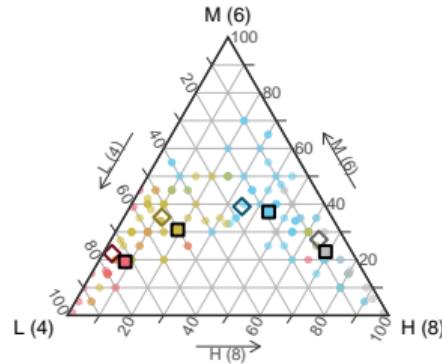
Medium ability (3, 5, 7), Different



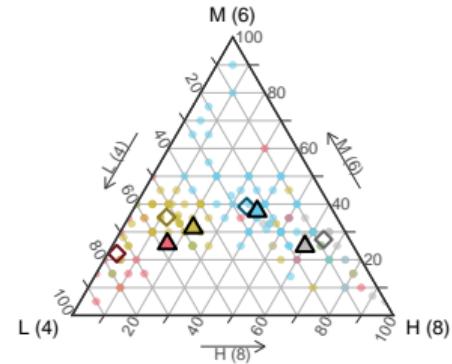
High ability (4, 6, 8), Individual



High ability (4, 6, 8), Same



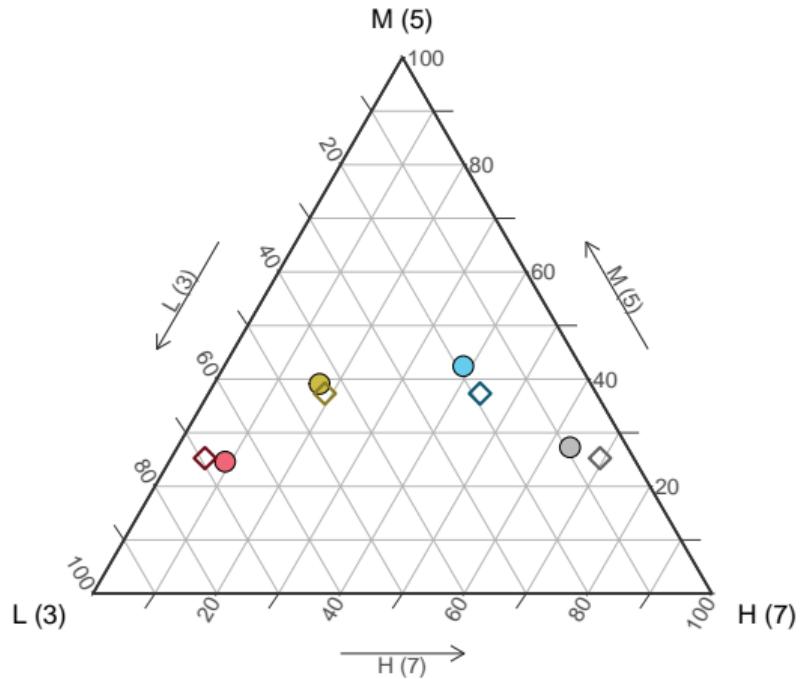
High ability (4, 6, 8), Different



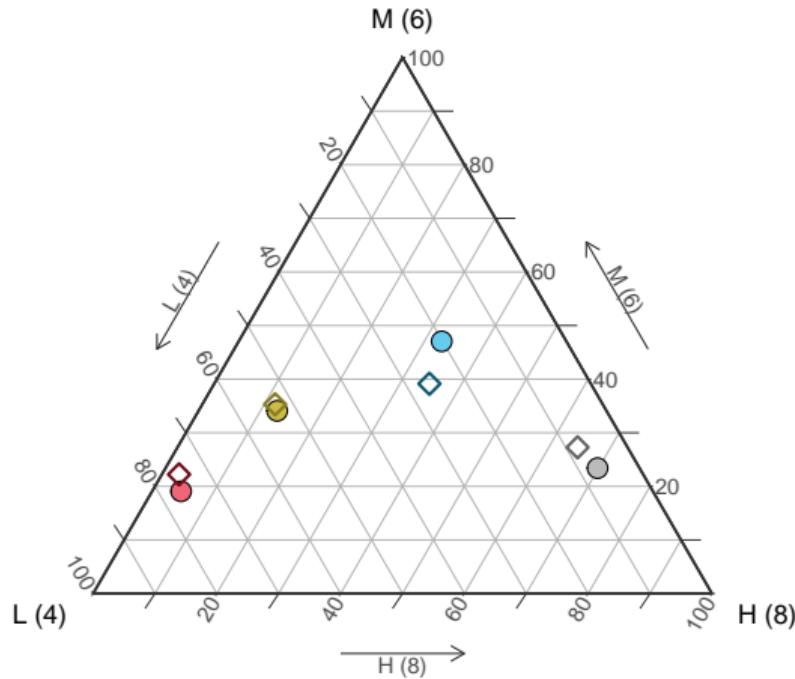
Signals ● 0/3 positive ○ 1/3 positive □ 2/3 positive ◑ 3/3 positive

