A Theory of Supply Function Choice and Aggregate Supply

Joel P. Flynn[*] — Yale
George Nikolakoudis[†] — Princeton
Karthik A. Sastry[‡] — Princeton

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Abstract

Many modern theories of the business cycle generate demand-driven fluctuations by assuming that monopolistic firms set a price in advance and commit to supplying the market-clearing quantity. In this paper, we enrich firms' supply decisions by allowing them to choose any *supply function*: a description of the price charged at each quantity of production. We characterize firms' optimal supply function choice in general equilibrium and analyze how this choice affects the slope of *aggregate supply*. We find that aggregate supply flattens when there is (i) lower inflation uncertainty, (ii) greater idiosyncratic demand uncertainty, and (iii) increased market power. Money is maximally non-neutral if firms' quantities are perfectly elastic to prices (price-setting) and neutral if only if firms' quantities are perfectly inelastic to prices (quantity-setting). When mapped to the data, our theory explains the long-run flattening of aggregate supply as well as its steepening in times of high inflation uncertainty (*e.g.*, the 1970s and 2020s) but not times of high real uncertainty (*e.g.*, the Great Recession).

[*]Yale University Department of Economics, 30 Hillhouse Avenue, New Haven, CT, 06511. Email: joel.flynn@yale.edu.

[†]Princeton University Department of Economics, Julis Romo Rabinowitz Building, Princeton, NJ 08544. Email: nikolakoudis@princeton.edu.

[‡]Princeton University Department of Economics, Julis Romo Rabinowitz Building, Princeton, NJ, 08544, Email: ksastry@princeton.edu.

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1 Introduction

At the heart of modern models of aggregate supply are monopolistic firms that make decisions under uncertainty. It is common to restrict these firms' supply decisions to an important but specific class: setting a price and committing to producing enough to meet *ex post* demand. For example, price-setting is assumed in classic models of aggregate supply based on exogenous, infrequent adjustment (Taylor, 1980; Calvo, 1983), menu costs (Barro, 1972; Caplin and Spulber, 1987; Golosov and Lucas, 2007), and limited information (Mankiw and Reis, 2002; Woodford, 2003a; Hellwig and Venkateswaran, 2009).

In this paper, we allow firms to choose any supply function: a mapping that describes the price charged at each quantity of production.[1] Supply function choice is a standard approach in microeconomic theory to model firms' ability to adjust decisions to realized demand, while remaining consistent with a foundation of information, contracting, or organizational frictions (*e.g.*, Grossman, 1981; Hart, 1985; Klemperer and Meyer, 1989; Vives, 2011, 2017; Pavan et al., 2022; Rostek and Yoon, 2023). Seen through this lens, committing to a fixed price is not generally the optimal choice. Our goal is to understand how allowing for richer behaviors at the microeconomic level affects our understanding of the macroeconomy.

We find that introducing supply functions in an otherwise standard monetary business cycle model yields a novel theory of an aggregate supply curve with an *endogenous* slope (defined as the ratio of the responses of the price level and real output to a monetary shock). Concretely, aggregate supply flattens under: (i) lower price-level uncertainty, (ii) greater idiosyncratic demand uncertainty, (iii) increased market power, and (iv) more hawkish monetary policy. When applied to the data, our model generates variation in the slope of aggregate supply that is consistent with empirical evidence on how this slope has changed over time in the US. Taken together, our analysis shows how changes in *monetary policy*—like a commitment to price stability—or changes in *market structure*—such as a secular rise in market power—may flatten the aggregate supply curve.

Supply Function Choice of a Single Firm. We begin our analysis in partial equilibrium. We study a firm that faces a constant-price-elasticity demand curve and operates a constant-returns-to-scale production function. It has log-normal uncertainty about its competitors' prices, demand, productivity, input prices, and the stochastic discount factor.

Given its beliefs, the firm chooses a *supply function* $f : \mathbb{R}^2_{++} \to \mathbb{R}$. This function defines the firm's supply curve as the locus of prices and quantities that solves $f(p, q) = 0$. Because the market clears, the firm produces and prices where the market demand curve intersects its

[1] This is different from *nonlinear pricing*, whereby firms transact different quantities at different prices. A supply function specifies the uniform price that everyone pays as a function of the total quantity sold.

supply curve. Internalizing this, the firm chooses its optimal, non-parametric supply function to maximize its expected real profits under the stochastic discount factor. In other words, we allow firms to implement the ECON 101 notion of a supply curve: a systematic relationship between the price that they charge and the quantity that they produce. Price-setting is nested by functions of the form $f(p) = 0$, or perfectly elastic supply. Quantity-setting is nested by functions of the form $f(q) = 0$, or perfectly inelastic supply (*e.g.*, as in Jaimovich and Rebelo, 2009; Angeletos and La'O, 2010, 2013; Benhabib et al., 2015). Thus, while we allow firms to set prices or quantities, we also allow them to choose potentially more preferable strategies.

We solve in closed-form for the optimal supply function and show that it is endogenously log-linear: $\log p = \alpha_0 + \alpha_1 \log q$. Thus, the firm's behavior in response to changes in market demand is described by its optimally chosen inverse supply elasticity, α_1: the percentage by which the firm increases prices in response to a one percent increase in production. In turn, this elasticity depends on the firm's market power and its relative uncertainty about demand, competitors' prices, and real marginal costs. These relationships arise because uncertainty and market power shape firms' relative desires to hedge against different types of shocks.

Three comparative statics are particularly important for our macroeconomic analysis. First, higher uncertainty about firm-level demand pushes toward a lower α_1, or firms' behaving more like price-setters. The limit case of price-setting perfectly insulates firms against demand shocks, as the optimal response of a firm to changing demand conditions is to set its relative price equal to a constant markup on its real marginal cost. Second, higher uncertainty about competitors' prices pushes toward a higher α_1, or firms' behaving more like quantity-setters. The limit case of quantity-setting perfectly insulates firms against shocks to competitors' prices as it allows the firm's relative price to adjust perfectly in response to such changes. Third, a lower elasticity of demand pushes toward a lower α_1, or firms behaving more like price-setters. More market power, thus defined, reduces the cost to the firm of setting the "wrong" price.

General Equilibrium: From Supply Functions to Aggregate Supply. To study the aggregate implications of supply-function choice, we next embed our framework in a monetary business-cycle model with incomplete information (Woodford, 2003a; Hellwig and Venkateswaran, 2009). In addition to exogenous microeconomic and macroeconomic uncertainty, the model generates endogenous macroeconomic uncertainty about firms' demand, aggregate prices, and real marginal costs.

We first characterize aggregate outcomes given fixed firm-level supply functions. This allows us to isolate the importance of supply functions for shock transmission, before studying equilibrium choice of supply functions. We show that, in the unique log-linear equilibrium, the price level and real output follow an *aggregate supply and aggregate demand* representation. There is a well-defined "slope of aggregate supply," which also corresponds to the relative responses of the price level and real output to an aggregate demand (money supply) shock. This slope depends critically on the slope of firms' supply functions. Aggregate supply is inelastic—or, money is neutral—if and only if firms are quantity-setters. Aggregate supply is maximally elastic—or, money is as non-neutral as possible—if firms are price-setters. Between those extremes, the slope of aggregate supply is monotone increasing in the slope of firm-level supply. Finally, a lower elasticity of demand (increased market power) flattens the aggregate supply curve. This effect is present as long as firms are *not* pure price-setters. Intuitively, a higher elasticity of demand increases how much a given change in the *aggregate* price level moves any given firm's demand curve.

We next characterize how the slope of aggregate supply is endogenously determined, via the fixed point relating macroeconomic uncertainty to firms' supply-function choice. This reveals feedback loops: uncertainty affects supply functions, which affects the slope of aggregate supply, and in turn shapes macroeconomic uncertainty.

Under an empirically reasonable parameter restriction that balances strategic complementarity (from aggregate demand externalities) with substitutability (from wage pressure), we can derive the slope of aggregate supply in closed form. This slope decreases in firms' *relative uncertainty* about idiosyncratic demand shocks *vs.* the money supply. For example, an economy with more hawkish monetary policy, defined as a less volatile money supply, features a flatter aggregate supply curve and therefore an endogenously smaller effect of aggregate demand shocks on the price level. In that sense, this economy has "more stable prices" for two reinforcing reasons. First, there are fewer aggregate demand shocks. Second, firms respond to more stable prices by flattening their supply curves, which endogenously reduces the responsiveness of prices to aggregate demand shocks. This observation is consistent with the narrative that more hawkish monetary policy in the United States (*e.g.*, during and after the tenure of Paul Volcker) achieved price stability by flattening the aggregate supply curve. Moreover, this implies the following trade-off for policymakers: maintaining a high degree of monetary discretion, which induces greater monetary uncertainty, endogenously makes monetary policy less effective at influencing the real economy.

An economy with higher idiosyncratic demand variation also features a flatter aggregate supply curve. Combining this with the empirical observation that firms' idiosyncratic uncertainty rises substantially in recessions (Bloom et al., 2018), our theory offers the following resolution to the puzzle of "missing disinflation" during the Great Recession: aggregate supply itself endogenously *flattened* in the face of a large jump in microeconomic uncertainty.

Away from the special case of balanced strategic interaction, the model makes richer predictions in which market power and the volatility of productivity shocks also affect the slope of aggregate supply. Our quantitative exercise will suggest that both forces play an important role in determining the slope of aggregate supply in the US.

Aggregate Supply in the Model and the Data. We finally study the model's implications for the slope of aggregate supply in the United States. To do so, we estimate time-varying uncertainty for macroeconomic aggregates using a Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model on aggregate time series for output, the price level, and real marginal costs. We combine these estimates with the model to generate a historical time series of the aggregate supply slope in the US.

The model helps explain three empirically documented phenomena related to the changing slope of aggregate supply in the US. First, the model explains a quantitatively significant portion of the steeping of aggregate supply from the 1960s to the 1970s and the flattening of aggregate supply from the 1970s to the Great Moderation (as estimated by, *e.g.*, Ball and Mazumder, 2011). The changes in the slope of aggregate supply are primarily driven by changes in inflation uncertainty. Concretely, when inflation uncertainty is low, firms choose flatter supply functions: pure price-setting becomes more favorable to firms both when relative price variation is small. This flattening at the micro level translates to a flatter aggregate supply curve. We also show how incorporating an upward trend in market power into our calibration implies an even more dramatic flattening.

Second, our model rationalizes why aggregate supply remained flat during the Great Recession (a period characterized by a spike in real, rather than nominal, uncertainty). This allows our model to rationalize both the missing disinflation (Coibion and Gorodnichenko, 2015) and missing inflation (Bobeica and Jarociński, 2019) puzzles—inflation did not fall by as much as implied by typical menu-cost or Calvo-pricing models during the Great Recession and did not rise by as much thereafter.

Finally, we find that the model explains the steepening of aggregate supply in the post-Covid period, as estimated by Cerrato and Gitti (2022). Our model rationalizes this as a consequence of a surge in inflation uncertainty—implying both that post-pandemic aggregate demand shocks may have had large effects on prices and that contractionary monetary policy might be able to rein in inflation with relatively low costs to output.

Related Literature. Our main methodological contribution is to derive aggregate supply in a business-cycle model from a foundation of supply function competition. Supply function competition has been extensively studied in microeconomic theory, industrial organization, and finance (Grossman, 1981; Hart, 1985; Klemperer and Meyer, 1989; Kyle, 1989; Vives, 2017; Pavan et al., 2022; Rostek and Yoon, 2023) but has not yet been applied in a macroeconomic setting. We contribute to this theoretical literature by analytically characterizing equilibrium supply functions with several new features: non-quadratic preferences; imperfect substitutability; multiple, correlated sources of uncertainty; and strategic interactions in both input and product markets.

In the macroeconomics literature, Lucas and Woodford (1993) and Eden (1994) study a different market structure of *ex ante* capacity investments and sequential transactions that generate behavior that is as if firms set supply schedules. In these models, as under supply function competition, prices vary with quantities according to the producer's plan. Nonetheless, by abstracting from a specific model for these adjustment dynamics and instead studying supply functions in reduced form, we obtain analytical solutions for firms' optimal supply function choices and equilibrium macroeconomic dynamics.

The closest analysis in the literature on firms' optimal supply decisions is performed by Reis (2006), who compares price-setting and quantity-setting for a rationally inattentive firm in a canonical macroeconomic setting. Our analysis differs from and builds on Reis' analysis by studying supply schedules beyond price-setting and quantity-setting, allowing for multiple, correlated shocks to the firm, and studying equilibrium implications.

Our finding that uncertainty shapes the slope of aggregate supply is shared with two literatures. First, and most relatedly, the classic "islands model" analysis of Lucas (1972, 1973, 1975). Unlike Lucas' model (1972), our model features monopolist producers engaging in supply-function competition instead of price-taking producers (competitive markets). The "inference problem" that links uncertainty to supply decisions in our model arises for a different reason, without reference to the migration or physically separated markets hypothesized by Phelps (1970). Second, menu cost models of price-setting allow the extent of *total* uncertainty to matter for the extent of monetary non-neutrality (see *e.g.*, Barro, 1972; Sheshinski and Weiss, 1977; Golosov and Lucas, 2007). By contrast, in our theory, *total* uncertainty does not affect the slope of aggregate supply, while *relative* uncertainty does.

Finally, by studying how the macroeconomic implications of firms' information and market power are mediated by supply functions, our analysis contributes to the literatures on information and aggregate supply (*e.g.*, Woodford, 2003a; Maćkowiak and Wiederholt, 2009; Afrouzi and Yang, 2021; Angeletos and Huo, 2021) and market power and aggregate supply (*e.g.*, Mongey, 2021; Wang and Werning, 2022; Fujiwara and Matsuyama, 2022).

Outline. Section 2 solves for the firm's optimal supply function in partial equilibrium. Section 3 introduces the monetary business cycle model in which we embed supply functions. Section 4 characterizes equilibrium with supply function choice and shows how supply function choices affect aggregate supply. Section 5 compares the model's predictions for the slope of aggregate supply to existing empirical evidence. Section 6 concludes.

2 Supply Function Choice in Partial Equilibrium

In this section, we introduce our model of supply function choice for a single firm making decisions under uncertainty. To build intuition, we begin by studying the simpler problem of choosing between just two supply functions: price-setting and quantity-setting. Then, we study the more general problem of choosing a non-parametric supply function. Our main result in this section shows that optimal supply functions are log-linear and characterizes their slope in terms of firms' uncertainty and market power.

2.1 The Firm's Problem

Set-up. A firm produces output $q \in \mathbb{R}_+$ via a constant-returns-to-scale production technology using a single input $x \in \mathbb{R}_+$:

$$q = \Theta x \tag{1}$$

where $\Theta \in \mathbb{R}_{++}$ is the firm's Hicks-neutral productivity. The firm can purchase the input at price $p_x \in \mathbb{R}_{++}$. The firm faces a constant-elasticity-of-demand demand curve given by:

$$\frac{p}{P} = \left(\frac{q}{\Psi}\right)^{-\frac{1}{\eta}} \tag{2}$$

where $p \in \mathbb{R}_+$ is the market price, $\Psi \in \mathbb{R}_{++}$ is a demand shifter, $P \in \mathbb{R}_{++}$ is the aggregate price level, and $\eta > 1$ is the price elasticity of demand. We interpret the elasticity of demand as an (inverse) measure of market power: when η is high, the quantity demanded is more sensitive to the price. The firm's profits are priced according to a real stochastic discount factor $\Lambda \in \mathbb{R}_{++}$. For simplicity, we define the firm's real marginal cost as $\mathcal{M} = P^{-1}\Theta^{-1}p_x$.

At the beginning of the decision period, the firm is uncertain about demand, costs, others' prices, and the stochastic discount factor (SDF). Specifically, they believe that the state $(\Psi, \mathcal{M}, P, \Lambda)$ follows a log-normal distribution with mean μ and variance Σ.[2] The firm's payoff is given by its expected real profits (revenue minus costs), as priced by the real SDF:

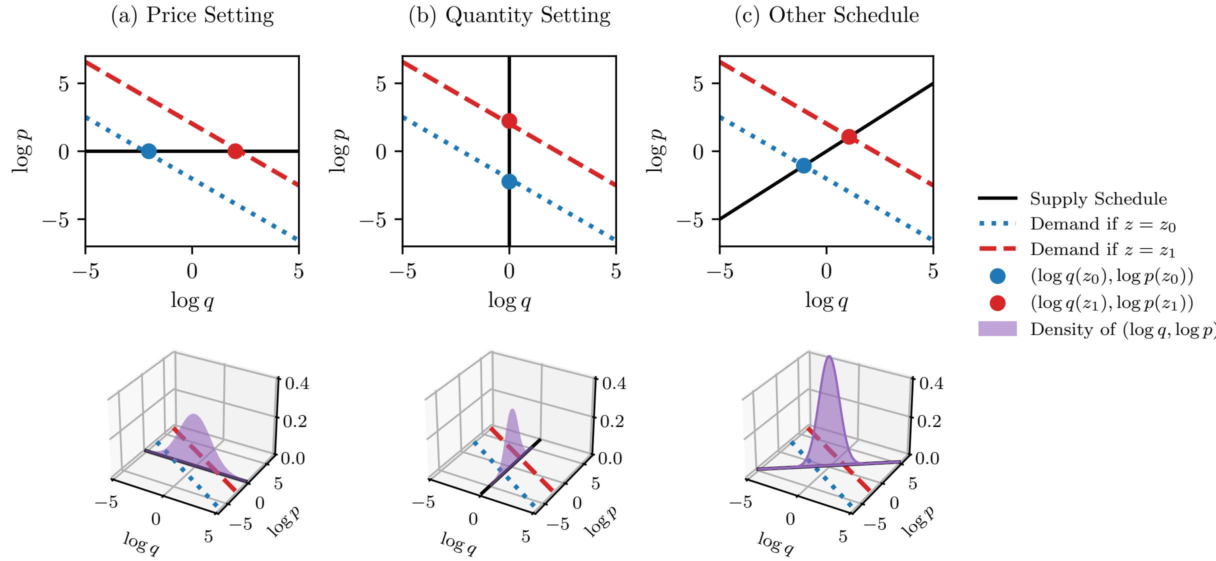
$$\mathbb{E}\left[\Lambda\left(\frac{p}{P} - \mathcal{M}\right)q\right] \tag{3}$$

where $\mathbb{E}[\cdot]$ is the firm's expectation given some joint beliefs about $(\Lambda, P, \mathcal{M}, p, q)$.

The firm commits to implementing price-quantity pairs described by the implicit equation $f(p, q) = 0$ where $f : \mathbb{R}^2_{++} \to \mathbb{R}$. We will refer to f as the supply function. Price-setting is nested as a case in which $f(p, q) \equiv f^P(p)$. Quantity-setting is nested as a case in which

[2] Of course, $\mathcal{M}$ is log-normal so long as P, Θ, and p_x are log-normal.

Figure 1: An Illustration of Supply-Function Choice



Note: The columns correspond to different supply functions. The top row illustrates *ex post* market clearing for two realizations of the demand curve. The bottom curve illustrates the induced joint distribution of quantities and prices given log-normal uncertainty about z. See the main text for more details.

$f(p, q) \equiv f^Q(q)$. More generally, we allow plans to be given by any non-parametric function f, allowing for possible non-monotonicity and discontinuities.

After choosing a supply function f, and following the realization of Ψ and P, the firm produces at a point where f intersects the demand curve. That is, the market clears. Toward making this rigorous, we define the *nominal demand state* $z = \Psi P^\eta$ and rewrite the demand curve as $q = zp^{-\eta}$. Thus, having set f and following the realization of z, the firm's price is given by some solution $\hat{p}$ to the equation $f(\hat{p}, z\hat{p}^{-\eta}) = 0$ with the realized quantity being $\hat{q} = z\hat{p}^{-\eta}$. We assume that the firm chooses the profit-maximizing selection from the set of solutions if there are many and does not produce if there is no solution. Given a supply function f, we let $H(f)$ be the induced joint distribution over $(\Lambda, P, \mathcal{M}, p, q)$ given the firm's prior beliefs. The firm's problem of choosing an optimal supply function is therefore:

$$\sup_{f:\mathbb{R}^2_{++}\to\mathbb{R}} \mathbb{E}_{H(f)} \left[\Lambda \left(\frac{p}{P} - \mathcal{M}\right) q\right] \tag{4}$$

To demonstrate how different supply functions may yield different payoffs, in Figure 1 we illustrate how different supply functions translate into different distributions over outcomes for the firm. The columns respectively correspond to a price-setting function, $\log p = 0$ or $f(p, q) = 1 - p$; a quantity-setting function, $\log q = 0$ or $f(p, q) = 1 - q$; and a flexible rule, $\log p = \log q$ or $f(p, q) = 1 - \frac{p}{q}$. In the first row, we show each supply function (solid black line) and two demand curves, corresponding to a large demand realization z_1 (red dashed line) and a low demand realization z_0 (blue dotted line). The dots indicate the respective intersections of supply and demand, or realized quantity-price pairs in these states. In the second row, we illustrate the induced joint distribution of quantity-price pairs in purple. The price- and quantity-setting policies fix uncertainty about one dimension, but induce uncertainty in the other. The flexible rule induces joint uncertainty in both variables. To evaluate a supply function, the firm must evaluate its payoffs given the induced joint distribution of prices and quantities with competitors' prices, real marginal costs, and the stochastic discount factor.

Interpreting Supply Functions. The idea behind a supply function is familiar from ECON 101: it is the price that the firm plans to charge given any level of production. In our analysis, as in the existing literature, we will treat the supply function as a metaphor for an unmodelled process by which firms transact with customers.

Klemperer and Meyer (1989) provide two particularly concrete examples of firms that *de facto* implement supply schedules. In the first, they describe how service providers (specifically, management consultants) vary the prices that they charge in response to the quantity of services provided.[3] In the second, they describe how airlines use computer software to put seats on discount depending on how many are currently sold. In this case, the firm explicitly uses technology to implement a supply schedule.

More abstractly, the general interpretation of supply schedules is that they represent and abstract from specific underlying processes of dynamic transactions, capacity choices, and price adjustments. To describe this, we borrow and extend an analogy from Reis (2006). Consider a bakery that must decide upon how much bread to produce and how to price its bread. A price-setting bakery fixes its price and keeps selling bread until it exhausts customer demand at that price. A quantity-setting bakery produces bread and sells it at the greatest price such that all of the bread produced is sold. Extending this analogy, a bakery that chooses a supply function can be thought of as observing how much bread is

[3] They write: "If a consulting firm sticks to a fixed rate per hour, it is fixing a price (perhaps subject to a capacity constraint). In fact, however, even when firms quote fixed rates, the real price often varies. When business is slack, more hours are worked on projects than are reported, but when the office is busy, marginally related training, travel time, and the time spent originally may all be charged to the client. Top management in effect commits to a supply function by choosing the number of employees and the rules and organizational values that determine how both the real price and the number of hours supplied adjust to demand—some firms hold the real price very close to the quoted one by choosing very rigid rules about accurately reporting the hours worked to the client, while others allow individual managers far more discretion."

being sold at a given price and then adjusting its price to maximize profits. Thus, if demand happens to be strong and they are selling a higher quantity than they expected, the baker may decide to raise the price of its bread. The responsiveness of the baker's production to its price is precisely its inverse supply elasticity. The supply function f captures this inverse supply elasticity as the slope of the locus of price-quantity pairs that it implies. Thus, supply functions allow us to more flexibly model the short-run responsiveness of the firm to demand fluctuations.

This argument is formalized by the papers of Lucas and Woodford (1993) and Eden (1994). In these models, trade is sequential and capacity is chosen *ex ante*. The resulting prices charged and quantities produced are responsive to the realized state of demand. Thus, in these models, it is as if firms choose supply schedules. At the cost of not emphasizing these specific microfoundations, supply schedules allow for significantly more tractable analysis.

2.2 Building Intuition: Prices *vs.* Quantities

To build intuition for how uncertainty and market power shape the optimal supply function, we first study the restricted problem of a firm that can either fix a price or a quantity.

Optimal Price-Setting. In this case, $f(p,q) = f^P(p)$ and the price-setting problem is given by:

$$V^P = \max_{p \in \mathbb{R}_+} \mathbb{E}\left[\Lambda\left(\frac{p}{P} - \mathcal{M}\right)\Psi\left(\frac{p}{P}\right)^{-\eta}\right] \tag{5}$$

Taking first-order conditions, the optimal price is given by:

$$p^* = \frac{\eta}{\eta - 1}\frac{\mathbb{E}\left[\Lambda\mathcal{M}P^\eta\Psi\right]}{\mathbb{E}\left[\Lambda P^{\eta-1}\Psi\right]} \tag{6}$$

where the numerator is the expected marginal benefit of charging higher prices in reducing costs and the denominator is the expected marginal cost of charging higher prices in increasing revenue. In the absence of uncertainty, this reduces to the statement that the optimal relative price is a constant markup of $\frac{\eta}{\eta-1}$ on real marginal costs. Substituting the optimal price into the firm's payoff function and rearranging, we obtain that:

$$V^P = \frac{1}{\eta - 1}\left(\frac{\eta}{\eta - 1}\right)^{-\eta}\mathbb{E}\left[\Lambda\mathcal{M}P^\eta\Psi\right]^{1-\eta}\mathbb{E}\left[\Lambda P^{\eta-1}\Psi\right]^\eta \tag{7}$$

Optimal Quantity-Setting. In this case, $f(p,q) = f^Q(q)$ and the quantity-setting problem is therefore:

$$V^Q = \max_{q \in \mathbb{R}_+} \mathbb{E}\left[\Lambda\left(\left(\frac{q}{\Psi}\right)^{-\frac{1}{\eta}} - \mathcal{M}\right)q\right] \tag{8}$$

and optimal quantity is given by:

$$q^* = \left(\frac{\eta}{\eta - 1} \frac{\mathbb{E}\left[\Lambda \mathcal{M}\right]}{\mathbb{E}\left[\Lambda \Psi^{\frac{1}{\eta}}\right]} \right)^{-\eta} \tag{9}$$

where the numerator is the expected marginal cost of expanding production and the denominator is the expected marginal revenue from expanding production. In the absence of uncertainty, this is the quantity that the firm sells by setting its relative price equal to a constant markup on its real marginal cost. Substituting the optimal quantity into the firm's payoff, we obtain:

$$V^Q = \frac{1}{\eta - 1} \left(\frac{\eta}{\eta - 1} \right)^{-\eta} \mathbb{E}\left[\Lambda \mathcal{M}\right]^{1-\eta} \mathbb{E}\left[\Lambda \Psi^{\frac{1}{\eta}}\right]^{\eta} \tag{10}$$

Result: When to Set Prices *vs.* Quantities. A cursory inspection of the values of price-setting and quantity-setting (Equations 7 and 10) reveals that they are not generally equal. Define the log-difference between the values of price-setting and quantity-setting as:

$$\Delta = \log V^P - \log V^Q \tag{11}$$

We obtain the following result for the proportional benefit of prices over quantities.

Proposition 1 (Prices *vs.* Quantities)**.** *The comparative advantage of price-setting over quantity-setting is given by:*

$$\Delta = \frac{1}{2}(\eta - 1) \left(\frac{1}{\eta} \sigma_\Psi^2 - \eta \sigma_P^2 - 2\sigma_{\Psi,\mathcal{M}} - 2\eta \sigma_{P,\mathcal{M}} \right) \tag{12}$$

This is increasing in demand uncertainty σ_Ψ^2. It is decreasing in price uncertainty σ_P^2, the covariance between demand and real marginal costs $\sigma_{\Psi,\mathcal{M}}$, and the covariance between prices and real marginal costs $\sigma_{P,\mathcal{M}}$. Moreover, when $\sigma_{P,\mathcal{M}} \geq -\frac{1}{2}\sigma_P^2$, sgn$(\Delta)$ is decreasing in η.

Proof. See Appendix A.1. □

The key idea is that price- and quantity-setting hedge the firm against different shocks, in analogy to price *vs.* quantity regulation in Weitzman (1974) or interest-rate *vs.* money supply targeting in Poole (1970).

