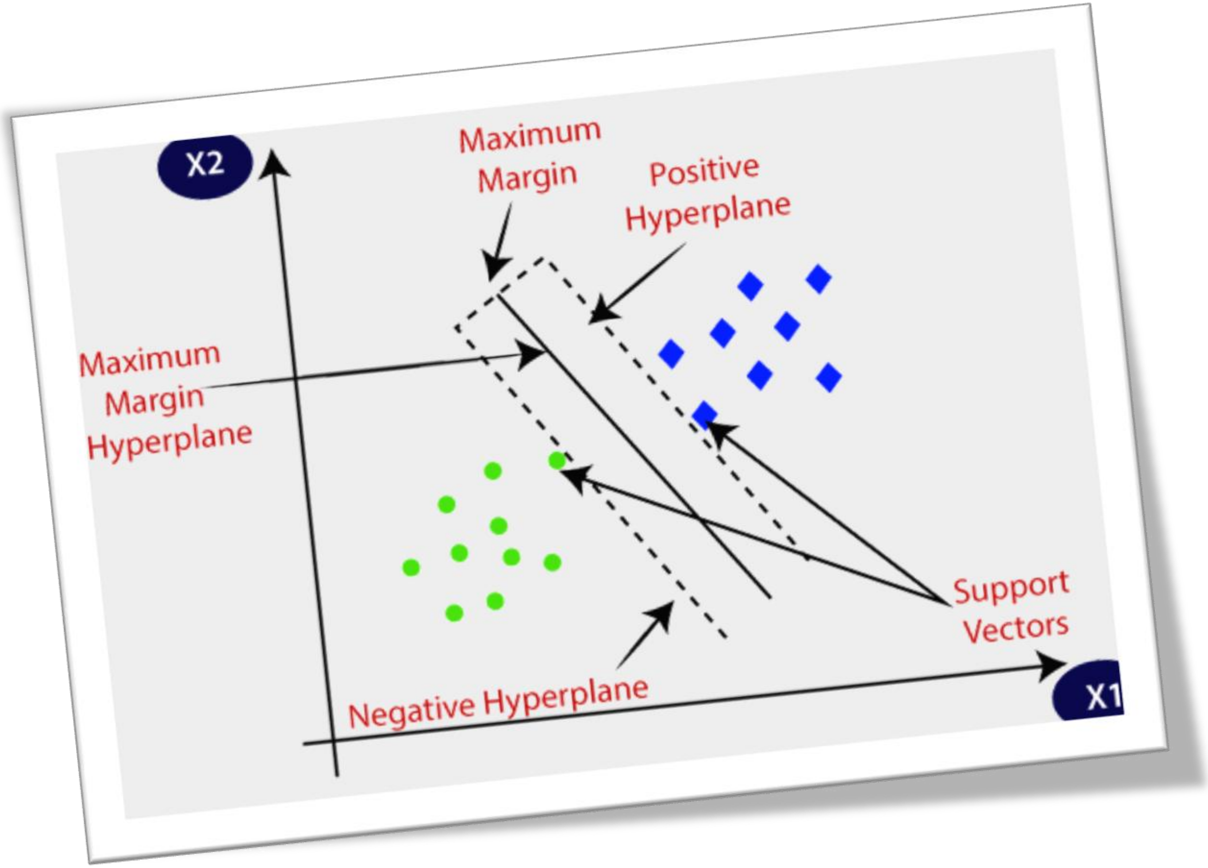
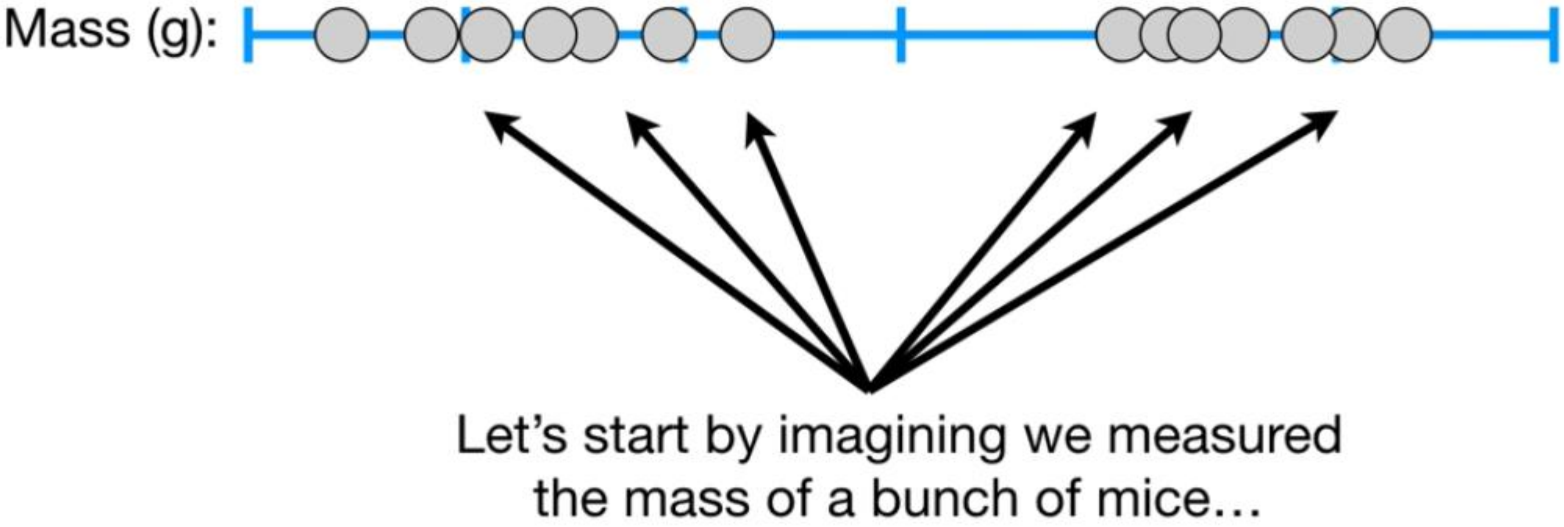
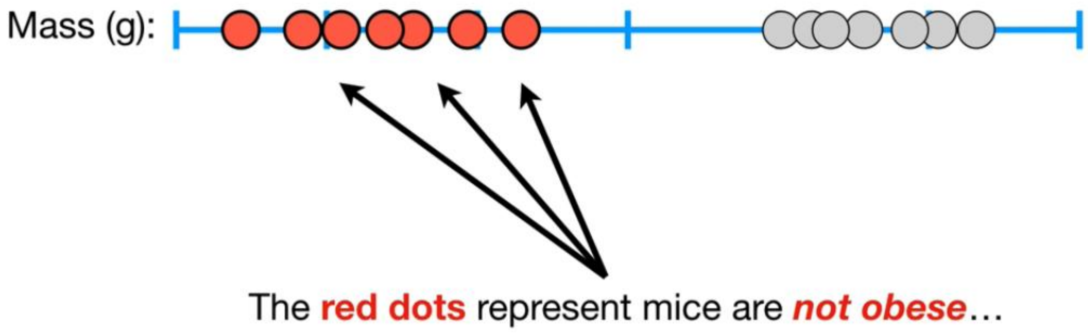


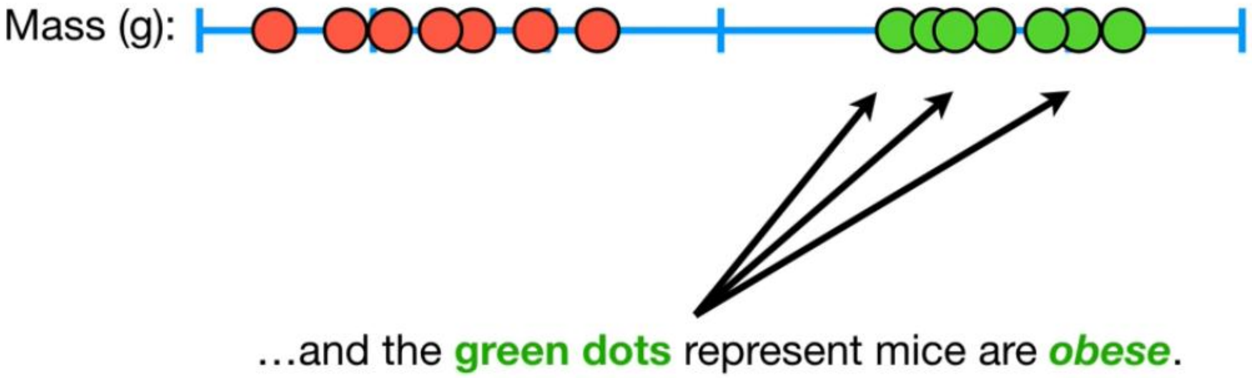
Support Vector Regression

Support Vector Machine

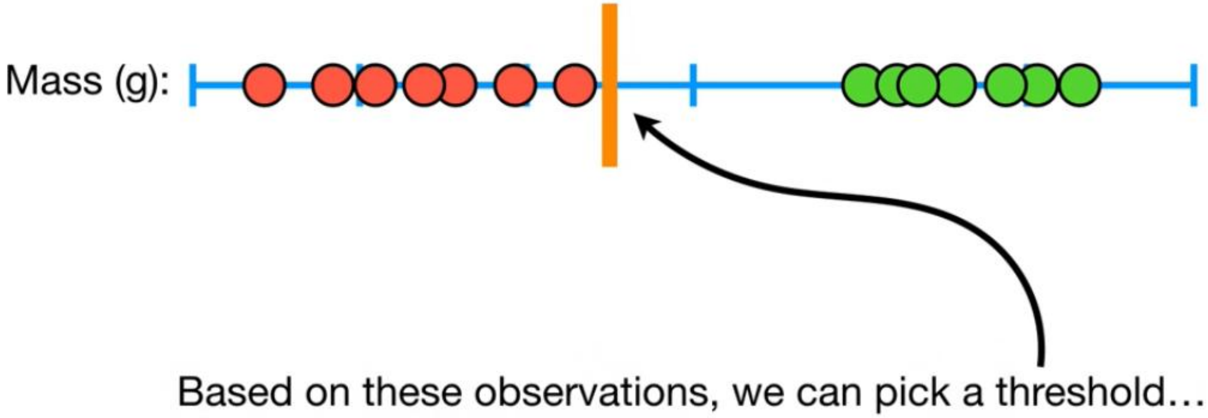


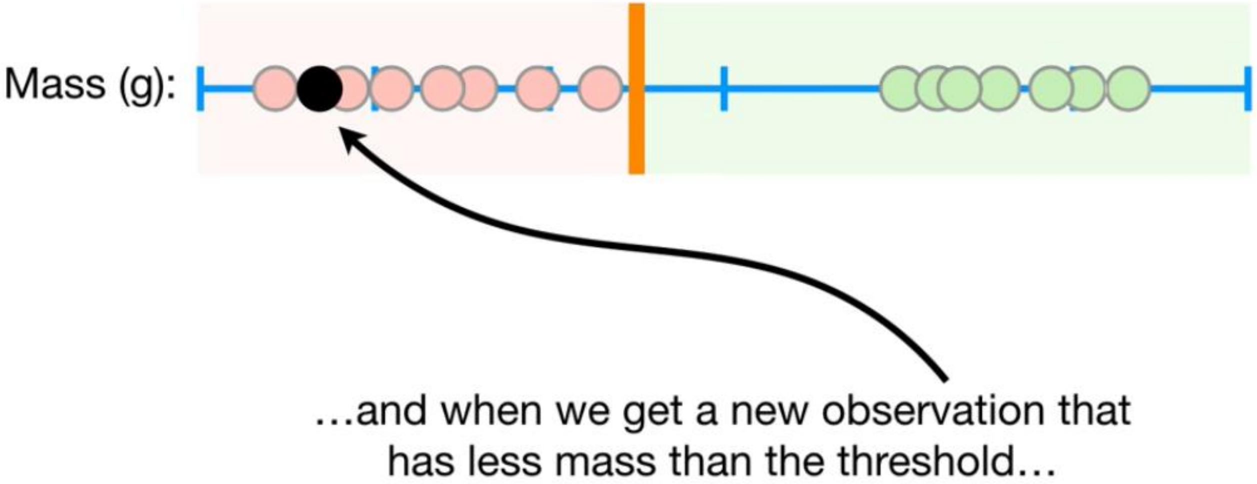
Support Vector Machine



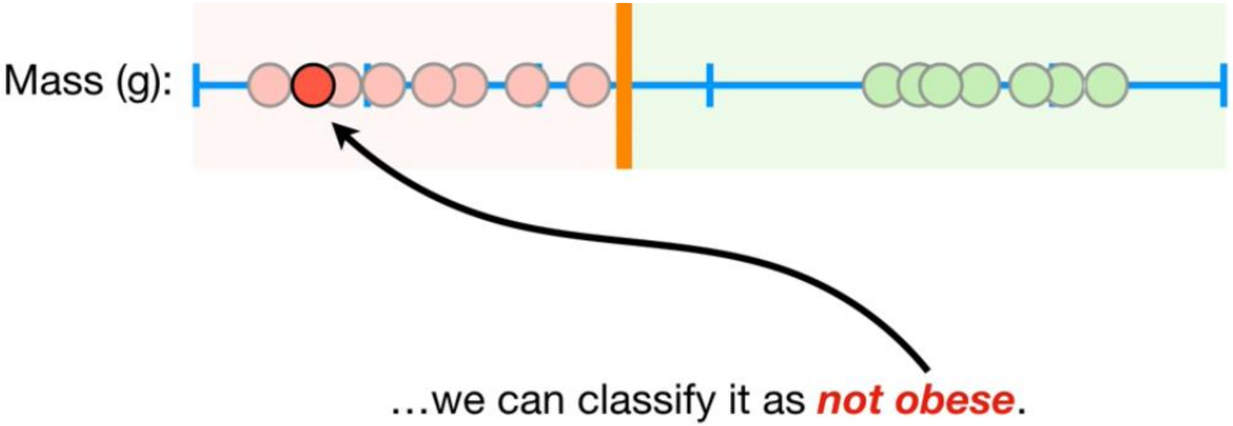


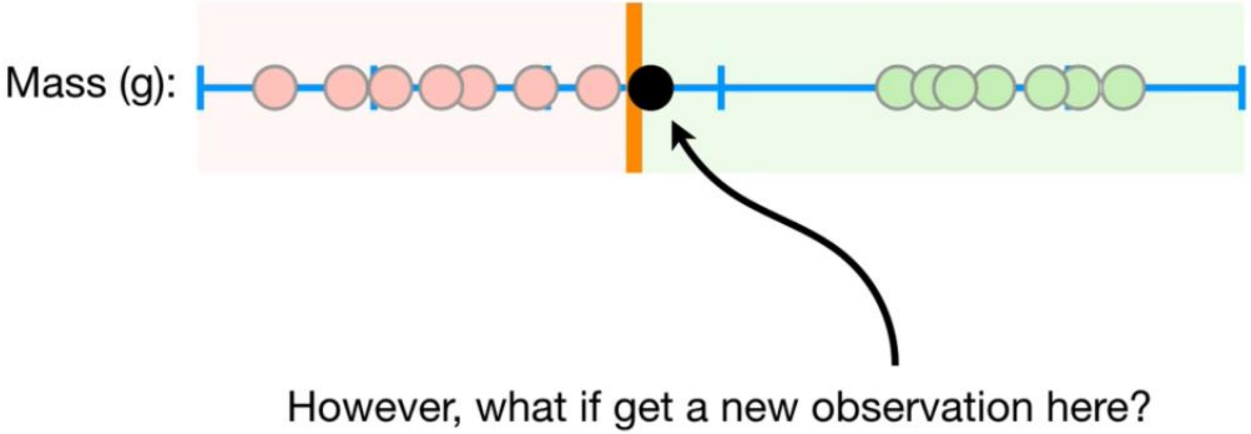
Support Vector Machine



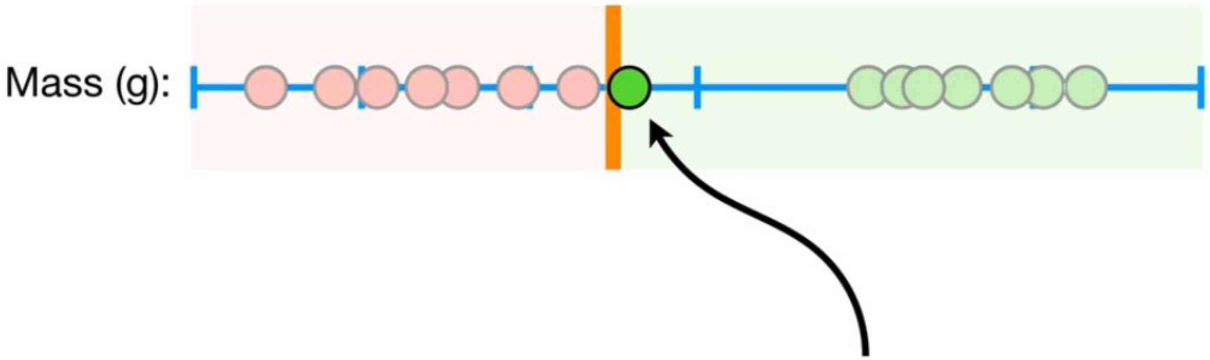


Support Vector Machine

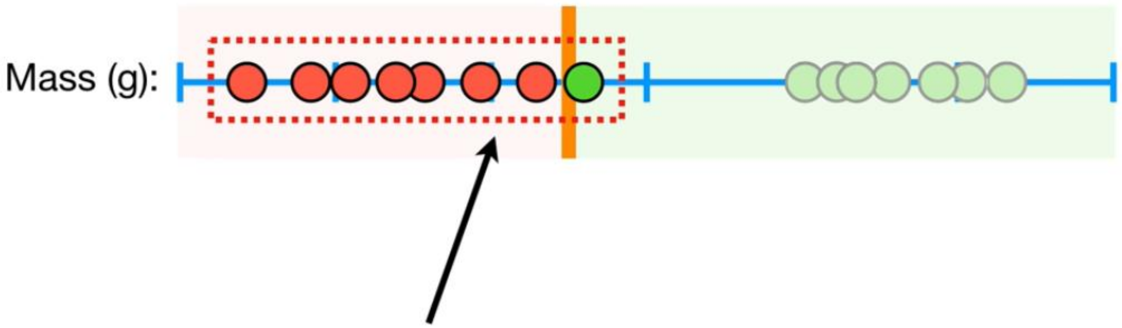




Support Vector Machine

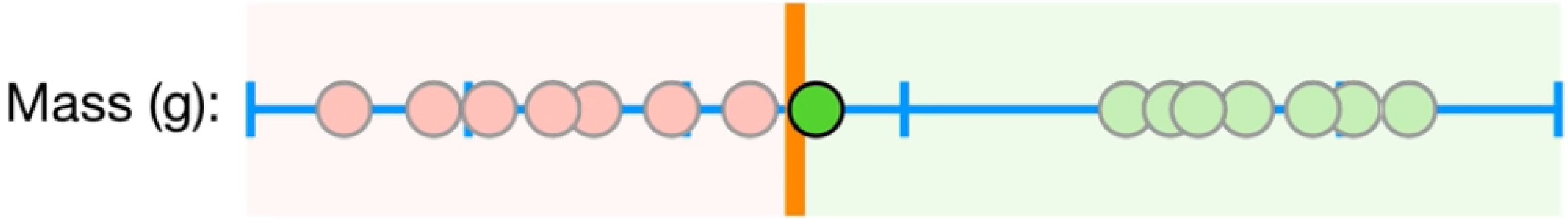


Because this observation has more mass than the threshold, we classify it as ***obese***.



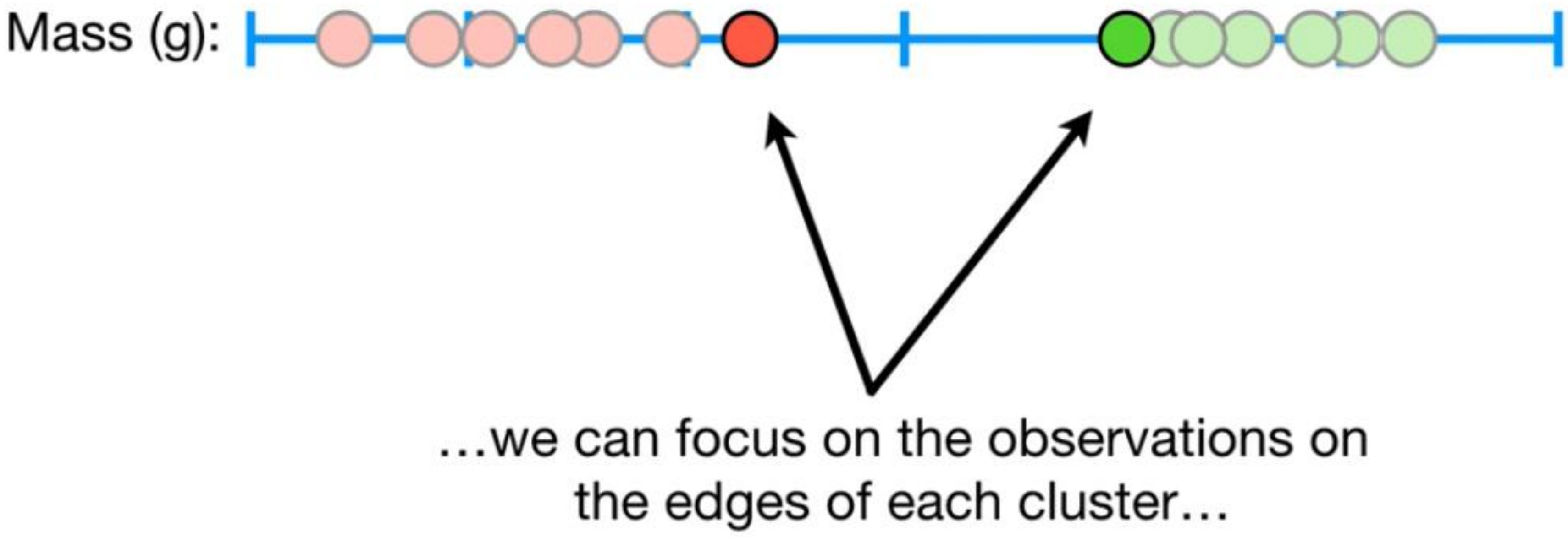
But that doesn't make sense, because it is much closer to the observations that are ***not obese***.

Support Vector Machine

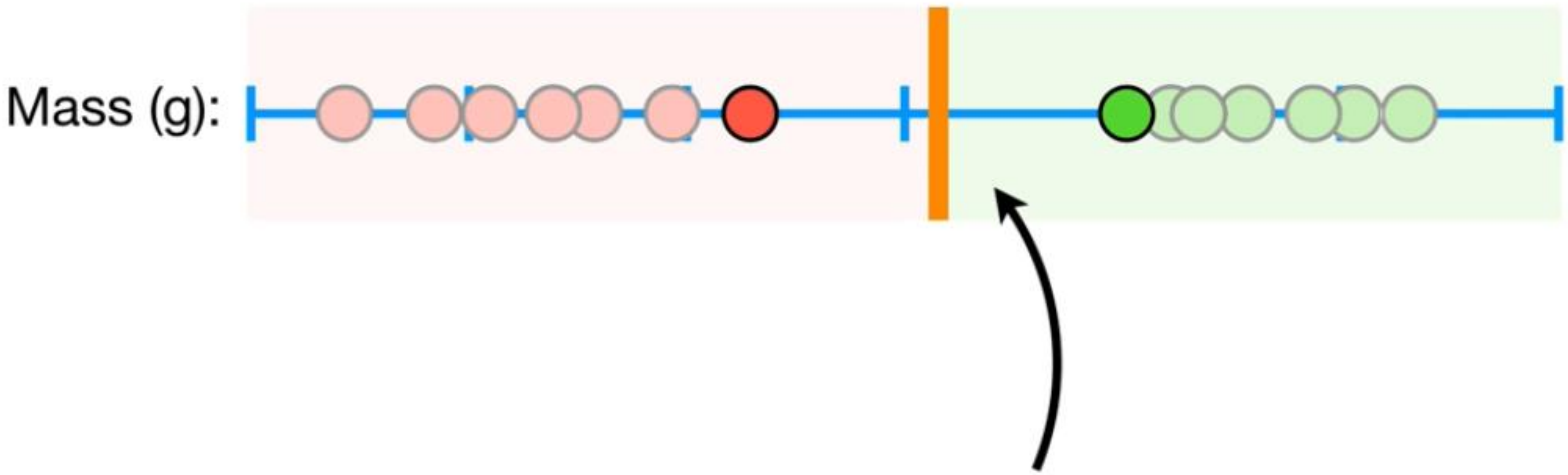


Can we do better?

Support Vector Machine

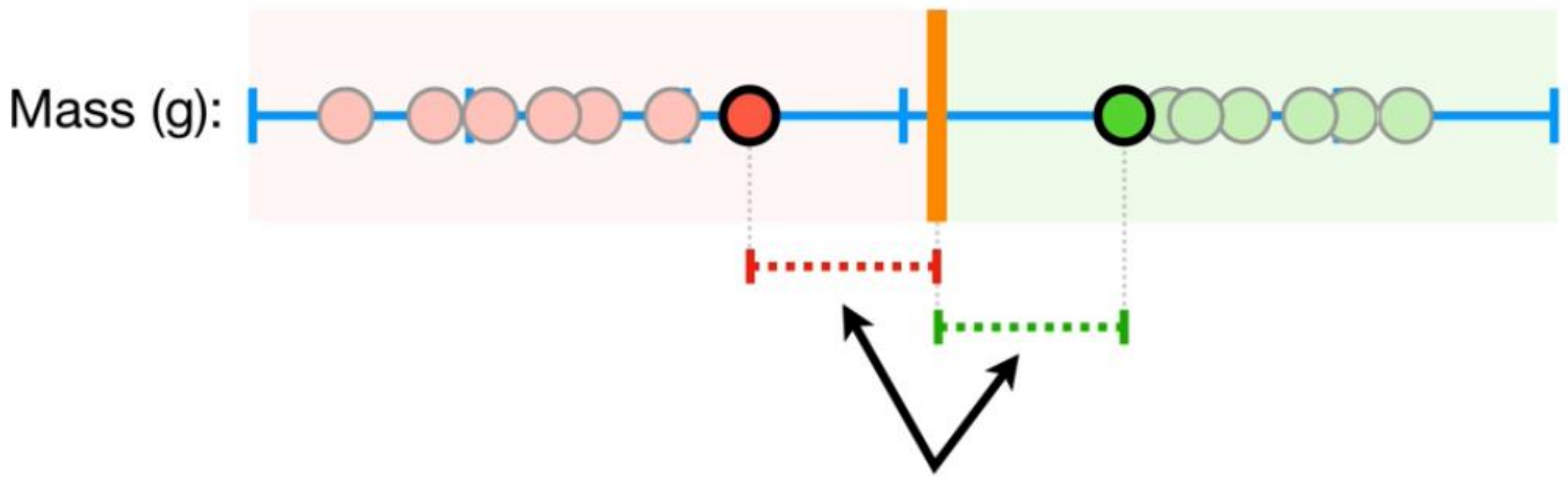


Support Vector Machine



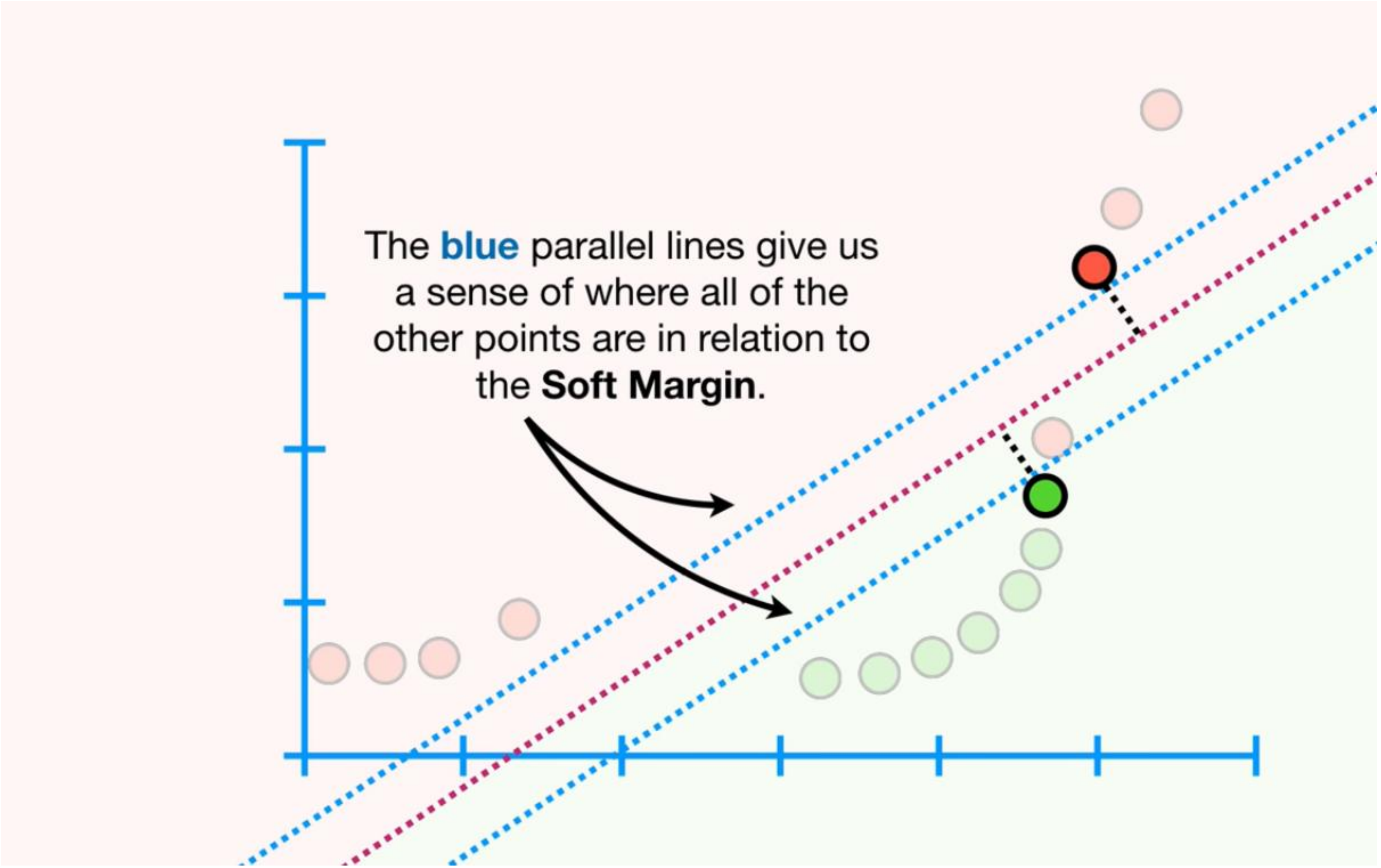
...and use the midpoint between
them as the threshold.

Support Vector Machine



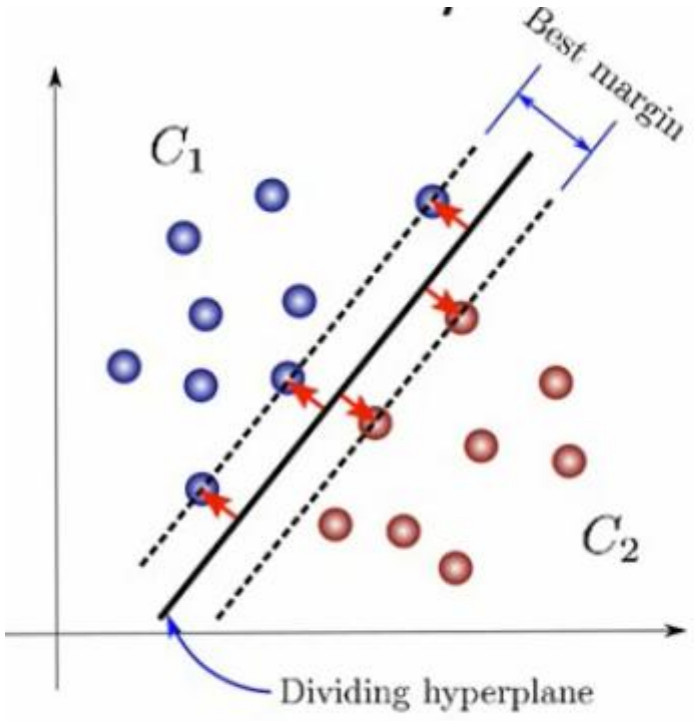
The shortest distance between the observations and the threshold is called the **margin**.

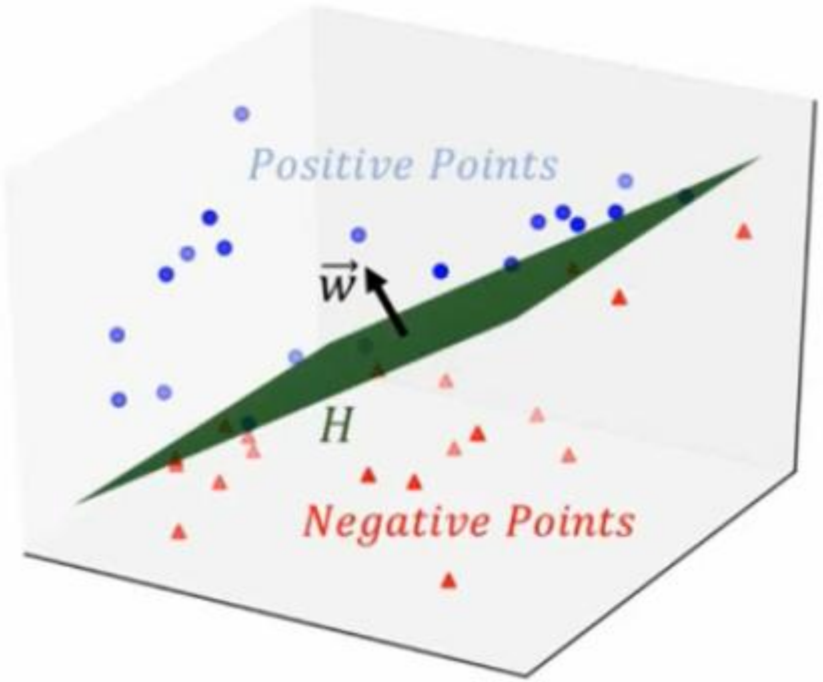
Support Vector Machine



Support Vector Machine

- SVM is used to solve regression and classification problems.
- The objective of the support vector machine is to find a hyper plane in N dimensional space (N- the Number of features) that distinctly classifies the data points.



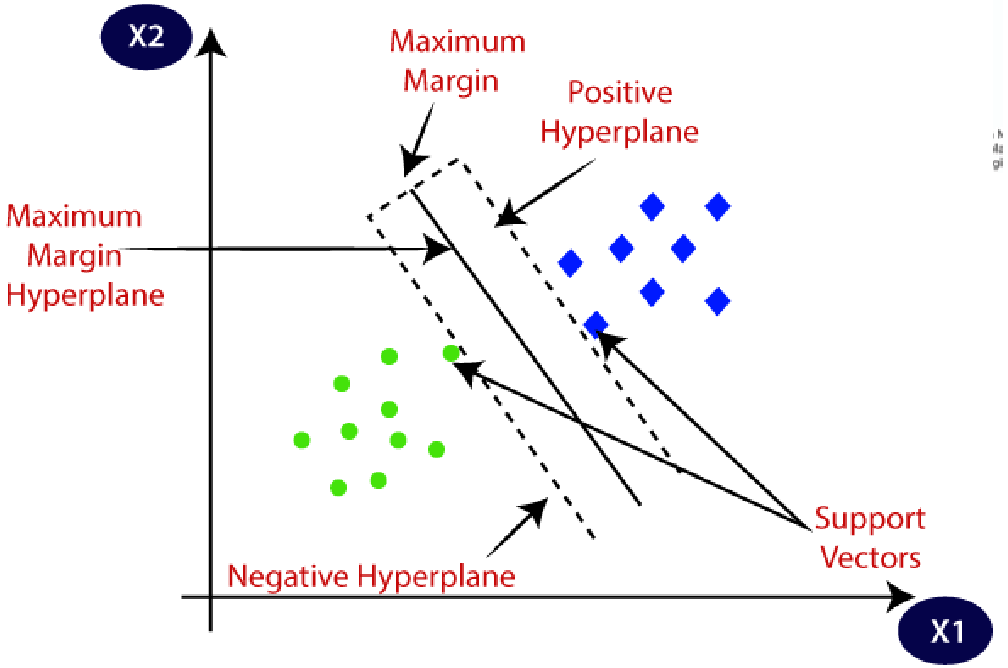


Support Vector Machine

- Support Vectors
- Hyperplane
- Marginal Distance
- Linear Separable
- Non Linear Separable

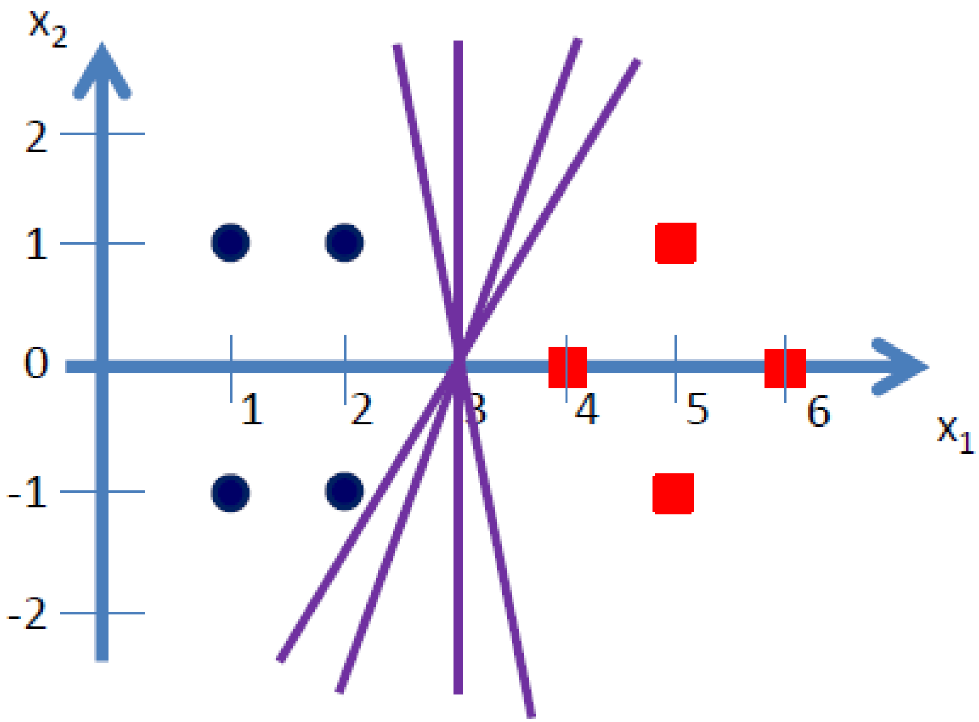
Support Vector Machine

- First require to find hyper plane and also create two parallel line(one line pass from at least one positive point and other line pass from at least one negative points).
- Select the hyper plane having maximum marginal distance.



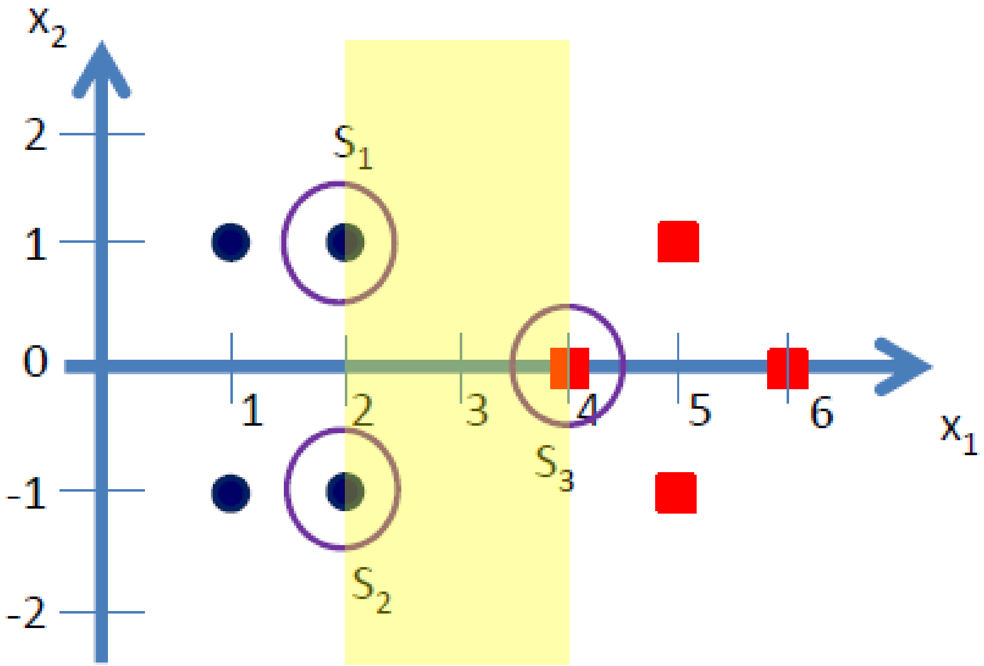
Support Vector Machine

- Consider we have data points that are linearly separable



Math intuition behind SVM

- We select 3 support vectors to start with



$$S_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$S_2 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$$S_3 = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$$

Support Vector Machine

- Here we will use vectors augmented with a 1 as a bias input, and for clarity we will differentiate these with an over-tilde

$$S_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$S_2 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

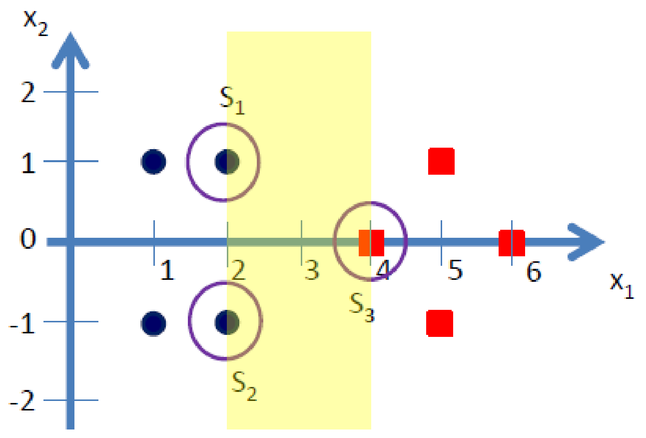
$$S_3 = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$$

$$\tilde{S_1} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$

$$\tilde{S_2} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

$$\tilde{S_3} = \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix}$$

Support Vector Machine



Now we need to find 3 parameters α_1, α_2, and α_3 based on the following 3 linear equations:

$$\alpha_1 \widetilde{S_1}.\widetilde{S_1} + \alpha_2 \widetilde{S_2}.\widetilde{S_1} + \alpha_3 \widetilde{S_3}.\widetilde{S_1} = -1 \;\; (-ve\ class)$$

$$\alpha_1 \widetilde{S_1}.\widetilde{S_2} + \alpha_2 \widetilde{S_2}.\widetilde{S_2} + \alpha_3 \widetilde{S_3}.\widetilde{S_2} = -1 \; (-ve\ class)$$

$$\alpha_1 \widetilde{S_1}.\widetilde{S_3} + \alpha_2 \widetilde{S_2}.\widetilde{S_3} + \alpha_3 \widetilde{S_3}.\widetilde{S_3} = +1 \;\; (+ve\ class)$$

Support Vector Machine

$$\alpha_1 \widetilde{S_1}.\widetilde{S_1} + \alpha_2 \widetilde{S_2}.\widetilde{S_1} + \alpha_3 \widetilde{S_3}.\widetilde{S_1} = -1 \ \ (-ve\ class)$$

$$\alpha_1 \widetilde{S_1}.\widetilde{S_2} + \alpha_2 \widetilde{S_2}.\widetilde{S_2} + \alpha_3 \widetilde{S_3}.\widetilde{S_2} = -1 \ (-ve\ class)$$

$$\alpha_1 \widetilde{S_1}.\widetilde{S_3} + \alpha_2 \widetilde{S_2}.\widetilde{S_3} + \alpha_3 \widetilde{S_3}.\widetilde{S_3} = +1 \ \ (+ve\ class)$$

Let's substitute the values for $\widetilde{S}_1$, $\widetilde{S}_2$ and $\widetilde{S}_3$ in the above equations.

$$\widetilde{S}_1 = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \qquad \widetilde{S}_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \qquad \widetilde{S}_3 = \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix}$$

$$\alpha_1 \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}.\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + \alpha_2 \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}.\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + \alpha_3 \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix}.\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = -1$$

$$\alpha_1 \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}.\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} + \alpha_2 \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}.\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} + \alpha_3 \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix}.\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} = -1$$

$$\alpha_1 \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}.\begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} + \alpha_2 \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}.\begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} + \alpha_3 \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix}.\begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} = +1$$

Support Vector Machine

$$\alpha_1 \begin{pmatrix}2\\1\\1\end{pmatrix} . \begin{pmatrix}2\\1\\1\end{pmatrix} + \alpha_2 \begin{pmatrix}2\\-1\\1\end{pmatrix} . \begin{pmatrix}2\\1\\1\end{pmatrix} + \alpha_3 \begin{pmatrix}4\\0\\1\end{pmatrix} . \begin{pmatrix}2\\1\\1\end{pmatrix} = -1$$

$$\alpha_1 \begin{pmatrix}2\\1\\1\end{pmatrix} . \begin{pmatrix}2\\-1\\1\end{pmatrix} + \alpha_2 \begin{pmatrix}2\\-1\\1\end{pmatrix} . \begin{pmatrix}2\\-1\\1\end{pmatrix} + \alpha_3 \begin{pmatrix}4\\0\\1\end{pmatrix} . \begin{pmatrix}2\\-1\\1\end{pmatrix} = -1$$

$$\alpha_1 \begin{pmatrix}2\\1\\1\end{pmatrix} . \begin{pmatrix}4\\0\\1\end{pmatrix} + \alpha_2 \begin{pmatrix}2\\-1\\1\end{pmatrix} . \begin{pmatrix}4\\0\\1\end{pmatrix} + \alpha_3 \begin{pmatrix}4\\0\\1\end{pmatrix} . \begin{pmatrix}4\\0\\1\end{pmatrix} = +1$$

After simplification we get:

$$6\alpha_1 + 4\alpha_2 + 9\alpha_3 = -1$$

$$4\alpha_1 + 6\alpha_2 + 9\alpha_3 = -1$$

$$9\alpha_1 + 9\alpha_2 + 17\alpha_3 = +1$$

Support Vector Machine

After simplification we get:

$$6\alpha_1 + 4\alpha_2 + 9\alpha_3 = -1$$

$$4\alpha_1 + 6\alpha_2 + 9\alpha_3 = -1$$

$$9\alpha_1 + 9\alpha_2 + 17\alpha_3 = +1$$

- After applying any method for to solve system equation all values we get

α_1=α_2= -3.25 and α_3= 3.5.

Support Vector Machine

$\alpha_1 = \alpha_2 = -3.25$ and $\alpha_3 = 3.5$.

$$\tilde{S_1} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$

$$\tilde{S_2} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

$$\tilde{S_3} = \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix}$$

- The hyper plane that discriminates the positive class from the negative class is given by:

$$\tilde{w} = \sum_i \alpha_i \tilde{S_i}$$

Substituting the values we get:

$$\tilde{w} = \alpha_1 \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + \alpha_2 \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} + \alpha_3 \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix}$$

$$\tilde{w} = (-3.25).\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + (-3.25).\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} + (3.5).\begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -3 \end{pmatrix}$$

Support Vector Machine

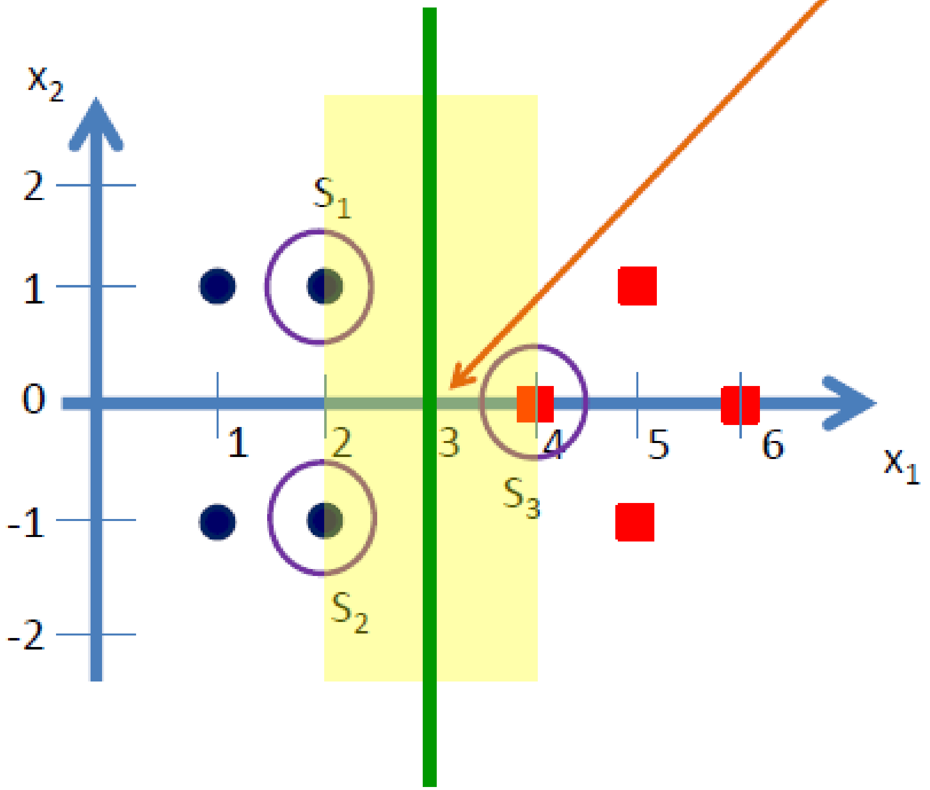
$$\tilde{w} = (-3.25).\begin{pmatrix}2\\1\\1\end{pmatrix} + (-3.25).\begin{pmatrix}2\\-1\\1\end{pmatrix} + (3.5).\begin{pmatrix}4\\0\\1\end{pmatrix} = \begin{pmatrix}1\\0\\-3\end{pmatrix}$$

- Our vectors are augmented with a bias.
- Hence we can equate the entry in $\tilde{W}$ as the hyper plane with an offset *b.*
- Therefore the separating hyper plane equation

$$y = wx + b \text{ with } w = \begin{pmatrix}1\\0\end{pmatrix} \text{ and offset } b = -3.$$

Support Vector Machine

$$y = wx + b \text{ with } w = \binom{1}{0} \text{ and offset } b = -3.$$



Support Vector Machine- Example1

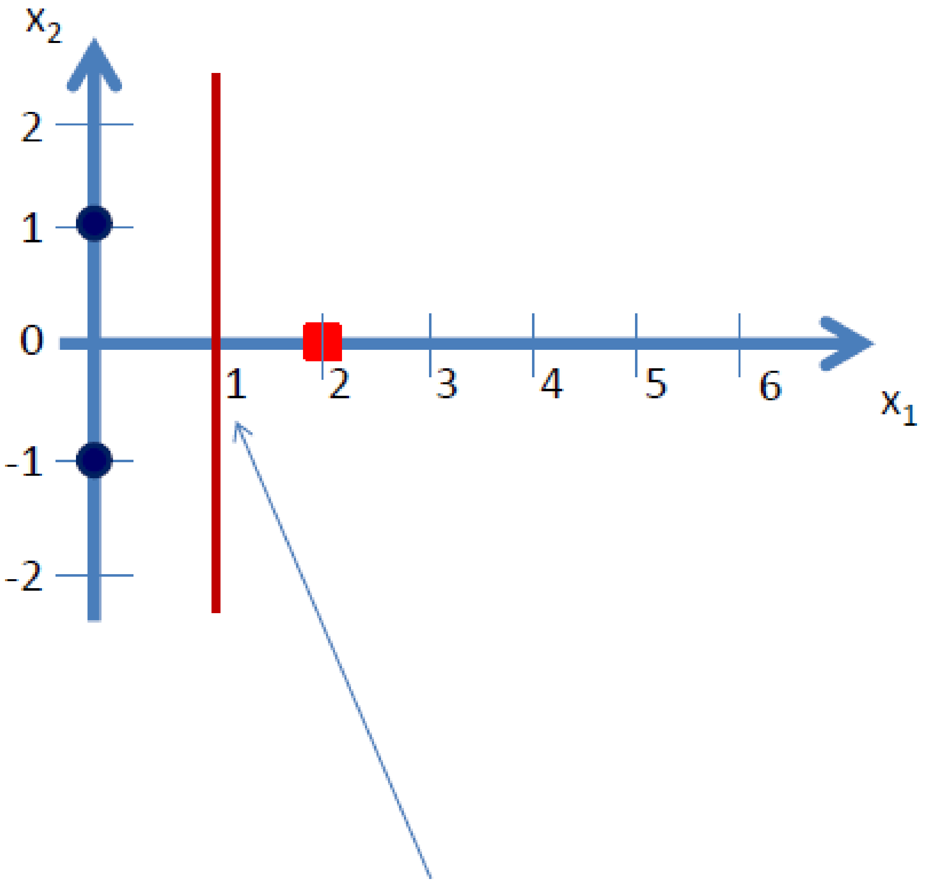
```
% 3 support vector version

s1 = [ 0 -1 1 ];
s2 = [ 0 1 1 ];
s3 = [ 2 0 1 ];

A = [ sum(s1.*s1) sum(s2.*s1) sum(s3.*s1) ;
sum(s1.*s2) sum(s2.*s2) sum(s3.*s2) ;
sum(s1.*s3) sum(s2.*s3) sum(s3.*s3) ]
Y = [ -1 -1 +1 ]
X = Y/A

p = X(1)
q = X(2)
r = X(3)

W = [ p*s1 + q*s2 + r*s3 ]
```



When you run you should get: $\tilde{w}$ = [1 0 -1]. This is a vertical line passing through x1=1.

Support Vector Machine- Example 2

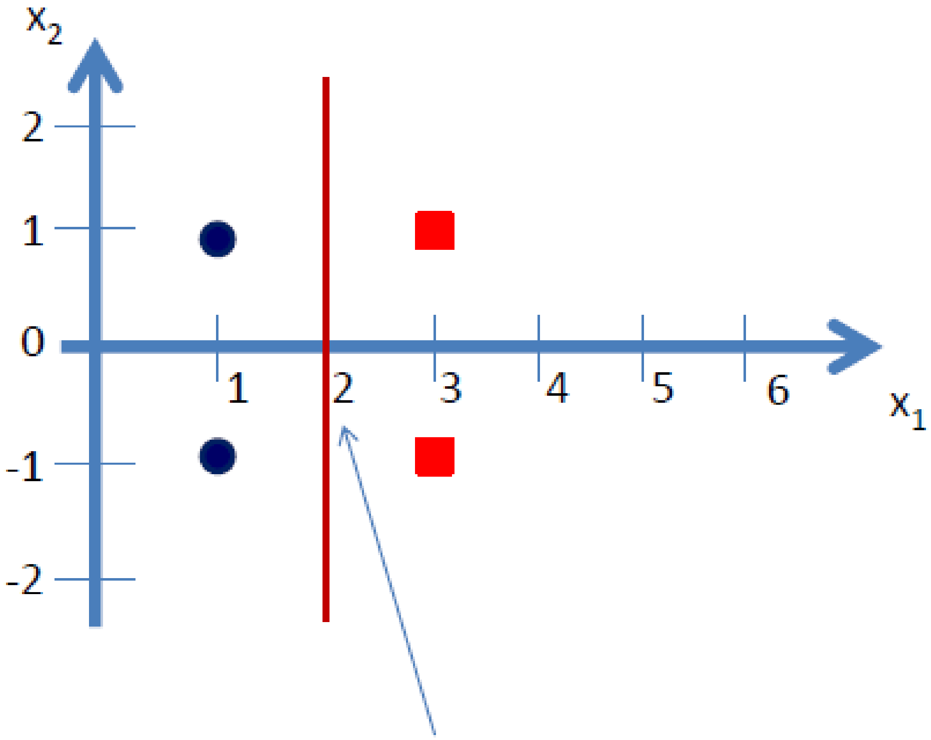
```
% 4 support vector version

s1 = [ 1 1 1 ];
s2 = [ 1 –1 1 ];
s3 = [ 3 –1 1 ];
s4 = [ 3 1 0 ];

A = [ sum(s1.*s1) sum(s2.*s1) sum(s3.*s1) sum(s4.*s1);
   sum(s1.*s2) sum(s2.*s2) sum(s3.*s2) sum(s4.*s1);
   sum(s1.*s3) sum(s2.*s3) sum(s3.*s3) sum(s4.*s3);
   sum(s1.*s4) sum(s2.*s4) sum(s3.*s4) sum(s4.*s4);]
Y = [ –1 –1 +1 +1 ]
X = Y/A

p = X(1)
q = X(2)
r = X(3)
s = X(4)

W = [ p*s1 + q*s2 + r*s3 + s*s4 ]
```



When you run you should get: $\tilde{w}$ = [1 0 -2]. This is a vertical line passing through x1=2.

Support Vector Machine- Example 3

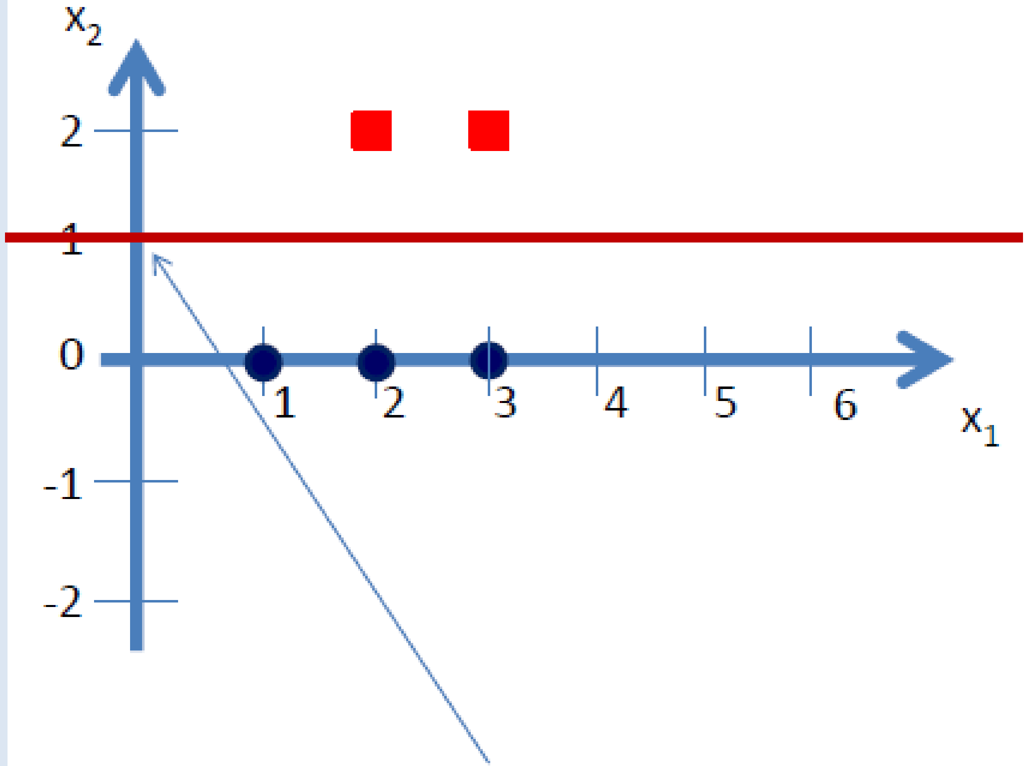
```
% 5 support vector version

s1 = [ 1 0 1 ];
s2 = [ 2 0 1 ];
s3 = [ 3 0 1 ];
s4 = [ 2 2 1 ];
s5 = [ 3 2 1 ];

A = [ sum(s1.*s1) sum(s2.*s1) sum(s3.*s1) sum(s4.*s1)
sum(s5.*s1);
   sum(s1.*s2) sum(s2.*s2) sum(s3.*s2) sum(s4.*s2) sum(s5.*s2);
   sum(s1.*s3) sum(s2.*s3) sum(s3.*s3) sum(s4.*s3) sum(s5.*s3);
   sum(s1.*s4) sum(s2.*s4) sum(s3.*s4) sum(s4.*s4) sum(s5.*s4);
   sum(s1.*s5) sum(s2.*s5) sum(s3.*s5) sum(s4.*s5) sum(s5.*s5)]
Y = [ -1 -1 -1 +1 +1 ]
X = Y/A

p = X(1)
q = X(2)
r = X(3)
s = X(4)
t = X(5)

W = [ p*s1 + q*s2 + r*s3 + s*s4 + t*s5 ]
```



When you run you should get: $\tilde{w}$ = [0 1 -1]. This is a horizontal line passing through x2=1.

Support Vector Machine- Classification Example

Let's take the 5 support vector version

$\tilde{w}$ = [0 1 -1]. This is a horizontal line passing through x2=1.

Let's classify the point (x1,x2)=(4,2).

$$w.x = \binom{0}{1}.\binom{4}{2} = 2 > 1$$

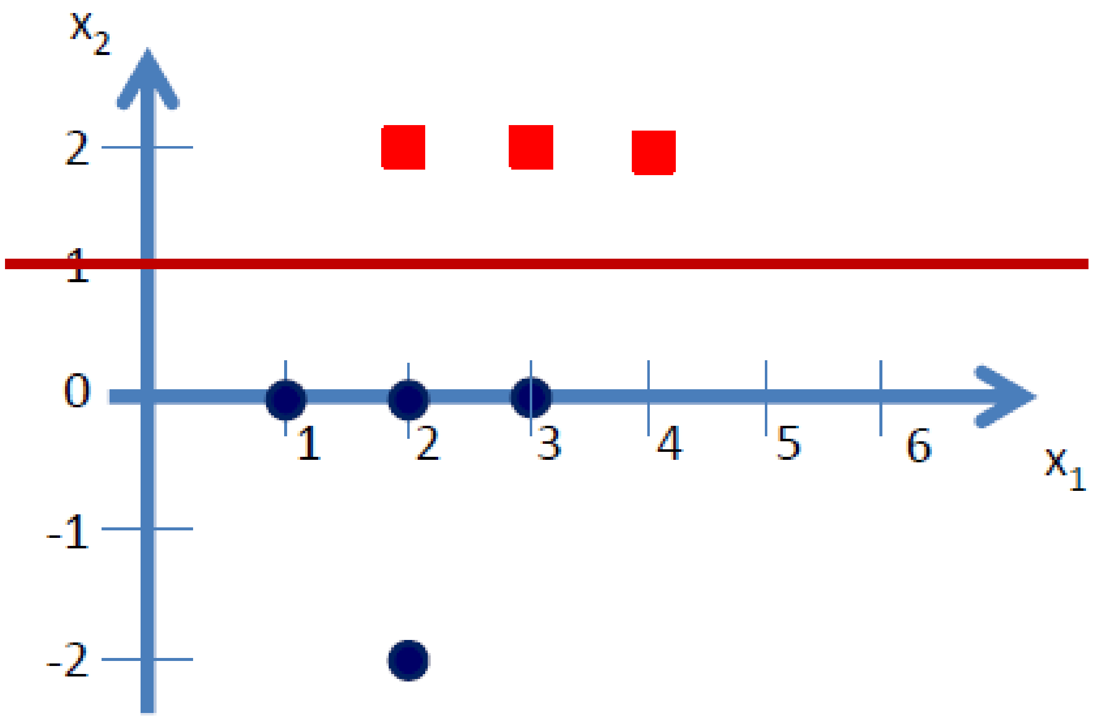
Hence this point belongs to the red class

Let's classify the point (x1,x2)=(2,-2).

$$w.x = \binom{0}{1}.\binom{2}{-2} = -2 < 1$$

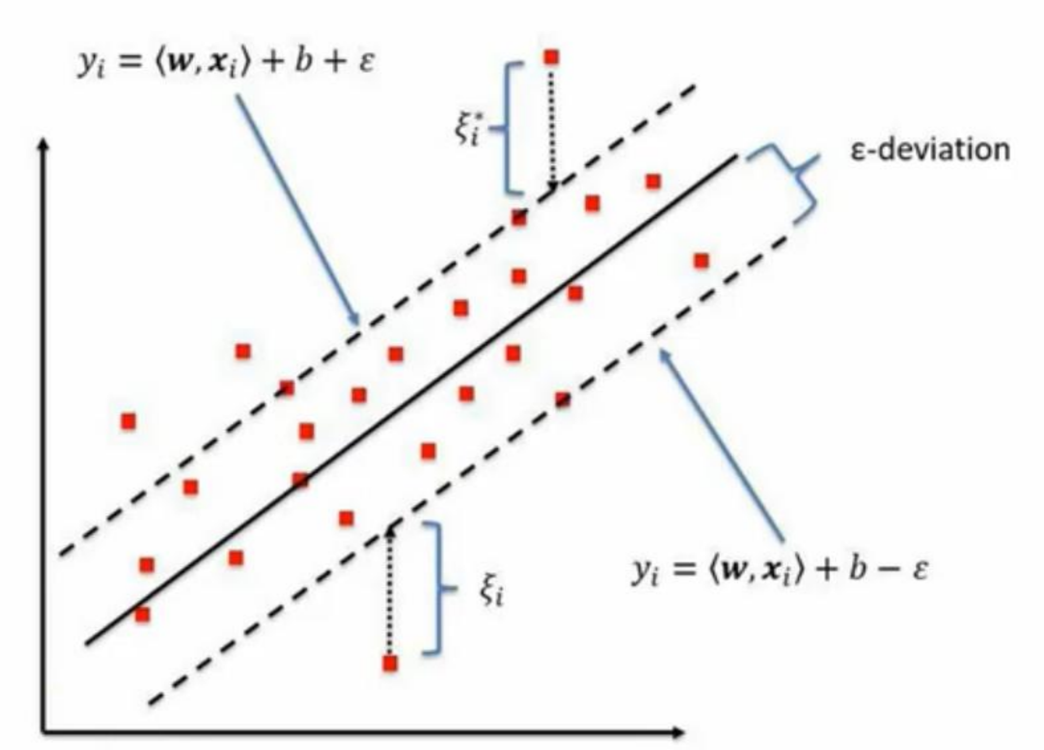
Hence this point belongs to the blue class

We can do the same for any new point.



Support Vector Regression

- Fig show how SVM is used for Regression.
- Hyper plane plotted with equal distance with extreme end and its used to predict the values



Support Vector Regression

- For Non Linear Separable data SVM used various kernels.
- SVM Kernels convert 2D (low dimensional to High dimensional).