Final belief in *Individual* treatment

Low group (scores of 3, 5, 7)



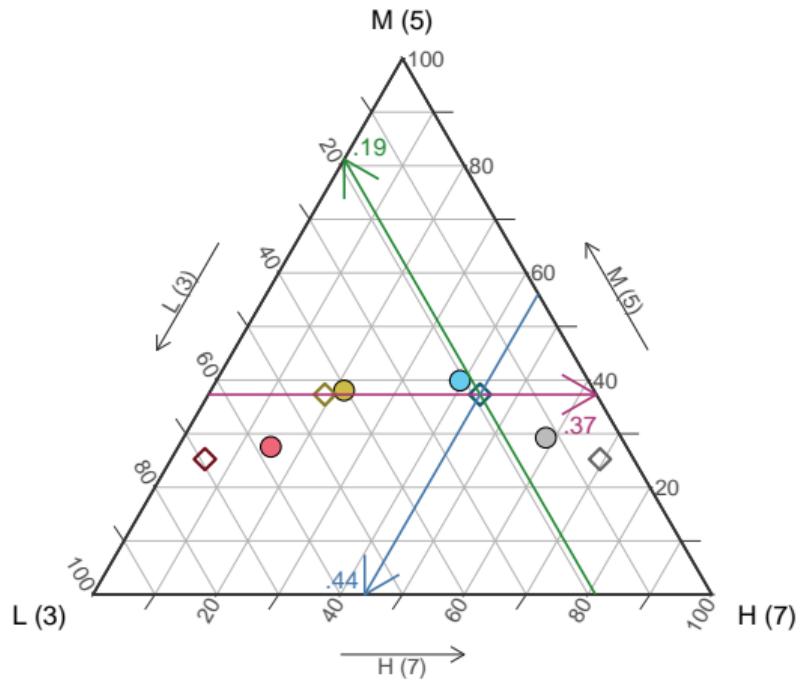
High group (scores of 4, 6, 8)



Signals ● 0/3 positive ● 1/3 positive ● 2/3 positive ● 3/3 positive

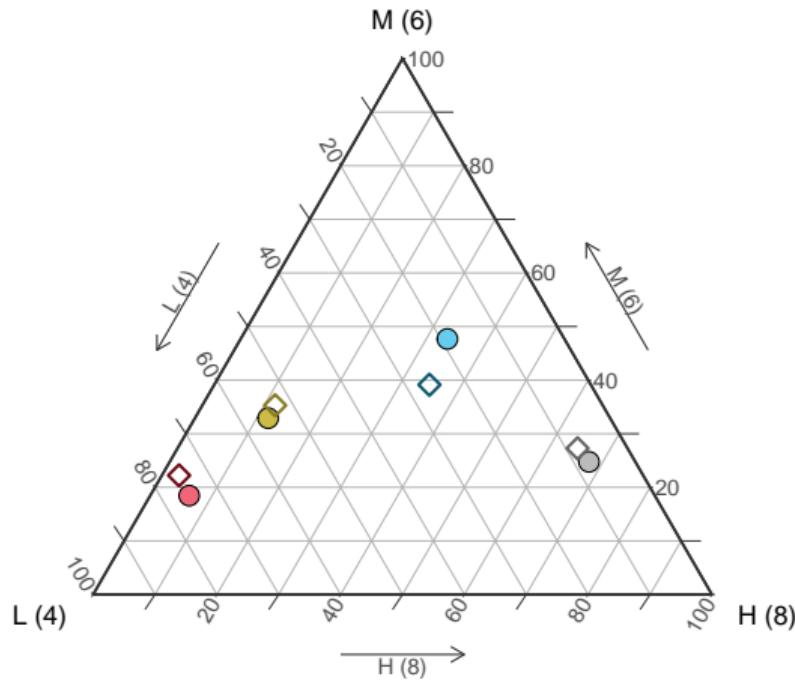
Final belief in *Individual* treatment

Medium ability group (scores of 3, 5, 7)