The Role of Uncertainty. To understand the intuition for the comparative statics, we go case by case. First, in the presence of demand shocks alone, setting relative prices equal to a constant markup on marginal costs coincides with the first-best. By contrast, fixing the quantity supplied induces losses. Thus, demand shocks favor price-setting. Second, in the face of aggregate price shocks, fixing an optimal quantity allows relative prices to adjust perfectly while fixing an optimal price leads the firm's price to diverge from the aggregate price and loses revenue. Thus, aggregate price shocks favor quantity-setting. Third and fourth, when demand and real marginal costs or aggregate prices and real marginal costs negatively covary, price-setting causes the firm to produce a large amount exactly when costs are low, favoring price-setting.

The Role of Market Power. The extent to which the firm's uncertainty matters is mediated by the price elasticity of demand (*i.e.*, the extent of market power). In particular, as long as the covariance between prices and real marginal costs is not sufficiently negative, lower market power (higher η) favors quantity-setting and greater market power (lower η) favors price-setting. Fixing marginal costs, as firms' demand curves become more flat, price-setting exposes the firm to significant risk of setting the "wrong" price and either making zero profit (if their price is higher than competitors') or a very negative profit (if their price is lower than competitors'). This is potentially counteracted by a strong negative correlation between costs and others' prices: if when others charge high prices your marginal cost is low, it is better to do price-setting as you undercut your competition precisely when it is profitable to do so. In practice, we will later estimate a positive correlation between the price level and real marginal costs, making this more of a theoretical curiosity.

2.3 The Optimal Supply Function

We now study the globally optimal supply function, the solution to Problem 4. We characterize the firm's optimal policy in closed form and illustrate comparative statics in the extent of uncertainty and the price elasticity of demand.

Optimal Supply Functions. We now derive the optimal plan in closed form using variational arguments.

Theorem 1 (The Optimal Supply Function). *Any optimal supply curve is almost everywhere given by:*

$$f(p,q) = \log p - \alpha_0 - \alpha_1 \log q \tag{13}$$

where the slope of the optimal price-quantity locus, $\alpha_1 \in \overline{\mathbb{R}}$, is given by:

$$\alpha_1 = \frac{\eta\sigma_P^2 + \sigma_{\mathcal{M},\Psi} + \sigma_{P,\Psi} + \eta\sigma_{\mathcal{M},P}}{\sigma_\Psi^2 - \eta\sigma_{\mathcal{M},\Psi} + \eta\sigma_{P,\Psi} - \eta^2\sigma_{\mathcal{M},P}} \tag{14}$$

Proof. See Appendix A.2. $\square$

To provide intuition for this result, it is helpful to first sketch its proof. A useful observation is that the problem of choosing an optimal supply function can be recast as a problem of choosing price-quantity pairs $(p(z), q(z))$ that are indexed by the realization of the nominal demand state $z = \Psi P^\eta$ and are such that the market clears: $(p(z), q(z)) = (p(z), zp(z)^{-\eta})$. Intuitively, when setting a supply schedule, the firm anticipates that it will produce where the demand curve hits the supply function. Thus, as the demand curve is indexed by z, it is as if the firm chooses a z-contingent price-quantity plan. Under price-setting, for instance, the price is fixed at $\bar{p} \in \mathbb{R}_{++}$ and the quantity adjusts to clear the market, $(p(z), q(z)) = (\bar{p}, z\bar{p}^{-\eta})$. Similarly, under quantity-setting, the quantity is fixed at $\bar{q} \in \mathbb{R}_{++}$ and the price adjusts to clear the market, $(p(z), q(z)) = (z^{1/\eta}\bar{q}, \bar{q})$. In a general problem of supply function choice, the only difference is that the contingency of prices and quantities on realized demand, which was also present under both price-setting and quantity-setting, is chosen optimally to maximize payoffs.

A necessary condition for optimality is that, for any given realization $z = t$, there is no local benefit to changing the price $p(t)$. Taking a simple first-order condition of the firm's maximization problem implies that the following equations must hold for almost all $t \in \mathbb{R}_{++}$:

$$p(t) = \frac{\eta}{\eta - 1} \frac{\mathbb{E}[\Lambda \mathcal{M} \mid z = t]}{\mathbb{E}[\Lambda P^{-1} \mid z = t]} \quad \text{and} \quad q(t) = tp(t)^{-\eta} \tag{15}$$

This optimality condition resembles the optimal price under price-setting (Equation 6), with the key difference that it conditions on nominal demand z. Outcomes under optimal rules therefore differ from optimal outcomes under price-setting (or quantity-setting) due to the firm's ability to make inferences about the stochastic discount factor, real marginal costs, and the price level. We are then able to solve for the optimal supply function in closed form, despite the infinite-dimensionality of Problem 4, because Equation 15 reduces to a log-linear relation between p and q given lognormality of the firm's beliefs.

This explains why the optimal supply function is log-linear. It remains to explain why the optimal inverse supply elasticity takes the form given in Equation 14. This specific form arises because α_1 is the relative rate at which the firm wants log prices and log quantities to increase with the nominal demand state $\log z$:

$$\alpha_1 = \frac{\mathrm{d}\log p}{\mathrm{d}\log z} \bigg/ \frac{\mathrm{d}\log q}{\mathrm{d}\log z} = \frac{\mathrm{Cov}[\log z, \log p^{**}]}{\mathrm{Cov}[\log z, \log q^{**}]} \tag{16}$$

where p^{**} and q^{**} are the optimal *ex post* prices and quantities that the firm would set with full information:

$$p^{**} = \frac{\eta}{\eta - 1}\mathcal{M}P \qquad \text{and} \qquad q^{**} = \left(\frac{\eta}{\eta - 1}\right)^{-\eta} \frac{z}{(\mathcal{M}P)^{\eta}} \tag{17}$$

An econometric metaphor illustrates why this is the optimal way to set α_1. By Equation 16, the firm's optimal policy is equivalent to running the following two-stage least squares (2SLS) regression: the firm estimates how its optimal price should change with its optimal quantity, using the nominal demand state z as an instrument for the optimal quantity. The supply function is steep ($|\alpha_1|$ is large) if nominal demand predicts large movements in the *ex post* optimal price. In the 2SLS metaphor, this corresponds to a large coefficient in the "reduced form" regression of p^{**} on z. The supply function is flat ($|\alpha_1|$ is small) if nominal demand predicts large movements in the *ex post* optimal quantity. In the 2SLS metaphor, this corresponds to a large coefficient in the "first stage" regression of q^{**} on z.

Turning to how the nature of uncertainty affects the firm's optimal inverse supply elasticity, we begin by focusing on the case in which the firm's supply schedule is upward-sloping. This occurs if $0 \leq \text{Cov}[\log z, \log(\mathcal{M}P)] \leq \frac{1}{\eta}\text{Var}[\log z]$: high demand predicts that nominal costs are higher, but not too much higher. In this case, greater price-level uncertainty (σ_P^2 increases) steepens the optimal supply schedule. The intuition is similar to the one we derived earlier: not knowing the prices of your competitors makes quantity-setting relatively more attractive relative to price-setting because quantity-setting allows your *relative* price to adjust *ex post*. On the other hand, greater demand uncertainty (σ_Ψ^2 increases) flattens the optimal supply schedule. Intuitively, as in the binary comparison of prices *vs.* quantities, demand uncertainty favors price-setting as it allows production to adjust to accommodate greater demand. Finally, greater covariances between real marginal costs and demand and real marginal costs and the price level increase the firm's inverse supply elasticity. Intuitively, when these covariances increase, the firm expects to produce more exactly when it is more costly. Thus, the firm optimally sets a steeper supply schedule to avoid over-producing in response to changes in demand.

We finally observe that a positively sloped supply function is not guaranteed: if nominal costs move sufficiently with nominal demand, then a monopolist may prefer a *downward* sloping supply function in order to hedge against high costs in high-demand states. In practice, however, we will find no empirical evidence for this condition, and it remains a theoretical curiosity.

Market Power and Supply Functions. The elasticity of demand plays two roles in determining the optimal (inverse) elasticity of supply. The first relates to *payoffs*: when η is high, *ex post* optimal quantities are more sensitive to changes in nominal marginal costs (holding fixed nominal demand). Intuitively, when goods are more substitutable, the firm's optimal policy depends dramatically on whether its marginal costs are above or below others' prices. The second role relates to *information*: when η is high, nominal demand contains relatively more information about the price level P and less about real demand Ψ. When studied in our general-equilibrium environment (Sections 3 and 4), these forces will open up the possibility that the slope of aggregate supply systematically depends on the extent of market power in the macroeconomy.

In general, the interaction of these two forces can make the optimal supply function steepen or flatten when η increases. But, below, we describe a sufficient condition under which steeper demand induces steeper supply:

Corollary 1 (Market Power and Optimal Supply)**.** *A sufficient condition for greater market power to lower the inverse supply elasticity, or* $\frac{\partial \alpha_1}{\partial \eta} > 0$*, is that each of the following three inequalities holds:*

$$\alpha_1 \geq 0,\ \sigma_{\mathcal{M},P} \geq 0,\ 2\eta\sigma_{\mathcal{M},P} + \sigma_{\mathcal{M},\Psi} \geq \sigma_{P,\Psi} \tag{18}$$

Proof. See Appendix A.3. □

The force of these conditions is to restrict the extent to which high nominal demand predicts low marginal costs. In this case, the dominant logic is the following. When there is fiercer competition, an upward-sloping aggregate supply function better allows a firm to index its prices relative to its nominal costs. As discussed earlier, this allows the firm to better hedge its risks from setting the "wrong" price when products are very substitutable.

Later, in our quantitative analysis (Section 5), we find that the condition of Corollary 1 always holds in US data since 1960 as long as $\eta > 3$. Thus, the empirically relevant case appears to be that market power flattens the firms' optimal supply function.

Price- and Quantity-Setting Revisited. The previous result and discussion make clear that pure price- and quantity-setting are "edge cases" in the larger space of supply functions. Nonetheless, we observe below that they are intuitively obtained in the limiting cases of extreme demand or price-level uncertainty:

Corollary 2 (A Foundation for Price-Setting and Quantity-Setting)**.** *The following statements are true:*

1. *As* $\sigma_P^2 \to \infty$, $|\alpha_1| \to \infty$ *and the optimal plan converges to quantity-setting.*
2. *As* $\sigma_\Psi^2 \to \infty$, $\alpha_1 \to 0$ *and the optimal plan converges to price-setting.*

This result provides a foundation for focusing on price- and quantity-setting when only one source of risk is dominant. In a macroeconomic environment, however, we may expect all sources of risk to be present in comparable orders of magnitude. In such a scenario, the extreme policies may perform poorly, for both the firm and the economic analyst.

Extensions: Multiple Inputs, Decreasing Returns to Scale, Monopsony, and Beyond Isoelastic Demand. In Appendix B, we generalize this analysis in two directions. First, we allow for a Cobb-Douglas production technology with multiple inputs, decreasing returns to scale, and convex costs of hiring additional inputs (capturing monopsony). We show that all of these forces change the analysis solely by introducing a single composite parameter that aggregates the decreasing returns and monopsony forces across inputs. We show in Proposition 6 that the optimal supply function remains optimally log-linear and uncertainty enters in a similar way. Moreover, as is perhaps intuitive, decreasing returns to scale and monopsony power both reduce the optimal supply elasticity of the firm and push the firm toward quantity-setting. Second, we allow for demand that is not iso-elastic and separates the firm's own-price elasticity of demand from the firm's cross-price elasticity of demand. We solve for the optimal supply curve in this case in Proposition 7. We show that uncertainty enters in a similar way but the optimal supply curve ceases to be log-linear as the optimal markup is endogenous to the scale of production.

3 A Monetary Macroeconomic Model

We now embed supply-function choice in a monetary macroeconomic model. We otherwise use intentionally standard microfoundations (see, *e.g.*, Woodford, 2003b; Hellwig and Venkateswaran, 2009; Drenik and Perez, 2020). In this context, we will be interested in understanding three things: i) how the microeconomic inverse supply elasticity maps into the elasticity of aggregate supply, ii) how equilibrium macroeconomic dynamics endogenously influence the optimal microeconomic supply elasticity, and iii) how these two channels interact to determine equilibrium macroeconomic dynamics.

3.1 Households

Time is discrete and infinite $t \in \mathbb{N}$. There is a continuum of differentiated goods indexed by $i \in [0,1]$, each of which is produced by a different firm.

A representative household has standard (Hellwig and Veldkamp, 2009; Golosov and Lucas, 2007) expected discounted utility preferences with discount factor $\beta \in (0,1)$ and per-period utility defined over consumption of each variety, C_{it}; holdings of real money balances,

$\frac{M_t}{P_t}$; and labor effort supplied to each firm, N_{it}:

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{C_t^{1-\gamma}}{1-\gamma}+\ln\frac{M_t}{P_t}-\int_{[0,1]}\phi_{it}N_{it}\,\mathrm{d}i\right)\right] \tag{19}$$

where $\gamma \geq 0$ indexes income effects in both money demand and labor supply and $\phi_{it} > 0$ is the marginal disutility of labor supplied to firm i at time t, which is an IID lognormal variable with time-dependent variance, or $\log\phi_{it} \sim N(\mu_\phi, \sigma^2_{\phi,t})$. The consumption aggregate C_t is a constant-elasticity-of-substitution aggregate of the individual consumption varieties with elasticity of substitution $\eta > 1$:

$$C_t = \left(\int_{[0,1]}\vartheta_{it}^{\frac{1}{\eta}}c_{it}^{\frac{\eta-1}{\eta}}\,\mathrm{d}i\right)^{\frac{\eta}{\eta-1}} \tag{20}$$

where ϑ_{it} is an IID preference shock that is also lognormal with time-dependent variance, or $\log\vartheta_{it} \sim N(\mu_\vartheta, \sigma^2_{\vartheta,t})$. We also define the corresponding ideal price index:

$$P_t = \left(\int_{[0,1]}\vartheta_{it}p_{it}^{1-\eta}\,\mathrm{d}i\right)^{\frac{1}{1-\eta}} \tag{21}$$

Households can save in either money or risk-free one-period bonds B_t (in zero net supply) that pay an interest rate of $(1+i_t)$. The household owns the firms in the economy, each of which has profits of Π_{it}. Thus, the household faces the following budget constraint at each time t:

$$M_t + B_t + \int_{[0,1]}p_{it}C_{it}\,\mathrm{d}i = M_{t-1} + (1+i_{t-1})B_{t-1} + \int_{[0,1]}w_{it}N_{it}\,\mathrm{d}i + \int_{[0,1]}\Pi_{it}\,\mathrm{d}i \tag{22}$$

where p_{it} is the price of variety of variety i and w_{it} is a variety-specific nominal wage.

The aggregate money supply follows an exogenous random walk with drift μ_M and time-dependent volatility σ_t^M:

$$\log M_t = \log M_{t-1} + \mu_M + \sigma_t^M\varepsilon_t^M \tag{23}$$

where the money innovation is an IID random variable that follows $\varepsilon_t^M \sim N(0,1)$. So that interest rates remain strictly positive, we assume that $\frac{1}{2}(\sigma_t^M)^2 \leq \mu_M$ for all $t \in \mathbb{N}$. We will say that monetary policy is more *hawkish* when monetary volatility σ_t^M is lower.

3.2 Firms

The production side of the model follows closely the model from Section 2. Each consumption variety is produced by a separate monopolist firm, also indexed by $i \in [0, 1]$. Each firm operates a production technology that is linear in labor:

$$q_{it} = \zeta_{it} A_t L_{it} \tag{24}$$

where L_{it} is the amount of labor employed, ζ_{it} is IID lognormal with time-dependent volatility $\sigma_{\zeta,t}$, or $\log \zeta_{it} \sim N(\mu_\zeta, \sigma^2_{\zeta,t})$, and $\log A_t$ follows an AR(1) with time-varying volatility σ^A_t:

$$\log A_t = \rho \log A_{t-1} + \sigma^A_t \varepsilon^A_t \tag{25}$$

where the productivity innovations are IID and follow $\varepsilon^A_t \sim N(0, 1)$. When the firm sells output at price p_{it} and hires labor at wage w_{it}, its nominal profits are given by $\Pi_{it} = p_{it} q_{it} - w_{it} L_{it}$. Since firms are owned by the representative household, their objective is to maximize expectations of real profits, discounted by a real stochastic discount factor Λ_t. Thus, the firm's payoff is $\frac{\Lambda_t}{P_t} \Pi_{it}$.

At the beginning of time period t, firms first observe A_{t-1} and M_{t-1}. Firms also receive private signals about aggregate productivity s^A_{it} and the money supply s^M_{it}:

$$\begin{aligned} s^A_{it} &= \log A_t + \sigma_{A,s,t} \varepsilon^{s,A}_{it} \\ s^M_{it} &= \log M_t + \sigma_{M,s,t} \varepsilon^{s,M}_{it} \end{aligned} \tag{26}$$

where the signal noise is IID and follows $\varepsilon^{s,A}_{it}, \varepsilon^{s,M}_{it} \sim N(0, 1)$. Firms are uncertain about the idiosyncratic productivity shock z_{it}, demand shock ϑ_{it}, and labor supply shock ϕ_{it}.[4]

3.3 Markets and Equilibrium

In each period, conditional on the aforementioned information set, firms choose a supply function. As in Section 2, firms make this decision under uncertainty about demand, costs, and the stochastic discount factor. But, as will become clear, this uncertainty is now partially about *endogenous* objects. After firms make their choices, the money supply, idiosyncratic demand shocks, and both aggregate and idiosyncratic productivity are realized. Finally, the household makes its consumption and savings decisions and any prices that were not fixed adjust to clear the market. Formally, we define an equilibrium as follows:

[4] It is not important that firms are fully uninformed about these quantities. The model's predictions would be identical if firms also received noisy signals about their idiosyncratic shocks.

Definition 1 (Supply-Function General Equilibrium)**.** *An equilibrium is a collection of variables*

$$\left\{\{p_{it}, q_{it}, C_{it}, N_{it}, L_{it}, w_{it}, \Pi_{it}\}_{i\in[0,1]}, C_t, P_t, M_t, A_t, B_t, N_t, \Lambda_t\right\}_{t\in\mathbb{N}}$$

and a sequence of supply functions $\left\{f_{it} : \mathbb{R}^2_{++} \to \mathbb{R}\right\}_{i\in[0,1],t\in\mathbb{N}}$ *such that, in all periods:*

1. *All firms choose their supply function* f_{it} *to maximize expected real profits under the household's stochastic discount factor.*
2. *The household chooses consumption* C_{it}*, labor supply* N_{it}*, money holdings* M_t*, and bond holdings* B_t *to maximize their expected utility subject to their lifetime budget constraint, while* Λ_t *is the household's marginal utility of consumption.*
3. *Money supply* M_t *and productivity* A_t *evolve exogenously via Equations 23 and 25.*
4. *Firms' and consumers' expectations are consistent with the equilibrium law of motion.*
5. *The markets for the intermediate goods, final good, labor varieties, bonds, and money balances all clear.*

We will also often be interested in describing equilibrium dynamics conditional on a (potentially suboptimal) supply function for firms. Formally, these *temporary equilibria* are equilibria in which we do not require statement (1) of Definition 1. In this spirit, in Appendix C, we also study equilibrium dynamics under the restriction that firms can only choose between two supply schedules, price-setting or quantity-setting, and recover similar insights.

4 Supply Function Choice and Aggregate Supply

We now study the model's equilibrium predictions, focusing on the equilibrium determination of the aggregate supply curve. We proceed in three steps. First, we solve for all equilibrium conditions except for the firm's supply-function decision. Second, we show that, fixing any log-linear supply schedule, the economy admits a unique log-linear equilibrium that has a simple Aggregate Supply and Aggregate Demand representation. Importantly, the slope of aggregate supply depends on the slope of firm-level supply, in conjunction with other parameters. Third, we combine this with our solution for optimal supply schedules from Theorem 1 and fully characterize equilibrium in terms of a single, scalar fixed-point equation for the firm-level supply elasticity. We study how strategic interactions, market power, and the combination of microeconomic demand uncertainty alongside aggregate productivity and monetary uncertainty affect the equilibrium aggregate supply elasticity.

4.1 Firms' Uncertainty in Equilibrium

We begin by deriving the general-equilibrium analogs of the four objects that were central to the firm's problem in Section 2: firm-specific demand shocks, firm-specific marginal costs, the price level, and the stochastic discount factor.

From the intratemporal Euler equation for consumption demand *vs.* labor supply, the household equates the marginal benefit of supplying additional labor $w_{it}C_t^{-\gamma}P_t^{-1}$ with its marginal cost ϕ_{it}. Thus, variety-specific wages are given by

$$w_{it} = \phi_{it} P_t C_t^{\gamma} \tag{27}$$

From the intertemporal Euler equation between consumption and money today, the cost of holding an additional dollar today equals the benefit of holding an additional dollar today plus the value of an additional dollar tomorrow:

$$C_t^{-\gamma}\frac{1}{P_t} = \frac{1}{M_t} + \beta\mathbb{E}_t\left[C_{t+1}^{-\gamma}\frac{1}{P_{t+1}}\right] \tag{28}$$

Further, from the intertemporal choice between bonds, the cost of saving an additional dollar today equals the nominal interest rate $1+i_t$ times the value of an additional dollar tomorrow:

$$C_t^{-\gamma}\frac{1}{P_t} = \beta(1+i_t)\mathbb{E}_t\left[C_{t+1}^{-\gamma}\frac{1}{P_{t+1}}\right] \tag{29}$$

By combining Equations 28 and 29, we obtain that aggregate consumption follows:

$$C_t = \left(\frac{i_t}{1+i_t}\right)^{\frac{1}{\gamma}}\left(\frac{M_t}{P_t}\right)^{\frac{1}{\gamma}} \tag{30}$$

which implies that aggregate consumption is increasing in real money balances, with elasticity given by $\frac{1}{\gamma}$. Intuitively, when consumption utility has greater curvature, income effects in money demand are larger and money demand is more responsive to changes in consumption. Thus, consumption responds less to real money balances when γ is large. The level of real money balances naturally depends on the opportunity cost of holding money i_t, and so money demand is lower when interest rates are high, all else equal.

Moreover, by substituting Equation 30 back into Equation 29, we obtain a recursion that interest rates must satisfy:

$$\frac{1+i_t}{i_t} = 1 + \beta\mathbb{E}_t\left[\frac{1+i_{t+1}}{i_{t+1}}\frac{M_t}{M_{t+1}}\right] \tag{31}$$

As money follows a random walk, solving this equation forward and employing the household's transversality condition, we obtain that:[5]

$$\frac{1+i_t}{i_t} = 1 + \beta \exp\left\{-\mu_M + \frac{1}{2}(\sigma_{t+1}^M)^2\right\}\left(1 + \sum_{i=1}^{\infty}\prod_{j=1}^{i}\beta \exp\left\{-\mu_M + \frac{1}{2}(\sigma_{t+j+1}^M)^2\right\}\right) \tag{33}$$

which is deterministic, but depends on the full future path of monetary volatility.

From the household's choice among varieties, the demand curve for each variety i is

$$\frac{p_{it}}{P_t} = \left(\frac{c_{it}}{\vartheta_{it} C_t}\right)^{-\frac{1}{\eta}} \tag{34}$$

Firm i faces strong demand when aggregate consumption is high, its competitors' prices are low, or its idiosyncratic demand is high. Moreover, η is the price elasticity of demand.

Summarizing the above, we have derived the following equilibrium mapping from endogenous objects to the objects that are relevant to the firm in partial equilibrium.

Proposition 2 (Firm-Level Shocks in General Equilibrium)**.** *In any temporary equilibrium, demand shocks, aggregate price shocks, stochastic discount factor shocks, and marginal cost shocks follow:*

$$\Psi_{it} = \vartheta_{it} C_t\,, \quad P_t = \frac{i_t}{1+i_t} C_t^{-\gamma} M_t\,, \quad \Lambda_t = C_t^{-\gamma}\,, \quad \mathcal{M}_{it} = \frac{\phi_{it} C_t^{\gamma}}{z_{it} A_t} \tag{35}$$

Proof. See Appendix A.4. □

The first expression says that demand shocks have two components: an idiosyncratic shock deriving from consumer preferences and an aggregate shock corresponding to the aggregate demand externality (Blanchard and Kiyotaki, 1987). The second expression says that the price level must increase in nominal money balances, increase in the nominal interest rate, and decrease in consumption to lie on this demand curve. The third expression says that marginal utility is higher when consumption is lower. The fourth expression says that costs increase when wages increase due to disutility shocks or income effects and when productivity decreases at the micro or macro level.

The uncertainty described in Proposition 2 concerns endogenous objects. This introduces strategic uncertainty (*i.e.*, payoff-relevant uncertainty about other firms' choices), familiar

[5] Observe also that in the case of time-invariant money volatility, interest rates follow the familiar equation:

$$1 + i_t = \beta^{-1} \exp\left\{\mu_M - \frac{1}{2}(\sigma^M)^2\right\} \tag{32}$$

from the analysis of Woodford (2003a) under price-setting but missing, for example, from the analysis of Lucas (1972). Moreover, firms' uncertainty is correlated across variables due to macroeconomic linkages in the product, money, and labor markets.

An important technical implication of Proposition 2 is that, if C_t is log-normal, then so too is $(\Psi_{it}, P_t, \Lambda_t, \mathcal{M}_{it})$. This follows from the fact that all four expressions are log-linear and all other fundamentals $(A_t, M_t, \vartheta_{it}, \phi_{it}, z_{it})$ are log-normal by assumption. Therefore, if we can find that C_t is log-normal in equilibrium, our Theorem 1 can be directly applied to determine the optimal supply function in general equilibrium in a fully non-linear setting. We will call an equilibrium in which $\log C_t$ is linear in $(\log A_t, \log M_t)$ a *log-linear equilibrium*.

4.2 From Supply to Aggregate Supply with Fixed Functions

We next characterize aggregate outcomes given a fixed supply function. This allows us to define the *aggregate supply elasticity* and describe how it depends on firms' supply functions.

In particular, we consider a class of economies in which firms' exogenously set log-linear supply functions of the form

$$\log p_{it} = \alpha^*_{0t,i}(\alpha_{1,t}) + \alpha_{1,t} \log q_{it} \tag{36}$$

where $\alpha_{1,t} \in \mathbb{R}$ is a fixed parameter and $\alpha^*_{0t,i}(\alpha_{1,t})$ is the profit-maximizing "intercept" conditional on this slope.[6] This optimal intercept depends on the slope $\alpha_{1,t}$, the firm's beliefs, and realized demand, but not (independently) on the realized quantity.

Conditional on these supply function choices, we guess and verify that there exists a unique equilibrium allocation in which aggregate consumption and the price level are log-linear in aggregate shocks:

$$\begin{aligned} \log P_t &= \chi_{0,t}(\alpha_{1,t}) + \chi_{A,t}(\alpha_{1,t}) \log A_t + \chi_{M,t}(\alpha_{1,t}) \log M_t \\ \log C_t &= \tilde{\chi}_{0,t}(\alpha_{1,t}) + \tilde{\chi}_{A,t}(\alpha_{1,t}) \log A_t + \tilde{\chi}_{M,t}(\alpha_{1,t}) \log M_t \end{aligned} \tag{37}$$

where, in each equation, the "intercepts" and "slopes" depend on $\alpha_{1,t}$.

Define the posterior weight on the firms' signals of productivity and the aggregate money supply as, respectively,

$$\kappa^A_t = \frac{1}{1 + \left(\frac{\sigma_{A,s,t}}{\sigma^A_t}\right)^2}, \qquad \kappa^M_t = \frac{1}{1 + \left(\frac{\sigma_{M,s,t}}{\sigma^M_t}\right)^2} \tag{38}$$

[6] We will later verify that all firms use a common slope in equilibrium. In light of Theorem 1, this is because all firms are exposed to uncertainty in the same way.

Moreover, define the slope of supply functions in $\log z_{it} = \eta \log P_t + \log \Psi_{it}$ as:[7]

$$\omega_{1,t} = \frac{\alpha_{1,t}}{1 + \eta \alpha_{1,t}} \tag{39}$$

We now characterize equilibrium macroeconomic dynamics with fixed supply functions:

Proposition 3 (Macroeconomic Dynamics with Supply Functions)**.** *If all firms use log-linear supply functions of the form in Equation 36, output in the unique log-linear temporary equilibrium follows:*

$$\log C_t = \tilde{\chi}_{0,t} + \frac{1}{\gamma} \frac{\kappa_t^A}{1 - \omega_{1,t}\left(\eta - \frac{1}{\gamma}\right)(1 - \kappa_t^A)} \log A_t + \frac{1}{\gamma} \frac{(1 - \kappa_t^M)(1 - \eta \omega_{1,t})}{1 - \omega_{1,t}\left(\eta - \frac{1}{\gamma}\right)(1 - \kappa_t^M)} \log M_t \tag{40}$$

and the aggregate price in the unique log-linear temporary equilibrium is given by:

$$\log P_t = \chi_{0,t} - \frac{\kappa_t^A}{1 - \omega_{1,t}\left(\eta - \frac{1}{\gamma}\right)(1 - \kappa_t^A)} \log A_t + \frac{\kappa_t^M + \frac{\omega_{1,t}}{\gamma}(1 - \kappa_t^M)}{1 - \omega_{1,t}\left(\eta - \frac{1}{\gamma}\right)(1 - \kappa_t^M)} \log M_t \tag{41}$$

where $\chi_{0,t}$ and $\tilde{\chi}_{0,t}$ are constants that depend only on parameters (including $\alpha_{1,t}$) and past shocks to the economy.

Proof. See Appendix A.5 □

To build intuition for this result, it is helpful to think of these dynamics as being generated by a simple Aggregate Demand and Aggregate Supply (AD/AS) model, in which productivity shocks shift the AS curve and money shocks shift the AD curve. Indeed, the dynamics derived in Proposition 3 are formally equivalent to those that we would have derived if we had written down the following AD/AS model.

Corollary 3 (AD/AS Representation)**.** *The dynamics generated in the unique log-linear temporary equilibrium are equivalent to those of the following "Aggregate Demand and Aggregate Supply" model:*

$$\log P_t = \log\left(\frac{i_t}{1 + i_t}\right) - \epsilon_t^D \log Y_t + \log M_t \tag{AD}$$

$$\log P_t = \left(\chi_{0,t} - \epsilon_t^S \tilde{\chi}_{0,t}\right) + \epsilon_t^S \log Y_t + (\chi_{A,t} - \epsilon_t^S \tilde{\chi}_{A,t}) \log A_t \tag{AS}$$

[7] So everything remains well defined, we will impose that $\omega_{1,t} \neq \left(\eta - \frac{1}{\gamma}\right)(1 - \kappa_t^x)$ for $x \in \{A, M\}$. Our analysis verifies that these values of $\omega_{1,t}$ cannot occur in log-linear equilibrium (see the proof of Theorem 2).

where the inverse supply and demand elasticities are given by:

$$\epsilon_t^S = \gamma \frac{\kappa_t^M + \frac{\omega_{1,t}}{\gamma}(1-\kappa_t^M)}{(1-\omega_{1,t}\eta)(1-\kappa_t^M)} \quad \textit{and} \quad \epsilon_t^D = \gamma \tag{42}$$

Proof. See Appendix A.6. □

In this representation, the aggregate demand curve is Equation 30, which combines the Euler equations for money and bonds with the transversality condition and implies that: i) the interest rate is a function of exogenous parameters and ii) aggregate consumption has an elasticity of $1/\gamma$ to changes in real money balances. Thus, the "inverse elasticity of aggregate demand" in our model is γ. The aggregate supply curve describes the equilibrium relationship between aggregate output and aggregate prices by aggregating firms' microeconomic pricing and production decisions conditional on a fixed inverse supply elasticity.

We illustrate this representation in Figure 2. An "aggregate demand shock," an increase of the money supply by $\log M_1 - \log M_0 = \Delta \log M > 0$, shifts up the AD curve. This has an effect of $\frac{\Delta \log M}{\epsilon^D + \epsilon^S}$ on real output and $\epsilon^S \frac{\Delta \log M}{\epsilon^D + \epsilon^S}$ on the price level. In particular, the price effect is larger and the quantity effect is smaller if ϵ^S is large. This calculation also makes clear that ϵ^S measures the relative effect of an aggregate demand shock on the price level versus real output.

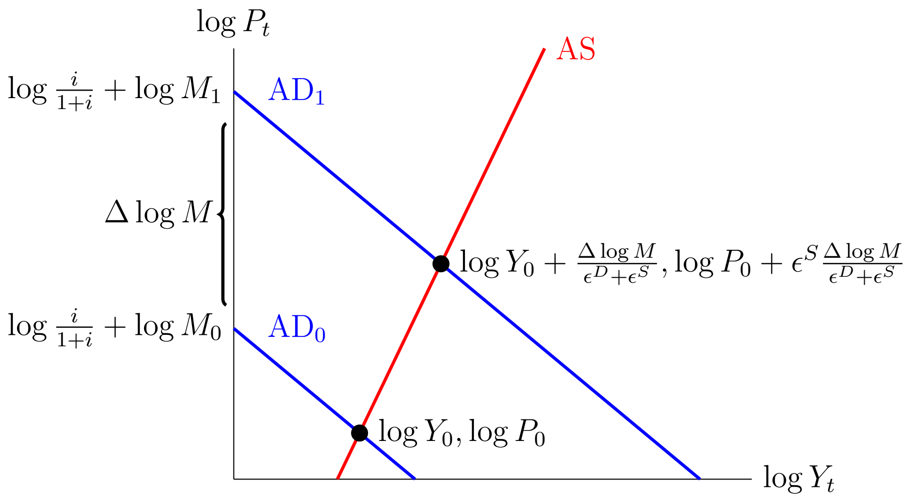
The Propagation of Demand Shocks. To obtain more intuition for the propagation of shocks via firms' supply schedules, we expand out the response of the price level to a money shock into a partial equilibrium effect and a series of higher-order general equilibrium effects:[8]

$$\frac{\Delta \log P}{\Delta \log M} = \frac{\epsilon_t^S}{\epsilon_t^D + \epsilon_t^S} = \underbrace{\left(\kappa_t^M + \frac{\omega_{1,t}}{\gamma}(1-\kappa_t^M)\right)}_{\text{Partial Equilibrium}} \times \underbrace{\sum_{j=0}^{\infty} \left(\omega_{1,t}\left(\eta - \frac{1}{\gamma}\right)(1-\kappa_t^M)\right)^j}_{\text{General Equilibrium}} \tag{43}$$

To understand the partial equilibrium (PE) effect, observe that when M goes up by 1%, all else equal, real money balances increase by 1%. From the household's optimality conditions, this increases their consumption demand by $\epsilon_t^{D^{-1}}\% = \frac{1}{\gamma}\%$. This has two effects. First, the firm experiences a $\frac{1}{\gamma}\%$ demand shock. As the firm has inverse supply elasticity of $\omega_{1,t}$, this leads the firm to increase its prices by $\frac{\omega_{1,t}}{\gamma}\%$. Second, from the household's labor supply condition, real marginal costs increase by $\gamma \times \frac{1}{\gamma}\% = 1\%$. As the firm wishes to set its relative price equal to a constant markup on its real marginal costs, this makes the firm

[8] This summation only converges when $\left|\omega_{1,t}\left(\eta - 1/\gamma\right)\left(1-\kappa_t^M\right)\right| < 1$. Our fixed point arguments establish that the claimed formulae hold more generally whenever $\left|\omega_{1,t}\left(\eta - 1/\gamma\right)\left(1-\kappa_t^M\right)\right| \neq 1$.

Figure 2: An Aggregate Supply and Demand Representation



Note: An aggregate supply and demand illustration of dynamics after a shock of size $\Delta \log M$ to the money supply (see Corollary 3).

want to increase prices by 1%. As they have already increased their prices by $\frac{\omega_{1,t}}{\gamma}\%$, they would achieve this 1% total price increase by increasing prices by $1 - \frac{\omega_{1,t}}{\gamma}\%$ in response to the 1% increase in real marginal costs. However, as firms receive imperfect signals of the money supply, their posterior means after the 1% shock increase in money and real marginal costs increase by only $\kappa_t^M\%$. Thus, on average, they respond to the increase in marginal costs by raising prices by $\kappa_t^M \times (1 - \frac{\omega_{1,t}}{\gamma})\%$. Thus, in partial equilibrium, the firms increase their prices on average by $\frac{\omega_{1,t}}{\gamma} + \kappa_t^M \times (1 - \frac{\omega_{1,t}}{\gamma})\% = \kappa_t^M + \frac{\omega_{1,t}}{\gamma}(1 - \kappa_t^M)\%$, which leads to an equal-sized effect on the aggregate price level.

To understand the general equilibrium effects, consider a 1% increase in the aggregate price level. This has three effects. First, as others' prices have risen by 1%, the firm experiences an $\eta\%$ demand shock. Second, as the price level has risen by 1%, real money balances fall and consumption demand falls by $\frac{1}{\gamma}\%$. Together, given the inverse supply elasticity of $\omega_{1,t}$, these effects lead firms to increase their prices by $\omega_{1,t} \times (\eta - \frac{1}{\gamma})\%$. Third, from the households' labor supply condition, the fall in real consumption demand induces a reduction in real marginal costs by $\frac{1}{\gamma} \times \gamma\% = 1\%$. With perfect information of the 1% increase in the price level, the firm would wish to reduce its price by $\omega_{1,t} \times (\eta - \frac{1}{\gamma})\%$, since this would imply that its *relative* price (which would fall by 1%) is maintained as a constant mark-up over real marginal costs (which has fallen by 1%). However, as firms are imperfectly informed of the monetary shock, if a money shock induced a 1% increase in prices, then they would only on average perceive a $\kappa_t^M\%$ increase in the price level. Thus, they reduce their prices by

$\kappa_t^M \times \omega_{1,t} \times (\eta - \frac{1}{\gamma})\%$. In total, out of a monetarily induced 1% increase in the aggregate price level, the average increase in firms' prices is therefore $\omega_{1,t} \times (\eta - \frac{1}{\gamma}) \times (1 - \kappa_t^M)\%$. Applying this logic to the initial $\kappa_t^M + \frac{\omega_{1,t}}{\gamma}(1 - \kappa_t^M)\%$ increase in prices from the PE effect and iterating it to all subsequent price increases in GE yields Equation 43.

Starkly, these general-equilibrium interactions would be absent if price-setting ($\omega_{1,t} = 0$) were exogenously assumed: the PE effect would be $\kappa_t^M\%$ and the GE effect would be 0%. More generally, a novel implication of our model is that general-equilibrium strategic interaction, known to depend on uncertainty via higher-order beliefs in price-setting models (*e.g.*, Woodford, 2003a) and quantity-setting models (*e.g.*, Angeletos and La'O, 2010), hinges critically on the slope of the supply function.

The Propagation of Supply Shocks. While our study is primarily focused on predictions for the aggregate supply curve and transmission of demand shocks, our model also makes predictions for the transmission of supply shocks. In the AD/AS representation, a positive shock to $\log A_t$ corresponds to an outward shift of the AS curve, which raises real output and lowers the price level. While the relative effect on the price level and on real output is fixed at $\epsilon_D = \gamma$, the level of these responses varies with $\omega_{1,t}$.

To understand the reason for this, we can, just as above, decompose the effect into partial and general equilibrium components:

$$\frac{\Delta \log P}{\Delta \log A} = \underbrace{\kappa_t^A}_{\text{PE}} \times \underbrace{\sum_{j=0}^{\infty} \left(\omega_{1,t} \left(\eta - \frac{1}{\gamma} \right) (1 - \kappa_t^A) \right)^j}_{\text{GE}} \tag{44}$$

The PE effect is immediate: firms perceive a $\kappa_t^A\%$ decrease in their real marginal costs and adjust their prices by an equal percentage. The GE effects of the change in the price level are identical other than that productivity uncertainty may differ from monetary uncertainty. Thus, strategic interactions are attenuated by a factor of $1 - \kappa_t^A$ rather than $1 - \kappa_t^M$. In sum, a key takeaway from our analysis is that the general equilibrium *transmission* of shocks crucially depends on the slope of microeconomic supply curves in the economy.

4.3 The Slope of Aggregate Supply in Temporary Equilibrium

Having demonstrated that the response of the economy to demand and supply shocks can be understood in terms of the slope of the aggregate supply curve, we now formally investigate how various microeconomic forces affect it. The following result, the proof of which follows immediately from differentiation of Equation 42, describes how the slope of aggregate supply depends on four key parameters (holding fixed the others):

Corollary 4 (How Microeconomic Forces Affect Aggregate Supply)**.** *If firms' supply curves are upward-sloping (i.e., $\omega_{1,t} \in [0, 1/\eta)$), then the following statements are true:*

1. *Steeper microeconomic supply steepens the AS curve:* $\partial\epsilon_t^S/\partial\omega_{1,t} \geq 0$*.*
2. *Precision of private information about money steepens the AS curve:* $\partial\epsilon_t^S/\partial\kappa_t^M \geq 0$*.*
3. *Income effects steepen the AS curve:* $\partial\epsilon_t^S/\partial\gamma \geq 0$*.*
4. *Market power flattens the AS curve:* $\partial\epsilon_t^S/\partial\eta \geq 0$*.*

To understand the first statement, observe that a steeper microeconomic supply function makes prices more responsive to realized quantities *ex post*. At the aggregate level, this implies that the price level is also more responsive to changes in output. Second, more precise private information about the money supply steepens the AS curve because firms respond to the perceived increase in the money supply by increasing *average* prices (as modulated through the intercept $\alpha_{0t,i}^*$). This reduces variation in *real* money balances, thereby attenuating the effect of demand shocks on aggregate output. Third, output responds less to money balances the higher is γ (cf. Proposition 2). Consequently, a higher γ steepens the AS curve.

Finally, market power flattens the AS curve. Crucially, this effect is non-zero if and only if $\omega_{1,t} \neq 0$, *i.e.*, firms do not undertake pure *price-setting*. This flattening operates through the general equilibrium transmission mechanisms of the model. When other firms raise their prices in response to a money supply shock, firm-level demand increases because the firm's *relative* price is now lower. The magnitude of this demand change is exactly parameterized by the elasticity of substitution η. If the responsiveness of prices to quantities at the firm level is non-zero, this demand increase generates an additional price level response. Consequently, higher market power flattens the AS curve by lowering the responsiveness of *firm-level* prices to *relative* price changes.

Prices *vs.* Quantities Revisited. We can illustrate some of these effects even more sharply by describing the slope of aggregate supply under the extreme assumptions of price-setting and quantity-setting:

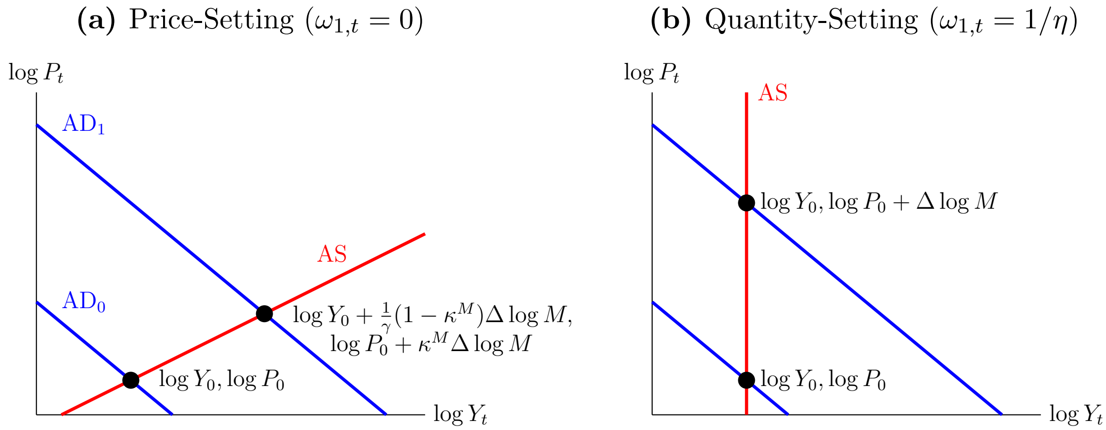
Corollary 5 (Aggregate Supply Under Price- and Quantity-Setting)**.** *If firms engage in price-setting* ($\omega_{1,t} = 0$)*, then:*

$$\epsilon_t^S = \gamma \frac{\kappa_t^M}{1 - \kappa_t^M} \tag{45}$$

If firms engage in quantity-setting ($\omega_{1,t} = \frac{1}{\eta}$)*, then:*

$$\epsilon_t^S = \infty \tag{46}$$

Figure 3: Aggregate Supply Under Price-Setting and Quantity-Setting



Note: An aggregate supply and demand illustration of dynamics after a shock of size $\Delta \log M$ to the money supply (see Corollary 3) under price-setting (panel a) and quantity-setting (panel b).

Since ϵ_t^S is increasing in $\omega_{1,t}$, the price-setting case provides a *lower bound* on the inverse elasticity of the aggregate supply curve and therefore maximizes the real effects of demand shocks. Moreover, as mentioned above, the slope is invariant to the elasticity of demand only in this case. In sharp contrast, the AS curve is vertical under quantity-setting and money has no real effects. This is not a foregone conclusion, but an equilibrium result. Indeed, quantity-setting firms could condition their production on their monetary signal and money would have real effects if they did so.[9] However, if firms set quantities, there is no equilibrium in which firms' quantities depend on the monetary signal as this would be suboptimal. We illustrate these two "extreme" predictions for aggregate supply and demand in Figure 3.

4.4 The Equilibrium Elasticity of Supply

We now endogenize the firm-level inverse supply elasticity as a best response to equilibrium macroeconomic dynamics. We have verified that if firms use log-linear supply functions, then aggregate dynamics are endogenously log-linear (by Proposition 3). Moreover, we have verified that if aggregate dynamics are log-linear, then firms' uncertainty is endogenously log-normal (by Proposition 2). Thus, we have shown that firms' supply curves are endogenously log-linear in a log-linear equilibrium (by Theorem 1). By combining all of these results, we reduce the determination of log-linear equilibrium in the full dynamic economy with functional supply decisions by firms to a single, scalar fixed-point equation for firms'

[9] As a simple example, setting $\log q_{it} = s_{it}^M$ is feasible for firms and this would allow money to have real effects: $\log C_t = \text{cons}_t + \log M_t$.

transformed inverse supply elasticities:

Theorem 2 (Equilibrium Supply Elasticity Characterization)**.** *All (and only all) solutions $\omega_{1,t} \in \mathbb{R}$ of the following equation correspond to transformed inverse supply elasticities in log-linear equilibrium:*

$$\omega_{1,t} = T_t(\omega_{1,t}) \equiv \frac{\frac{\left(\eta-\frac{1}{\gamma}\right)\kappa_t^A}{1-\omega_{1,t}\left(\eta-\frac{1}{\gamma}\right)\left(1-\kappa_t^A\right)}(\sigma_{t|s}^A)^2 + \frac{\frac{1}{\gamma}+\left(\eta-\frac{1}{\gamma}\right)\kappa_t^M}{1-\omega_{1,t}\left(\eta-\frac{1}{\gamma}\right)\left(1-\kappa_t^M\right)}(\sigma_{t|s}^M)^2}{\sigma_{\vartheta,t}^2 + \left(\frac{\left(\eta-\frac{1}{\gamma}\right)\kappa_t^A}{1-\omega_{1,t}\left(\eta-\frac{1}{\gamma}\right)\left(1-\kappa_t^A\right)}\right)^2(\sigma_{t|s}^A)^2 + \left(\frac{\frac{1}{\gamma}+\left(\eta-\frac{1}{\gamma}\right)\kappa_t^M}{1-\omega_{1,t}\left(\eta-\frac{1}{\gamma}\right)\left(1-\kappa_t^M\right)}\right)^2(\sigma_{t|s}^M)^2} \tag{47}$$

where $\left(\sigma_{t|s}^A\right)^2 = \left(1-\kappa_t^A\right)\left(\sigma_t^A\right)^2$ *and* $\left(\sigma_{t|s}^M\right)^2 = \left(1-\kappa_t^M\right)\left(\sigma_t^M\right)^2$.

Proof. See Appendix A.7. □

This fixed-point equation incorporates the variances and covariances that enter the optimal supply function as a function of equilibrium macroeconomic dynamics when firms use supply functions with transformed inverse supply elasticities $\omega_{1,t}$. This depends on the responsiveness of aggregate prices and output to aggregate productivity and monetary shocks as well as the conditional uncertainty about these shocks when firms set their supply functions. Firms' idiosyncratic uncertainty about demand matters, but firms' uncertainty about idiosyncratic productivity and factor prices do not as the variance of marginal costs *per se* does not matter for the choice of an optimal supply function.

In the remainder of this section, we will study this equation to understand equilibrium dynamics. First, we can use this result to establish log-linear equilibrium existence and provide a bound on the number of equilibria by rewriting the fixed-point equation as a quintic polynomial in $\omega_{1,t}$:

Proposition 4 (Existence and Number of Equilibria)**.** *There exists a log-linear equilibrium. There exist at most five log-linear equilibria.*

Proof. See Appendix A.8 □

We now study how uncertainty, strategic interactions, and market power shape the aggregate supply elasticity in equilibrium.

A Simple Characterization Under Balanced Strategic Interactions. We first characterize the elasticity of aggregate supply under the parametric condition $\eta\gamma = 1$. Recall from our discussion in Section 4.2 that η parameterizes the strength of *strategic complementarities*: the additional increase in demand a firm faces from an increase in the aggregate price level due to a change in *relative* prices. In contrast, $1/\gamma$ parameterizes the strength of *strategic substitutabilities*: the reduction in demand a firm faces from an increase in the aggregate price level due to a reduction in *aggregate consumption* (that results from the reduction in real money balances). Hence, $\eta\gamma = 1$ considers the case when these two strategic interactions exactly balance. This allows us to simplify the fixed point in Equation 47 considerably.

Corollary 6 (Idiosyncratic *vs.* Aggregate Demand Uncertainty)**.** *When $\eta\gamma = 1$, the unique inverse elasticity of aggregate supply is*

$$\epsilon_t^S = \gamma \frac{\kappa_t^M}{1-\kappa_t^M}\left(1 + \frac{1}{\gamma^2 \rho_t^2 \kappa_t^M}\right) \tag{48}$$

where $\rho_t = \frac{\sigma_{\vartheta,t}}{\sigma_{t|s}^M}$ is the relative uncertainty about demand vs. the money supply.

Proof. See Appendix A.9 □

First, observe that uncertainty about *aggregate* productivity does not enter the elasticity of aggregate supply as $\eta\gamma = 1$. This is because a perceived increase in aggregate productivity induces all firms to decrease their prices. In the absence of additional strategic interactions, firms will not respond to other firms' price reductions. Hence, the demand state z (Equation 15) is not useful for conducting inference about *nominal* marginal costs and κ_t^A does not enter the fixed point. The same is not true for uncertainty about the money supply, as it induces *direct* variation on the demand state z by changing aggregate consumption through real money balances. Consequently, firms can condition on the demand state z to learn about their real marginal costs when the money supply is uncertain.

Second, as $\rho_t \to \infty$, the inverse elasticity of aggregate supply approaches $\frac{1}{\gamma}\frac{\kappa_t^M}{1-\kappa_t^M}$. This is the AS curve slope under price-setting (Equation 45). Intuitively, *idiosyncratic* demand conditions do not affect a given firm's marginal cost. Hence, as idiosyncratic demand becomes relatively more volatile, the firm optimally sets a constant price to keep its markup over marginal cost constant. Had the firm chosen $\omega_{1,t} \neq 0$, the firm would induce unprofitable variation in its price by responding to idiosyncratic demand conditions.

Third, as $\rho_t \to 0$, the inverse elasticity of aggregate supply becomes vertical. Consequently, money has no real no effects on output. This is the AS curve that arises from quantity-setting ($\omega_{1,t} = \frac{1}{\eta}$). Intuitively, as uncertainty about the money supply – and therefore the aggregate price level – increases, firms find it optimal to keep their quantities constant and let their *relative* price adjust to demand.

This discussion highlights that *relative* uncertainty about idiosyncratic *vs.* aggregate demand shocks is a crucial determinant of the responsiveness of output to monetary shocks. Moreover, this feature only becomes relevant once firms are allowed to optimally choose their supply functions. As Corollary 5 demonstrates, if one were to exogenously impose price-setting or quantity-setting, the slope of aggregate supply is independent of any feature of idiosyncratic or aggregate uncertainty other than the signal-to-noise ratio for the money supply.

Thus, supply function choice implies, as a positive matter, a thorny trade-off for monetary policymakers. If the central bank wishes to maintain discretion to use monetary policy to affect the real economy, this will increase uncertainty about the money supply. In turn, this will steepen the equilibrium aggregate supply curve and make money less effective in guiding real economic outcomes. In this sense, discretionary monetary policy may be, at least partially, self-defeating.

Equilibrium Under Dominant-Uncertainty Limits. To better understand how each source of uncertainty matters, we next characterize how equilibria behave as each source of uncertainty becomes dominant.[10] These results hold away from the case in which $\eta\gamma = 1$.

Corollary 7 (Dominant-Shock Limits)**.** *The following statements are true:*

1. *As $\sigma_{\vartheta,t} \to \infty$, in any equilibrium $\omega_{1,t} \to 0$ (price-setting)*
2. *As $\sigma^M_{t|s} \to \infty$, in any equilibrium $\omega_{1,t} \to \frac{1}{\eta}$ (quantity-setting)*
3. *As $\sigma^A_{t|s} \to \infty$ and $\eta\gamma \neq 1$, in any equilibrium $\omega_{1,t} \to \frac{1}{\eta - \frac{1}{\gamma}}$*

Proof. See Appendix A.10 □

The intuition for this result mirrors that of Corollary 6. As idiosyncratic uncertainty about demand becomes dominant, firms find it optimal to set prices to keep their markup over real marginal costs constant. As prior uncertainty about the money supply becomes dominant, firms become more uncertain about the aggregate price level. Consequently, firms find it optimal to set quantities and let their relative prices adjust to meet demand. Finally, as uncertainty about aggregate productivity becomes dominant, firms use the demand state z to make inferences solely about the realization of aggregate productivity. Under perfect information, a 1% decrease in productivity would imply that firms raise their prices by 1%. This translates to an $(\eta - \frac{1}{\gamma})$% increase in demand for a given firm. Since firms would optimally like to keep their mark-up over marginal cost constant, they infer that this implies a 1% reduction in productivity, and decrease their prices by $\left[1/(\eta - \frac{1}{\gamma})\right]$%. Observe that this force implies a downward-sloping supply curve whenever $\eta\gamma < 1$. Intuitively, if $\eta\gamma < 1$, income effects in labor supply are weak and the firm expects a lower real marginal cost after a positive demand shock.

[10]Formally, we take these limits for $\sigma^x_{t|s}$ and $x \in \{M, A\}$ by scaling $\sigma_{x,s,t}$ and σ^x_t by a common factor.

The (Absent) Role of Total Uncertainty. We have so far seen that the nature of uncertainty (idiosyncratic *vs.* aggregate and demand *vs.* productivity) matters. Thus, the *presence* of uncertainty is of central importance to our analysis. However, a distinguishing feature of the theory that we have developed is that the total *level* of uncertainty does not matter. To make this claim formal, fix a scalar $\lambda \geq 0$ and scale all uncertainty in the economy according to:

$$(\sigma_{\vartheta,t}, \sigma_{z,t}, \sigma_{\phi,t}, \sigma_{A,t}, \sigma_{A,s,t}, \sigma_{M,t}, \sigma_{M,s,t}) \mapsto (\lambda\sigma_{\vartheta,t}, \lambda\sigma_{z,t}, \lambda\sigma_{\phi,t}, \lambda\sigma_{A,t}, \lambda\sigma_{A,s,t}, \lambda\sigma_{M,t}, \lambda\sigma_{M,s,t}) \quad (49)$$

In this sense, λ is a measure of the total level of uncertainty faced by firms. Define the correspondence $\mathcal{E}_t^S : \mathbb{R}_+ \rightrightarrows \bar{\mathbb{R}}$, where $\mathcal{E}_t^S(\lambda)$ is the set of equilibrium inverse supply elasticities for the level of uncertainty λ. We observe the following:

Proposition 5 (Invariance to Uncertainty and Discontinuity in the Limit)**.** *For $\lambda > 0$, $\mathcal{E}_t^S(\lambda)$ is constant and the equilibrium supply elasticity is invariant to the level of uncertainty. Moreover, $\mathcal{E}_t^S(0) = \{\infty\}$. Therefore, the equilibrium supply elasticity is discontinuous in the zero uncertainty limit:*

$$\lim_{\lambda \to 0} \mathcal{E}_t^S(\lambda) \neq \mathcal{E}_t^S(0) \quad (50)$$

Proof. See Appendix A.11 □

Thus, the predictions that our theory makes for the slope of the aggregate supply curve are invariant to the level of uncertainty. This distinguishes our theory from models of costly adjustment of prices or quantities, such as menu cost models (Barro, 1972; Sheshinski and Weiss, 1977). In these theories, the level of uncertainty matters because it both affects the probability that firms hit their adjustment thresholds and firms' optimal adjustment thresholds. For example, in the standard menu cost model with quadratic payoffs and Gaussian shocks, greater risk increases the chance that firms adjust their prices and makes the aggregate supply curve steeper.

Moreover, our model is discontinuous in the zero uncertainty limit. Indeed, $\mathcal{E}_t^S(\lambda)$ is typically neither upper hemi-continuous nor lower hemi-continuous at $\lambda = 0$. Thus, even a vanishingly small level of uncertainty can have significant effects on firm and aggregate behavior.

5 Model Meets Data: Aggregate Supply in the US

In this final section, we study our model's implications for the slope of aggregate supply in the United States. We employ a sufficient-statistics approach, which allows us to bypass any issues of equilibrium selection. We find that our model can explain a quantitatively significant portion of the secular flattening of aggregate supply from the 1980s to the Great Moderation due to changing relative uncertainty about inflation versus demand. The model explains even more of this flattening if we allow for an upward trend in market power. Our model remains consistent with a relatively flat aggregate supply curve in the Great Recession, since this period is characterized by a spike in real rather than nominal uncertainty (at the micro and macro levels), and a pronounced increase in the supply curve's slope in the post-Covid period, due to a resurgence of inflation and cost shocks.

5.1 Mapping the Data to the Model

Our model provides explicit formulae for the firm-level inverse supply elasticity $\omega_{1,t}$ (Theorem 1) and the inverse elasticity of aggregate supply ϵ_t^S (Corollary 3). To obtain estimates of these quantities, we need to know two sets of objects: the structural parameters (η, γ) and firms' time-varying uncertainty. We summarize our calibration in Table 1 and describe the details below.

Structural Parameters. We set the price elasticity of demand at $\eta = 9$, based on the study of Broda and Weinstein (2006). These authors use comprehensive panel data on US imports to estimate demand curves at the level of disaggregated products. This is, usefully for our purposes, *direct* evidence for the slope of demand curves, as opposed to indirect evidence from matching average product markups under the assumption that firms are full-information price setters. Later, in an extension, we consider an alternative calibration with a secular downward trend in η (*i.e.*, a secular upward trend in market power) over our studied time period (1960-2022). We set the size of income effects at $\gamma = 0.095$, based on the calibration in Flynn and Sastry (2022) for the cyclicality of US real wages. This is the relevant moment in our model given Proposition 2. We will later perform a full sensitivity analysis of these choices (see Figure 17).

Time-Varying Uncertainty. We next need to estimate firms' time-varying uncertainty about aggregate prices P_t, demand Ψ_{it}, and real marginal costs $\mathcal{M}_{it}$. To our knowledge, there are no datasets that directly measure firms' potentially correlated uncertainty about both microeconomic and macroeconomic objects. Our approach is to proxy for this using time-varying *statistical* uncertainty about macroeconomic aggregates implied by a GARCH model and assumptions, based on the existing literature, about the systematic relationship between macroeconomic and microeconomic uncertainty. This gives us estimates of $(\sigma^2_{P,t}, \sigma^2_{\Psi,t}, \sigma_{\mathcal{M},\Psi,t}, \sigma_{P,\Psi,t}, \sigma_{\mathcal{M},P,t})$. We then choose a time-invariant value of κ^M, the quality of firms' signal of the money supply, to target a sample-average aggregate supply slope of 0.15.

To estimate our model of macroeconomic uncertainty, we use quarterly-frequency US data on real GDP, the GDP deflator, and capacity-utilization adjusted total factor productivity (TFP) (Basu et al., 2006; Fernald, 2014) from 1960 Q1 to 2022 Q4. Thus, our mapping from model to data considers quarterly-frequency decisions.

We map these variables to our general equilibrium model from Section 3 as follows. First, by Proposition 2, the model-implied demand shock is $\Psi_{it} = Y_t \vartheta_{it}$, where Y_t is aggregate real GDP (*i.e.*, "aggregate demand") and ϑ_{it} is a firm-specific demand shock that is, by construction, orthogonal to aggregate conditions. Thus, we can decompose $\sigma^2_{\Psi,t} = \sigma^2_{Y,t} + \sigma^2_{\vartheta,t}$, where the latter two terms are respectively the perceived variances of $\log Y_t$ and $\log \vartheta_{it}$. Second, we may use Proposition 2 to obtain that real marginal costs are $\mathcal{M}_{it} = (\phi_{it} Y^{\gamma})/(\zeta_{it} A_t)$. However, as the firm-level factor price shock ϕ_{it} and productivity shock ζ_{it} are idiosyncratic and we only need to measure the covariances of $\mathcal{M}_{it}$ with Ψ_{it} and P_t, we do not need to measure ϕ_{it} or ζ_{it}. Thus, it is sufficient for us to measure the common component of real marginal costs $\mathcal{M}_t = Y_t^{\gamma}/A_t$. We use capacity-adjusted TFP as our measure of A_t.

Finally, we assume that uncertainty about idiosyncratic demand is directly proportional to uncertainty about aggregate marginal costs, or $\sigma^2_{\vartheta,t} = R^2 \sigma^2_{\mathcal{M},t}$. We justify this based on the finding of Bloom et al. (2018) that the stochastic volatility of TFPR among manufacturing firms ("micro volatility") is well modeled as directly proportional to stochastic volatility in aggregate conditions ("macro volatility"). This justification relies on an assumption that TFPR dispersion is primarily driven by demand shocks. This assumption is consistent with the findings of Foster et al. (2008) that cross-firm variation in revenue total factor productivity (TFPR) derives almost exclusively from demand differences rather than marginal cost differences within specific industries. Based on the quantitative findings of Bloom et al. (2018), we take $R = 6.5$ as a baseline. In an extension, to examine robustness to this proportionality assumption, we directly use (annual) data on TFPR dispersion from Bloom et al. (2018) to perform our calculation and find very similar results.

We next estimate time-varying uncertainties regarding inflation, real output, and real marginal costs using a multivariate GARCH model. In particular, letting Z_t denote the vector $(\Delta \log P_t, \Delta \log Y_t, \Delta \log \mathcal{M}_t)$, we model

$$Z_t = A Z_{t-1} + \varepsilon_t, \qquad \varepsilon_t \sim N(0, \Sigma_t), \qquad \Sigma_t = D_t^{\frac{1}{2}} C D_t^{\frac{1}{2}} \tag{51}$$

where A is a matrix of AR(1) coefficients, D_t is a diagonal matrix of time-varying variances (and $D_t^{\frac{1}{2}}$ is a diagonal matrix of standard deviations), and C is a static matrix of correlations.

We assume that each diagonal element of D_t, denoted as $\sigma^2_{i,t}$, evolves according to:

$$\sigma^2_{i,t} = s_i + \alpha_i \varepsilon^2_{i,t-1} + \beta_i \sigma^2_{i,t-1} \tag{52}$$

with unknown constant s_i and coefficients (α_i, β_i). Formally, this is a GARCH(1,1) model with constant conditional correlations (Bollerslev, 1990). We estimate all of the parameters via joint maximum likelihood.

Our goal is to capture the broad trends in *relative* uncertainty about macroeconomic variables. There are, of course, many possible statistical models for macroeconomic uncertainty. We use the GARCH approach as it is a standard approach in the literature.[11] In the GARCH model we estimate, uncertainty is high when there is a large prediction error in one of the equations (if $\alpha_i > 0$) and if uncertainty was high previously (if $\beta_i > 0$). The restriction to constant correlations restricts the covariances in Equation 12 to move in proportion to the variances, and thus rules out the possibility that the correlation structure among output, prices, and marginal costs varies over time. In return, this significantly reduces the number of estimated parameters and improves the convergence of the maximum likelihood algorithm.

Using the GARCH estimates, we derive maximum-likelihood point estimates of every element of Σ_t, which correspond to the variances in the (Gaussian) conditional forecast of Z_t. Letting $\hat{\sigma}_t$ denote the point estimates of specific elements of that matrix, we directly obtain $\hat{\sigma}^2_{P,t}$ and $\hat{\sigma}^2_{\mathcal{M},P,t}$ from the GARCH model and then we compute:

$$\hat{\sigma}^2_{\Psi,t} = \hat{\sigma}^2_{Y,t} + R^2 \hat{\sigma}^2_{\mathcal{M},t}\,, \quad \hat{\sigma}_{\mathcal{M},\Psi,t} = \hat{\sigma}_{\mathcal{M},Y,t}\,, \quad \hat{\sigma}_{\Psi,P,t} = \hat{\sigma}_{Y,P,t} \tag{53}$$

We plot our estimates of these objects in Figure 13 in the Appendix. We observe that our estimates of demand uncertainty are an order of magnitude larger than our estimates of other uncertainties. This is natural given our large assumed value of R. But this does not necessarily imply that demand uncertainty is the only influential force shaping the slope of microeconomic or macroeconomic supply, since uncertainties enter our formulas in interaction with the elasticity of demand η. We will return to this point when presenting our results.

Estimates of Model Objects. Armed with these estimates, we can calculate our empirical proxies for the firm-level inverse supply elasticity as simple plug-in estimates:

$$\hat{\alpha}_{1,t} = \frac{\eta \hat{\sigma}^2_{P,t} + \hat{\sigma}_{\mathcal{M},\Psi,t} + \hat{\sigma}_{P,\Psi,t} + \eta \hat{\sigma}_{\mathcal{M},P,t}}{\hat{\sigma}^2_{\Psi,t} - \eta \hat{\sigma}_{\mathcal{M},\Psi,t} + \eta \hat{\sigma}_{P,\Psi,t} - \eta^2 \hat{\sigma}_{\mathcal{M},P,t}} \quad \text{and} \quad \hat{\omega}_{1,t} = \frac{\hat{\alpha}_{1,t}}{1 + \eta \hat{\alpha}_{1,t}} \tag{54}$$

[11] Another option would have been to employ a latent state model that allows volatility to be directly affected by shocks (*e.g.*, Primiceri, 2005; Cogley and Sargent, 2005; Jurado et al., 2015). As Jurado et al. (2015) find very similar uncertainty series in latent-state and GARCH models in the US data both "qualitatively and quantitatively," we employ a GARCH approach for simplicity.

Table 1: Model Parameters and Estimation Approach

Parameter	Interpretation	Method	Value
η	Price elas. of demand	Match Broda and Weinstein (2006)	9
γ	Income effects	Match Flynn and Sastry (2022)	0.095
κ^M	Prec. of monetary info.	Match average slope of aggregate supply	0.40
$\sigma^2_{P,t}$	Price uncertainty	GARCH model	Fig. 13
$\sigma^2_{\Psi,t}$	Demand uncertainty	GARCH + match Bloom et al. (2018)	Fig. 13
$\sigma_{\mathcal{M},\Psi,t}$	Cost-demand covariance	GARCH model	Fig. 13
$\sigma_{P,\Psi,t}$	Pice-demand covariance	GARCH model	Fig. 13
$\sigma_{\mathcal{M},P,t}$	Cost-price covariance	GARCH model	Fig. 13

Note: Description of model parameters, how we interpret them, how we estimate them, and their values. The time series for the time-varying uncertainties are presented in Figure 13. The text of Section 5.1 describes our methods in full detail.

Using this sufficient statistics formula, we do not need to estimate a stochastic process for the underlying structural shocks and we can bypass any issues of equilibrium selection.

Our calculation captures uncertainty about outcomes realized in quarter t and is measurable in data from quarter $t-1$ and prior. It therefore describes incentives of a decisionmaker fixing a choice for quarter t based on their uncertainty at the end of quarter $t-1$. We similarly compute our estimate of the inverse elasticity of aggregate supply:

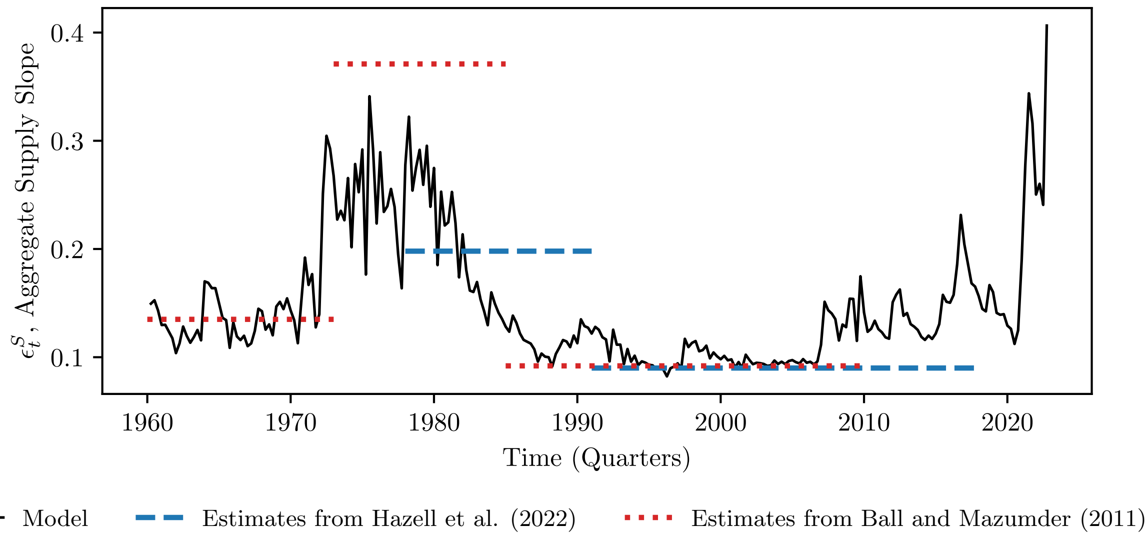
$$\hat{\epsilon}^S_t = \gamma \frac{\kappa^M + \frac{\hat{\omega}_{1,t}}{\gamma}(1-\kappa^M)}{(1-\hat{\omega}_{1t}\eta)(1-\kappa^M)} \tag{55}$$

Given our earlier estimates, this is pinned down given a single unknown parameter, the precision of private information about the money supply, κ^M. As mentioned earlier, we set κ^M so that the average $\hat{\epsilon}^S_t$ from 1960 to 2022 is 0.15. This yields a value of $\kappa^M = 0.40$.

5.2 Results: Aggregate Supply Over Time

Figure 4 plots our main, quarterly-frequency estimates from Equation 55 from 1960 Q1 to 2022 Q4. Aggregate supply was relatively flat in the 1960s, steepened in the 1970s and 1980s, before flattening again during the Volcker disinflation. Furthermore, aggregate supply was notably flat during the Great Moderation leading up to the financial crisis and then steepened again after the Covid-19 pandemic. Figure 14 plots our secondary, annual-frequency estimates that directly incorporate data on microeconomic uncertainty from Bloom et al. (2018). These estimates imply an even more dramatic flattening from the 1970s to the Great Moderation, although we cannot use them to make predictions for the 1960s or 2020s

Figure 4: The Slope of Aggregate Supply Over Time



Note: This Figure plots estimates of the inverse elasticity of aggregate supply as measured by Equations 54 and 55. The blue dashed line indicates the estimates of Hazell et al. (2022), based on state-level estimates of consumer prices and unemployment and an identification strategy that isolates local demand shocks (columns 3 and 4, panel B, of Table II). The red dotted line indicates the estimates of Ball and Mazumder (2011), based on aggregate data (column 4 of Table 3).

as the estimates from Bloom et al. (2018) do not cover these periods.

Our estimates are consistent with external estimates of the time-varying slope of aggregate supply. In Figure 4, we indicate estimates of the slope of aggregate supply by Hazell et al. (2022) with a blue dashed line and estimates by Ball and Mazumder (2011) with a red dotted line. The former authors use state-level data on unemployment and inflation and an instrumental variables (IV) strategy based on isolating state-level demand shocks. The latter authors use aggregate data on the output gap and inflation and measure their unconditional relationship. In the presence of confounding supply shocks, this should systematically understate the slope of aggregate supply.

In Table 2, we quantitatively compare our model's estimates for the flattening of the AS curve during the Great Moderation with those of the aforementioned references. The model with changing uncertainties but fixed structural parameters can explain 51% of the flattening estimated by Hazell et al. (2022) and 84% of the flattening estimated by Ball and Mazumder (2011). By an equivalent calculation, our model can also explain 54% of the latter authors' estimated *steepening* of aggregate supply between 1960-1972 and 1973-1984.

Estimating the change in Aggregate Supply following the Covid Crisis is the focus of an

Table 2: The Flattening Aggregate Supply Curve: Theory *vs.* Evidence

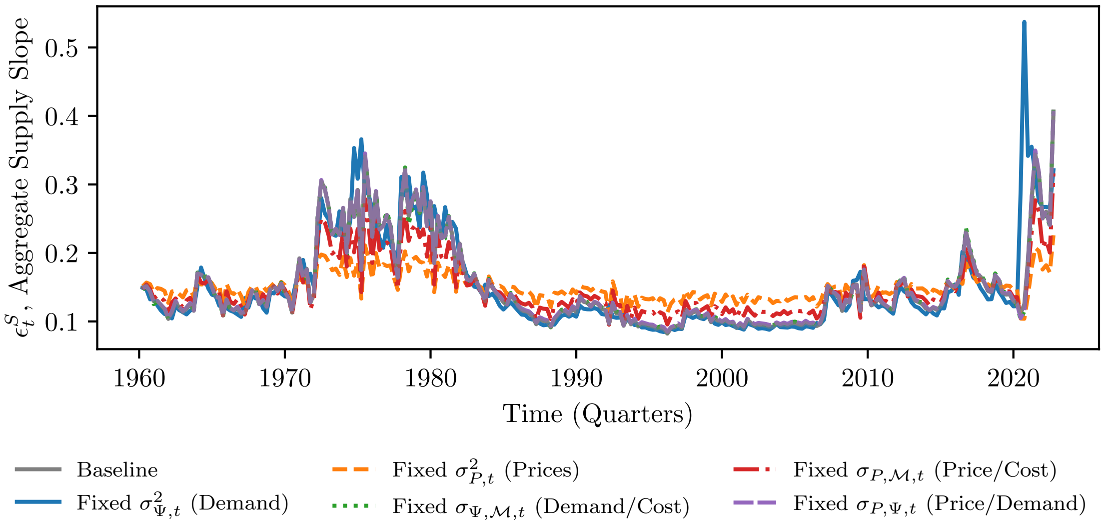
		HHNS (2022)	BM (2011)
	Period	1978-1990	1973-1984
Pre-Period	Data	0.198	0.371
	Model: Uncertainty Only	0.166	0.223
	Model: + Mkt. Power Trend	0.175	0.254
	Period	1991-2018	1985-2007
Post-Period	Data	0.090	0.136
	Model: Uncertainty Only	0.119	0.104
	Model: + Mkt. Power Trend	0.103	0.092
	Data	55%	63%
Flattening	Model: Uncertainty Only	28%	53%
	Model: + Mkt. Power Trend	41%	64%

Note: This table compares the model predictions and literature estimates for the long-run flattening of the aggregate supply curve from Hazell et al. (2022) and Ball and Mazumder (2011). The "Uncertainty Only" model is the baseline with fixed elasticity of demand $\eta = 9$ (Figure 4). The "+ Market Power Trend" model is the "Small Change" scenario (Figure 6), which introduces a linear trend from $\eta = 12$ in 1960 to $\eta = 6$ in 2022. "Flattening" is $100 \cdot (1 - \text{SlopePost}/\text{SlopePre})$.

emerging literature. Using MSA-level data, Cerrato and Gitti (2022) estimate that supply steepened by a factor of 3.4 between the "pre-Covid" period of January 1990 to February 2020 and the "post-Covid" period of March 2021 to the present. The comparable number generated by our model is 2.5, or about 3/4 of this estimate. More qualitatively, our estimating steepening is consistent with both large effects of aggregate demand shocks on inflation and with a relatively "soft landing" for monetary policy—that is, disinflation at relatively low output cost.

Mechanisms: Which Uncertainties Matter More? The time variation in our estimate of ϵ_t^S arises from time-varying uncertainty about several objects. To better understand the role of each component of the calculation, we perform a variant calculation in which we hold each uncertainty term fixed at its sample average, one by one. We plot the results in Figure 5. The combination of time-varying uncertainty about the price-level and time-varying uncertainty about the relationship between prices and (real) marginal costs helps quantitatively explain the overall flattening from the 1970s into the Great Moderation. While uncertainty about demand is large, and features significant high-frequency variation (see Appendix Figure 13), it is not essential for our low-frequency predictions. By contrast, incorporating demand uncertainty is essential to avoid predicting a large spike in the aggregate supply slope in the first several quarters of the Covid-19 lockdown.

Figure 5: Deconstructing the Slope of Aggregate Supply: Which Uncertainties Matter?



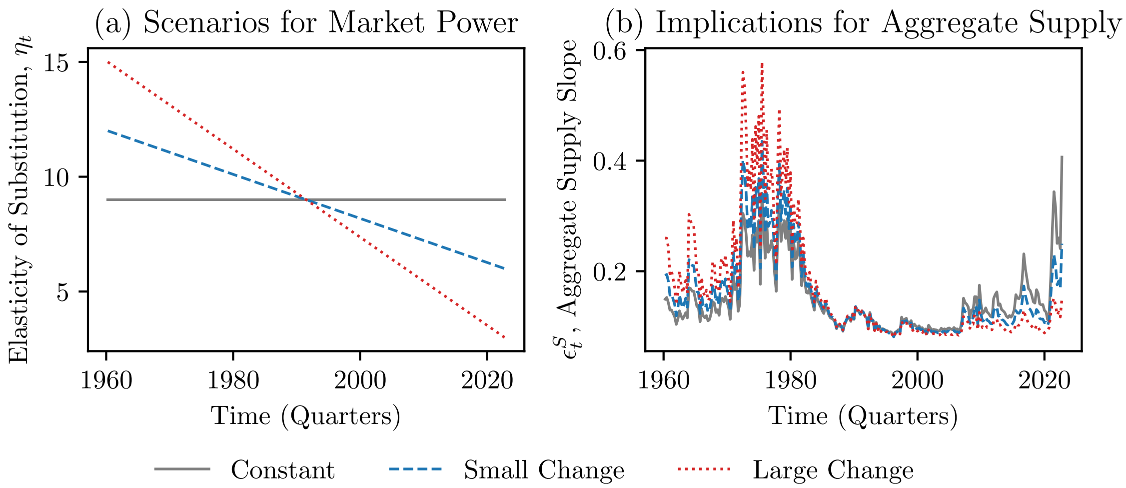
Note: This Figure plots estimates of the inverse elasticity of aggregate supply as measured by Equations 54 and 55, holding fixed one component of uncertainty at a time. The grey solid line corresponds to the baseline estimate from Figure 4.

Market Power and the Flattening Supply Curve. A recent literature has suggested that market power, as measured by rising mark-ups, has risen throughout time (De Loecker et al., 2020; Edmond et al., 2023; Demirer, 2020). Combined with our theoretical finding that increased market power flattens aggregate supply under plausible parameter values, this suggests another potentially relevant culprit for the long-run flattening of supply.

To study this possibility, we consider alternative calibrations of the slope of aggregate supply in which we allow a secular downward trend in the elasticity of demand. We consider a "small change" in which η linearly declines from 12 to 6 and a more exaggerated "large change" in which η linearly declines from 15 to 3. These exercises are *not* counterfactuals, which would require fully estimating the model and accounting for the effects of market power on macroeconomic uncertainty. Instead, they are alternative calibrations that would be more appropriate than our baseline if the elasticity of demand has truly fallen over time, which is a prominent hypothesis.

Introducing a decline in market power increases the slope of aggregate supply in the 1970s and decreases the slope in modern times (Figure 6). Calibrating to the "small change" in market power allows us to fit three-quarters of the flattening measured by Hazell et al. (2022) and all of the flattening measured by Ball and Mazumder (2011) (Table 2). The more extreme scenario for market power allows our model to explain a greater flattening,

Figure 6: Rising Market Power and Flattening Aggregate Supply



Note: This Figure plots the inverse elasticity of aggregate supply under different scenarios of declining market power (Panel b). The solid line keeps η constant at 9 and corresponds to our baseline estimates. The blue, dashed line ("small change") assumes a linear decline in η from 12 to 6 over the time sample. The red, dotted line assumes a linear decline in η from 15 to 3 over the time sample.

but potentially incorrectly predicts a relatively flat aggregate supply curve in the 2020s.

Supply Over a Longer Time Period. In our main analysis, we focus on the period after 1960. In an extension, we consider all data since World War II. This necessitates estimating a different GARCH model for macroeconomic uncertainty, so in principle, it could affect our estimates for the entire sample. We plot the results in Appendix Figure 15. We find very comparable estimates from 1960 onward, and a very large and volatile slope of aggregate supply between 1947 and 1960. The latter finding is consistent with there being large volatility in the money supply and the price level in the wake of the War and the Bretton Woods agreement.

Robustness and Sensitivity Analysis. In Appendix Figure 17 we report the robustness of these findings to different calibrations of η, R, and γ. Specifically, we re-calibrate the model to match the average slope of 0.15 and check how different assumptions affect our model's predictions for the long-run flattening during the Great Moderation. We predict a larger flattening under larger assumed values of η, which exacerbate the effect of changing inflation uncertainty; smaller R, which allows inflation uncertainty to play a larger role in the calculation; and larger γ, which makes real wages more cyclical, especially in the 1970s.

In Appendix Figure 16, we report results from a pseudo-out-of-sample variant of our main calculation, in which we use data up to quarter $t-1$ to forecast the conditional variance of variables at quarter t. We find very similar broad patterns, although estimates in the early part of the sample are, unsurprisingly, noisier.

6 Conclusion

In this paper, we enrich firms' supply decisions by allowing them to choose an arbitrary supply function that describes the price charged at each quantity of production. We show how to model supply functions in a macroeconomic setting and characterize how the optimal supply function depends on the elasticity of demand and the nature of uncertainty that firms face. Our framework yields rich implications when embedded in an otherwise standard monetary business cycle model. We find that greater market power and increased uncertainty about the price level relative to demand endogenously steepen the slope of aggregate supply. When mapped to the data, our theory explains the long-run flattening of the aggregate supply curve as an outcome of more hawkish monetary policy and rising market power.

On the basis of our analysis, we argue that supply schedules warrant serious consideration as an alternative model of firm conduct in macroeconomics for three core reasons. First, most existing work assumes that firms set a price in advance and commit to supply at the market-clearing quantity. Our results emphasize that this is not generally an optimal way for a firm to behave and that the macroeconomic conclusions that one draws about the effects of uncertainty, the propagation of monetary and productivity shocks, and the role of market power are highly sensitive to this choice. For example, the price-setting assumption maximizes the degree of monetary non-neutrality and leaves no role for market power. Second, we have shown that working with supply schedules is analytically tractable under the standard assumptions in the literature. Finally, taking the supply-schedule perspective yields economic predictions that are consistent with broad trends in US aggregate supply over the last 60 years.

Within the context of supply schedules and the macroeconomy, our study is only a first exploration; there remains much to examine, both empirically and theoretically. We highlight two particularly salient implications of our analysis that we leave open to future work. First, our work highlights the importance of firm-level supply elasticities as a critical moment for the business cycle. Empirical work that measures such elasticities and investigates how they systematically vary would be highly valuable for disciplining richer models with supply schedules. Such work would require detailed firm-level data on both prices and quantities. Second, it would be interesting to examine the conduct of optimal monetary policy in a setting with supply schedule choice. We have shown that more volatile monetary policy (perhaps associated with more discretionary monetary policy) can be self-defeating by making the economy endogenously less responsive to monetary stimulus. A complete normative analysis of this issue is an interesting avenue for future research.

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Appendices

A Omitted Proofs

A.1 Proof of Proposition 1

To derive the optimal price, we take the first-order condition of Equation 5. This yields:

$$\eta {p^*}^{-\eta-1}\mathbb{E}\left[\Lambda \mathcal{M} P^\eta \Psi\right] = (\eta-1){p^*}^{-\eta}\mathbb{E}\left[\Lambda P^{\eta-1}\Psi\right] \tag{56}$$

which rearranges to Equation 6. Substituting p^* into Equation 5, we obtain Equation 7:

$$\begin{aligned}
V^P &= \mathbb{E}\left[\Lambda\left(\frac{p^*}{P}-\mathcal{M}\right)\Psi\left(\frac{p^*}{P}\right)^{-\eta}\right] \\
&= \mathbb{E}\left[\Lambda\left(\frac{1}{P}\frac{\eta}{\eta-1}\frac{\xi^P}{\zeta^P}-\mathcal{M}\right)\Psi P^\eta\left(\frac{\eta}{\eta-1}\right)^{-\eta}\left(\frac{\xi^P}{\zeta^P}\right)^{-\eta}\right] \\
&= \left(\frac{\eta}{\eta-1}\right)^{-\eta}\left[\frac{\eta}{\eta-1}\left(\frac{\xi^P}{\zeta^P}\right)^{1-\eta}\zeta^P-\left(\frac{\xi^P}{\zeta^P}\right)^{-\eta}\xi^P\right] \\
&= \left(\frac{\eta}{\eta-1}\right)^{-\eta}\left(\frac{\eta}{\eta-1}-1\right){\xi^P}^{1-\eta}{\zeta^P}^{\eta} = \frac{1}{\eta-1}\left(\frac{\eta}{\eta-1}\right)^{-\eta}\mathbb{E}\left[\Lambda\mathcal{M}P^\eta\Psi\right]^{1-\eta}\mathbb{E}\left[\Lambda P^{\eta-1}\Psi\right]^{\eta}
\end{aligned} \tag{57}$$

where $\xi^P = \mathbb{E}\left[\Lambda\mathcal{M}P^\eta\Psi\right]$ and $\zeta^P = \mathbb{E}\left[\Lambda P^{\eta-1}\Psi\right]$.

To derive the optimal quantity, we take the first-order condition of Equation 8. This yields:

$$\mathbb{E}\left[\Lambda\mathcal{M}\right] = \frac{\eta-1}{\eta}{q^*}^{-\frac{1}{\eta}}\mathbb{E}[\Lambda\Psi^{\frac{1}{\eta}}] \tag{58}$$

which rearranges to Equation 9. Substituting q^* into Equation 8, we obtain Equation 10:

$$\begin{aligned}
V^Q &= \mathbb{E}\left[\Lambda\left(\left(\frac{x}{\Psi}\right)^{-\frac{1}{\eta}}-\mathcal{M}\right)q\right] \\
&= \mathbb{E}\left[\Lambda\left(\frac{\eta}{\eta-1}\frac{\xi^Q}{\zeta^Q}\Psi^{\frac{1}{\eta}}-\mathcal{M}\right)\left(\frac{\eta}{\eta-1}\right)^{-\eta}\left(\frac{\xi^Q}{\zeta^Q}\right)^{-\eta}\right] \\
&= \left(\frac{\eta}{\eta-1}\right)^{-\eta}\left[\frac{\eta}{\eta-1}\left(\frac{\xi^Q}{\zeta^Q}\right)^{1-\eta}\zeta^Q-\left(\frac{\xi^Q}{\zeta^Q}\right)^{-\eta}\xi^Q\right] \\
&= \left(\frac{\eta}{\eta-1}\right)^{-\eta}\left[\frac{\eta}{\eta-1}-1\right]{\xi^Q}^{1-\eta}{\zeta^Q}^{\eta} = \frac{1}{\eta-1}\left(\frac{\eta}{\eta-1}\right)^{-\eta}\mathbb{E}\left[\Lambda\mathcal{M}\right]^{1-\eta}\mathbb{E}[\Lambda\Psi^{\frac{1}{\eta}}]^{\eta}
\end{aligned} \tag{59}$$

where $\xi^Q = \mathbb{E}\left[\Lambda\mathcal{M}\right]$ and $\zeta^Q = \mathbb{E}[\Lambda\Psi^{\frac{1}{\eta}}]$.

To find Δ, we first write $V^P - V^Q$ as:

$$\Delta = \eta(\log\zeta^P - \log\zeta^Q) - (\eta-1)(\log\xi^P - \log\xi^Q) \tag{60}$$

Thus, it suffices to compute $(\zeta^P, \zeta^Q, \xi^P, \xi^Q)$. Given log-normality of $(\Psi, P, \Lambda, \mathcal{M})$, we may write:

$$\begin{pmatrix} \log \Psi \\ \log P \\ \log \Lambda \\ \log \mathcal{M} \end{pmatrix} \sim N\left(\begin{pmatrix} \mu_\Psi \\ \mu_P \\ \mu_\Lambda \\ \mu_\mathcal{M} \end{pmatrix}, \begin{pmatrix} \sigma^2_\Psi & \sigma_{\Psi,P} & \sigma_{\Psi,\Lambda} & \sigma_{\Psi,\mathcal{M}} \\ \sigma_{\Psi,P} & \sigma^2_P & \sigma_{P,\Lambda} & \sigma_{P,\mathcal{M}} \\ \sigma_{\Psi,\Lambda} & \sigma_{P,\Lambda} & \sigma^2_\Lambda & \sigma_{\Lambda,\mathcal{M}} \\ \sigma_{\Psi,\mathcal{M}} & \sigma_{P,\mathcal{M}} & \sigma_{\Lambda,\mathcal{M}} & \sigma^2_\mathcal{M} \end{pmatrix}\right) \tag{61}$$

To compute the second term of Equation 60, we compute:

$$\begin{aligned} \log \xi^P &= \log \mathbb{E}\left[\Lambda \mathcal{M} P^\eta \Psi\right] = \log \mathbb{E}\left[\exp\{\log \Lambda + \log \mathcal{M} + \eta \log P + \log \Psi\}\right] \\ &= \mu_\Lambda + \mu_\mathcal{M} + \eta\mu_P + \mu_\Psi + \frac{1}{2}\left(\sigma^2_\Lambda + \sigma^2_\mathcal{M} + \eta^2\sigma^2_P + \sigma^2_\Psi\right) \\ &\quad + \sigma_{\Lambda,\mathcal{M}} + \eta\sigma_{\Lambda,P} + \sigma_{\Lambda,\Psi} + \eta\sigma_{\mathcal{M},P} + \sigma_{\mathcal{M},\Psi} + \eta\sigma_{P,\Psi} \end{aligned} \tag{62}$$

and

$$\begin{aligned} \log \xi^Q &= \log \mathbb{E}[\Lambda \mathcal{M}] = \log \mathbb{E}[\exp\{\log \Lambda + \log \mathcal{M}\}] \\ &= \mu_\Lambda + \mu_\mathcal{M} + \frac{1}{2}\left(\sigma^2_\Lambda + \sigma^2_\mathcal{M}\right) + \sigma_{\Lambda,\mathcal{M}} \end{aligned} \tag{63}$$

Thus, the second term of Equation 60 is given by:

$$\begin{aligned} (\eta - 1)(\log \xi^P - \log \xi^Q) = (\eta - 1)\Bigg[&\eta\mu_P + \mu_\Psi + \frac{1}{2}\left(\eta^2\sigma^2_P + \sigma^2_\Psi\right) \\ &+ \eta\sigma_{\Lambda,P} + \sigma_{\Lambda,\Psi} + \eta\sigma_{\mathcal{M},P} + \sigma_{\mathcal{M},\Psi} + \eta\sigma_{P,\Psi}\Bigg] \end{aligned} \tag{64}$$

To compute the first term of Equation 60, we compute:

$$\begin{aligned} \log \zeta^P &= \log \mathbb{E}\left[\Lambda P^{\eta-1}\Psi\right] = \log \mathbb{E}[\exp\{\log \Lambda + (\eta - 1)\log P + \log \Psi\}] \\ &= \mu_\Lambda + (\eta - 1)\mu_P + \mu_\Psi + \frac{1}{2}\left(\sigma^2_\Lambda + (\eta - 1)^2\sigma^2_P + \sigma^2_\Psi\right) \\ &\quad + (\eta - 1)\sigma_{\Lambda,P} + \sigma_{\Lambda,\Psi} + (\eta - 1)\sigma_{P,\Psi} \end{aligned} \tag{65}$$

and

$$\begin{aligned} \log \zeta^Q &= \log \mathbb{E}\left[\Lambda \Psi^{\frac{1}{\eta}}\right] = \log \mathbb{E}\left[\exp\left\{\log \Lambda + \frac{1}{\eta}\log \Psi\right\}\right] \\ &= \mu_\Lambda + \frac{1}{\eta}\mu_\Psi + \frac{1}{2}\left(\sigma^2_\Lambda + \frac{1}{\eta^2}\sigma^2_\Psi\right) + \frac{1}{\eta}\sigma_{\Lambda,\Psi} \end{aligned} \tag{66}$$

Thus, the first term of Equation 60 is given by:

$$\begin{aligned}\eta(\log\zeta^P - \log\zeta^Q) = \eta\Bigg[&(\eta-1)\mu_P + \frac{\eta-1}{\eta}\mu_\Psi + \frac{1}{2}\left((\eta-1)^2\sigma_P^2 + \left(1-\frac{1}{\eta^2}\right)\sigma_\Psi^2\right) \\ &+ (\eta-1)\sigma_{\Lambda,P} + \frac{\eta-1}{\eta}\sigma_{\Lambda,\Psi} + (\eta-1)\sigma_{P,\Psi}\Bigg]\end{aligned} \tag{67}$$

Taking the difference between the two terms, we obtain Equation 12:

$$\begin{aligned}\Delta &= \frac{1}{2}\left[\left(\eta(\eta-1)^2 - \eta^2(\eta-1)\right)\sigma_P^2 + \left(\eta\left(1-\frac{1}{\eta^2}\right) - (\eta-1)\right)\sigma_\Psi^2\right] \\ &\quad - \eta(\eta-1)\sigma_{\mathcal{M},P} - (\eta-1)\sigma_{\mathcal{M},\Psi} \\ &= \frac{1}{2}\left(\frac{\eta-1}{\eta}\sigma_\Psi^2 - \eta(\eta-1)\sigma_P^2 - 2(\eta-1)\sigma_{\mathcal{M},\Psi} - 2\eta(\eta-1)\sigma_{\mathcal{M},P}\right)\end{aligned} \tag{68}$$

This completes the derivation of Equation 12. The comparative statics in the variance terms are immediate from the expression. The comparative statics for $\text{sgn}(\Delta)$ come from inspection of the expression for $\Delta/(\eta-1)$, which has the same sign as Δ under the maintained assumption that $\eta > 1$.

A.2 Proof of Theorem 1

Proof. Fix a supply function f. The realized price of the firm in state z solves $f(\hat{p}(z), z\hat{p}(z)^{-\eta}) = 0$. As we placed no restrictions on f, it is equivalent to think of the firm as choosing $\hat{p}$ directly. For a given choice of $\hat{p}$, the firm's payoff is given by:

$$J(\hat{p}) = \int_{\mathbb{R}^4_{++}} \Lambda\left(\frac{\hat{p}(z)}{P} - \mathcal{M}\right) z\hat{p}(z)^{-\eta}\,\mathrm{d}G\left(\Lambda, P, \mathcal{M}, z\right) \tag{69}$$

where G is the cumulative distribution function representing the firm's beliefs. We therefore study the problem:

$$\sup_{\hat{p}:\mathbb{R}_+\to\mathbb{R}_{++}} J(\hat{p}) \tag{70}$$

Given a solution $\hat{p}$ for how firms optimally adapt their prices to demand, we will recover the optimal plan f for how firms optimally set a supply function.

We first derive Equation 15 using variational methods. Consider a variation $\tilde{p}(z) = p(z) + \varepsilon h(z)$. The expected payoff under this variation is:

$$J(\varepsilon; h) = \int_{\mathbb{R}^4_{++}} \Lambda\left(\frac{p(z)+\varepsilon h(z)}{P} - \mathcal{M}\right) z\left(p(z)+\varepsilon h(z)\right)^{-\eta}\mathrm{d}G\left(\Lambda, P, \mathcal{M}, z\right) \tag{71}$$

A necessary condition for the optimality of a function p is that $J_\varepsilon(0;h)=0$ for all $F-$measurable h. Taking this derivative and setting $\varepsilon=0$, we obtain:

$$0=\int_{\mathbb{R}^4_{++}}\left[\Lambda\frac{h(z)}{P}zp(z)^{-\eta}-\eta\Lambda h(z)\left(\frac{p(z)}{P}-\mathcal{M}\right)zp(z)^{-\eta-1}\right]\mathrm{d}G\left(\Lambda,P,\mathcal{M},z\right) \tag{72}$$

Consider h functions given by the Dirac delta functions on each z, $h(z)=\delta_z$. This condition becomes:

$$0=\int_{\mathbb{R}^3_{++}}\left[\Lambda\frac{1}{P}tp(t)^{-\eta}-\eta\Lambda\left(\frac{p(t)}{P}-\mathcal{M}\right)tp(t)^{-\eta-1}\right]g(\Lambda,P,\mathcal{M},t)\,\mathrm{d}\Lambda\,\mathrm{d}P\,\mathrm{d}\mathcal{M} \tag{73}$$

for all $t\in\mathbb{R}_{++}$. This is equivalent to:

$$\begin{aligned}0&=\int_{\mathbb{R}^3_{++}}\left[\Lambda\frac{1}{P}tp(t)^{-\eta}-\eta\Lambda\left(\frac{p(t)}{P}-\mathcal{M}\right)tp(t)^{-\eta-1}\right]g(\Lambda,P,\mathcal{M}|t)\,\mathrm{d}\Lambda\,\mathrm{d}P\,\mathrm{d}\mathcal{M}\\&=(1-\eta)\mathbb{E}\left[\Lambda\frac{1}{P}|z=t\right]tp(t)^{-\eta}+\eta\mathbb{E}\left[\Lambda\mathcal{M}|z=t\right]tp(t)^{-\eta-1}\end{aligned} \tag{74}$$

Thus, we have that an optimal solution necessarily follows:

$$p(t)=\frac{\eta}{\eta-1}\frac{\mathbb{E}[\Lambda\mathcal{M}|z=t]}{\mathbb{E}[\Lambda P^{-1}|z=t]} \tag{75}$$

as claimed in Equation 15.

We now evaluate the expectations. Using log-normality,

$$\begin{aligned}\mathbb{E}[\Lambda\mathcal{M}|z=t]&=\exp\left\{\mu_{\Lambda|z}(t)+\mu_{\mathcal{M}|z}(t)+\frac{1}{2}\sigma^2_{\Lambda|z}+\frac{1}{2}\sigma^2_{\mathcal{M}|z}+\sigma_{\Lambda,\mathcal{M}|z}\right\}\\\mathbb{E}[\Lambda P^{-1}|z=t]&=\exp\left\{\mu_{\Lambda|z}(t)-\mu_{P|z}(t)+\frac{1}{2}\sigma^2_{\Lambda|z}+\frac{1}{2}\sigma^2_{P|z}-\sigma_{\Lambda,P|z}\right\}\end{aligned} \tag{76}$$

where $\mu_{X|z}=\mathbb{E}[\log X\,|\log z]$ and $\sigma_{X,Y|z}=\mathrm{Cov}[\log X,\log Y\,|\log z]$. Thus,

$$\frac{\mathbb{E}[\Lambda\mathcal{M}|z=t]}{\mathbb{E}[\Lambda P^{-1}|z=t]}=\exp\left\{\mu_{\mathcal{M}|z}(t)+\mu_{P|z}(t)+\frac{1}{2}\sigma^2_{\mathcal{M}|z}-\frac{1}{2}\sigma^2_{P|z}+\sigma_{\Lambda,\mathcal{M}|z}+\sigma_{\Lambda,P|z}\right\} \tag{77}$$

Using standard formulas for Gaussian conditional expectations,

$$\begin{aligned} \mu_{\mathcal{M}|z}(t) &= \mu_{\mathcal{M}} + \frac{\sigma_{\mathcal{M},z}}{\sigma_z^2}(\log t - \mu_z) & \mu_{P|z}(t) &= \mu_P + \frac{\sigma_{P,z}}{\sigma_z^2}(\log t - \mu_z) \\ \sigma^2_{\mathcal{M}|z} &= \sigma^2_{\mathcal{M}} - \frac{\sigma^2_{\mathcal{M},z}}{\sigma_z^2} & \sigma^2_{P|z} &= \sigma^2_P - \frac{\sigma^2_{P,z}}{\sigma_z^2} \\ \sigma_{\Lambda,\mathcal{M}|z} &= \sigma_{\Lambda,\mathcal{M}} - \frac{\sigma_{\Lambda,z}\sigma_{\mathcal{M},z}}{\sigma_z^2} & \sigma_{\Lambda,P|z} &= \sigma_{\Lambda,P} - \frac{\sigma_{\Lambda,z}\sigma_{P,z}}{\sigma_z^2} \end{aligned} \tag{78}$$

where:

$$\begin{aligned} \sigma_z^2 &= \sigma_\Psi^2 + \eta^2\sigma_P^2 + 2\eta\sigma_{\Psi,P} & \sigma_{P,z} &= \sigma_{P,\Psi} + \eta\sigma_P^2 \\ \sigma_{\mathcal{M},z} &= \sigma_{\mathcal{M},\Psi} + \eta\sigma_{\mathcal{M},P} & \sigma_{\Lambda,z} &= \sigma_{\Lambda,\Psi} + \eta\sigma_{\Lambda,P} \end{aligned} \tag{79}$$

We now combine these expressions with Equation 75 to derive the optimal supply function. We first observe that

$$\log p = \omega_0 + \omega_1 \log t \tag{80}$$

where:

$$\omega_0 = \log\frac{\eta}{\eta-1} + \mu_{\mathcal{M}} + \mu_P - \omega_1\mu_z + \frac{1}{2}\sigma^2_{\mathcal{M}|z} - \frac{1}{2}\sigma^2_{P|z} + \sigma_{\Lambda,\mathcal{M}|z} + \sigma_{\Lambda,P|z} \tag{81}$$

$$\omega_1 = \frac{\sigma_{\mathcal{M},z} + \sigma_{P,z}}{\sigma_z^2} = \frac{\sigma_{\mathcal{M},\Psi} + \eta\sigma_{\mathcal{M},P} + \sigma_{P,\Psi} + \eta\sigma_P^2}{\sigma_\Psi^2 + \eta^2\sigma_P^2 + 2\eta\sigma_{\Psi,P}} \tag{82}$$

Next, using the demand curve, we observe that $z = qp^\eta$. Therefore, $\log t = \log q - \eta \log p$. Substituting this into Equation 80, and re-arranging, we obtain

$$\log p = \alpha_0 + \alpha_1 \log q \tag{83}$$

where:

$$\alpha_0 = \frac{\omega_0}{1-\eta\omega_1}, \qquad \alpha_1 = \frac{\omega_1}{1-\eta\omega_1} \tag{84}$$

We finally derive the claimed expression for α_1,

$$\begin{aligned} \alpha_1 &= \frac{\frac{\sigma_{\mathcal{M},\Psi}+\eta\sigma_{\mathcal{M},P}+\sigma_{P,\Psi}+\eta\sigma_P^2}{\sigma_\Psi^2+\eta^2\sigma_P^2+2\eta\sigma_{\Psi,P}}}{1-\eta\frac{\sigma_{\mathcal{M},\Psi}+\eta\sigma_{\mathcal{M},P}+\sigma_{P,\Psi}+\eta\sigma_P^2}{\sigma_\Psi^2+\eta^2\sigma_P^2+2\eta\sigma_{\Psi,P}}} \\ &= \frac{\sigma_{\mathcal{M},\Psi} + \eta\sigma_{\mathcal{M},P} + \sigma_{P,\Psi} + \eta\sigma_P^2}{\sigma_\Psi^2 + \eta\sigma_{\Psi,P} - \eta\sigma_{\mathcal{M},\Psi} - \eta^2\sigma_{\mathcal{M},P}} \end{aligned} \tag{85}$$

Completing the proof.

□

A.3 Proof of Corollary 1

Proof. If $2\eta\sigma_{\mathcal{M},P} + \sigma_{\mathcal{M},\Psi} \geq \sigma_{P,\Psi}$, then the denominator of Equation 14 is decreasing in η. Moreover, if $\sigma_{\mathcal{M},P} \geq 0$, the numerator is increasing in η. Hence, α_1 is increasing in η whenever $\alpha_1 > 0$. ☐

A.4 Proof of Proposition 2

Proof. We first derive Equation 30. From Equations 28 and 29, we obtain:

$$\frac{1}{M_t} + \beta\mathbb{E}_t\left[C_{t+1}^{-\gamma}\frac{1}{P_{t+1}}\right] = \beta(1+i_t)\mathbb{E}_t\left[C_{t+1}^{-\gamma}\frac{1}{P_{t+1}}\right] \tag{86}$$

It follows that:

$$\frac{1}{M_t} = \beta i_t\mathbb{E}_t\left[C_{t+1}^{-\gamma}\frac{1}{P_{t+1}}\right] = \frac{i_t}{1+i_t}C_t^{-\gamma}\frac{1}{P_t} \tag{87}$$

where the second equality uses Equation 29 once again. This rearranges to Equation 30.

We next derive Equation 33. Substituting equation 30 into Equation 29, we obtain:

$$\frac{1+i_t}{i_t}\frac{1}{M_t} = \beta(1+i_t)\mathbb{E}_t\left[\frac{1+i_{t+1}}{i_{t+1}}\frac{1}{M_{t+1}}\right] \tag{88}$$

Dividing both sides by $(1+i_t)$, multiplying by M_t, and then adding one, we obtain:

$$\frac{1+i_t}{i_t} = 1 + \beta\mathbb{E}_t\left[\frac{1+i_{t+1}}{i_{t+1}}\frac{M_t}{M_{t+1}}\right] = 1 + \beta\mathbb{E}_t\left[\exp\{-\mu_M - \sigma_{t+1}^M\varepsilon_{t+1}^M\}\frac{1+i_{t+1}}{i_{t+1}}\right] \tag{89}$$

where the second equality exploits the fact that M_t follows a random walk with drift. If we guess that i_t is deterministic and define $x_t = \frac{1+i_t}{i_t}$, then we obtain that:

$$x_t = 1 + \delta_t x_{t+1} \tag{90}$$

where:

$$\delta_t = \beta\exp\left\{-\mu_M + \frac{1}{2}(\sigma_{t+1}^M)^2\right\} \tag{91}$$

We observe that $\delta_t \in [0, \beta]$ for all t due to the assumption that $\frac{1}{2}(\sigma_t^M)^2 \leq \mu_M$. Solving this equation forward, we obtain that for $T \geq 2$:

$$x_t = 1 + \delta_t\left(1 + \sum_{i=1}^{T-1}\prod_{j=1}^{i}\delta_{t+j}\right) + \delta_t\left(\prod_{j=1}^{T}\delta_{t+j}\right)x_{t+T+1} \tag{92}$$

Taking the limit $T \to \infty$, this becomes:

$$x_t = 1 + \delta_t \left(1 + \sum_{i=1}^{\infty} \prod_{j=1}^{i} \delta_{t+j}\right) + \delta_t \lim_{T\to\infty} \left(\prod_{j=1}^{T} \delta_{t+j}\right) x_{t+T+1} \tag{93}$$

where the final term can be bounded using the fact that $\delta_t \in [0, \beta]$:

$$0 \leq \delta_t \lim_{T\to\infty} \left(\prod_{j=1}^{T} \delta_{t+j}\right) x_{t+T+1} \leq \lim_{T\to\infty} \beta^{T+1} x_{t+T+1} \tag{94}$$

The household's transversality condition ensures that this upper bound is zero. Formally, the transversality condition (necessary for the optimality of the household's choices) is that:

$$\lim_{T\to\infty} \beta^T \frac{C_T^{-\gamma}}{P_T} (M_T + (1 + i_T) B_T) = 0 \tag{95}$$

Moreover, as $B_t = 0$ for all $t \in \mathbb{N}$, this reduces to $\lim_{T\to\infty} \beta^T \frac{C_T^{-\gamma}}{P_T} M_T = 0$. By Equation 87, we have that $\frac{x_t}{M_t} = \frac{C_t^{-\gamma}}{P_t}$. Thus, the transversality condition reduces to $\lim_{T\to\infty} \beta^T x_T = 0$. Combining this with Equation 94, we have that $\lim_{T\to\infty} \left(\prod_{j=1}^{T} \delta_{t+j}\right) x_{t+T+1} = 0$. Equation 33 follows:

$$\frac{1 + i_t}{i_t} = 1 + \beta \exp\left\{-\mu_M + \frac{1}{2}(\sigma_{t+1}^M)^2\right\} \left(1 + \sum_{i=1}^{\infty} \prod_{j=1}^{i} \beta \exp\left\{-\mu_M + \frac{1}{2}(\sigma_{t+j+1}^M)^2\right\}\right) \tag{96}$$

The formulae in Equation 35 then follow. In particular, $\Psi_{it} = \vartheta_{it} C_t$ follows from comparing Equations 2 and 34. $P_t = \frac{i_t}{1+i_t} C_t^{-\gamma} M_t$ follows from Equation 30. $\Lambda_t = C_t^{-\gamma}$ is the households marginal utility from consumption. Finally, $\mathcal{M}_{it} = \frac{1}{z_{it} A_t} \frac{w_{it}}{P_t} = \frac{\phi_{it} C_t^{\gamma}}{z_{it} A_t}$ follows from Equation 27. $\square$

A.5 Proof of Proposition 3

Proof. We suppress dependence on t for ease of notation. Consider a plan:

$$\log p_i = \log \tilde{\alpha}_{0,i} + \alpha_1 \log q_i \tag{97}$$

where $\tilde{\alpha}_{0,i} = e^{\alpha_{0,i}}$. The demand-supply relationship that the firm faces is:

$$\log p_i = -\frac{1}{\eta} (\log q_i - \log \Psi) + \log P \tag{98}$$

The realized quantity therefore is:

$$\log q_i = \frac{-\eta}{1+\eta\alpha_1} \log \tilde{\alpha}_{0,i} + \frac{1}{1+\eta\alpha_1} \log \Psi_i P^\eta \tag{99}$$

and the realized price is:

$$\log p_i = \frac{1}{1+\eta\alpha_1} \log \tilde{\alpha}_{0,i} + \frac{\alpha_1}{1+\eta\alpha_1} \log \Psi_i P^\eta \tag{100}$$

It is useful to make the change of variables $\omega_1 = \frac{\alpha_1}{1+\eta\alpha_1}$, which implies that we may write

$$\log p_i = (1-\eta\omega_1) \log \tilde{\alpha}_{0,i} + \omega_1 \log \Psi_i P^\eta \tag{101}$$

Our goal is to express dynamics only as a function of ω_1. We first find the optimal $\alpha_{0,i}$ in terms of ω_1. The firm therefore solves:

$$\max_{\tilde{\alpha}_{0,i}} \mathbb{E}_i \left[\Lambda \left(\frac{p_i}{P} - \mathcal{M}_i \right) \left(\frac{p_i}{P} \right)^{-\eta} \Psi_i \right] \tag{102}$$

Substituting for the realized price using the demand-supply relationship yields:

$$\max_{\tilde{\alpha}_{0,i}} \mathbb{E} \left[\Lambda \left(\frac{\tilde{\alpha}_{0,i}^{1-\eta\omega_1}}{P} \left(\Psi_i P^\eta\right)^{\omega_1} - \mathcal{M}_i \right) \tilde{\alpha}_{0,i}^{\eta^2\omega_1-\eta} \left(\Psi_i P^\eta\right)^{1-\eta\omega_1} \right] \tag{103}$$

The optimal $\tilde{\alpha}_{0,i}$ is:

$$\tilde{\alpha}_{0,i}^{1-\eta\omega_1} = \frac{\eta}{\eta-1} \frac{\mathbb{E}_i[\Lambda \mathcal{M}_i \left(\Psi_i P^\eta\right)^{1-\eta\omega_1}]}{\mathbb{E}_i[\frac{\Lambda}{P} \left(\Psi_i P^\eta\right)^{1-\eta\omega_1+\omega_1}]} \tag{104}$$

Substituting back into the realized price yields:

$$p_i = \frac{\eta}{\eta-1} \frac{\mathbb{E}_i[\Lambda \mathcal{M}_i \left(\Psi_i P^\eta\right)^{1-\eta\omega_1}]}{\mathbb{E}_i[\frac{\Lambda}{P} \left(\Psi_i P^\eta\right)^{1-\eta\omega_1+\omega_1}]} \left(\Psi_i P^\eta\right)^{\omega_1} \tag{105}$$

We may express this only in terms of P by using Proposition 2, where we let $I = \frac{1+i}{i}$ for ease of notation:

$$\begin{aligned} p_i = \frac{\eta}{\eta-1} & \frac{\mathbb{E}_i \left[\phi(z_i A)^{-1} \left(\vartheta_i I^{-\frac{1}{\gamma}} P^{-\frac{1}{\gamma}} M^{\frac{1}{\gamma}} P^\eta \right)^{1-\eta\omega_1} \right]}{\mathbb{E}_i \left[I^{1-\frac{1}{\gamma}(1+\omega_1-\eta\omega_1)} M^{\frac{1}{\gamma}(1+\omega_1-\eta\omega_1)-1} \vartheta^{1+\omega_1-\eta\omega_1} P^{\left(\eta-\frac{1}{\gamma}\right)(1+\omega_1-\eta\omega_1)} \right]} \\ & \times \left(\vartheta_i I^{-\frac{1}{\gamma}} M^{\frac{1}{\gamma}} P^{\eta-\frac{1}{\gamma}} \right)^{\omega_1} \end{aligned} \tag{106}$$

Given the ideal price index formula (Equation 21), P must satisfy the aggregation:

$$P^{1-\eta} = \mathbb{E}\left[\vartheta_i p_i^{1-\eta}\right] \tag{107}$$

where the expectation is over the cross-section of firms. We guess and verify that the aggregate price is log-linear in aggregates

$$\log P = \chi_0 + \chi_A \log A + \chi_M \log M \tag{108}$$

Moreover, if the p_i are log-normally distributed (we will verify this below), then:

$$\log P = \mathbb{E}[\log p_i] + \frac{1}{2(1-\eta)}\mathrm{Var}((1-\eta)\log p_i) + \text{constants} \tag{109}$$

We first simplify the numerator of the first term by collecting all the terms involving s_i^A and s_i^M:

$$\begin{aligned}\log \mathbb{E}_i\left[\phi_i(z_i A)^{-1}\left(\vartheta I^{-\frac{1}{\gamma}} P^{-\frac{1}{\gamma}} M^{\frac{1}{\gamma}} P^{\eta}\right)^{1-\eta\omega_1}\right] &= \left[-\kappa^A + \kappa^A\left(\eta - \frac{1}{\gamma}\right)\chi_A(1-\eta\omega_1)\right] s_i^A \\ + \left[\chi_M\left(\eta - \frac{1}{\gamma}\right)(1-\eta\omega_1)\kappa^M + \frac{1}{\gamma}(1-\eta\omega_1)\kappa^M\right] s_i^M + \text{constants} &\end{aligned} \tag{110}$$

where the constants are independent of signals. We similarly simplify the denominator of the second term:

$$\begin{aligned}&\log \mathbb{E}_i\left[I^{1-\frac{1}{\gamma}(1+\omega_1-\eta\omega_1)} M^{\frac{1}{\gamma}(1+\omega_1-\eta\omega_1)-1} \vartheta^{1+\omega_1-\eta\omega_1} P^{\left(\eta-\frac{1}{\gamma}\right)(1+\omega_1-\eta\omega_1)}\right] = \\ &\left[\chi_A\left(\eta-\frac{1}{\gamma}\right)(1+\omega_1-\eta\omega_1)\kappa^A\right] s_i^A \\ &+ \left[\left[\frac{1}{\gamma}(1+\omega_1-\eta\omega_1)-1\right]\left(\kappa^M\right) + \chi_M\left(\eta-\frac{1}{\gamma}\right)(1+\omega_1-\eta\omega_1)\left(\kappa^M\right)\right] s_i^M \\ &+ \text{constants}\end{aligned} \tag{111}$$

where the constants are again independent of signals. Finally, we can simplify the last term:

$$\log\left(\vartheta_i I^{-\frac{1}{\gamma}} M^{\frac{1}{\gamma}} P^{\eta-\frac{1}{\gamma}}\right)^{\omega_1} = \omega_1 \chi_A\left(\eta - \frac{1}{\gamma}\right)\log A + \omega_1\left[\chi_M\left(\eta-\frac{1}{\gamma}\right) + \frac{1}{\gamma}\right]\log M + \text{constants} \tag{112}$$

where the constants are independent of the aggregate shocks. Hence, $\log p_i$ is indeed normally distributed and its variance is independent of the realization of aggregate shocks. We can now collect terms to verify our log-linear guess. Substituting the resulting expression for

$\log p_i$ and our guess for $\log P$ from Equation 108 into Equation 109, and solving for χ_A by collecting coefficients on $\log A$ yields:

$$\chi_A = -\frac{\kappa^A}{1 - \omega_1 \left(\eta - \frac{1}{\gamma}\right)(1 - \kappa^A)} \tag{113}$$

We may similarly solve for χ_M:

$$\chi_M = \frac{\kappa^M + \frac{\omega_1}{\gamma}(1 - \kappa^M)}{1 - \omega_1 \left(\eta - \frac{1}{\gamma}\right)(1 - \kappa^M)} \tag{114}$$

This proves the dynamics for the price level. The dynamics for consumption then follow from Proposition 2. □

A.6 Proof of Corollary 3

Proof. Using Equation 40 and market clearing $C_t = Y_t$, we have:

$$\log M_t = \frac{1}{\tilde{\chi}_{M,t}} \left(\log Y_t - \tilde{\chi}_{A,t} \log A_t - \tilde{\chi}_{0,t}\right) \tag{115}$$

Substituting for $\log M_t$ in Equation 41 then yields Equation AS. Doing a similar substitution for $\log A_t$ in Equation 40 then yields Equation AD. Note that this can also be derived by taking logarithms of

$$P_t = \frac{i_t}{1 + i_t} C_t^{-\gamma} M_t \tag{116}$$

in Proposition 2. □

A.7 Proof of Theorem 2

Proof. We suppress dependence on t for ease of notation. We have χ_M and χ_A as a function of ω_1 from Proposition 3. We also know that:

$$\omega_1 = \frac{\sigma_{\mathcal{M}_i,z} + \sigma_{P,z}}{\sigma_z^2} \tag{117}$$

from Equation 82. As $z_i = \vartheta_i \left(\frac{i}{1+i}\right)^{\frac{1}{\gamma}} M^{\frac{1}{\gamma}} P^{\eta-\frac{1}{\gamma}}$ and $\mathcal{M}_i = \phi_i (z_i A)^{-1} \frac{i}{1+i} \frac{M}{P}$, we have that:

$$\begin{aligned}
\sigma_{\mathcal{M}_i,z} &= \mathrm{Cov}\left(-(1+\chi_A)\log A + (1-\chi_M)\log M, \left(\eta-\frac{1}{\gamma}\right)\chi_A \log A + \left(\frac{1}{\gamma} + \left(\eta-\frac{1}{\gamma}\right)\chi_M\right)\log M\right) \\
&= -\left(\eta-\frac{1}{\gamma}\right)\chi_A(1+\chi_A)\sigma_A^2 + (1-\chi_M)\left(\frac{1}{\gamma}+\left(\eta-\frac{1}{\gamma}\right)\chi_M\right)\sigma_M^2 \\
\sigma_{P,z} &= \mathrm{Cov}\left(\chi_A \log A + \chi_M \log M, \left(\eta-\frac{1}{\gamma}\right)\chi_A \log A + \left(\frac{1}{\gamma}+\left(\eta-\frac{1}{\gamma}\right)\chi_M\right)\log M\right) \\
&= \left(\eta-\frac{1}{\gamma}\right)\chi_A^2\sigma_A^2 + \chi_M\left(\frac{1}{\gamma}+\left(\eta-\frac{1}{\gamma}\right)\chi_M\right)\sigma_M^2 \\
\sigma_z^2 &= \sigma_\vartheta^2 + \left(\eta-\frac{1}{\gamma}\right)^2\chi_A^2\sigma_A^2 + \left(\frac{1}{\gamma}+\left(\eta-\frac{1}{\gamma}\right)\chi_M\right)^2\sigma_M^2
\end{aligned} \tag{118}$$

Thus:

$$\omega_1 = \frac{-(\eta-\frac{1}{\gamma})\chi_A\sigma_A^2 + (\frac{1}{\gamma}+(\eta-\frac{1}{\gamma})\chi_M)\sigma_M^2}{\sigma_\vartheta^2 + (\eta-\frac{1}{\gamma})^2\chi_A^2\sigma_A^2 + (\frac{1}{\gamma}+(\eta-\frac{1}{\gamma})\chi_M)^2\sigma_M^2} \tag{119}$$

Note that the optimal ω_1 is common across all firms i. We may express this in fully reduced form as:

$$\omega_1 = T(\omega_1) = \frac{(\eta-\frac{1}{\gamma})\frac{\kappa_A}{1-\omega_1\left(\eta-\frac{1}{\gamma}\right)(1-\kappa_A)}\sigma_A^2 + (\frac{1}{\gamma}+(\eta-\frac{1}{\gamma})\frac{\kappa_M+\frac{\omega_1}{\gamma}(1-\kappa_M)}{1-\omega_1\left(\eta-\frac{1}{\gamma}\right)(1-\kappa_M)})\sigma_M^2}{\sigma_\vartheta^2 + (\eta-\frac{1}{\gamma})^2\left(\frac{\kappa_A}{1-\omega_1\left(\eta-\frac{1}{\gamma}\right)(1-\kappa_A)}\right)^2\sigma_A^2 + (\frac{1}{\gamma}+(\eta-\frac{1}{\gamma})\frac{\kappa_M+\frac{\omega_1}{\gamma}(1-\kappa_M)}{1-\omega_1\left(\eta-\frac{1}{\gamma}\right)(1-\kappa_M)})^2\sigma_M^2} \tag{120}$$

or

$$\omega_1 = T(\omega_1) = \frac{\frac{\left(\eta-\frac{1}{\gamma}\right)\kappa_A}{1-\omega_1\left(\eta-\frac{1}{\gamma}\right)(1-\kappa_A)}\sigma_A^2 + \frac{\frac{1}{\gamma}+\left(\eta-\frac{1}{\gamma}\right)\kappa_M}{1-\omega_1\left(\eta-\frac{1}{\gamma}\right)(1-\kappa_M)}\sigma_M^2}{\sigma_\vartheta^2 + \left(\frac{\left(\eta-\frac{1}{\gamma}\right)\kappa_A}{1-\omega_1\left(\eta-\frac{1}{\gamma}\right)(1-\kappa_A)}\right)^2\sigma_A^2 + \left(\frac{\frac{1}{\gamma}+\left(\eta-\frac{1}{\gamma}\right)\kappa_M}{1-\omega_1\left(\eta-\frac{1}{\gamma}\right)(1-\kappa_M)}\right)^2\sigma_M^2} \tag{121}$$

□

A.8 Proof of Proposition 4

Proof. We first establish equilibrium existence. First, we observe that T_t is a continuous function. The only possible points of discontinuity are: $\omega_{1,t}^M = \frac{1}{(\eta-\frac{1}{\gamma})(1-\kappa_t^M)}$ and $\omega_{1,t}^A = \frac{1}{(\eta-\frac{1}{\gamma})(1-\kappa_t^A)}$. However, at these points $\lim_{\omega_{1,t}\to\omega_{1,t}^M} T_t(\omega_{1,t}) = \lim_{\omega_{1,t}\to\omega_{1,t}^A} T_t(\omega_{1,t}) = T_t(\omega_{1,t}^M) = T_t(\omega_{1,t}^A) = 0$. Second, we observe that $\lim_{\omega_{1,t}\to-\infty} T_t(\omega_{1,t}) = \lim_{\omega_{1,t}\to\infty} T_t(\omega_{1,t}) = 0$. Consider now the function $W_t(\omega_{1,t}) = \omega_{1,t} - T_t(\omega_{1,t})$. This is a continuous function, $\lim_{\omega_{1,t}\to-\infty} W_t(\omega_{1,t}) = -\infty$, and $\lim_{\omega_{1,t}\to\infty} W_t(\omega_{1,t}) = \infty$. Thus, by the intermediate value theorem, there exists an $\omega_{1,t}^*$

such that $W_t(\omega^*_{1,t}) = 0$. By Theorem 2, $\omega^*_{1,t}$ defines a log-linear equilibrium.

We now show that there are at most five log-linear equilibria. For $\omega_{1,t} \neq \omega^A_{1,t}, \omega^M_{1,t}$ (neither of which can be a fixed point), we can rewrite Equation 47 as:

$$\begin{aligned}
&\omega_{1,t}\Bigg[\sigma^2_{\vartheta,t}\left(1-\omega_{1,t}\left(\eta-\frac{1}{\gamma}\right)(1-\kappa^A_t)\right)^2\left(1-\omega_{1,t}\left(\eta-\frac{1}{\gamma}\right)(1-\kappa^M_t)\right)^2 \\
&\quad + \left(\sigma^A_{t|s}\right)^2\left(\eta-\frac{1}{\gamma}\right)\kappa^A_t\left(1-\omega_{1,t}\left(\eta-\frac{1}{\gamma}\right)(1-\kappa^M_t)\right)^2 \\
&\quad + \left(\sigma^M_{t|s}\right)^2\left(\frac{1}{\gamma}+\left(\eta-\frac{1}{\gamma}\right)\kappa^M_t\right)\left(1-\omega_{1,t}\left(\eta-\frac{1}{\gamma}\right)(1-\kappa^A_t)\right)^2\Bigg] \\
&= \left(\sigma^A_{t|s}\right)^2\left(\eta-\frac{1}{\gamma}\right)\kappa^A_t\left(1-\omega_{1,t}\left(\eta-\frac{1}{\gamma}\right)(1-\kappa^M_t)\right)^2\left(1-\omega_{1,t}\left(\eta-\frac{1}{\gamma}\right)(1-\kappa^A_t)\right) \\
&+ \left(\sigma^M_{t|s}\right)^2\left(\frac{1}{\gamma}+\left(\eta-\frac{1}{\gamma}\right)\kappa^M_t\right)\left(1-\omega_{1,t}\left(\eta-\frac{1}{\gamma}\right)(1-\kappa^A_t)\right)^2\left(1-\omega_{1,t}\left(\eta-\frac{1}{\gamma}\right)(1-\kappa^M_t)\right)
\end{aligned} \tag{122}$$

This is a quintic polynomial in $\omega_{1,t}$, which has at most five real roots. Thus, by Theorem 2, there are at most five log-linear equilibria. □

A.9 Proof of Corollary 6

Proof. We drop time subscripts for ease of notation. Substituting $\eta = \frac{1}{\gamma}$ in Equation 47 yields:

$$\omega_1 = \frac{\frac{1}{\gamma}}{\rho^2 + \left(\frac{1}{\gamma}\right)^2} \tag{123}$$

Substituting this into Equation 42 yields:

$$\epsilon^S_t = \gamma\frac{\kappa^M_t}{(1-\kappa^M_t)} + \frac{1}{\gamma\rho^2(1-\kappa^M_t)} \tag{124}$$

□

A.10 Proof of Corollary 7

We drop time subscripts for ease of notation. The first statement follows directly from Equation 47. Furthermore, using Equation 47, as $\sigma^M_{t|s} \to \infty$, ω_1 must solve:

$$\begin{aligned}
\omega_1 &= \frac{1 - \omega_1 \left(\eta - \frac{1}{\gamma}\right)\left(1 - \kappa^M\right)}{\frac{1}{\gamma} + \left(\eta - \frac{1}{\gamma}\right)\kappa^M} \\
&= \frac{\gamma}{1 + (\eta\gamma - 1)\,\kappa^M} + \left(1 - \frac{\eta\gamma}{1 + (\eta\gamma - 1)\kappa^M}\right)\omega_1 \\
&= \frac{1}{\eta}
\end{aligned} \tag{125}$$

This proves the second statement. As $\sigma^A_{t|s} \to \infty$ and $\eta\gamma \neq 1$, ω_1 must solve:

$$\begin{aligned}
\omega_1 &= \frac{1 - \omega_1 \left(\eta - \frac{1}{\gamma}\right)\left(1 - \kappa^A\right)}{\left(\eta - \frac{1}{\gamma}\right)\kappa^A} \\
&= \frac{\gamma}{(\eta\gamma - 1)\,\kappa^A} + \left(1 - \frac{1}{\kappa^A}\right)\omega_1 \\
&= \frac{1}{\eta - \frac{1}{\gamma}}
\end{aligned} \tag{126}$$

This proves the third statement.

A.11 Proof of Proposition 5

Proof. By Theorem 2, The map describing equilibrium $\omega_{1,t}$ is invariant to λ for $\lambda > 0$. Thus, $\mathcal{E}^S_t(\lambda)$ is constant for $\lambda > 0$. If $\lambda = 0$, there are potentially many equilibria in supply functions. Nevertheless, from the proof of Theorem 1, we have that firms set $p_{it}/P_t = \frac{\eta}{\eta-1}\mathcal{M}_{it} = \frac{\eta}{\eta-1}C^\gamma_t/A_t$ under any optimal supply function. This implies that $\frac{\eta}{\eta-1}C^\gamma_t/A_t = 1$, and so money has no real effects, which implies that $\epsilon^S_t = \infty$. $\square$

B Supply Function Choice with Multiple Inputs, Decreasing Returns, Monopsony, and Beyond Isoelastic Demand

In this appendix, we generalize the firm's partial-equilibrium supply schedule problem in two ways. First, we enrich both its technology and inputs by allowing for many inputs, decreasing returns to scale, and monopsony power. Second, we enrich the demand it faces by decoupling the own-price elasticity and the cross-price elasticity and allowing for non-isoelastic demand curves that feature endogenous markups (allowing for Marshall's Second and Third laws of demand). In both cases, we show that our core insights generalize. In the interests of brevity, we leave embedding these generalizations in general equilibrium to future research, though it is clear to see how one could do this by leveraging our main analysis.[12]

B.1 Multiple Inputs, Decreasing Returns, and Monopsony

In this section, we generalize our baseline model of supply function choice to allow for multiple inputs, decreasing returns, and monopsony. We find that: (i) supply functions remain endogenously log-linear and (ii) decreasing returns and monopsony flatten the optimal supply schedule.

Primitives. Consider the baseline model from Section 2 with two modifications. First, the production function uses multiple inputs with different input shares and possibly features decreasing returns-to-scale:

$$q = \Theta \prod_{i=1}^{I} x_i^{a_i} \tag{127}$$

where $x_i \in \mathbb{R}_+$, $a_i \geq 0$, and $\sum_{i=1}^{I} a_i \leq 1$. Moreover, suppose that the producer potentially has monopsony power and faces an upward-sloping factor price curve such that the price of acquiring any input i when the firm demands x_i units is given by $\tilde{p}_i(x_i) = p_{xi} x_i^{b_i - 1}$, where $p_{xi} \in \mathbb{R}_{++}$ and $b_i \geq 1$. The case of no monopsony, or price-taking in the input market, occurs when $b_i = 1$. Thus, the cost of acquiring each type of input is given by:

$$c_i(x_i) = p_{xi} x_i^{b_i} \tag{128}$$

The firm believes that $(\Psi, P, \Lambda, \Theta, p_x)$ is jointly log-normal.

[12] The only complication with endogenous markups would be the endogenous non-log-linearity of the optimal supply curve. This would have to be dealt with via either approximation arguments or numerical methods, or both.

The Firm's Problem. We begin by solving the firm's cost minimization problem:

$$K(q;\Theta,p_x)=\min_x \sum_{i=1}^{I} p_{x_i}x_i^{b_i} \quad \text{s.t.} \quad q=\Theta\prod_{i=1}^{I} x_i^{a_i} \tag{129}$$

This has first-order condition given by:

$$\lambda=\frac{b_i p_{xi}}{a_i}x_i^{b_i}q^{-1} \tag{130}$$

Which implies that:

$$K(q;\Theta,p_x)=\lambda q\sum_{i=1}^{I}\frac{a_i}{b_i} \tag{131}$$

Moreover, fixing i, the FOC implies that we may write for all $j\neq i$:

$$x_j=\left(\frac{\frac{b_i p_{x_i}}{a_i}}{\frac{b_j p_{x_j}}{\alpha_j}}\right)^{\frac{1}{b_j}} x_i^{\frac{b_i}{b_j}} \tag{132}$$

By substituting this into the production function we have that:

$$q=\Theta x_i^{a_i+b_i\sum_{j\neq i}\frac{a_j}{b_j}}\prod_{j\neq i}\left(\frac{\frac{b_i p_{x_i}}{a_i}}{\frac{b_j p_{x_j}}{\alpha_j}}\right)^{\frac{\alpha_j}{b_j}} \tag{133}$$

which implies that:

$$x_i=\left(\frac{q}{\Theta\prod_{j\neq i}\left(\frac{\frac{b_i p_{x_i}}{a_i}}{\frac{b_j p_{x_j}}{\alpha_j}}\right)^{\frac{\alpha_j}{b_j}}}\right)^{\frac{1}{a_i+b_i\sum_{j\neq i}\frac{a_j}{b_j}}} \tag{134}$$

Returning to the FOC, we have that the Lagrange multiplier is given by:

$$\lambda=q^{-1+\frac{1}{\sum_{i=1}^{I}\frac{a_i}{b_i}}}\frac{b_i p_{xi}}{a_i}\left(\Theta\prod_{j\neq i}\left(\frac{\frac{b_i p_{x_i}}{a_i}}{\frac{b_j p_{x_j}}{\alpha_j}}\right)^{\frac{\alpha_j}{b_j}}\right)^{\frac{-1}{\sum_{i=1}^{I}\frac{a_j}{b_j}}} \tag{135}$$

Which then yields the cost function:

$$K(q;\Theta,p_x)=\mathcal{M}Pq^{\frac{1}{\delta}} \tag{136}$$

where:

$$\delta = \sum_{i=1}^{I} \frac{a_i}{b_i} \quad \text{and} \quad \mathcal{M} = P^{-1} \left(\Theta \prod_{i=1}^{I} \left(\frac{b_i p_{x_i}}{\alpha_i} \right)^{\frac{\alpha_j}{b_j}} \right)^{\frac{1}{\sum_{i=1}^{I} \frac{a_i}{b_i}}} \sum_{i=1}^{I} \frac{a_i}{b_i} \tag{137}$$

and we observe that $\mathcal{M}$ is log-normal given the joint log-normality of (Θ, p_x).

Turning to the firm's payoff function, we therefore have:

$$\mathbb{E}\left[\Lambda \left(\frac{p}{P} q - \mathcal{M} q^{\frac{1}{\delta}}\right)\right] \tag{138}$$

Thus, the problem with multiple inputs, monopsony, and decreasing returns modifies the firms' original payoff by only introducing the parameter δ. Helpfully, observe that $\delta = 1$ when: (i) there are constant returns to scale $\sum_{i=1}^{I} a_i = 1$ and (ii) there is no monopsony $b_i = 1$ for all i.

Given this, we can write the firm's objective as:

$$J(\hat{p}) = \int_{\mathbb{R}^4_{++}} \Lambda \left(\frac{\hat{p}(z)^{1-\eta}}{P} z - \mathcal{M} z^{\frac{1}{\delta}} \hat{p}(z)^{-\frac{\eta}{\delta}} \right) \mathrm{d}G\left(\Lambda, P, \mathcal{M}, z\right) \tag{139}$$

And, as before, we study the problem:

$$\sup_{\hat{p}:\mathbb{R}_+ \to \mathbb{R}_{++}} J(\hat{p}) \tag{140}$$

By doing this, we obtain a modified formula for the optimal supply function:

Proposition 6 (Optimal Supply Schedule With Multiple Inputs, Decreasing Returns, and Monopsony)**.** *Any optimal supply schedule is almost everywhere given by:*

$$f(p, q) = \log p - \frac{\omega_0 - \log \delta}{1 - \eta\omega_1} - \frac{\eta\left(\omega_1 + \frac{1-\delta}{\delta}\right)}{1 - \eta\omega_1} \log q \tag{141}$$

where ω_0 and ω_1 are the same as those derived in Theorem 1. Thus, the optimal inverse supply elasticity is given by:

$$\hat{\alpha}_1 = \frac{\eta\sigma_P^2 + \sigma_{\mathcal{M},\Psi} + \sigma_{P,\Psi} + \eta\sigma_{\mathcal{M},P}}{\sigma_\Psi^2 - \eta\sigma_{\mathcal{M},\Psi} + \eta\sigma_{P,\Psi} - \eta^2\sigma_{\mathcal{M},P}} \left(1 + \frac{1-\delta}{\delta} \frac{\sigma_\Psi^2 + \eta^2\sigma_P^2 + 2\eta\sigma_{\Psi,P}}{\sigma_{\mathcal{M},\Psi} + \eta\sigma_{\mathcal{M},P} + \sigma_{P,\Psi} + \eta\sigma_P^2}\right) \tag{142}$$

Proof. Applying the same variational arguments as in the Proof of Theorem 1, we obtain that $\hat{p}(t)$ must solve:

$$(\eta - 1)\mathbb{E}[\Lambda P^{-1} | z = t] t \hat{p}(t)^{-\eta} = \frac{\eta}{\delta} \mathbb{E}[\Lambda\mathcal{M} | z = t] t^{\frac{1}{\delta}} \hat{p}(z)^{-\frac{\eta}{\delta} - 1} \tag{143}$$

Which yields:

$$\hat{p}(t) = \left(\delta^{-1}\frac{\eta}{\eta-1}\frac{\mathbb{E}[\Lambda\mathcal{M}|z=t]}{\mathbb{E}[\Lambda P^{-1}|z=t]}\right)^{\frac{1}{1+\eta\left(\frac{1-\delta}{\delta}\right)}} t^{\frac{\frac{1-\delta}{\delta}}{1+\eta\left(\frac{1-\delta}{\delta}\right)}} \tag{144}$$

Thus, we have that:

$$\log p = \frac{1}{1+\eta\left(\frac{1-\delta}{\delta}\right)}\left(\omega_0 - \log\delta\right) + \frac{1}{1+\eta\left(\frac{1-\delta}{\delta}\right)}\left(\omega_1 + \frac{1-\delta}{\delta}\right)\log z \tag{145}$$

where ω_0 and ω_1 are as in Theorem 1. Rewriting as a supply schedule, we obtain:

$$\log p = \frac{\frac{1}{1+\eta\left(\frac{1-\delta}{\delta}\right)}\left(\omega_0 - \log\delta\right)}{1-\frac{\eta}{1+\eta\left(\frac{1-\delta}{\delta}\right)}\left(\omega_1+\frac{1-\delta}{\delta}\right)} + \frac{\frac{1}{1+\eta\left(\frac{1-\delta}{\delta}\right)}\left(\omega_1+\frac{1-\delta}{\delta}\right)}{1-\frac{\eta}{1+\eta\left(\frac{1-\delta}{\delta}\right)}\left(\omega_1+\frac{1-\delta}{\delta}\right)}\log q \tag{146}$$

Which reduces to the claimed formula. □

Thus, when the supply curve is initially upward-sloping ($\omega_1 \in [0, \eta^{-1}]$), the introduction of decreasing returns and/or monopsony unambiguously reduces the supply elasticity and makes firms closer to quantity-setting.

B.2 Beyond Isoelastic Demand

Isoelastic demand imposes both that the firm's own price elasticity of demand and its cross-price elasticity of demand are constant. In this appendix, we show how to derive optimal supply functions in closed form when the firm's own price elasticity of demand varies. This allows the demand curve to satisfy Marshall's second law of demand that the price elasticity of demand is increasing in the price as well as Marshall's third law of demand that the rate of increase of the price elasticity goes down with the price. We show that uncertainty about demand, prices, and marginal costs continue to operate in a very similar fashion. However, due to endogeneity of the optimal markup, the optimal supply schedule now ceases to be log-linear.

To capture these features, suppose that demand is *multiplicatively separable*: $d(p,\Psi,P) = z(\Psi,P)\phi(p)$ for some function ϕ such that $p\phi''(p)/\phi'(p) < -2$. This latter condition is satisfied by isoelastic demand exactly under the familiar condition that $\eta > 1$ and ensures the existence of a unique optimal price. We further assume that $z(\Psi,P) = \nu_0\Psi^{\nu_1}P^{\nu_2}$ for $\nu_0,\nu_1,\nu_2 \in \mathbb{R}\setminus\{0\}$. This makes firms' uncertainty about the location of their demand curve log-normal. This assumption does rule out non-separable demand, such as the demand system proposed by Kimball (1995). However, it is important to note that this demand system is motivated by evidence on the firm's own price elasticity, which is governed by

ϕ, and not the cross-price elasticity, which is governed by ν_2. Thus, our proposed demand system is equally able to capture facts about the firms' own price elasticity as the one proposed in Kimball (1995) (or the richer structures proposed by Fujiwara and Matsuyama, 2022; Wang and Werning, 2022).

Under this demand system, we can derive a modified formula for the optimal supply curve which is now no longer log-linear, but continues to be governed by similar forces:

Proposition 7. *If demand is multiplicatively separable, then any optimal supply function is almost everywhere given by:*

$$f(p,q) = \log q + \hat{\alpha}_0 - \log\left(\phi(p)\left\{p\left[1+\frac{\phi(p)}{p\phi'(p)}\right]\right\}^{\frac{1}{\hat{\omega}_1}}\right) \tag{147}$$

where:

$$\hat{\omega}_1 = \frac{\nu_1(\sigma_{\mathcal{M},\Psi}+\sigma_{P,\Psi})+\nu_2(\sigma_P^2+\sigma_{\mathcal{M},P})}{\nu_1^2\sigma_\Psi^2+\nu_2^2\sigma_P^2+2\nu_1\nu_2\sigma_{\Psi,P}} \tag{148}$$

Proof. Applying the same variational arguments as in Theorem 1, we obtain that:

$$\hat{p}(z) + \frac{\phi(\hat{p}(z))}{\phi'(\hat{p}(z))} = \frac{\mathbb{E}[\Lambda\mathcal{M}|z]}{\mathbb{E}[\Lambda P^{-1}|z]} \tag{149}$$

where the condition $p\phi''(p)/\phi'(p) < -2$ yields strict concavity of the objective and makes $\hat{p}(z)$ the unique maximizer. Taking logarithms of both sides and evaluating the conditional expectations as per Theorem 1, we obtain that:

$$\log\left(\hat{p}(z)\left[1+\frac{\phi(\hat{p}(z))}{\hat{p}(z)\phi'(\hat{p}(z))}\right]\right) = \hat{\omega}_0 + \hat{\omega}_1\log z \tag{150}$$

where $\hat{\omega}_1 = \frac{\sigma_{\mathcal{M},z}+\sigma_{P,z}}{\sigma_z^2}$, which yields Equation 148. Using $\log z = \log q - \log\phi(p)$ and rearranging yields Equation 147. □

Demand uncertainty and price uncertainty enter the same way as before, via $\hat{\omega}_1$, and the intuition is the same. However, there are now two distinct notions of market power and they therefore operate in a more subtle way. First, consider the role of the cross-price elasticity of demand ν_2. When ν_2 is higher, the firm's price is *ex post* more responsive to changes in others' prices. Second, consider the role of the own-price elasticity of demand $\left(\frac{p\phi'(p)}{\phi(p)}\right)^{-1}$. This induces non-linearity of the optimal supply schedule to the extent that it is not constant. This is because the firm's optimal markup changes as it moves along its demand curve.

C Prices *vs.* Quantities in General Equilibrium

C.1 Model and Equilibrium

All of the model primitives are as in Section 3. However, we now restrict firms to follow either price-setting or quantity-setting at all times. We define equilibrium in two steps. We first fix firms' "choice of choices" at each date t to define a rational expectations *temporary equilibrium*:

Definition 2 (Temporary Equilibrium)**.** *A temporary equilibrium is a partition of* $\mathbb{N}$ *into two sets* $\mathcal{T}^P$ *and* $\mathcal{T}^Q$ *and a collection of variables*

$$\left\{\{p_{it}, q_{it}, C_{it}, N_{it}, L_{it}, w_{it}, \Pi_{it}\}_{i\in[0,1]}, C_t, P_t, M_t, A_t, B_t, N_t, \Lambda_t\right\}_{t\in\mathbb{N}}$$

such that:

1. *In periods* $t \in \mathcal{T}^P$*, all firms choose their prices* p_{it} *to maximize expected real profits under the household's real stochastic discount factor.*
2. *In periods* $t \in \mathcal{T}^Q$*, all firms choose their quantities* q_{it} *to maximize expected real profits under the household's real stochastic discount factor.*
3. *In all periods, the household chooses consumption* C_{it}*, labor supply* N_{it}*, money holdings* M_t*, and bond holdings* B_t *to maximize their expected utility subject to their lifetime budget constraint, while* Λ_t *is the household's marginal utility of consumption.*
4. *In all periods, money supply* M_t *and productivity* A_t *and evolve exogenously via Equations 23 and 25.*
5. *In all periods, firms' and consumers' expectations are consistent with the equilibrium law of motion.*
6. *In all periods, the markets for the intermediate goods, final good, labor varieties, bonds, and money balances all clear.*

In a temporary equilibrium, firms set either prices or quantities, but the choice between the two is not necessarily optimal. We define an *equilibrium* as a temporary equilibrium in which the choice between price and quantity-setting is optimal at all times:

Definition 3 (Equilibrium)**.** *An equilibrium is a temporary equilibrium in which:*

1. *If* $t \in \mathcal{T}^P$*, all firms find price-setting optimal. That is, expected real profits under the household's real stochastic discount factor are weakly higher under price-setting than quantity-setting.*

2. *If $t \in \mathcal{T}^Q$, all firms find quantity-setting optimal. That is, expected real profits under the household's real stochastic discount factor are weakly higher under price-setting than quantity-setting.*

We now study the equilibrium properties of the model.

C.2 Prices *vs.* Quantities in Equilibrium: Incentives and Strategic Interactions

We first describe dynamics in temporary equilibria (Definition 2) in which all firms set prices or quantities *by assumption*. To this end, we use Proposition 3 to express the dynamics of consumption and prices under pure price-setting and pure quantity-setting. Concretely, we assume that the dynamics for consumption are log-linear in the aggregate shocks:

$$\log C_t = \tilde{\chi}^X_{0,t} + \chi^X_{A,t} \log A_t + \chi^X_{M,t} \log M_t \tag{151}$$

where we allow the coefficients to be *regime-specific* $X \in \{Q, P\}$. We then verify that dynamics take a log-linear form.

Corollary 8. *Output and the price level under pure price-setting follow in the unique log-linear equilibrium of the economy follow:*

$$\log C_t = \chi^P_{0,t} + \frac{1}{\gamma}\kappa^A_t \log A_t + \frac{1}{\gamma}(1-\kappa^M_t)\log M_t \tag{152}$$

$$\log P_t = \tilde{\chi}^P_{0,t} - \kappa^A_t \log A_t \tag{153}$$

Output and the price level under pure quantity-setting in the unique log-linear equilibrium of the economy follow:

$$\log C_t = \chi^Q_{0,t} + \frac{1}{\gamma}\frac{\kappa^A_t}{1-\left(1-\frac{1}{\eta\gamma}\right)(1-\kappa^A_t)}\log A_t \tag{154}$$

$$\log P_t = \tilde{\chi}^Q_{0,t} - \frac{\kappa^A_t}{1-\left(1-\frac{1}{\eta\gamma}\right)(1-\kappa^A_t)}\log A_t + \log M_t \tag{155}$$

where $\chi^P_{0,t}$, $\tilde{\chi}^P_{0,t}$, $\chi^Q_{0,t}$, and $\tilde{\chi}^Q_{0,t}$ are constants independent of A_t and M_t.

Proof. Setting $\omega_{1,t} = 0$ in Proposition 3 yields dynamics under pure price-setting. Setting $\omega_{1,t} = 1/\eta$ in Proposition 3 yields dynamics under pure quantity-setting. □

Observe that money is neutral under pure quantity-setting, but not under pure price-setting, as discussed in Corollary 5. Moreover, productivity shocks have a larger effect on output in a quantity-setting regime relative to a price-setting regime if and only if $\eta\gamma > 1$. Intuitively, it is the elasticity of substitution that mediates how much firms will adjust their output in response to a *perceived* increase in productivity under a quantity-setting regime. In contrast, it is real money balances (that is independent of η) that mediates the output response in a pure price-setting regime.

We now ask when would firms prefer to set prices or quantities in equilibrium. To study this, we first derive an expression for Δ in terms of uncertainty about equilibrium objects. We combine Proposition 1 with (i) Proposition 2 and (ii) the observation that consumption is log-linear in both temporary equilibria to derive that:

$$\Delta_t = \frac{1}{2}(\eta - 1)\left(\frac{1}{\eta}\sigma^2_{\vartheta,t} + \frac{1}{\eta}\left(1 - \eta\gamma\right)^2 \sigma^2_{C,t} - \eta(\sigma^M_t)^2 + 2(1 - \eta\gamma)\sigma_{C,A,t}\right) \tag{156}$$

where $\sigma^2_{C,t}$ is the firm's posterior variance for output, $\sigma_{C,A,t}$ is the firm's posterior covariance for output and productivity, $\sigma^2_{\vartheta,t}$ is the variance of idiosyncratic demand shocks, and $(\sigma^M_t)^2$ is the variance of money supply innovations. Higher uncertainty about idiosyncratic demand shocks and lower uncertainty about the money supply provide *exogenous* incentives for price-setting. Higher uncertainty about consumption provides an *endogenous* incentive that unambiguously favors price-setting. This is the net effect of two forces that favor price-setting – increasing demand uncertainty and decreasing the covariance of prices and marginal costs – with two forces that favor quantity-setting – increasing price uncertainty and increasing the covariance between demand and marginal costs. Higher covariance between consumption and productivity favors price-setting if $\eta\gamma < 1$ and quantity-setting otherwise. In the former case, the dominant effect of this covariance is to lower the covariance of marginal costs and demand (favoring price-setting); in the latter case, the dominant effect is to raise the covariance of marginal costs and the price level (favoring quantity-setting). Finally, the variance of idiosyncratic productivity shocks and the variance of idiosyncratic labor supply (factor price) shocks drop out, because they do not induce *covariance* between marginal costs and either demand or the price level.

We now combine the previous observation with the equilibrium dynamics (Corollary 8) to fully describe Δ_t in terms of primitives in each regime:

Lemma 1 (Prices *vs.* Quantities in Equilibrium)**.** *If all firms set quantities, then the com-*

parative advantage of price-setting is:

$$\Delta_t^Q = \frac{1}{2}(\eta - 1)\left(\frac{1}{\eta}\sigma_{\vartheta,t}^2 - \eta\kappa_t^M\sigma_{M,s}^2 \right. \\ \left. + \left(\frac{1}{\eta}(1-\eta\gamma)\frac{\eta\kappa_t^A}{1-\kappa_t^A(1-\eta\gamma)} + 2\right)(1-\eta\gamma)\frac{\eta\left(\kappa_t^A\right)^2}{1-\kappa_t^A(1-\eta\gamma)}\sigma_{A,s}^2\right) \tag{157}$$

Moreover, all firms can set quantities in equilibrium at time t if and only if $\Delta_t^Q \leq 0$. If all firms set prices, then the comparative advantage of price-setting is:

$$\Delta_t^P = \frac{1}{2}(\eta - 1)\left(\frac{1}{\eta}\sigma_{\vartheta,t}^2 + \left(-\eta + \frac{1}{\eta}(1-\eta\gamma)^2\left(\frac{1-\kappa_t^M}{\gamma}\right)^2\right)\kappa_t^M\sigma_{M,s}^2 \right. \\ \left. + \left(\frac{1}{\eta}(1-\eta\gamma)\frac{\kappa_t^A}{\gamma} + 2\right)(1-\eta\gamma)\frac{\left(\kappa_t^A\right)^2}{\gamma}\sigma_{A,s}^2\right) \tag{158}$$

Proof. From Proposition 1, we have that:

$$\Delta_t = \frac{1}{2}(\eta - 1)\left(\frac{1}{\eta}\sigma_{\Psi,t}^2 - \eta\sigma_{P,t}^2 - 2\sigma_{\Psi,\mathcal{M},t} - 2\eta\sigma_{P,\mathcal{M},t}\right) \tag{159}$$

where, now, all the variances are time-dependent. Applying Proposition 2 to obtain expressions for $(\Psi, P, \mathcal{M})$ in equilibrium, and exploiting the log-linearity of each expression, we have that:

$$\begin{aligned} \sigma_{\Psi,t}^2 &= \sigma_{\vartheta,t}^2 + \sigma_{C,t}^2 \\ \sigma_{P,t}^2 &= \gamma^2\sigma_{C,t}^2 + (\sigma_t^M)^2 - 2\gamma\sigma_{C,M,t} \\ \sigma_{\Psi,\mathcal{M},t} &= \gamma\sigma_{C,t}^2 - \sigma_{C,A,t} \\ \sigma_{P,\mathcal{M},t} &= \gamma\sigma_{C,A,t} - \gamma^2\sigma_{C,t}^2 + \gamma\sigma_{C,M,t} \end{aligned} \tag{160}$$

where $\sigma_{X,t}^2$ denotes the firm's posterior variance of variable X at time t and $\sigma_{X,Y,t}$ denotes the firm's posterior covariance of variables X and Y at time t. Substituting these formulae, we obtain Equation 156:

$$\Delta_t = \frac{1}{2}(\eta - 1)\left(\frac{1}{\eta}\sigma_{\vartheta,t}^2 + \frac{1}{\eta}(1-\eta\gamma)^2\sigma_{C,t}^2 - \eta(\sigma_t^M)^2 + 2(1-\eta\gamma)\sigma_{C,A,t}\right) \tag{161}$$

Moreover, applying Proposition 3, we have that these variances for the firm in each of the price-setting and quantity-setting regimes are given in each regime $X \in \{Q, P\}$ by

$$\begin{aligned} (\sigma_{C,t}^X)^2 &= (\chi_{A,t}^X)^2 \sigma_{A|s,t}^2 + (\chi_{M,t}^X)^2 \sigma_{M|s,t}^2 \\ (\sigma_{C,A,t}^X)^2 &= \chi_{A,t}^X \sigma_{A|s,t}^2 \end{aligned} \tag{162}$$

Substituting Equation 162 into Equation 156, we obtain the following expression for Δ_t indexed by the regime $X \in \{Q, P\}$:

$$\begin{aligned} \Delta_t^X = \frac{1}{2}(\eta - 1)\Bigg(&\frac{1}{\eta}\sigma_{\vartheta,t}^2 + \left(-\eta + \frac{1}{\eta}(1-\eta\gamma)^2 (\chi_{M,t}^X)^2\right) \sigma_{M|s,t}^2 \\ &+ \left(\frac{1}{\eta}(1-\eta\gamma)\chi_{A,t}^X + 2\right)(1-\eta\gamma)\chi_{A,t}^X \sigma_{A|s,t}^2 \Bigg) \end{aligned} \tag{163}$$

We now derive the two desired expressions for Δ_t, splitting the calculation into the quantity-setting and price-setting cases.

Quantity-Setting. Substituting $\chi_{A,t}^Q$ and $\chi_{M,t}^Q$ (quantity-setting) from Corollary 8 and exploiting the fact that the conditional variances are given by $\sigma_{A|s,t}^2 = \kappa_t^A \sigma_{A,s}^2$ and $\sigma_{M|s,t}^2 = \kappa_t^M \sigma_{M,s}^2$, we obtain:

$$\begin{aligned} \Delta_t^Q = \frac{1}{2}(\eta - 1)\Bigg(&\frac{1}{\eta}\sigma_{\vartheta,t}^2 - \eta\kappa_t^M \sigma_{M,s}^2 \\ &+ \left(\frac{1}{\eta}(1-\eta\gamma)\frac{\eta\kappa_t^A}{1-\kappa_t^A(1-\eta\gamma)} + 2\right)(1-\eta\gamma)\frac{\eta\kappa_t^A}{1-\kappa_t^A(1-\eta\gamma)}\kappa_t^A \sigma_{A,s}^2 \Bigg) \end{aligned} \tag{164}$$

as desired.

Price-Setting. Mirroring the steps above using the coefficients from Corollary 8, we obtain

$$\begin{aligned} \Delta_t^P = \frac{1}{2}(\eta - 1)\Bigg(&\frac{1}{\eta}\sigma_{\vartheta,t}^2 + \left(-\eta + \frac{1}{\eta}(1-\eta\gamma)^2 \left(\frac{1-\kappa_t^M}{\gamma}\right)^2\right) \kappa_t^M \sigma_{M,s}^2 \\ &+ \left(\frac{1}{\eta}(1-\eta\gamma)\frac{\kappa_t^A}{\gamma} + 2\right)(1-\eta\gamma)\frac{\kappa_t^A}{\gamma}\kappa_t^A \sigma_{A,s}^2 \Bigg) \end{aligned} \tag{165}$$

yielding the claimed expressions.

Next, consider the comparative statics for Δ_t^P. First,

$$\frac{\partial \Delta_t^P}{\partial \kappa_t^M} = \sigma_{M,s}^2 \left(-\eta + \frac{1}{\eta}(1-\eta\gamma)^2 \left(\frac{1-\kappa_t^M}{\gamma}\right)^2 + \frac{2}{\eta\gamma^2}(1-\eta\gamma)^2(1-\kappa_t^M)\kappa_t^M\right) \tag{166}$$

The condition $\frac{\partial \Delta_t^P}{\partial \kappa_t^M} > 0$ corresponds to

$$\left(\frac{\eta\gamma}{1-\eta\gamma}\right)^2 < (1-\kappa_t^M)^2 + 2(1-\kappa_t^M)\kappa_t^M = 1-(\kappa_t^M)^2 \tag{167}$$

Re-arranging gives, as desired, $\kappa_t^M < \sqrt{1-\left(\frac{\eta\gamma}{1-\eta\gamma}\right)^2}$. Next, Δ_t^P is strictly increasing in κ_t^A if and only if:

$$\left(\frac{1}{\eta}(1-\eta\gamma)\frac{\kappa_t^A}{\gamma}+2\right)(1-\eta\gamma)\frac{\left(\kappa_t^A\right)^2}{\gamma} \tag{168}$$

is strictly increasing in κ_t^A. As argued above, this condition holds if $\eta\gamma < 1$ and does not if $\eta\gamma > 1$.

□

Importantly, since $\Delta_t^Q \neq \Delta_t^P$ in general, others' choice of whether to set prices or quantities affects any given firm's incentives to set prices or quantities. Does the fact that others set prices (quantities) increase or decrease my own desire to set prices (quantities)? Strikingly, we find that these decisions are always *strategic complements*. That is, when all other firms set prices, a given firm has stronger incentives to set prices:

Proposition 8 (Complementarity in Choices of Choices)**.** *The decision to set a price or a quantity is one of strategic complements, i.e, $\Delta_t^P \geq \Delta_t^Q$, with strict inequality whenever $\eta\gamma \neq 1$.*

Proof. Define $\Delta\Delta_t = \Delta_t^P - \Delta_t^Q$ and observe that:

$$\begin{aligned}\Delta\Delta_t = \frac{1}{2}(\eta-1)\Bigg[&\frac{1}{\eta}(1-\eta\gamma)^2\left(\frac{1-\kappa_t^M}{\gamma}\right)^2\kappa_t^M\sigma_{M,s}^2 \\ &+\left(\frac{1}{\eta}(1-\eta\gamma)^2\left(\chi_{A,t}^{P^2}-\chi_{A,t}^{Q^2}\right)+2(1-\eta\gamma)(\chi_{A,t}^P-\chi_{A,t}^Q)\right)\kappa_t^A\sigma_{A,s}^2\Bigg]\end{aligned} \tag{169}$$

First, when $\eta\gamma = 1$, we have that $\Delta\Delta_t = 0$. Second, suppose that $\eta\gamma < 1$. We observe that the first term in brackets is strictly positive. Turning to the second term, as $\eta\gamma < 1$, we have that $\Delta\Delta_t > 0$ if and only if $\chi_{A,t}^P > \chi_{A,t}^Q$. This inequality is equivalent to:

$$\frac{\kappa_t^A}{\gamma} > \frac{\eta\kappa_t^A}{1-\kappa_t^A(1-\eta\gamma)} \tag{170}$$

As $\eta\gamma < 1$ and $\kappa_t^A \in (0,1)$, we have that the denominator on the right-hand side is positive.

Thus, we can re-express this required inequality as:

$$1 - \eta\gamma > \kappa_t^A (1 - \eta\gamma) \tag{171}$$

which is true as $\eta\gamma < 1$ and $\kappa_t^A \in (0, 1)$. Thus, $\Delta\Delta_t > 0$ when $\eta\gamma < 1$. Third, suppose that $\eta\gamma > 1$. Once again, the first term in brackets is strictly positive. Thus, it suffices to show that:

$$\frac{1}{\eta}(1 - \eta\gamma)^2 \left(\chi_{A,t}^{P^2} - \chi_{A,t}^{Q^2} \right) + 2(1 - \eta\gamma)(\chi_{A,t}^P - \chi_{A,t}^Q) > 0 \tag{172}$$

See that we can factor the left-hand side of this expression as:

$$(1 - \eta\gamma)(\chi_{A,t}^P - \chi_{A,t}^Q) \left(\frac{1}{\eta}(1 - \eta\gamma)(\chi_{A,t}^P + \chi_{A,t}^Q) + 2 \right) \tag{173}$$

By the reverse of the logic in part two, we have that $\chi_{A,t}^P < \chi_{A,t}^Q$. Thus, the expression in question is strictly positive if and only if:

$$2 > \frac{1}{\eta}(\eta\gamma - 1)(\chi_{A,t}^P + \chi_{A,t}^Q) \tag{174}$$

We now observe that $\chi_{A,t}^P + \chi_{A,t}^Q < 2\chi_{A,t}^Q$. Moreover, $\chi_{A,t}^Q$ is an increasing function of κ_t^A and is therefore bounded above by $\frac{\eta}{1+\eta\gamma-1} = \frac{1}{\gamma}$. Thus, we have that:

$$\frac{1}{\eta}(\eta\gamma - 1)(\chi_{A,t}^P + \chi_{A,t}^Q) < \frac{2}{\eta\gamma}(\eta\gamma - 1) = 2 - \frac{2}{\eta\gamma} < 2 \tag{175}$$

This establishes that $\Delta\Delta_t > 0$ if $\eta\gamma > 1$. Taken together, we have shown that $\Delta\Delta_t \geq 0$ and $\Delta\Delta_t > 0$ if and only if $\eta\gamma \neq 1$, establishing the claim. □

To give the intuition for this result, we first consider the case when $\eta\gamma < 1$. In this case, it can easily be seen that consumption responds more to productivity shocks under price-setting. Moreover, regardless of the value of $\eta\gamma$, consumption responds more to monetary shocks under price-setting. Therefore, others being price-setters increases both the variance of consumption and the covariance of consumption with productivity. Both of these forces favor price-setting, as shown in Equation 156. In summary, others setting prices induces aggregate volatility which makes it more attractive for any given firm to also set a price. In the case of $\eta\gamma \geq 1$, consumption is more responsive to monetary shocks but less responsive to productivity shocks under price-setting versus quantity-setting. In the proof, we show how these effects net out in Equation 156 in the direction of making price-setting more attractive when other firms set prices.

We now use this result to show the existence of equilibria in which all firms *optimally* choose to set prices or quantities:

Corollary 9 (Existence of Pure Equilibria)**.** *There exists an equilibrium in which, at each date t, either all firms set prices or all firms set quantities.*

To prove this result, we consider two cases at each date t. First, suppose that firms prefer to set prices if others set quantities, or $\Delta_t^Q \geq 0$. In this case, they even more strongly prefer to set prices if others set prices, or $\Delta_t^P \geq \Delta_t^Q \geq 0$. Therefore, choosing to set prices is consistent with equilibrium. Conversely, suppose that firms prefer to set quantities when others set quantities, $\Delta_t^Q < 0$. In this case, choosing to set quantities is consistent with equilibrium. As these cases are exhaustive, a pure equilibrium exists. Note that this logic heavily relies on our finding that the decision to set prices was one of complements; if it were one of substitutes, then pure equilibria could fail to exist.[13]

C.3 Time-Varying Uncertainty and Regime Switches

As shown in Lemma 1, the comparative advantage of price-setting *vs.* quantity-setting changes over time because firms' uncertainty about microeconomic and macroeconomic variables changes over time. This observation, combined with Corollary 9, implies the existence of equilibria in which time-varying volatility induces time-varying uncertainty and *regime changes* between price- and quantity-setting. These regime changes, in turn, affect the propagation of aggregate shocks as summarized in Corollary 8. Thus, "uncertainty shocks" that affect exogenous volatility have further effects on the volatility of endogenous outcomes (income, prices) due to the endogenous "choice of choices."

To better understand these forces, we now study the comparative statics of (Δ_t^Q, Δ_t^P) in the parameters for time-varying volatility. We start by studying uncertainty about the aggregate productivity state A_t. Higher aggregate productivity uncertainty pushes toward either price- or quantity-setting depending on the parameter condition $\eta\gamma \gtrless 1$:

Corollary 10. *If $\eta\gamma > 1$, then both Δ_t^Q and Δ_t^P are decreasing in κ_t^A and in σ_t^A. If $\eta\gamma < 1$, then both Δ_t^Q and Δ_t^P are increasing in κ_t^A and in σ_t^A. If $\eta\gamma = 1$, then Δ_t^Q and Δ_t^P are equal and invariant to κ_t^A and σ_t^A.*

Proof. We consider the three cases $\eta\gamma = 1$, $\eta\gamma < 1$, and $\eta\gamma > 1$ separately.

1. $\eta\gamma = 1$. By Lemma 1, we have that $\Delta_t^Q = \Delta^Q(0)$ and $\Delta_t^P = \Delta^Q(0)$, which are both independent of κ_t^A.

[13] A mixed equilibrium would always exist.

2. $\eta\gamma < 1$. By Lemma 1, we have that Δ_t^Q is strictly increasing in κ_t^A if and only if

$$\left(\frac{1}{\eta}(1-\eta\gamma)\frac{\eta\kappa_t^A}{1-\kappa_t^A(1-\eta\gamma)}+2\right)(1-\eta\gamma)\frac{\eta\left(\kappa_t^A\right)^2}{1-\kappa_t^A(1-\eta\gamma)} \tag{176}$$

is strictly increasing in κ_t^A. As $\eta\gamma < 1$ and $\frac{\eta\kappa_t^A}{1-\kappa_t^A(1-\eta\gamma)}$ is strictly increasing in κ_t^A and strictly positive, we have that the term in parentheses is strictly positive and strictly increasing. The term outside parentheses is strictly increasing and strictly positive for the same reasons. Moreover, Δ_t^P is strictly increasing in κ_t^A if and only if:

$$\left(\frac{1}{\eta}(1-\eta\gamma)\frac{\kappa_t^A}{\gamma}+2\right)(1-\eta\gamma)\frac{\left(\kappa_t^A\right)^2}{\gamma} \tag{177}$$

is strictly increasing in κ_t^A. As $\eta\gamma < 1$, this is immediate.

3. $\eta\gamma > 1$. By Lemma 1, we have that Δ_t^Q is strictly decreasing in κ_t^A if and only if Expression 176 is strictly decreasing in κ_t^A. Define $\omega = 1-\eta\gamma$ and observe that we need to show that:

$$\left(\frac{\omega\kappa_t^A}{1-\omega\kappa_t^A}+2\right)\frac{\omega\left(\kappa_t^A\right)^2}{1-\omega\kappa_t^A} \tag{178}$$

is a strictly decreasing function of κ_t^A. Taking the derivative of this expression and rearranging, we require that:

$$\omega\kappa_t^A\left(\omega^2\left(\kappa_t^A\right)^2-3\omega\kappa_t^A+4\right)<0 \tag{179}$$

As $\omega < 0$, we require that $\omega^2\left(\kappa_t^A\right)^2-3\omega\kappa_t^A+4>0$. This is positive if the quadratic on the left-hand side has no real roots. As $9\omega^2-16\omega^2<0$, the quadratic indeed has no real roots and so Δ_t^Q is strictly decreasing in κ_t^A.

Δ_t^P is strictly decreasing in κ_t^A if and only:

$$\left(\frac{\omega}{1-\omega}\kappa_t^A+2\right)\frac{\omega}{1-\omega}\left(\kappa_t^A\right)^2 \tag{180}$$

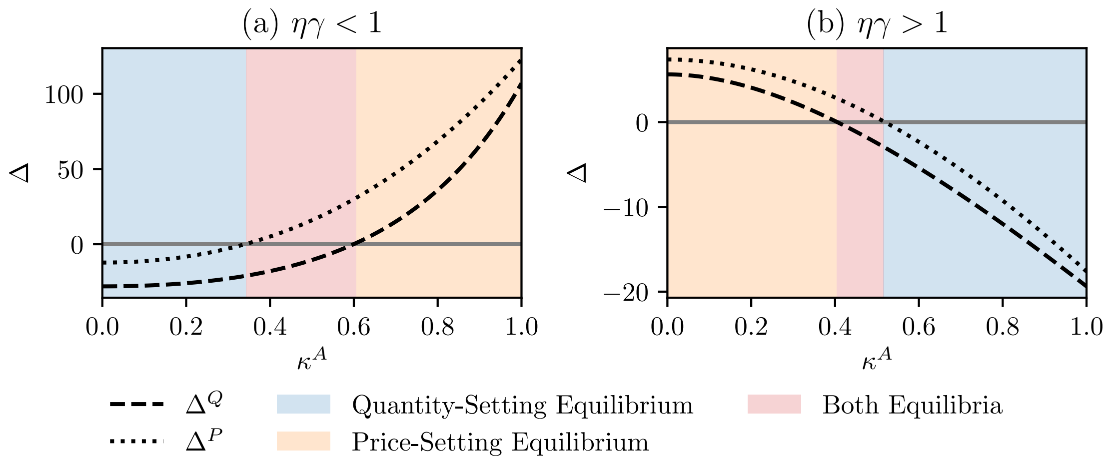
is strictly decreasing in κ_t^A. Taking the derivative of this expression and rearranging, we require that:

$$\kappa_t^A < \frac{4}{3}\frac{\omega-1}{\omega} \tag{181}$$

which is always satisfied as $\omega < 0$.

As κ_t^A is increasing in σ_t^A, this establishes the result. $\square$

Figure 7: Equilibrium with Changing Productivity Uncertainty

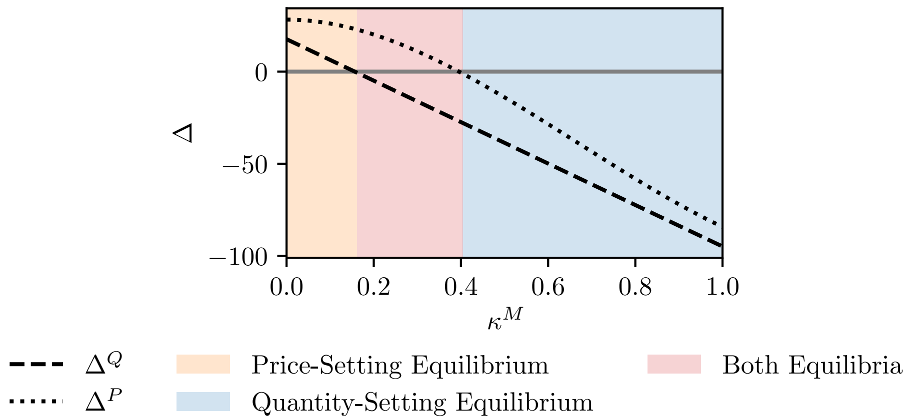


Note: This figure illustrates firms' equilibrium incentives for price-setting as uncertainty about productivity changes. In each panel, we plot Δ^Q (dashed line) and Δ^P (dotted line) as a function of κ^A, fixing all other parameter values. In Example A, we use parameters such that $\eta\gamma < 1$. In Example B, we use parameters such that $\eta\gamma > 1$. We shade the region with only a quantity-setting equilibrium blue, the region with only a price-setting equilibrium orange, and the region with both equilibria red.

When $\eta\gamma < 1$, the dominant effects of productivity uncertainty are to increase aggregate demand uncertainty and to lower the covariance between demand and marginal costs. When $\eta\gamma > 1$, the dominant effect is to increase the covariance between marginal costs and the price level. Finally, in the special case in which $\eta\gamma = 1$, these forces net out to zero.

We illustrate this result and its implications for equilibrium regime-switching in a numerical example. In Figure 7, we plot Δ^Q and Δ^P as a function of κ^A for two different calibrations, corresponding to $\eta\gamma < 1$ and $\eta\gamma > 1$. We shade regions of the parameter space in which only one equilibrium exists (blue for quantity-setting and orange for price-setting) and in which both equilibria exist (red). In the economies corresponding to each parameter case, as κ_t^A moves exogenously (because of underlying movements in σ_t^A), the equilibrium transitions between quantity-setting and price-setting. For example, in the left panel with $\eta\gamma < 1$, periods of high productivity uncertainty (high κ_t^A) correspond to price-setting and periods of low productivity uncertainty (high κ_t^A) correspond to quantity-setting. If κ_t^A lies in the middle, red-shaded region in any period t, there exists an equilibrium in which firms set prices in that period as well as one in which they set quantities in that period. In this way, even if κ_t^A were constant over time but lying in this multiple-equilibrium region, there could be self-fulfilling macroeconomic volatility that arises from endogenous regime shifts.

Figure 8: Equilibrium with Changing Money-Supply Uncertainty



Note: This figure illustrates firms' equilibrium incentives for price-setting as uncertainty about the money supply changes. We plot Δ^Q (dashed line) and Δ^P (dotted line) as a function of κ^M, fixing all other parameter values. We use parameters such that $\eta\gamma > \frac{1}{2}$, so both functions are monotone decreasing (see Corollary 12). We shade the region with only a quantity-setting equilibrium blue, the region with only a price-setting equilibrium orange, and the region with both equilibria red.

We next study the role of idiosyncratic uncertainty. We find that idiosyncratic demand uncertainty unambiguously favors quantity-setting, while idiosyncratic uncertainty about productivity and factor prices (via labor supply) do not matter:

Corollary 11. *Both Δ_t^Q and Δ_t^P are increasing in $\sigma^2_{\vartheta,t}$ and neither depends on $\sigma^2_{z,t}$ or $\sigma^2_{\phi,t}$.*

This result is immediate from inspection of the formulas in Lemma 1 and is not intermediated by equilibrium forces. Economically, it implies that "uncertainty shocks" that increase idiosyncratic variation in firms' demand unambiguously push the economy toward price-setting. In light of empirical evidence that (i) idiosyncratic volatility in firms' revenue TFP rises dramatically in recessions (*e.g.*, Bloom et al., 2018) and (ii) a majority of revenue TFP variation arises from demand rather than productivity shocks (Foster et al., 2008), Corollary 11 suggests a powerful force for regime switches that line up with the business cycle. By contrast, uncertainty about idiosyncratic productivity and factor prices does *not* behave symmetrically to uncertainty about demand. This follows from our original observation that the uncertainty about marginal costs matters only through its covariance with demand and the price level, and not through its variance (Proposition 1).

We finally study uncertainty about the money supply M_t. As with uncertainty about productivity, understanding its effect requires disciplining opposing equilibrium forces:

Corollary 12. *Δ_t^Q is always decreasing in κ_t^M and σ_t^M. If $\eta\gamma \geq \frac{1}{2}$, then Δ_t^P is strictly decreasing in κ_t^M and σ_t^M. If $\eta\gamma < \frac{1}{2}$, then there exists a $\bar{\kappa}^M \in [0, 1/3]$ such that Δ_t^P is increasing for $\kappa_t^M < \bar{\kappa}^M$ and decreasing for $\kappa_t^M > \bar{\kappa}^M$.*

Proof. The fact that Δ_t^Q is decreasing in κ_t^M is immediate from Lemma 1. Moreover, from Lemma 1, Δ_t^P is decreasing in κ_t^M if and only if

$$\left(-\eta + \frac{1}{\eta}(1-\eta\gamma)^2 \left(\frac{1-\kappa_t^M}{\gamma}\right)^2\right) \kappa_t^M \tag{182}$$

is decreasing in κ_t^M. Taking the derivative of this expression, this is equivalent to

$$\frac{(1-\eta\gamma)^2}{(\eta\gamma)^2} \left(\frac{1-2\eta\gamma}{(1-\eta\gamma)^2} - 4\kappa_t^M + 3(\kappa_t^M)^2\right) < 0 \tag{183}$$

This is a strictly convex quadratic. Hence, if we show that this expression is weakly negative evaluated at $\kappa_t^M = 0$ and $\kappa_t^M = 1$, it will be strictly negative for all $\kappa_t^M \in (0, 1)$. A sufficient condition for this expression to be weakly negative at $\kappa_t^M = 0$ is that

$$\frac{1-2\eta\gamma}{(1-\eta\gamma)^2} \leq 0 \tag{184}$$

which occurs if and only if $\eta\gamma \geq 1/2$. It is easily verified that $\eta\gamma \geq 1/2$ also makes the expression strictly negative at $\kappa_t^M = 1$. This proves that Δ_t^P is strictly decreasing for all $\kappa_t^M \in (0, 1)$ whenever $\eta\gamma \geq 1/2$.

We next study the case in which $\eta\gamma < \frac{1}{2}$. We re-arrange condition 183 to

$$\kappa_t^M(4 - 3\kappa_t^M) > \frac{1-2\eta\gamma}{(1-\eta\gamma)^2} \tag{185}$$

We first observe that this condition always holds at $\kappa_t^M = 1$, as the left-hand-side is 1 and the right-hand-side, given $\eta\gamma < 1/2$, is bounded above by 1. Therefore, the critical value $\bar{\kappa}^M$ is the smaller root of the quadratic equation $\kappa_t^M(4 - 3\kappa_t^M) - \frac{1-2\eta\gamma}{(1-\eta\gamma)^2} = 0$. By direct calculation, this is

$$\bar{\kappa}^M = \frac{1}{3}\left(2 - \sqrt{4 - 3\frac{1-2\eta\gamma}{(1-\eta\gamma)^2}}\right) \tag{186}$$

$$= \frac{1}{3}\left(2 - \sqrt{1 - \left(\frac{\eta\gamma}{1-\eta\gamma}\right)^2}\right) \tag{187}$$

$$= \frac{2}{3} - \sqrt{\frac{1}{9} + \frac{1}{3}\left(\frac{\eta\gamma}{1-\eta\gamma}\right)^2} \tag{188}$$

where, in the second equality, we use the fact that $\frac{1-2\eta\gamma}{(1-\eta\gamma)^2} = 1 - \left(\frac{\eta\gamma}{1-\eta\gamma}\right)^2$. We finally note that $\bar{\kappa}^M$ is monotone increasing in $\eta\gamma$, is minimized at 0 when $\eta\gamma = 1/2$, and is maximized at $\frac{1}{3}$ when $\eta\gamma = 0$. □

Under quantity-setting, because monetary shocks are neutral for output, increasing the volatility of money-supply shocks (lowering κ_t^M) serves only to increase the volatility of the price level and further favor quantity-setting. Under price-setting, because monetary shocks are not neutral for output, there is a countervailing effect from increasing the volatility of aggregate demand. Therefore, when $\eta\gamma$ is sufficiently low, the effect of money-supply uncertainty is ambiguous.

We illustrate this result numerically in Figure 8. We focus on a calibration in which $\eta\gamma > \frac{1}{2}$, so both Δ_t^Q and Δ_t^P are monotone decreasing in κ_t^M. In periods of low money-supply uncertainty, firms have stronger incentives to set prices; in periods of high money-supply uncertainty, firms have stronger incentives to set quantities. Moreover, in the quantity-setting regimes, the aggregate price level responds more to money-supply innovations (Corollary 8) which further sharpens the incentives for quantity-setting. This positive feedback loop underlies our comparative statics result.

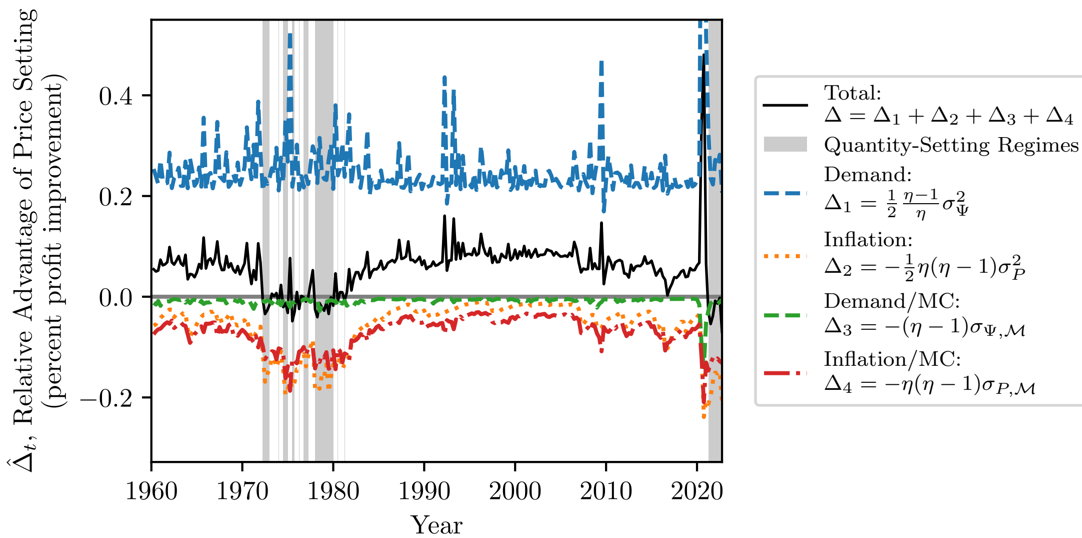
C.4 Prices *vs.* Quantities in the Data

In this final Appendix subsection, we introduce methods to empirically measure the relative advantage of price- and quantity-setting over recent US history.

Methods. Using our GARCH model from the main text and Proposition 1, we can compute an empirical estimate for when firms should prefer price-setting:

$$\hat{\Delta}_t = \frac{1}{2}(\eta - 1)\left(\frac{1}{\eta}\hat{\sigma}_{\Psi,t}^2 - \eta\hat{\sigma}_{P,t}^2 - 2\hat{\sigma}_{\Psi,\mathcal{M},t} - 2\eta\hat{\sigma}_{P,\mathcal{M},t}\right) \tag{189}$$

Figure 9: The Relative Benefit of Price-Setting in US Data



Note: This figure plots our empirical estimate of $\hat{\Delta}_t$ (the comparative advantage of price-setting relative to quantity-setting) and its components, as defined in Proposition 1 (Equation 12). The black line plots $\hat{\Delta}_t$, in units of expected percent profit improvement (100 times log points). The blue (dashed), orange (dotted), green (dashed), and red (dash-dotted) lines plot each of the four components of $\hat{\Delta}_t$, corresponding to uncertainty about different variables. The grey shading denotes periods in which $\hat{\Delta}_t < 0$ and thus, according to Proposition 1, quantity-setting is optimal for firms. As described in Section 5, the calculation uses estimates of time-varying volatilities from a CCC GARCH(1,1) model and a calibrated demand elasticity of $\eta = 9$. The demand component exceeds the scale of the figure in Q2 and Q3 of 2020.

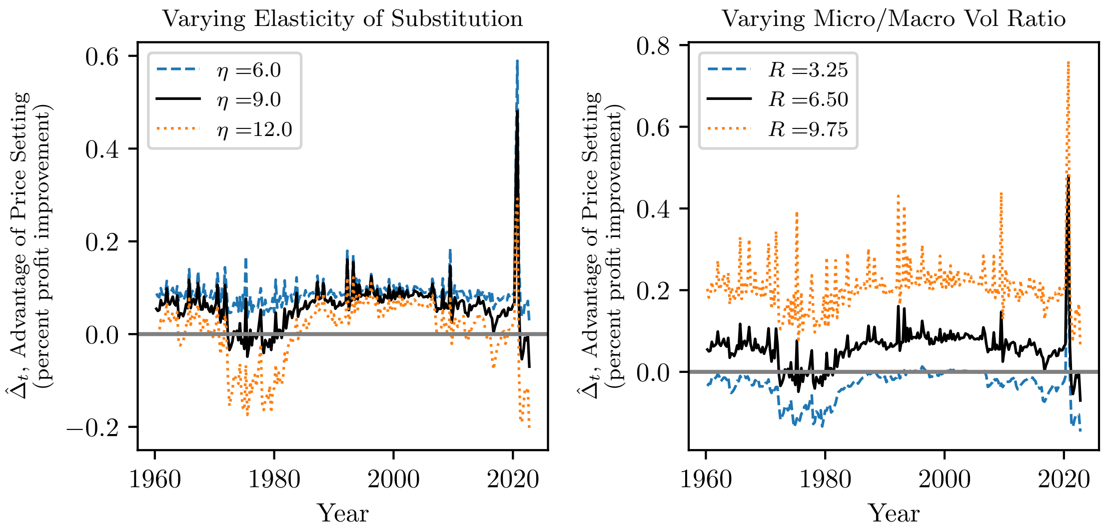
Our calculation captures uncertainty about outcomes realized in quarter t, and is measurable in data from quarter $t-1$ and earlier. It therefore describes incentives of a decisionmaker fixing a choice for quarter t based on their uncertainty at the beginning of the quarter, before data are realized.

We plot our calculation of $\hat{\Delta}_t$ in Figure 9. We show our overall calculation in black and each component in color. We shade periods which favor quantity-setting, or for which $\hat{\Delta}_t < 0$.

Strikingly, both quantity- and price-setting are optimal at different points in the sample. Thus, viewed through the lens of our model and its mapping to the data, firms may be either price- or quantity-setters depending on the macroeconomic context. Moreover, through the same lens, this evidence rules out the conventional assumption that firms always choose prices or always choose quantities.

Price-setting is optimal in most of the sample, or 219 of 251 quarters. This notably comprises the 1960s and the Great Moderation, in which both inflation and demand variance

Figure 10: The Relative Benefit of Price-Setting Under Alternative Parameters



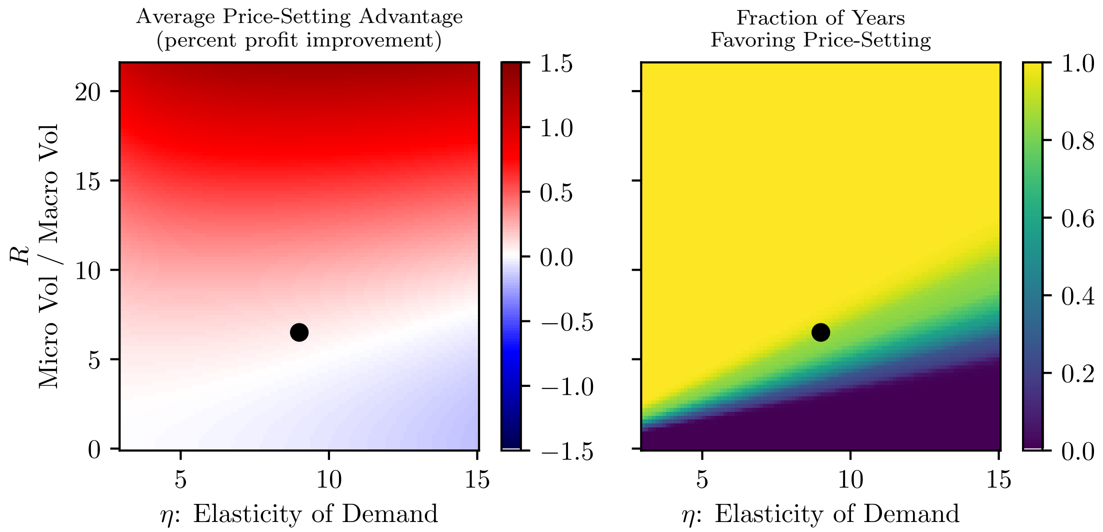
Note: Both panels plot our empirical estimate of $\hat{\Delta}_t$ defined in Proposition 1 (Equation 12) under alternative assumptions for the elasticity of substitution η (left) and the micro-to-macro volatility ratio R (right). In both plots, our baseline estimate corresponds to the solid black line.

were relatively tame, and the Great Recession and the onset of the Covid-19 Lockdown Recession (Q2 2020), when demand variance abruptly spiked.

Quantity-setting is optimal intermittently between 1972 Q2 and 1981 Q2, for a total of 25 of the possible 37 quarters in this period, and continuously between 2021 Q2 and the end of the sample. These all correspond to periods of particularly high contributions of the terms corresponding to inflation variance and inflation-marginal-cost covariance. Through the lens of the model, firms would prefer to set quantities in these periods to hedge against the increase in uncertainty about joint movements in inflation and marginal costs. Our calculation weighs this consideration against demand risk, which favors price-setting and may also be elevated in recessions. For example, in 1975 Q2 and 2021 Q1, demand uncertainty is sufficiently high to outweigh elevated inflation and inflation-marginal-cost uncertainty, and our calculation favors price-setting on net ($\hat{\Delta}_t > 0$).

We finally note that the advantage of one method over another is always relatively small in payoff terms. In our sample, this advantage peaks at 0.48% (0.0048 log points) in Q3 of 2020. In all periods excluding Q2 and Q3 of 2020, the difference peaks at 0.16%. This is a striking juxtaposition with the model prediction that a change in firm behavior between price- and quantity-setting can have large effects on equilibrium outcomes.

Figure 11: The Relative Benefit of Price-Setting Under Alternative Parameters



Note: This figure summarizes the relative advantage of price-setting for alternative values of the elasticity of demand (horizontal axis) and the ratio of microeconomic to macroeconomic volatility (vertical axis). The left panel plots the average advantage of price-setting over the sample, in units of 100 times log points (percent). The right panel plots the fraction of the sample in which price-setting is optimal, or in which $\hat{\Delta}_t > 0$. In both panels, our baseline calibration is indicated with a solid dot.

Robustness to Parameter Values and Measurement Strategies. Two parameters that were central to our calculation, but difficult to pin down in the data, were the price elasticity of demand ($\eta = 9$) and the ratio of micro to macro volatility ($R = 6.5$). In Figure 10, we plot the implied time series for Δ under specific alternative assumptions for each parameter. In Figure 11, we vary both parameters continuously over a larger grid and plot "heat maps" for the average value of $\hat{\Delta}_t$ and the percentage of the sample with $\hat{\Delta}_t > 0$.

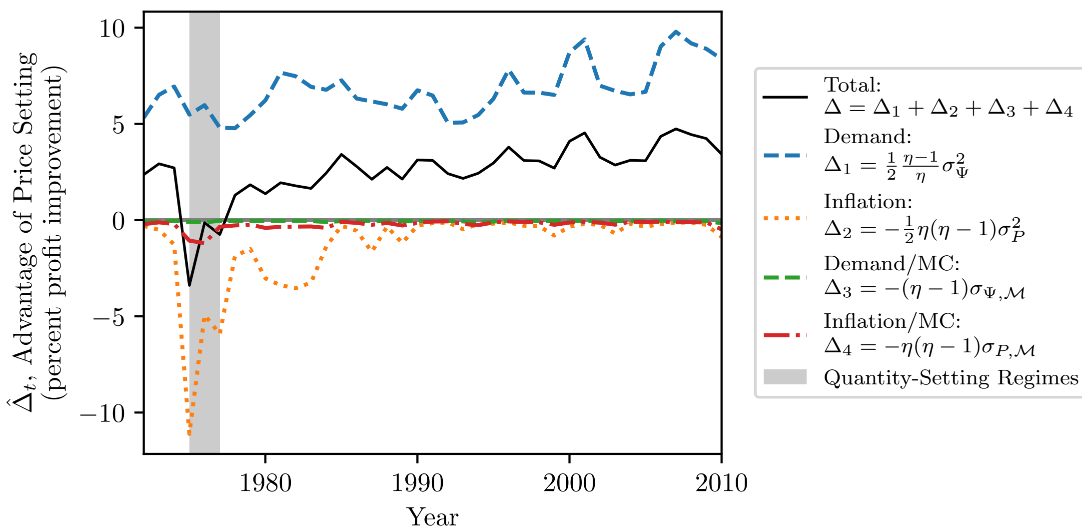
Decreasing the elasticity of demand favors price-setting, while increasing the elasticity of demand favors quantity-setting (left panel). The primary reason, quantitatively, is that highly inelastic demand curves amplify the effects of demand shocks on prices for fixed quantities, and hence increase potential losses from quantity-setting. In the data, this further pushes toward price-setting, especially in time periods with especially high demand volatility.

Increasing demand risk favors price-setting by construction (right panel). In particular, increasing the extent of microeconomic volatility by 50% favors price-setting in all periods (orange dotted line), while decreasing this parameter by 50% implies quantity-setting in a majority of periods (blue dashed line). As noted by Bloom et al. (2018), calibrating this parameter on the basis of observed variances in *measured* firm-level fundamentals requires modeling choices. In particular, one must take a stand on what fraction of measured volatility corresponds to measurement error and what fraction of volatility from an econometrician's perspective is unknown to firm managers, who likely have superior information.

As an alternative strategy to measure the contribution of idiosyncratic volatility, we can use the direct measurements of Bloom et al. (2018) based on annual data from manufacturing establishments from 1972 to 2010, along with assumptions about measurement error and observability of shocks. To accommodate this variant calculation, we re-estimate the VAR(1) CCC GARCH(1,1) model on annual data for the same macro time series. We then use the Bloom et al. (2018) estimates of the cross-sectional standard deviation of manufacturing TFPR along with those authors' quantitative assumption that 45.4% of this measured volatility (standard deviation) corresponds to measurement error. We make the intentionally extreme assumption that all of this remaining variance is unforecastable by firms. Figure 12 shows our results. This calculation echoes the conclusion that the 1970s were favorable to quantity-setting due to the relatively high inflation volatility and relatively low demand volatility.

Comparison to External Evidence. An alternative way to gauge the plausibility of firms' entertaining both price- and quantity-setting plans is via direct survey evidence. As observed by Reis (2006), Aiginger (1999) collected data on this topic. In a survey of managers of Austrian manufacturing firms, he asked: "What is your main strategic variable: do you decide to produce a specific quantity, thereafter permitting demand to decide upon price conditions, or do you set the price, with competitors and the market determining the quantity sold?" Among managers, 32% said that they use the quantity plan and 68% said that they use the price plan. We interpret this as additional evidence that neither price nor quantity plans are obviously favored in practice.

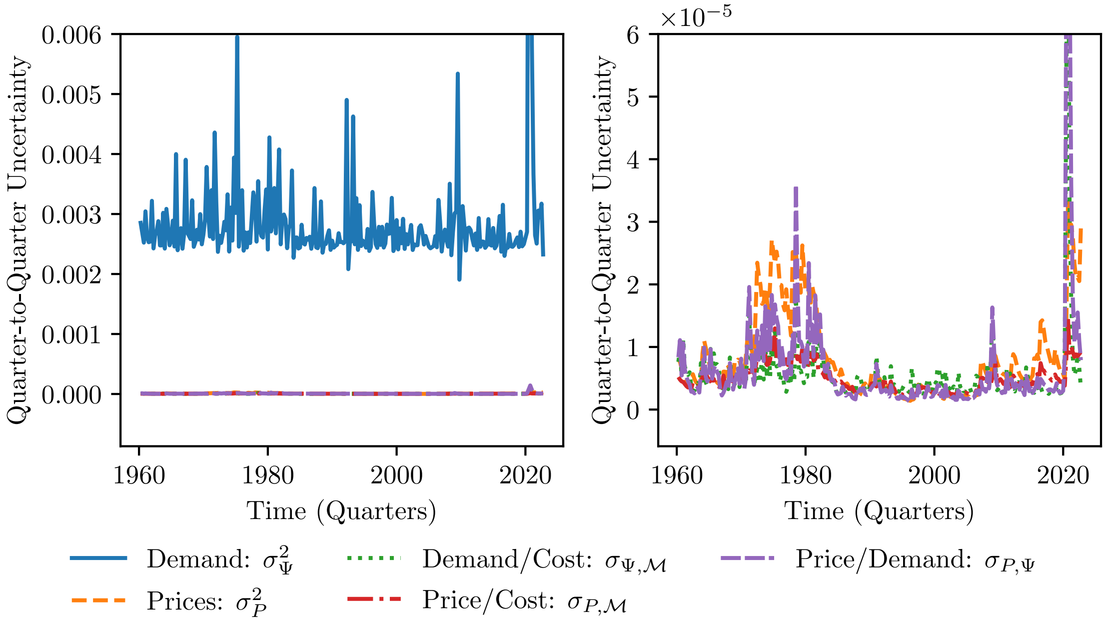
Figure 12: The Relative Benefit of Price-Setting in an Alternative, Annual Calculation



Note: This figure plots our empirical estimate of $\hat{\Delta}_t$ (the comparative advantage of price-setting relative to quantity-setting) and its components, as defined in Proposition 1 (Equation 12), under a variant method with annual-frequency data and direct measurement of micro volatility from Bloom et al. (2018). