

# SUPPORT VECTOR MACHINES

**Scholastic Video Book Series**

Part 1

Linear Support Vector Machines (LSVM)

(with English Narrations)

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# **Support Vector Machines - #1**

## **Liner Support Vector Machines (LSVM)**

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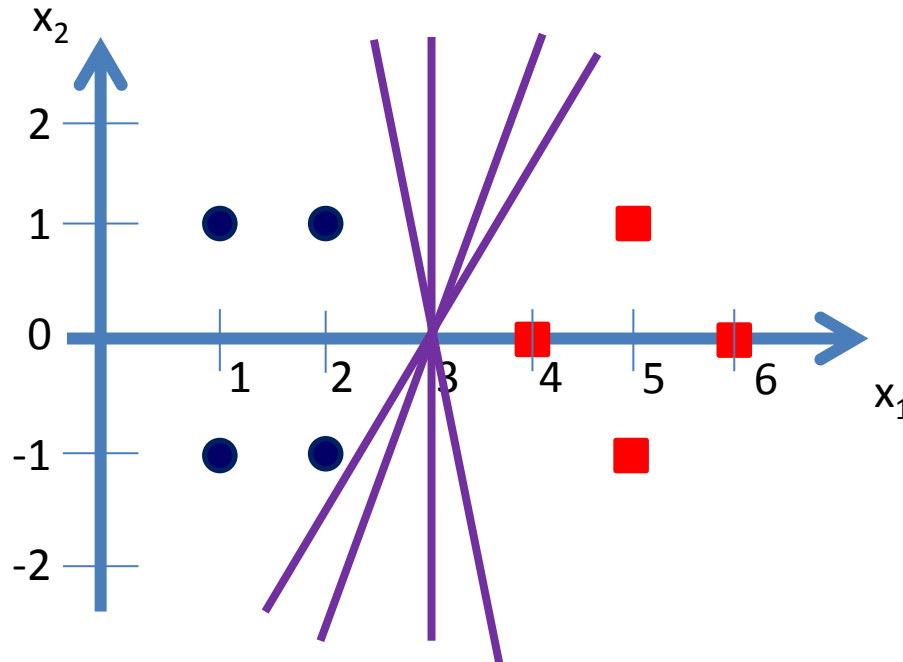
<http://youtu.be/LXGaYVXkGtg>

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(SVM-001)

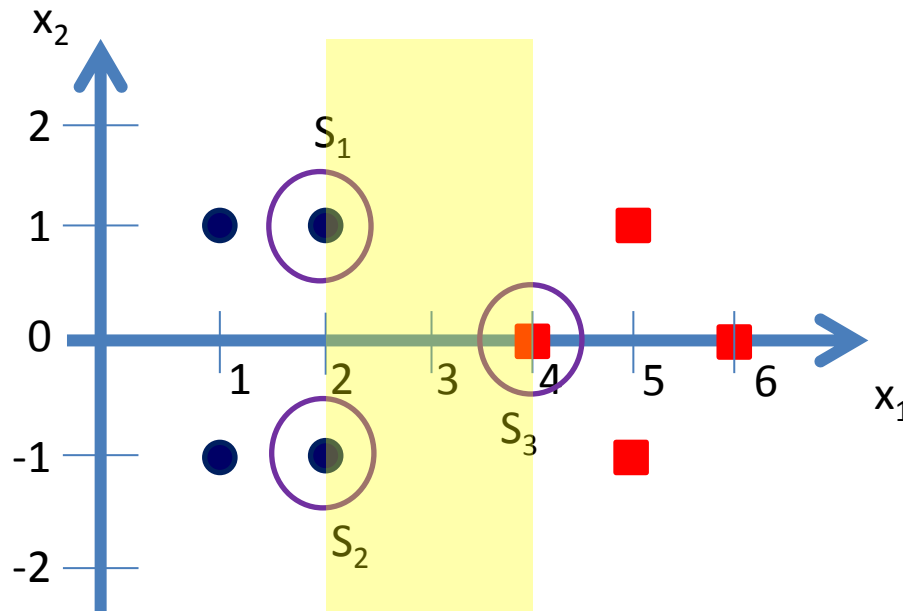
## Support Vector Machines

- Support Vector Machine (SVM) algorithms are used in Classification.
- Classification can be viewed as the task of separating classes in feature space.



# Support Vector Machines

- Here we select 3 Support Vectors to start with.
- They are  $S_1$ ,  $S_2$  and  $S_3$ .



$$S_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$S_2 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$$S_3 = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$$

Click here to see the video

<http://youtu.be/LXGaYVXkGtg>

## Support Vector Machines

- Here we will use vectors augmented with a 1 as a bias input, and for clarity we will differentiate these with an over-tilde.

That is:

$$s_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

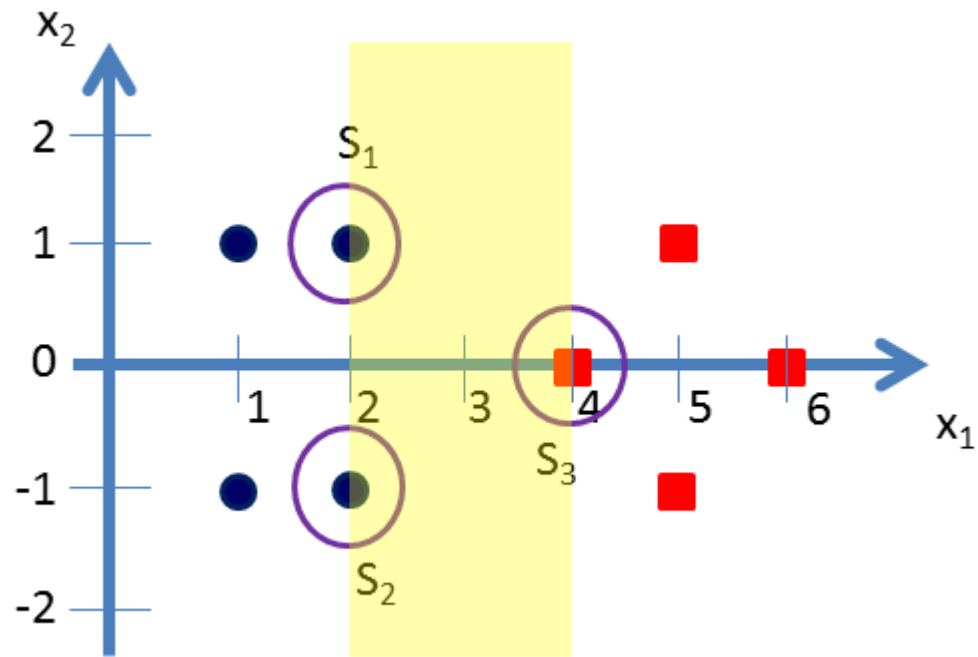
$$s_2 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$$s_3 = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$$

$$\tilde{s}_1 = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$

$$\tilde{s}_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

$$\tilde{s}_3 = \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix}$$



- Now we need to find 3 parameters  $\alpha_1$ ,  $\alpha_2$ , and  $\alpha_3$  based on the following 3 linear equations:

$$\alpha_1 \widetilde{S}_1 \cdot \widetilde{S}_1 + \alpha_2 \widetilde{S}_2 \cdot \widetilde{S}_1 + \alpha_3 \widetilde{S}_3 \cdot \widetilde{S}_1 = -1 \quad (-ve \text{ class})$$

$$\alpha_1 \widetilde{S}_1 \cdot \widetilde{S}_2 + \alpha_2 \widetilde{S}_2 \cdot \widetilde{S}_2 + \alpha_3 \widetilde{S}_3 \cdot \widetilde{S}_2 = -1 \quad (-ve \text{ class})$$

$$\alpha_1 \widetilde{S}_1 \cdot \widetilde{S}_3 + \alpha_2 \widetilde{S}_2 \cdot \widetilde{S}_3 + \alpha_3 \widetilde{S}_3 \cdot \widetilde{S}_3 = +1 \quad (+ve \text{ class})$$

$$\alpha_1 \widetilde{S}_1 \cdot \widetilde{S}_1 + \alpha_2 \widetilde{S}_2 \cdot \widetilde{S}_1 + \alpha_3 \widetilde{S}_3 \cdot \widetilde{S}_1 = -1 \text{ (-ve class)}$$

$$\alpha_1 \widetilde{S}_1 \cdot \widetilde{S}_2 + \alpha_2 \widetilde{S}_2 \cdot \widetilde{S}_2 + \alpha_3 \widetilde{S}_3 \cdot \widetilde{S}_2 = -1 \text{ (-ve class)}$$

$$\alpha_1 \widetilde{S}_1 \cdot \widetilde{S}_3 + \alpha_2 \widetilde{S}_2 \cdot \widetilde{S}_3 + \alpha_3 \widetilde{S}_3 \cdot \widetilde{S}_3 = +1 \text{ (+ve class)}$$

- Let's substitute the values for  $\widetilde{S}_1$ ,  $\widetilde{S}_2$  and  $\widetilde{S}_3$  in the above equations.

$$\widetilde{S}_1 = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \quad \widetilde{S}_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \quad \widetilde{S}_3 = \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix}$$

$$\alpha_1 \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + \alpha_2 \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + \alpha_3 \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = -1$$

$$\alpha_1 \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} + \alpha_2 \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} + \alpha_3 \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} = -1$$

$$\alpha_1 \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} + \alpha_2 \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} + \alpha_3 \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} = +1$$

$$\alpha_1 \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + \alpha_2 \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + \alpha_3 \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = -1$$

$$\alpha_1 \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} + \alpha_2 \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} + \alpha_3 \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} = -1$$

$$\alpha_1 \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} + \alpha_2 \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} + \alpha_3 \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} = +1$$

- After simplification we get:

$$6\alpha_1 + 4\alpha_2 + 9\alpha_3 = -1$$

$$4\alpha_1 + 6\alpha_2 + 9\alpha_3 = -1$$

$$9\alpha_1 + 9\alpha_2 + 17\alpha_3 = +1$$

- Simplifying the above 3 simultaneous equations we get:  $\alpha_1 = \alpha_2 = -3.25$  and  $\alpha_3 = 3.5$ .



$$\alpha_1 = \alpha_2 = -3.25 \text{ and } \alpha_3 = 3.5$$

$$\begin{aligned}\tilde{S}_1 &= \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \\ \tilde{S}_2 &= \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \\ \tilde{S}_3 &= \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix}\end{aligned}$$

- The hyper plane that discriminates the positive class from the negative class is give by:

$$\tilde{w} = \sum_i \alpha_i \tilde{S}_i$$

- Substituting the values we get:

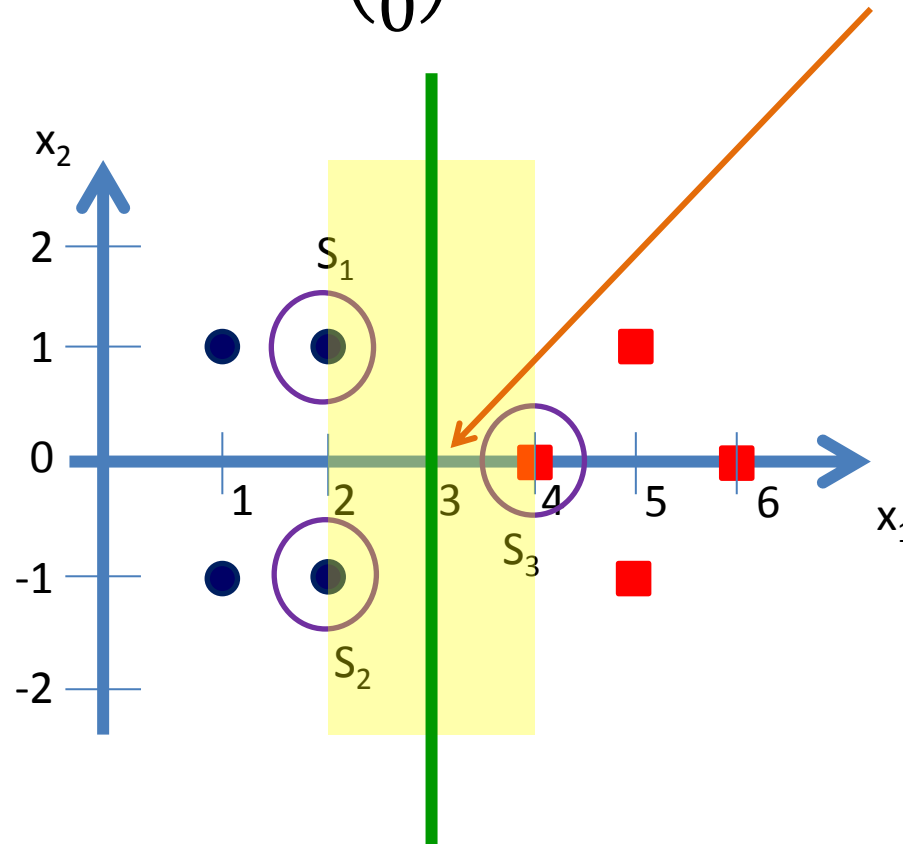
$$\begin{aligned}\tilde{w} &= \alpha_1 \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + \alpha_2 \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} + \alpha_3 \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} \\ \tilde{w} &= (-3.25) \cdot \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + (-3.25) \cdot \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} + (3.5) \cdot \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -3 \end{pmatrix}\end{aligned}$$

$$\tilde{w} = (-3.25) \cdot \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + (-3.25) \cdot \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} + (3.5) \cdot \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -3 \end{pmatrix}$$

- Our vectors are augmented with a bias.
- Hence we can equate the entry in  $\tilde{w}$  as the hyper plane with an offset  $b$ .
- Therefore the separating hyper plane equation  $y = wx + b$  with  $w = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$  and offset  $b = -3$ .

## Support Vector Machines

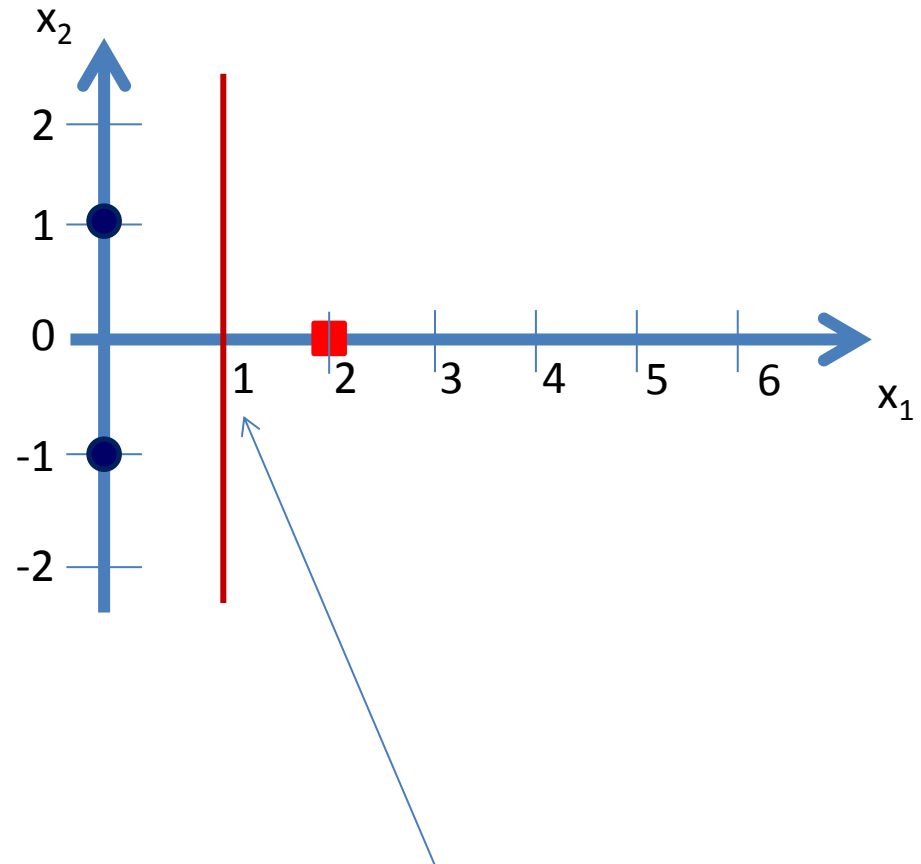
- $y = wx + b$  with  $w = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$  and offset  $b = -3$ .



- This is the expected decision surface of the LSVM.

# Support Vector Machines – Programme Code Example 1

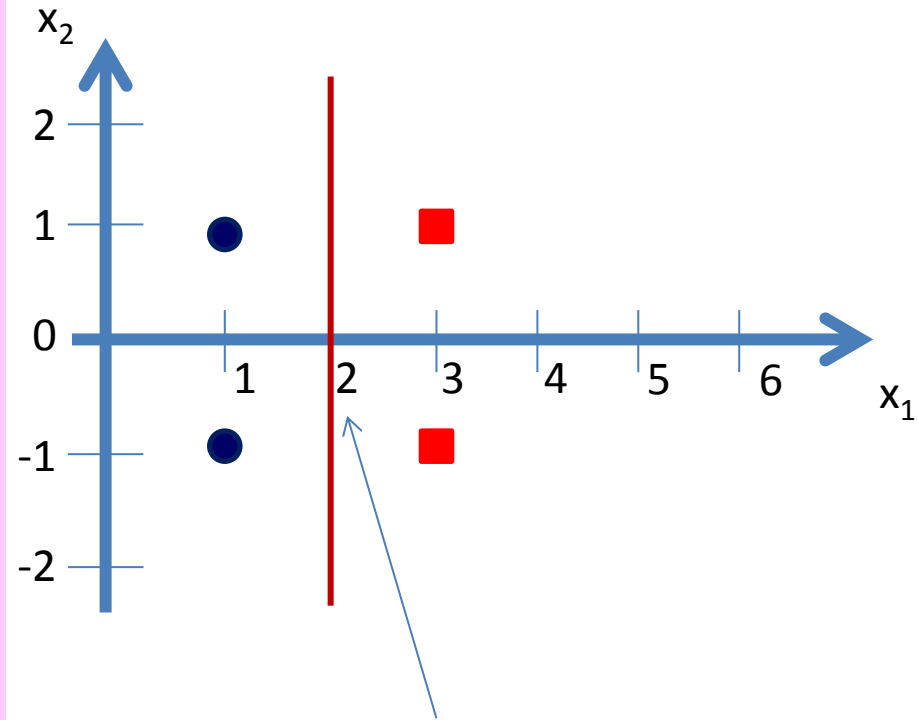
```
% 3 support vector version
•
•
• s1 = [ 0 -1 1 ];
• s2 = [ 0 1 1 ];
• s3 = [ 2 0 1 ];
•
• A = [ sum(s1.*s1) sum(s2.*s1) sum(s3.*s1) ;
•       sum(s1.*s2) sum(s2.*s2) sum(s3.*s2) ;
•       sum(s1.*s3) sum(s2.*s3) sum(s3.*s3) ]
• Y = [ -1 -1 +1 ]
• X = Y/A
•
• p = X(1)
• q = X(2)
• r = X(3)
•
• W = [ p*s1 + q*s2 + r*s3 ]
```



When you run you should get:  $\tilde{W} = [ 1 \ 0 \ -1 ]$ . This is a vertical line passing through  $x_1=1$ .

# Support Vector Machines - Programme Code Examples 2

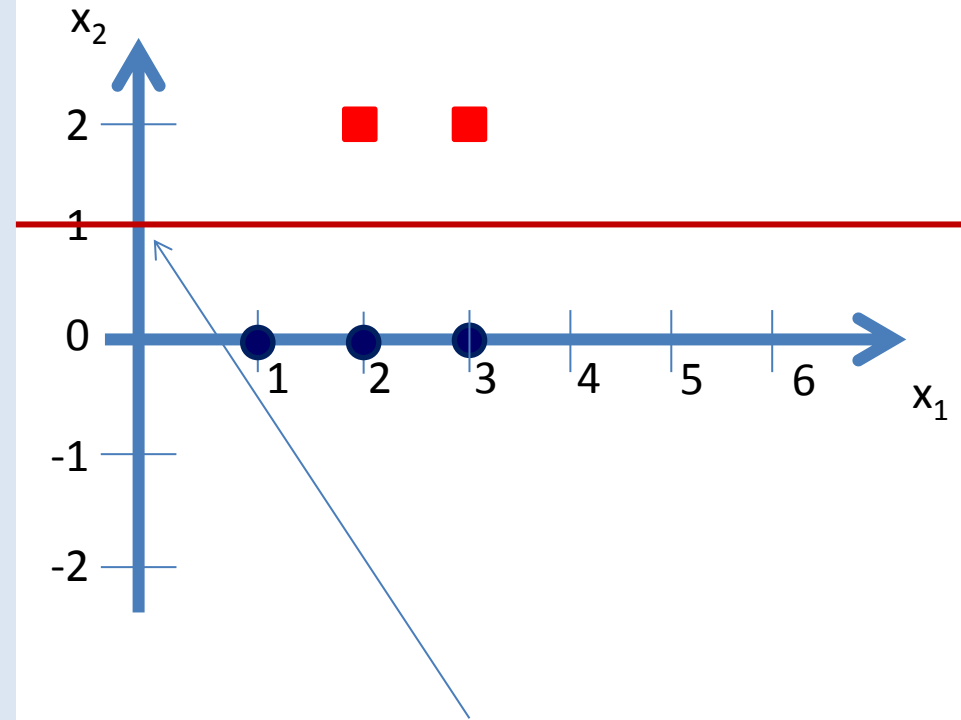
```
% 4 support vector version
•
•
• s1 = [ 1 1 1 ];
• s2 = [ 1 -1 1 ];
• s3 = [ 3 -1 1 ];
• s4 = [ 3 1 0 ];
•
• A = [ sum(s1.*s1) sum(s2.*s1) sum(s3.*s1) sum(s4.*s1);
•       sum(s1.*s2) sum(s2.*s2) sum(s3.*s2) sum(s4.*s1);
•       sum(s1.*s3) sum(s2.*s3) sum(s3.*s3) sum(s4.*s3);
•       sum(s1.*s4) sum(s2.*s4) sum(s3.*s4) sum(s4.*s4);]
• Y = [ -1 -1 +1 +1 ]
• X = Y/A
•
• p = X(1)
• q = X(2)
• r = X(3)
• s = X(4)
•
• W = [ p*s1 + q*s2 + r*s3 + s*s4 ]
```



When you run you should get:  $\tilde{w} = [ 1 \ 0 \ -2 ]$ . This is a vertical line passing through  $x_1=2$ .

# Support Vector Machines - Programme Code Example 3

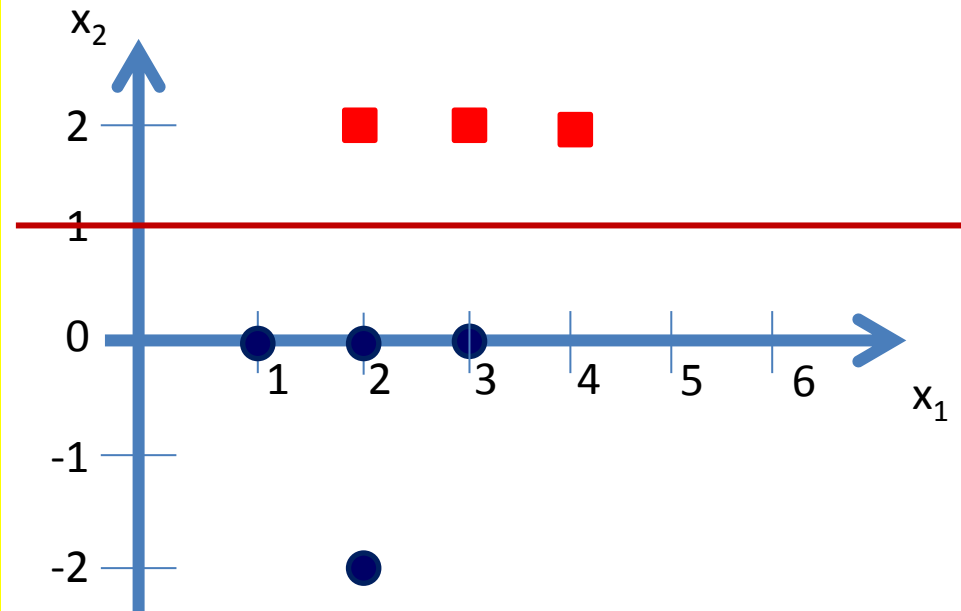
```
% 5 support vector version
.
.
s1 = [ 1 0 1 ];
s2 = [ 2 0 1 ];
s3 = [ 3 0 1 ];
s4 = [ 2 2 1 ];
s5 = [ 3 2 1 ];
.
A = [ sum(s1.*s1) sum(s2.*s1) sum(s3.*s1) sum(s4.*s1)
      sum(s5.*s1);
      sum(s1.*s2) sum(s2.*s2) sum(s3.*s2) sum(s4.*s2) sum(s5.*s2);
      sum(s1.*s3) sum(s2.*s3) sum(s3.*s3) sum(s4.*s3) sum(s5.*s3);
      sum(s1.*s4) sum(s2.*s4) sum(s3.*s4) sum(s4.*s4) sum(s5.*s4);
      sum(s1.*s5) sum(s2.*s5) sum(s3.*s5) sum(s4.*s5) sum(s5.*s5) ];
Y = [ -1 -1 -1 +1 +1 ]
X = Y/A
.
p = X(1)
q = X(2)
r = X(3)
s = X(4)
t = X(5)
.
W = [ p*s1 + q*s2 + r*s3 + s*s4 + t*s5 ]
```



When you run you should get:  $\tilde{w} = [0 \ 1 \ -1]$ . This is a horizontal line passing through  $x_2=1$ .

# Support Vector Machines – Classification Examples

- Let's take the 5 support vector version
- $\tilde{w} = [0 \ 1 \ -1]$ . This is a horizontal line passing through  $x_2=1$ .
- Let's classify the point  $(x_1, x_2)=(4, 2)$ .
- $w \cdot x = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 2 \end{pmatrix} = 2 > 1$
- Hence this point belongs to the red class
- Let's classify the point  $(x_1, x_2)=(2, -2)$ .
- $w \cdot x = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -2 \end{pmatrix} = -2 < 1$
- Hence this point belongs to the blue class
- We can do the same for any new point.



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Linear Support Vector Machines (LSVM)

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