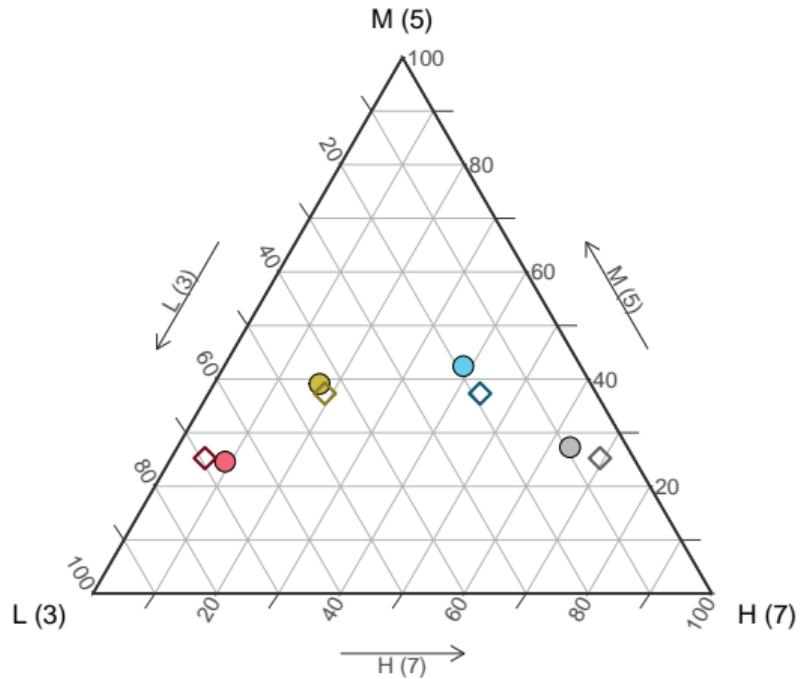
Signals ● 0/3 positive ● 1/3 positive ● 2/3 positive ● 3/3 positive

High ability group (scores of 4, 6, 8)

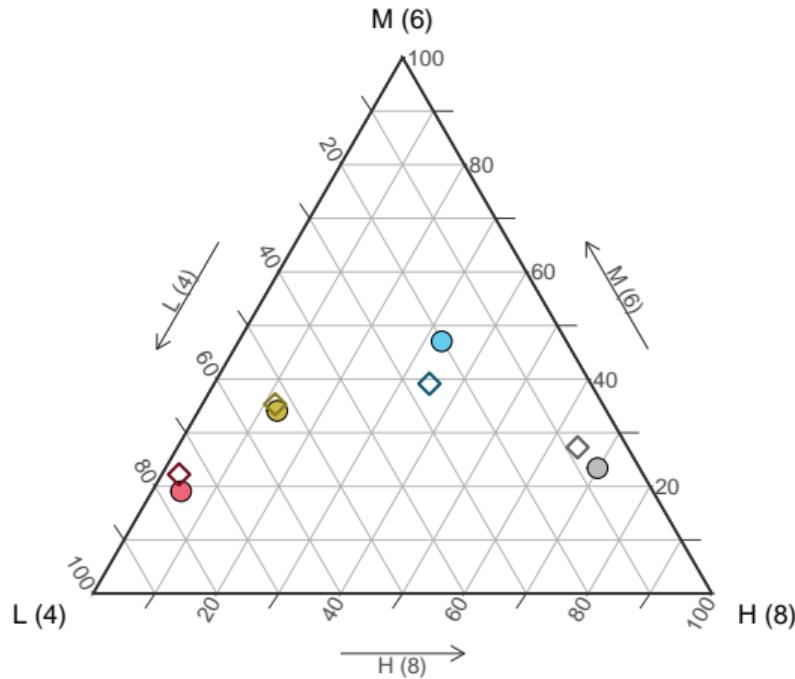


Final belief in *Individual* treatment

Low group (scores of 3, 5, 7)



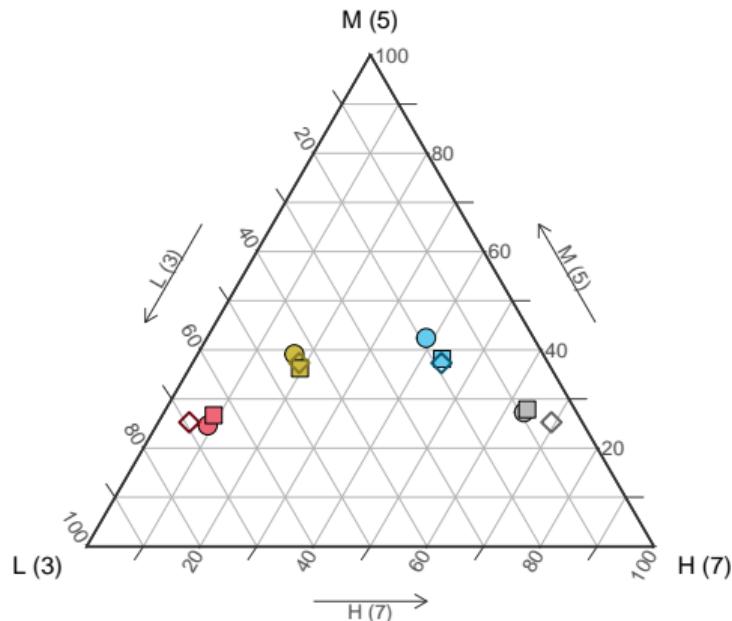
High group (scores of 4, 6, 8)



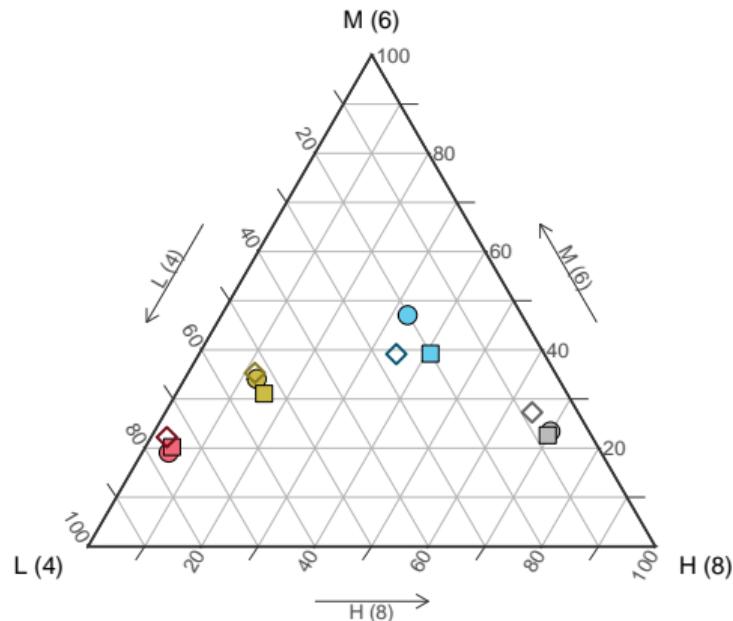
Signals ● 0/3 positive ● 1/3 positive ● 2/3 positive ● 3/3 positive

Effect of simultaneous evaluation on final belief

Low group (scores of 3, 5, 7)



High group (scores of 4, 6, 8)

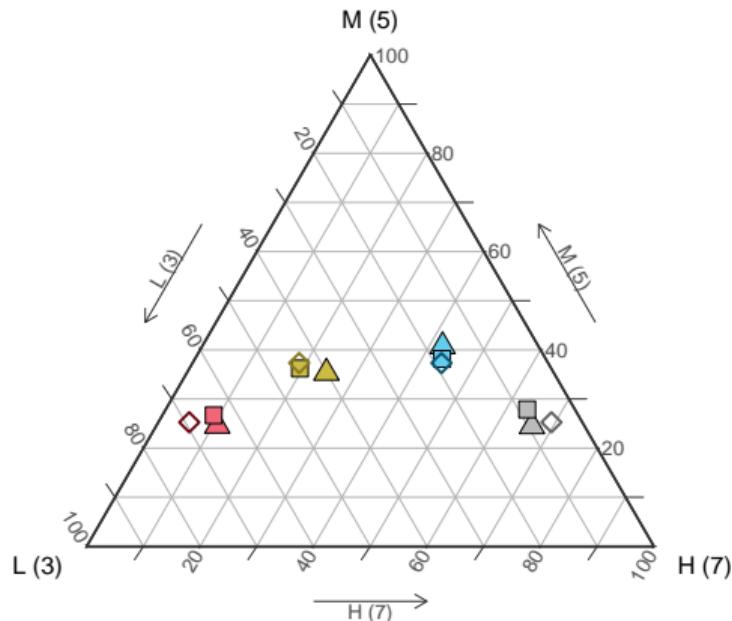


Signals ● 0/3 positive ○ 1/3 positive □ 2/3 positive ◇ 3/3 positive

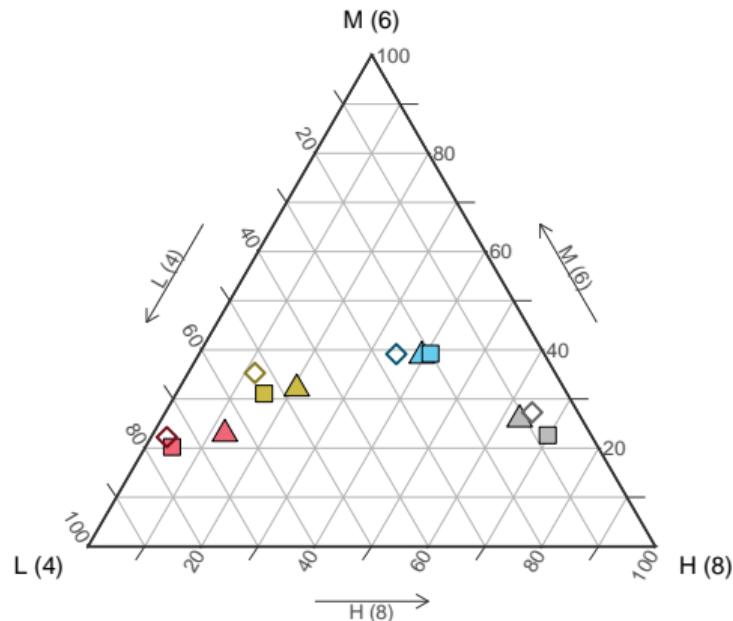
Treatment ○ Individual □ Same ◇ Bayesian

Effect of partner from different group on final belief

Low group (scores of 3, 5, 7)



High group (scores of 4, 6, 8)

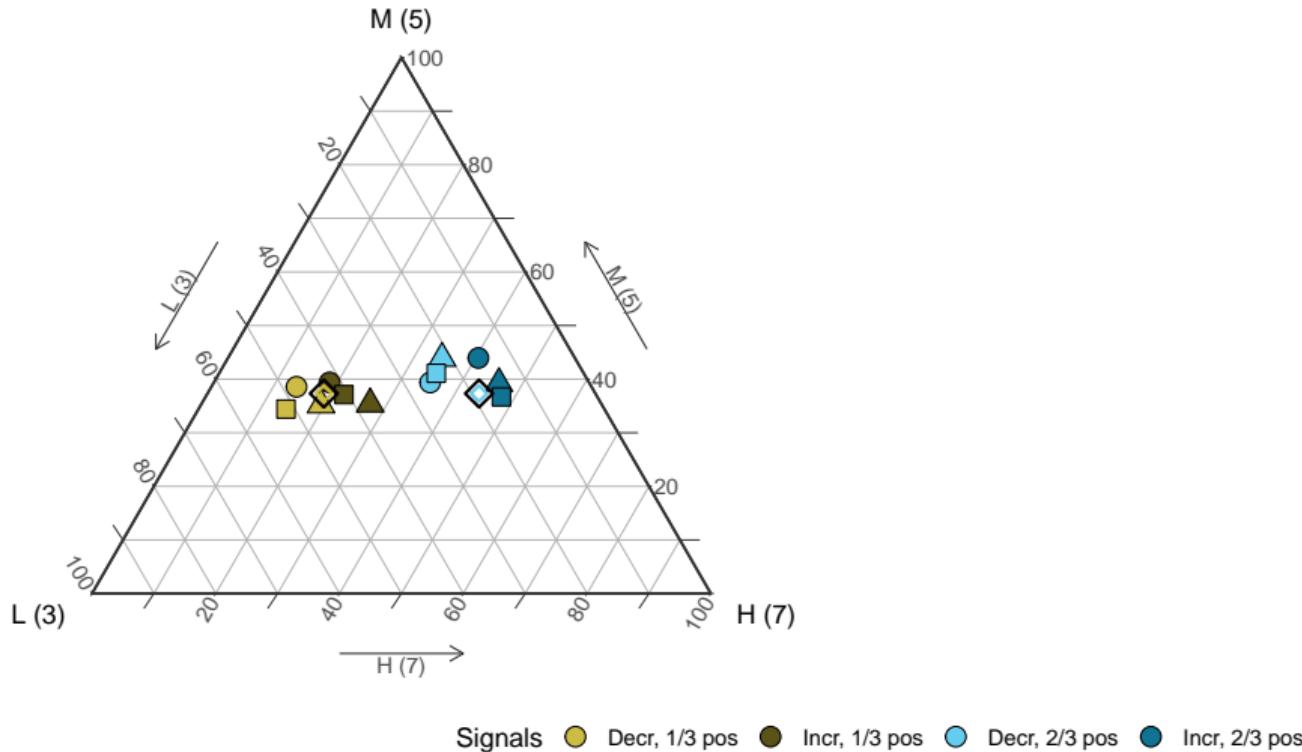


Signals ● 0/3 positive ○ 1/3 positive ▲ 2/3 positive ● 3/3 positive

Treatment □ Same △ Different ◇ Bayesian

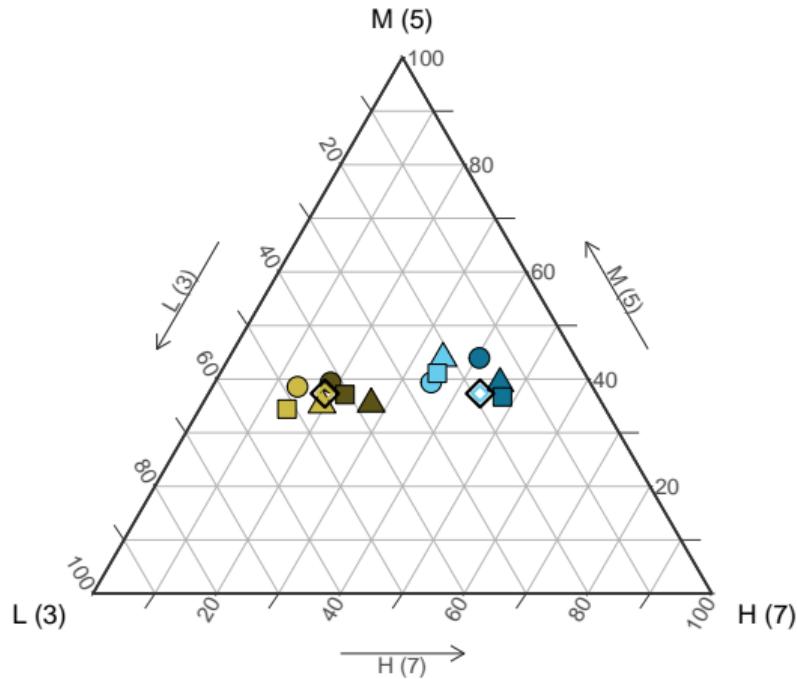
Generally, beliefs higher after increasing sequence.

Low group (scores of 3, 5, 7)

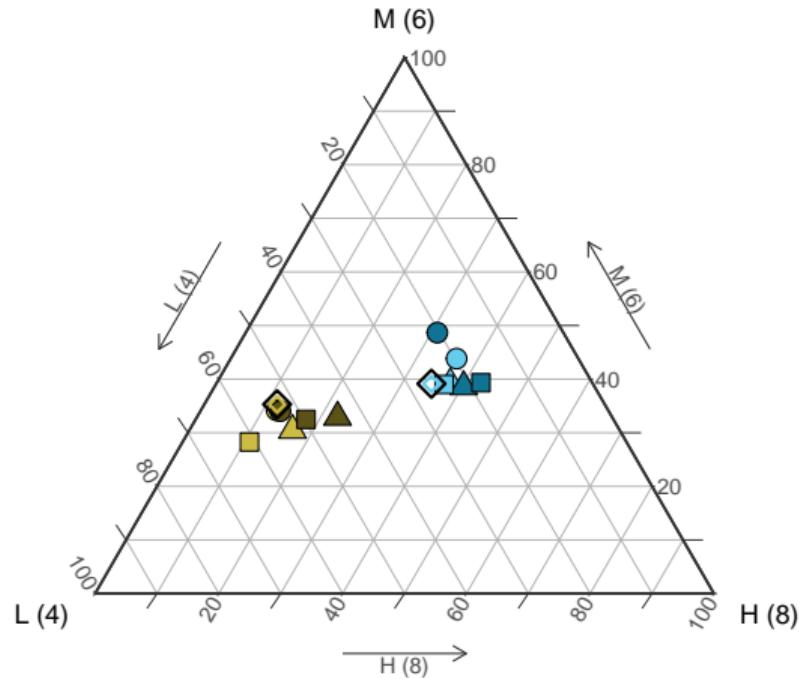


Generally, beliefs higher after increasing sequence.

Low group (scores of 3, 5, 7)



High group (scores of 4, 6, 8)



Signals ● Decr, 1/3 pos ● Incr, 1/3 pos ○ Decr, 2/3 pos ● Incr, 2/3 pos

Appendix

Interface

Figures

Final belief: simplexes

Effect of signal order

Endogeneity

Other

Order effect in *Individual* treatment by group.

	(1) P(L)	(2) P(M)	(3) P(H)
High ability (4, 6, 8)	-0.083** (0.032)	0.027 (0.027)	0.036 (0.034)
Increasing signals	-0.094*** (0.030)	0.038*** (0.012)	0.056** (0.025)
High ability \times increasing	0.120*** (0.038)	-0.020 (0.025)	-0.100** (0.039)
Constant	0.016 (0.049)	-1.143*** (0.346)	0.028 (0.038)
Effect of incr signals in high ability group (Incr + High \times incr)	0.026	0.018	-0.044
Control for Bayesian belief	✓	✓	✓
Observations	240	240	240

Order effect in individual vs pair treatments by group.

	Medium ability (3, 5, 7)			High ability (4, 6, 8)		
	(1) P(L)	(2) P(M)	(3) P(H)	(4) P(L)	(5) P(M)	(6) P(H)
Increasing signals	-0.094*** (0.030)	0.038*** (0.012)	0.056** (0.025)	0.026 (0.023)	0.018 (0.022)	-0.044 (0.030)
<i>Pair</i>	-0.021 (0.033)	0.006 (0.026)	0.014 (0.031)	0.045* (0.024)	-0.063*** (0.023)	0.018 (0.027)
Increasing \times <i>Pair</i>	0.023 (0.040)	-0.057*** (0.018)	0.034 (0.037)	-0.108*** (0.036)	-0.001 (0.025)	0.110** (0.043)
Effect of incr in <i>Pair</i> (Incr + Incr \times <i>Pair</i>)	-0.071***	-0.020	0.090***	-0.082***	0.017	0.066**
Control for Bayesian belief	✓	✓	✓	✓	✓	✓
Observations	360	360	360	360	360	360

Appendix

Interface

Figures

Final belief: simplexes

Effect of signal order

Endogeneity

Other

Endogeneity

- ▶ Evaluator i reports belief about worker w in time t .
- ▶ For a given worker, t goes from 1 to 3.
- ▶ Suppose true model is each i has fixed weights on prior & data:

$$\begin{aligned} \log \left(\frac{P(score_j|signal)_{i,w,t}}{P(score_k|signal)_{i,w,t}} \right) = \\ \beta_{Ci} * 1[s_t = \text{correct}] * \log \underbrace{\left(\frac{P(\text{correct}|score_j)_{i,w,t}}{P(\text{correct}|score_k)_{i,w,t}} \right)}_{\text{LLR if signal in } t \text{ incorrect}} \\ + \beta_{Ii} * 1[s_t = \text{incorrect}] * \log \left(\frac{P(\text{incorrect}|score_j)_{i,w,t}}{P(\text{incorrect}|score_k)_{i,w,t}} \right) \\ + \delta_i * \overbrace{\log \left(\frac{P(score_j)_{i,w,t-1}}{P(score_k)_{i,w,t-1}} \right)}^{\text{log prior odds}} + \eta_{i,w,t} \end{aligned}$$

- ▶ In round 1, log prior odds exogenously set to 0. In round 2 and 3, use lagged reported posterior.
- ▶ ϵ is i.i.d mean zero noise.

Endogeneity

$$\begin{aligned} & \overbrace{\log \left(\frac{P(score_j|signal)_{i,w,t}}{P(score_k|signal)_{i,w,t}} \right)}^{\text{log posterior odds}} = \\ & \beta_{Ci} * 1[s_t = \text{correct}] * \overbrace{\log \left(\frac{P(correct|score_j)_{i,w,t}}{P(correct|score_k)_{i,w,t}} \right)}^{\text{LLR if signal in } t \text{ correct}} \\ & + \beta_{li} * 1[s_t = \text{incorrect}] * \overbrace{\log \left(\frac{P(incorrect|score_j)_{i,w,t}}{P(incorrect|score_k)_{i,w,t}} \right)}^{\text{LLR if signal in } t \text{ incorrect}} \\ & + \delta_i * \overbrace{\log \left(\frac{P(score_j)_{i,w,t-1}}{P(score_k)_{i,w,t-1}} \right)}^{\text{log prior odds}} + \eta_{i,w,t} \end{aligned}$$

- ▶ In round 1, log prior odds exogenously set to 0.
- ▶ In round 2 and 3, use lagged reported posterior.

Endogeneity

- ▶ Say that we just run the population regression:

$$\begin{aligned} \log \left(\frac{P(score_j|signal)_{i,w,t}}{P(score_k|signal)_{i,w,t}} \right) = \\ \beta_C * 1[s_t = correct] * \log \left(\frac{P(correct|score_j)_{i,w,t}}{P(correct|score_k)_{i,w,t}} \right) \\ + \beta_I * 1[s_t = incorrect] * \log \left(\frac{P(incorrect|score_j)_{i,w,t}}{P(incorrect|score_k)_{i,w,t}} \right) \\ + \delta * \log \left(\frac{P(score_j)_{i,w,t-1}}{P(score_k)_{i,w,t-1}} \right) + \epsilon_{i,w,t} \end{aligned}$$

- ▶ In this case, $\epsilon_{i,w,t}$ contains δ_i , β_{Ci} , β_{Ii} .
- ▶ In rounds 2 and 3 about a worker, δ_i , β_{Ci} , β_{Ii} will not be randomly assigned with respect to the prior: this is because use lagged posterior, which itself contains δ_i , β_{Ci} , β_{Ii} .
- ▶ Means $E[\epsilon_{i,w,t} | \mathbf{X}] \neq 0$.

Appendix

Interface

Figures

Final belief: simplexes

Effect of signal order

Endogeneity

Other

$$\begin{aligned}
& \left(\frac{1}{NT} \sum_i \sum_t \begin{bmatrix} s_{it}^2 & s_{it} p_{it} \\ s_{it} p_{it} & p_{it}^2 \end{bmatrix} \right)^{-1} \times \frac{1}{NT} \sum_i \sum_t [s_{it} \quad p_{it}] y_{it} = \\
& \left(\frac{1}{NT} \sum_i \sum_t \begin{bmatrix} s_{it}^2 & s_{it} p_{it} \\ s_{it} p_{it} & p_{it}^2 \end{bmatrix} \right)^{-1} \times \frac{1}{NT} \sum_i \sum_t \begin{bmatrix} s_{it} \\ p_{it} \end{bmatrix} \begin{bmatrix} s_{it} & p_{it} \\ p_{it} & \beta_i + \beta_t \end{bmatrix} \neq \\
& \left(\frac{1}{NT} \sum_i \sum_t \begin{bmatrix} s_{it}^2 & s_{it} p_{it} \\ s_{it} p_{it} & p_{it}^2 \end{bmatrix} \right)^{-1} \times \frac{1}{NT} \sum_i \sum_t \begin{bmatrix} s_{it} \\ p_{it} \end{bmatrix} [s_{it} \quad p_{it}] \frac{1}{NT} \sum_i \sum_t \begin{bmatrix} \beta_i + \beta_t \\ \delta_i + \delta_t \end{bmatrix}
\end{aligned}$$

- If $X'X$ only contains signals, this will factor.

IV doesn't fix it

$$\left(\frac{1}{NT} \sum_i \sum_t \begin{bmatrix} s_{it}^2 & s_{it} \textcolor{blue}{piv}_{it} \\ s_{it} \textcolor{blue}{piv}_{it} & \textcolor{blue}{piv}_{it}^2 \end{bmatrix} \right)^{-1} \times \frac{1}{NT} \sum_i \sum_t \begin{bmatrix} s_{it} \\ \textcolor{blue}{piv}_{it} \end{bmatrix} \begin{bmatrix} s_{it} & \textcolor{blue}{piv}_{it} \\ \textcolor{blue}{piv}_{it} & \delta_i + \delta_t \end{bmatrix}$$

- ▶ This is true whether piv_{it} comes from instrumenting with Bayesian belief, or instrumenting i 's prior today with i 's belief in $t - 1$.

▶ Back

Example ASVAB questions

Math

How many 15 passenger vans will it take to drive all 52 members of the football team to the stadium?

- 2
- 3
- 4
- 5

Verbal

Banal most nearly means

- commonplace
- forceful
- tranquil
- indifferent

Science

When water is taken apart by electricity, what two substances are formed?

- Carbon and oxygen
- Hydrogen and oxygen
- Oxygen and nitrogen
- Hydrogen and nitrogen