SUPPORT VECTOR MACHINES

Scholastic Video Book Series

Part 2

Non-Linear Support Vector Machines

(with English Narrations)

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Support Vector Machines - #2

Non-Liner Support Vector Machines

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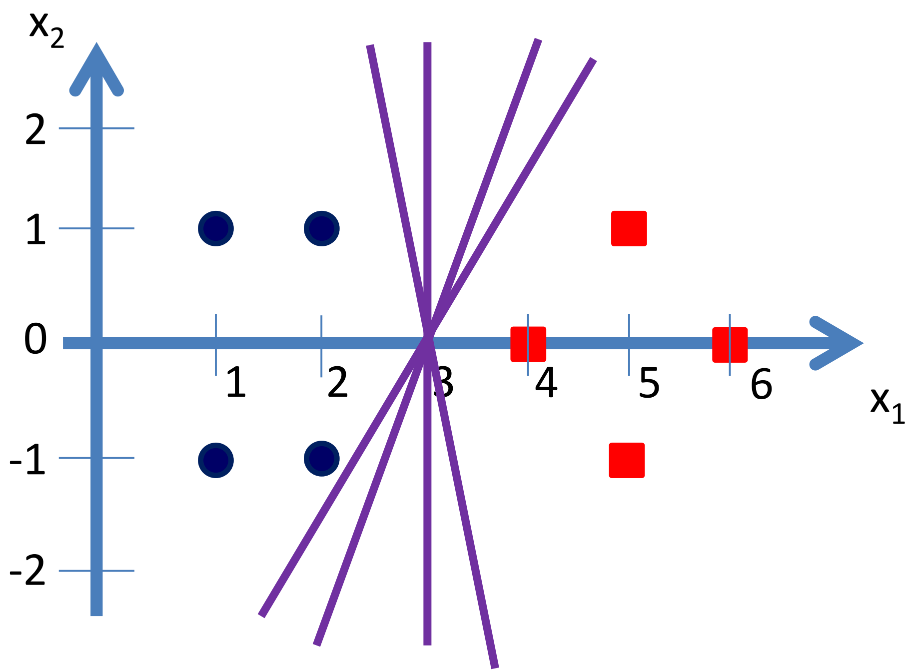
Support Vector Machines

- Support Vector Machine (SVM) algorithms are used in Classification.
- Classification can be viewed as the task of separating classes in feature space.



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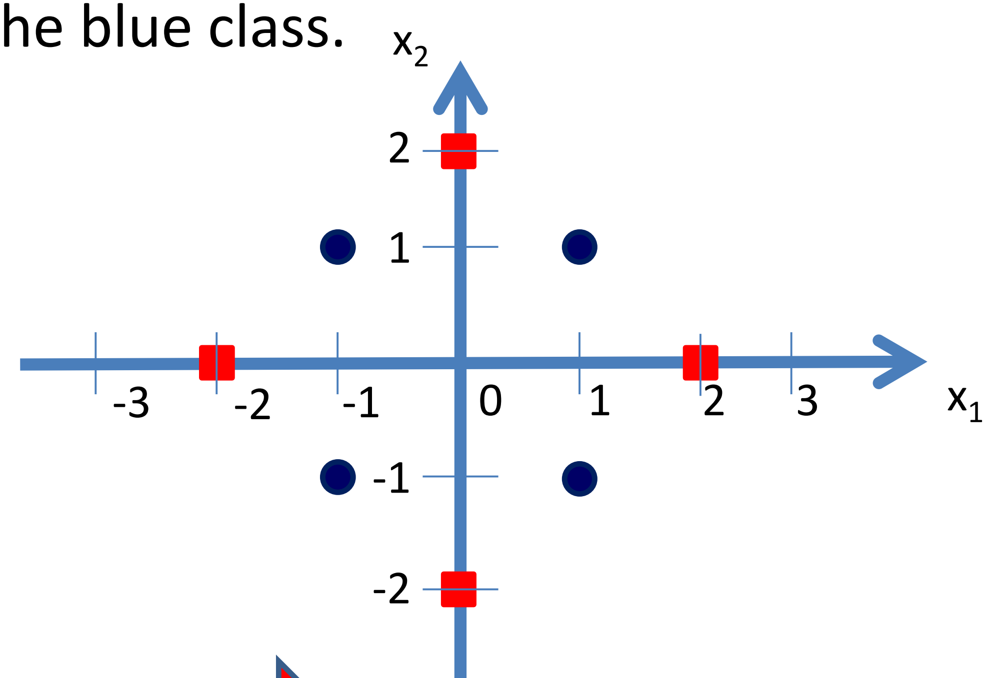
Support Vector Machines

- We looked at the linear type SVM in Part I of our SVM lesson series. Here we will look at an example like the one given below and find out how to carry out the classification.
- Obviously there is no clear separating hyperplane between the red class and the blue class.



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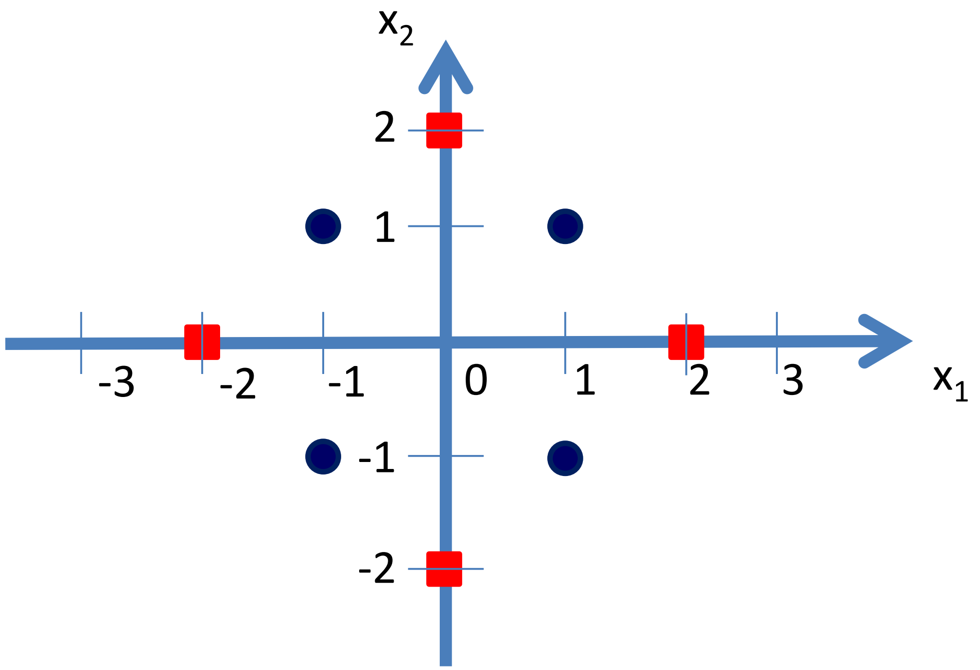
Support Vector Machines



- Blue class vectors are: $\begin{pmatrix}1\\1\end{pmatrix}, \begin{pmatrix}-1\\1\end{pmatrix}, \begin{pmatrix}-1\\-1\end{pmatrix}, \begin{pmatrix}1\\-1\end{pmatrix}$
- Red class vectors are: $\begin{pmatrix}2\\0\end{pmatrix}, \begin{pmatrix}0\\2\end{pmatrix}, \begin{pmatrix}-2\\0\end{pmatrix}, \begin{pmatrix}0\\-2\end{pmatrix}$

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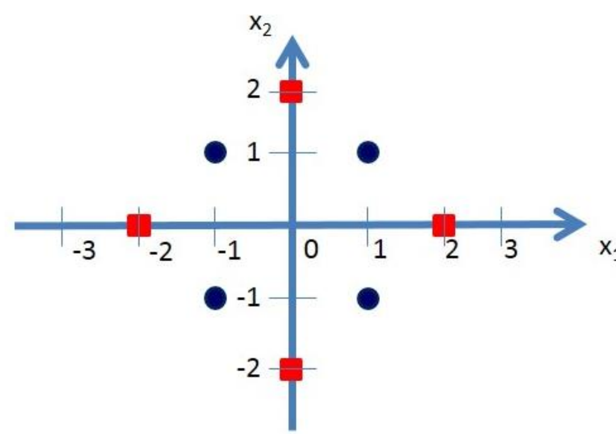
Support Vector Machines

- Here we need to find a non-linear mapping function Φ which can transform these data in to a new feature space where a seperating hyperplane can be found.
- Let us consider the following mapping function.
- $$\Phi\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{cases} \begin{pmatrix} 6 - x_1 + (x_1 - x_2)^2 \\ 6 - x_2 + (x_1 - x_2)^2 \end{pmatrix} & \text{if } \sqrt{x_1^2 + x_2^2} \geq 2 \\ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} & \text{otherwise} \end{cases}$$



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Support Vector Machines

- Now let us transform the blue and red calss vectors using the non-linear mapping function Φ .

- $$\Phi\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{cases} \begin{pmatrix} 6 - x_1 + (x_1 - x_2)^2 \\ 6 - x_2 + (x_1 - x_2)^2 \end{pmatrix} & \text{if } \sqrt{x_1^2 + x_2^2} \geq 2 \\ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} & \text{otherwise} \end{cases}$$

- Blue class vectors are: $\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ no change since $\sqrt{x_1^2 + x_2^2} < 2$ for all the vectors

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Support Vector Machines

- $\Phi\begin{pmatrix}x_1\\x_2\end{pmatrix} = \begin{cases}\begin{pmatrix}6-x_1+(x_1-x_2)^2\\6-x_2+(x_1-x_2)^2\end{pmatrix} & \text{if } \sqrt{x_1^2+x_2^2} \geq 2\\ \begin{pmatrix}x_1\\x_2\end{pmatrix} & \text{otherwise}\end{cases}$
- Let us take Red class vectors : $\begin{pmatrix}2\\0\end{pmatrix}, \begin{pmatrix}0\\2\end{pmatrix}, \begin{pmatrix}-2\\0\end{pmatrix}, \begin{pmatrix}0\\-2\end{pmatrix}$

- $\Phi\begin{pmatrix}x_1\\x_2\end{pmatrix} = \Phi\begin{pmatrix}2\\0\end{pmatrix} = \begin{pmatrix}6-2+(2-0)^2\\6-0+(2-0)^2\end{pmatrix} = \begin{pmatrix}8\\10\end{pmatrix}$
- $\Phi\begin{pmatrix}x_1\\x_2\end{pmatrix} = \Phi\begin{pmatrix}0\\2\end{pmatrix} = \begin{pmatrix}6-0+(0-2)^2\\6-2+(0-2)^2\end{pmatrix} = \begin{pmatrix}10\\8\end{pmatrix}$
- $\Phi\begin{pmatrix}x_1\\x_2\end{pmatrix} = \Phi\begin{pmatrix}-2\\0\end{pmatrix} = \begin{pmatrix}6+2+(-2-0)^2\\6-0+(-2-0)^2\end{pmatrix} = \begin{pmatrix}12\\10\end{pmatrix}$
- $\Phi\begin{pmatrix}x_1\\x_2\end{pmatrix} = \Phi\begin{pmatrix}0\\-2\end{pmatrix} = \begin{pmatrix}6-0+(0+2)^2\\6+2+(0+2)^2\end{pmatrix} = \begin{pmatrix}10\\12\end{pmatrix}$

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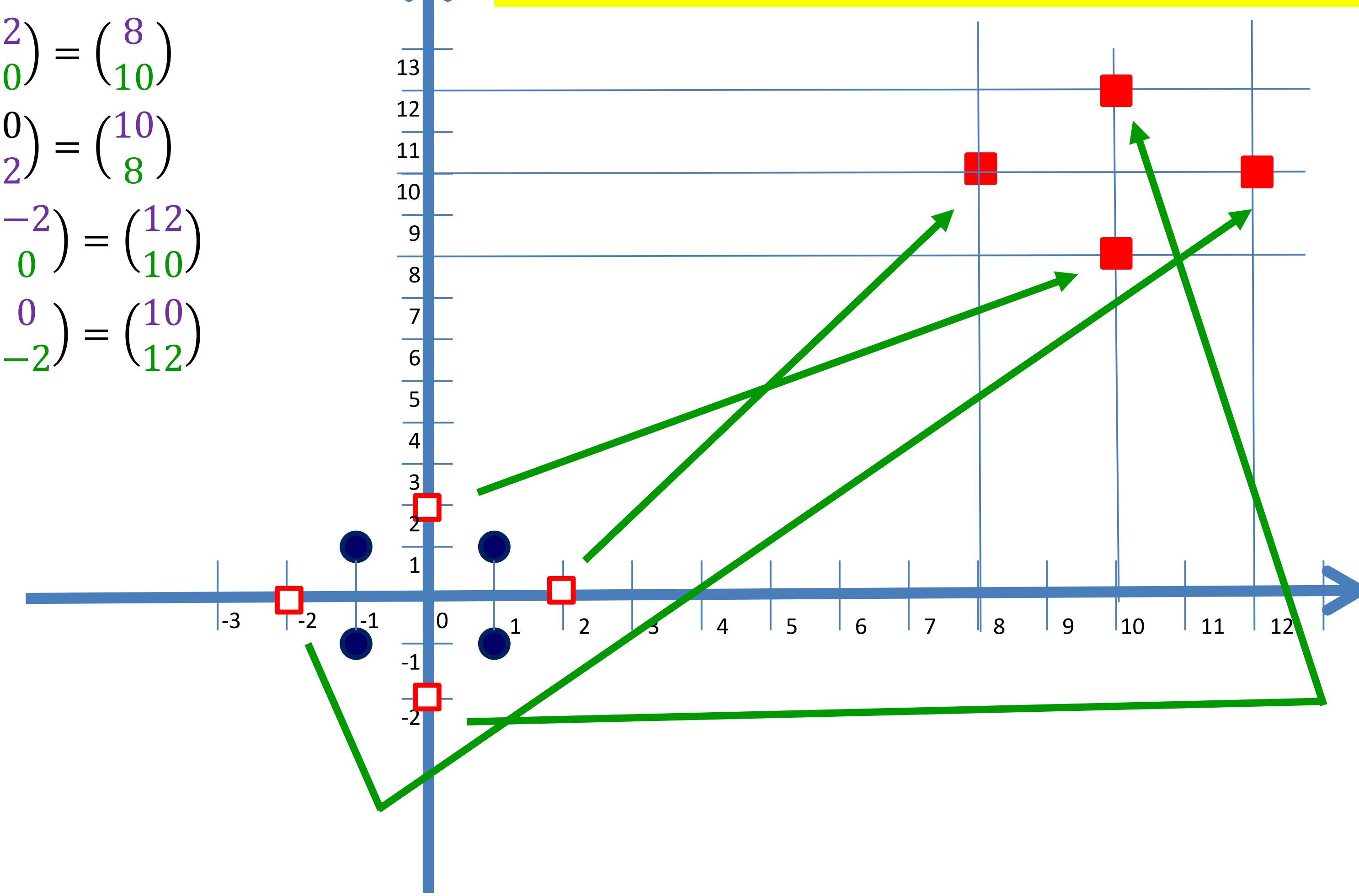
Vectors After Transforming

$$\Phi\begin{pmatrix}x_1\\x_2\end{pmatrix} = \begin{cases}\begin{pmatrix}6 - x_1 + (x_1 - x_2)^2\\6 - x_2 + (x_1 - x_2)^2\end{pmatrix} & \text{if } \sqrt{x_1^2 + x_2^2} \geq 2\\ \begin{pmatrix}x_1\\x_2\end{pmatrix} & \text{otherwise}\end{cases}$$

- $\Phi\begin{pmatrix}x_1\\x_2\end{pmatrix} = \Phi\begin{pmatrix}2\\0\end{pmatrix} = \begin{pmatrix}8\\10\end{pmatrix}$
- $\Phi\begin{pmatrix}x_1\\x_2\end{pmatrix} = \Phi\begin{pmatrix}0\\2\end{pmatrix} = \begin{pmatrix}10\\8\end{pmatrix}$
- $\Phi\begin{pmatrix}x_1\\x_2\end{pmatrix} = \Phi\begin{pmatrix}-2\\0\end{pmatrix} = \begin{pmatrix}12\\10\end{pmatrix}$
- $\Phi\begin{pmatrix}x_1\\x_2\end{pmatrix} = \Phi\begin{pmatrix}0\\-2\end{pmatrix} = \begin{pmatrix}10\\12\end{pmatrix}$



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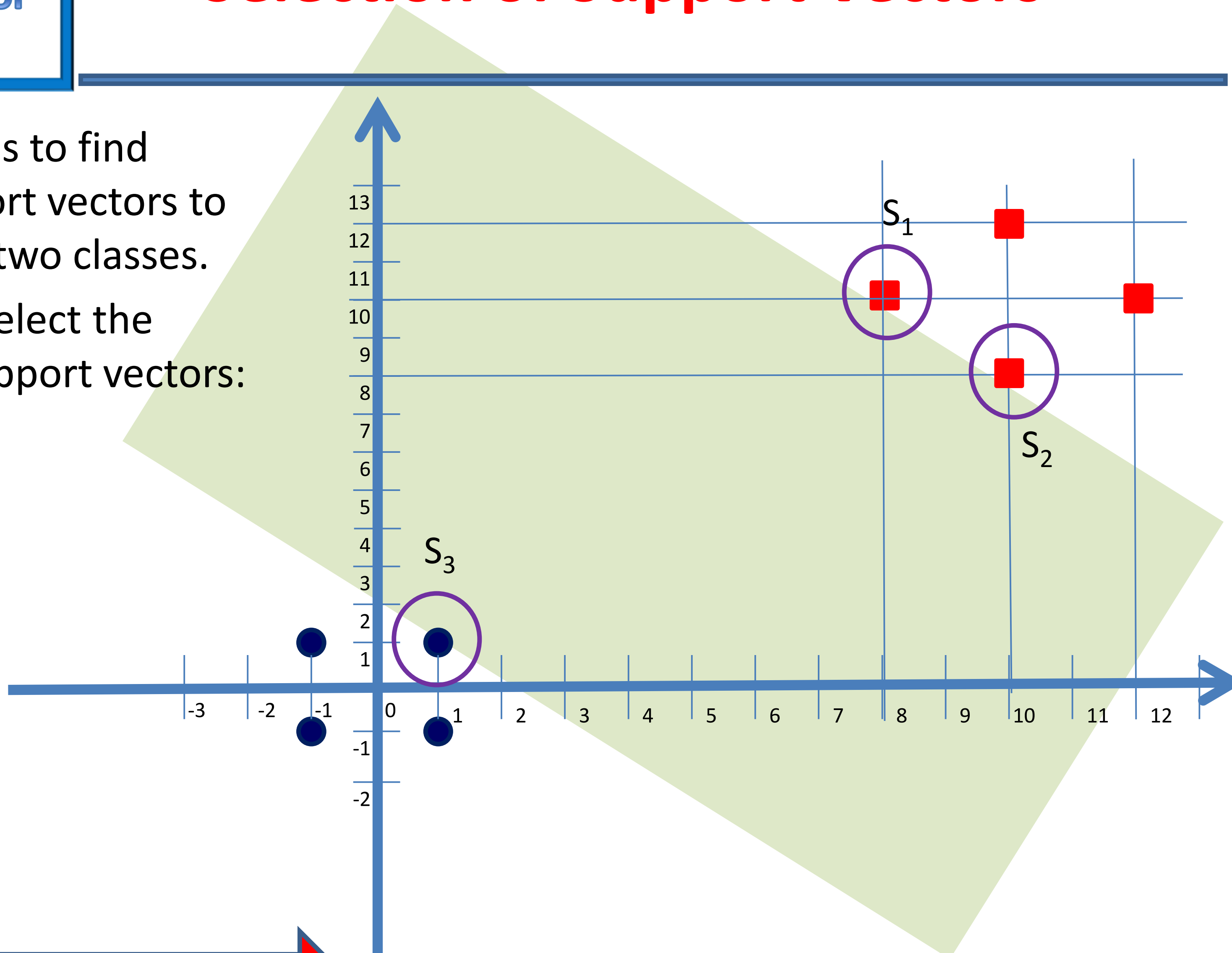
Selection of Support Vectors

- Now our task is to find suitable support vectors to classify these two classes.
- Here we will select the following 3 support vectors:
- $S_1 = \begin{pmatrix} 8 \\ 10 \end{pmatrix}$,
- $S_2 = \begin{pmatrix} 10 \\ 8 \end{pmatrix}$,
- and $S_3 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$



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Augmented Support Vectors

- Here we will use vectors augmented with a 1 as a bias input, and for clarity we will differentiate these with an over-tilde. That is:

$$S_1 = \begin{pmatrix} 8 \\ 10 \end{pmatrix} \qquad \widetilde{S}_1 = \begin{pmatrix} 8 \\ 10 \\ 1 \end{pmatrix}$$

$$S_2 = \begin{pmatrix} 10 \\ 8 \end{pmatrix} \qquad \widetilde{S}_2 = \begin{pmatrix} 10 \\ 8 \\ 1 \end{pmatrix}$$

$$S_3 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad \widetilde{S}_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

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Finding Parameters

- Now we need to find 3 parameters α_1, α_2, and α_3 based on the following 3 linear equations:

$$\alpha_1 \widetilde{S_1}.\widetilde{S_1} + \alpha_2 \widetilde{S_2}.\widetilde{S_1} + \alpha_3 \widetilde{S_3}.\widetilde{S_1} = +1 \ (+ve\ class)$$

$$\alpha_1 \widetilde{S_1}.\widetilde{S_2} + \alpha_2 \widetilde{S_2}.\widetilde{S_2} + \alpha_3 \widetilde{S_3}.\widetilde{S_2} = +1 \ (+ve\ class)$$

$$\alpha_1 \widetilde{S_1}.\widetilde{S_3} + \alpha_2 \widetilde{S_2}.\widetilde{S_3} + \alpha_3 \widetilde{S_3}.\widetilde{S_3} = -1 \ (-ve\ class)$$

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$$\alpha_1 \widetilde{S_1}.\widetilde{S_1} + \alpha_2 \widetilde{S_2}.\widetilde{S_1} + \alpha_3 \widetilde{S_3}.\widetilde{S_1} = +1 \ (+ve\ class)$$

$$\alpha_1 \widetilde{S_1}.\widetilde{S_2} + \alpha_2 \widetilde{S_2}.\widetilde{S_2} + \alpha_3 \widetilde{S_3}.\widetilde{S_2} = +1 \ (+ve\ class)$$

$$\alpha_1 \widetilde{S_1}.\widetilde{S_3} + \alpha_2 \widetilde{S_2}.\widetilde{S_3} + \alpha_3 \widetilde{S_3}.\widetilde{S_3} = -1 \ (-ve\ class)$$

- Let's substitute the values for $\widetilde{S}_1$, $\widetilde{S}_2$ and $\widetilde{S}_3$ in the above equations.

$$\widetilde{S_1} = \begin{pmatrix} 8 \\ 10 \\ 1 \end{pmatrix} \quad \widetilde{S_2} = \begin{pmatrix} 10 \\ 8 \\ 1 \end{pmatrix} \quad \widetilde{S_3} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\alpha_1 \begin{pmatrix} 8 \\ 10 \\ 1 \end{pmatrix} . \begin{pmatrix} 8 \\ 10 \\ 1 \end{pmatrix} + \alpha_2 \begin{pmatrix} 10 \\ 8 \\ 1 \end{pmatrix} . \begin{pmatrix} 8 \\ 10 \\ 1 \end{pmatrix} + \alpha_3 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} . \begin{pmatrix} 8 \\ 10 \\ 1 \end{pmatrix} = +1$$

$$\alpha_1 \begin{pmatrix} 8 \\ 10 \\ 1 \end{pmatrix} . \begin{pmatrix} 10 \\ 8 \\ 1 \end{pmatrix} + \alpha_2 \begin{pmatrix} 10 \\ 8 \\ 1 \end{pmatrix} . \begin{pmatrix} 10 \\ 8 \\ 1 \end{pmatrix} + \alpha_3 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} . \begin{pmatrix} 10 \\ 8 \\ 1 \end{pmatrix} = +1$$

$$\alpha_1 \begin{pmatrix} 8 \\ 10 \\ 1 \end{pmatrix} . \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \alpha_2 \begin{pmatrix} 10 \\ 8 \\ 1 \end{pmatrix} . \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \alpha_3 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} . \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = -1$$

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$$\alpha_1\begin{pmatrix}8\\10\\1\end{pmatrix}.\begin{pmatrix}8\\10\\1\end{pmatrix}+\alpha_2\begin{pmatrix}10\\8\\1\end{pmatrix}.\begin{pmatrix}8\\10\\1\end{pmatrix}+\alpha_3\begin{pmatrix}1\\1\\1\end{pmatrix}.\begin{pmatrix}8\\10\\1\end{pmatrix}=+1$$

$$\alpha_1\begin{pmatrix}8\\10\\1\end{pmatrix}.\begin{pmatrix}10\\8\\1\end{pmatrix}+\alpha_2\begin{pmatrix}10\\8\\1\end{pmatrix}.\begin{pmatrix}10\\8\\1\end{pmatrix}+\alpha_3\begin{pmatrix}1\\1\\1\end{pmatrix}.\begin{pmatrix}10\\8\\1\end{pmatrix}=+1$$

$$\alpha_1\begin{pmatrix}8\\10\\1\end{pmatrix}.\begin{pmatrix}1\\1\\1\end{pmatrix}+\alpha_2\begin{pmatrix}10\\8\\1\end{pmatrix}.\begin{pmatrix}1\\1\\1\end{pmatrix}+\alpha_3\begin{pmatrix}1\\1\\1\end{pmatrix}.\begin{pmatrix}1\\1\\1\end{pmatrix}=-1$$

- After multiplication we get:

$$165\,\alpha_1+161\,\alpha_2+19\,\alpha_3=+1$$

$$161\,\alpha_1+165\,\alpha_2+19\,\alpha_3=+1$$

$$19\,\alpha_1+19\,\alpha_2+3\,\alpha_3=-1$$

- Simplifying the above 3 simultaneous equations we get: $\alpha_1=\alpha_2= 0.859$ and $\alpha_3= -1.4219$.

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Discriminating Hyper Plane

- The hyper plane that discriminates the positive class from the negative class is given by:

$$\tilde{w} = \sum_i \alpha_i \tilde{S}_i$$

- Substituting the values we get:

$$\tilde{w} = \alpha_1 \begin{pmatrix} 8 \\ 10 \\ 1 \end{pmatrix} + \alpha_2 \begin{pmatrix} 10 \\ 8 \\ 1 \end{pmatrix} + \alpha_3 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\tilde{w} = (0.0859).\begin{pmatrix} 8 \\ 10 \\ 1 \end{pmatrix} + (0.0859).\begin{pmatrix} 10 \\ 8 \\ 1 \end{pmatrix} + (-1.4219).\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0.1243 \\ 0.1243 \\ -1.2501 \end{pmatrix}$$

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Equation for Discriminating Hyper Plane

- Our vectors are augmented with a bias.
- Hence we can equate the entry in $\widetilde{w}$ as the hyper plane with an offset b.
- Therefore the separating hyper plane equation

$y = wx + b$ with $w = \begin{pmatrix} 0.1243/0.1243 \\ 0.1243/0.1243 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

and an offset $b = -\frac{1.2501}{0.1243} = -10.057$.

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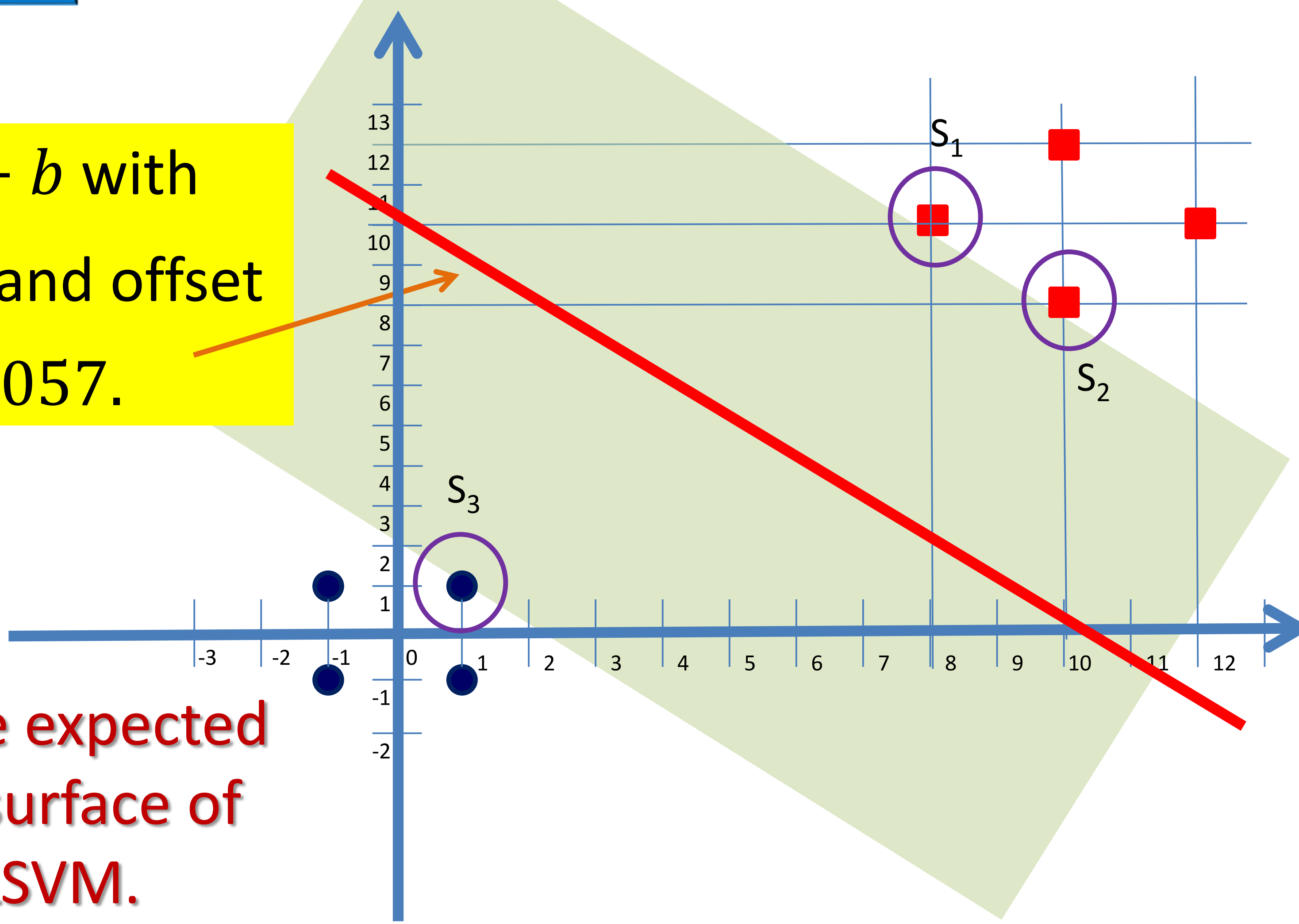
Hyper Plane Seperating the Two Classes

- $y = wx + b$ with $w = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and offset $b = -10.057$.



- This is the expected decision surface of the Non LSVM.

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Support Vector Machines – Programme Code Example 1

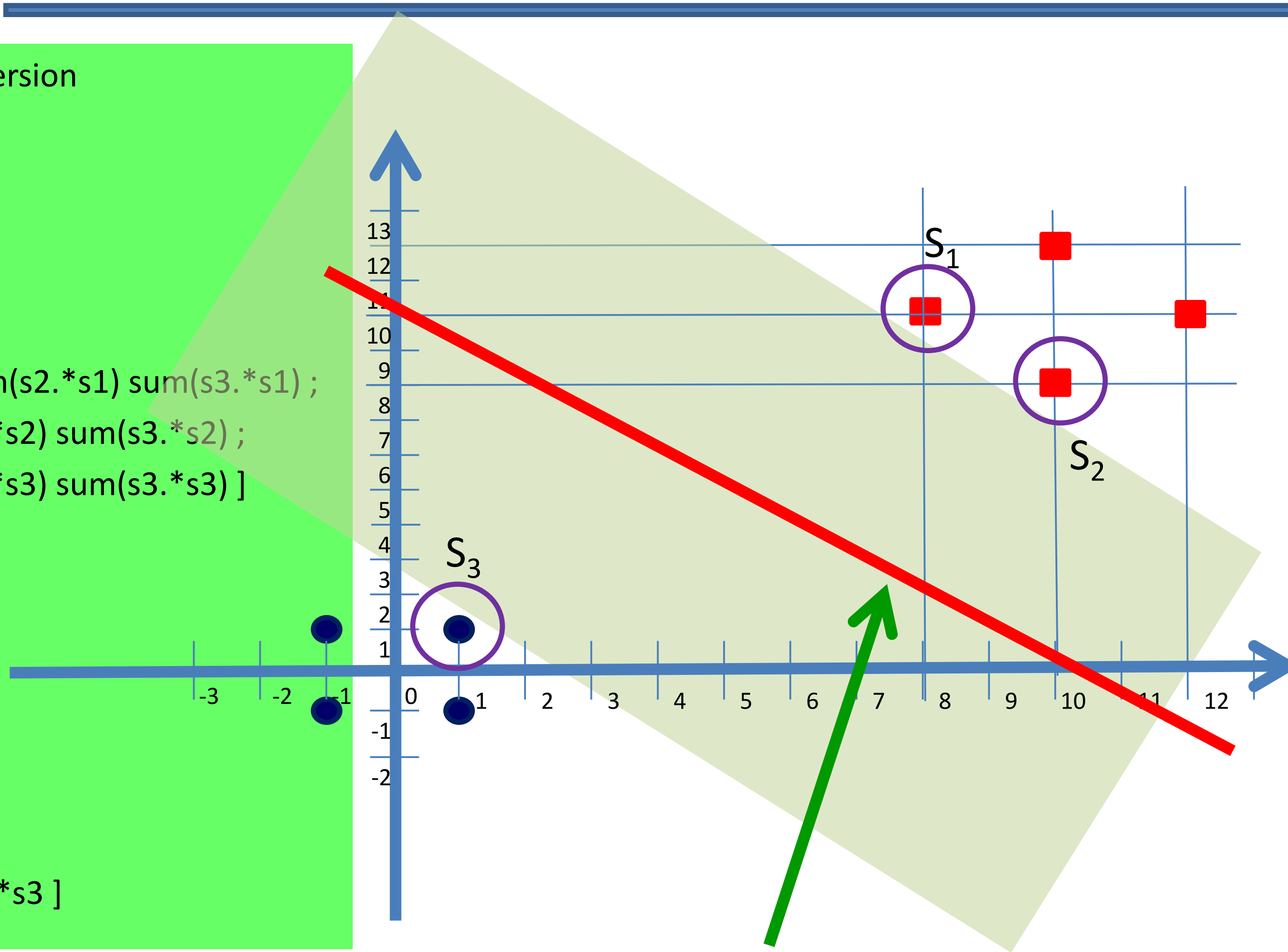
```
% 3 support vector version

s1 = [ 8 10 1 ];
s2 = [ 10 8 1 ];
s3 = [ 1 1 1 ];

A = [ sum(s1.*s1) sum(s2.*s1) sum(s3.*s1) ;
sum(s1.*s2) sum(s2.*s2) sum(s3.*s2) ;
sum(s1.*s3) sum(s2.*s3) sum(s3.*s3) ]
Y = [ -1 -1 +1 ]
X = Y/A

p = X(1)
q = X(2)
r = X(3)

W = [ p*s1 + q*s2 + r*s3 ]
```



When you run you should get: $\tilde{w}$ = [0.125 0.125 -1.25]. This is the line shown passing through x1=1.25/0.125=10 with a gradient -0.125/0.125=-1. (Note the values here are approximated)

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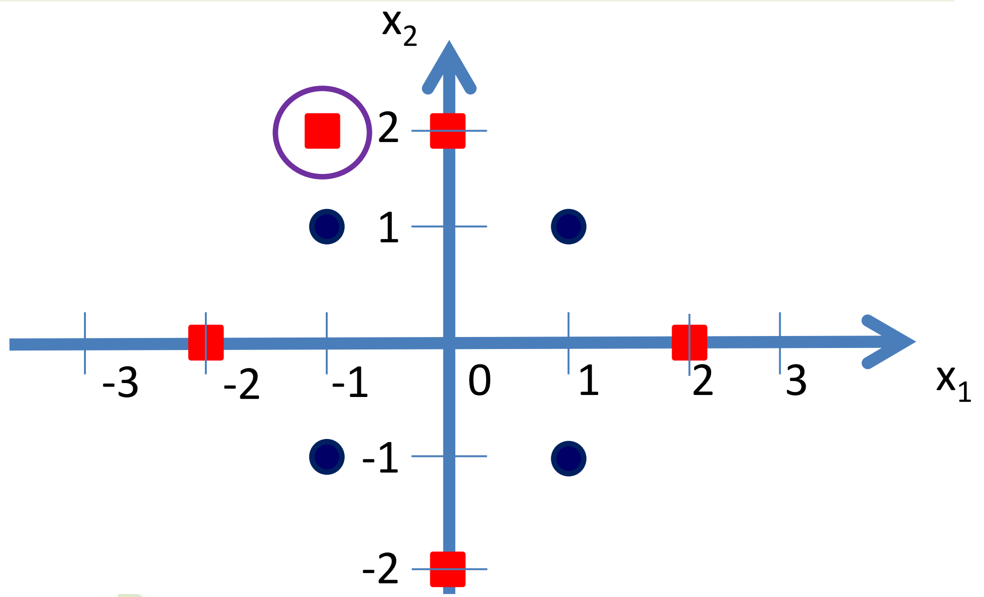
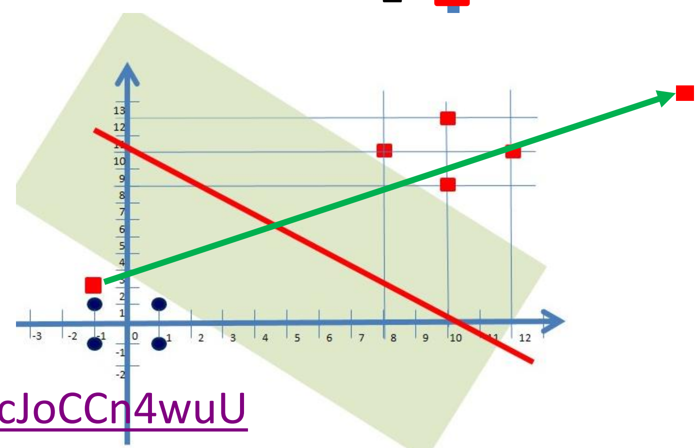
Non-Linear Support Vector Machines – Classification Examples

$$\Phi\begin{pmatrix}x_1\\x_2\end{pmatrix} = \begin{cases}\begin{pmatrix}6-x_1+(x_1-x_2)^2\\6-x_2+(x_1-x_2)^2\end{pmatrix} & \text{if } \sqrt{x_1^2+x_2^2}\geq 2\\ \begin{pmatrix}x_1\\x_2\end{pmatrix} & \text{otherwise}\end{cases}$$

- Let's consider a classification example here.
- Let's classify the point $(x_1,x_2)=(-1,2)$.
- $\Phi\begin{pmatrix}x_1\\x_2\end{pmatrix} = \Phi\begin{pmatrix}-1\\2\end{pmatrix} = \begin{pmatrix}6+1+(-1-2)^2\\6-2+(-1-2)^2\end{pmatrix} = \begin{pmatrix}16\\13\end{pmatrix}$
- $w.\Phi(x) = \begin{pmatrix}1\\1\end{pmatrix}.\begin{pmatrix}16\\13\end{pmatrix} = 29 > 10$
- Hence this point belongs to the red class.





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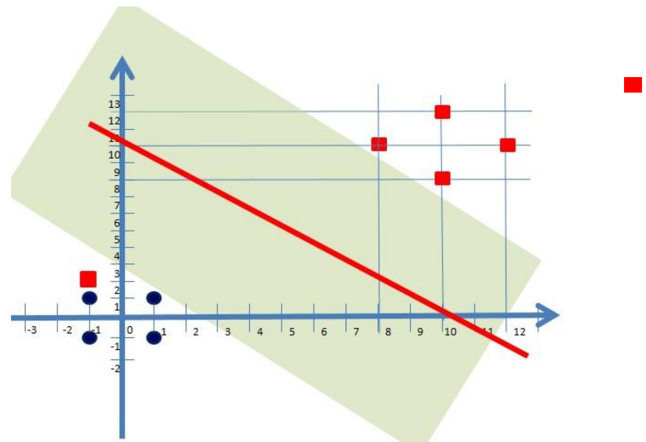
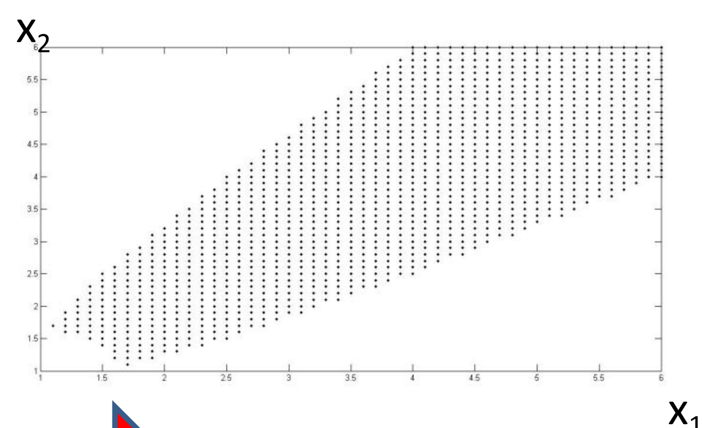
Important Note

$$\Phi\begin{pmatrix}x_1\\x_2\end{pmatrix}=\begin{cases}\begin{pmatrix}6-x_1+(x_1-x_2)^2\\6-x_2+(x_1-x_2)^2\end{pmatrix} & \text{if } \sqrt{x_1^2+x_2^2}\geq 2\\ \begin{pmatrix}x_1\\x_2\end{pmatrix} & \text{otherwise}\end{cases}$$

- Please note that due to the nature of the non linear transfer function and the selected support vectors used, here not all the points with $\sqrt{x_1^2+x_2^2}\geq 2$ will transform above the current classification boundary.
- The Figure shows (in dots) the region that will **not correctly** classify using the current mapping function.
- Feature vectors that lie in all the other regions inluding the vectors in negative quadrents can be classsified correctly using this non-linear SVM approach.





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(Non Linear SVM)

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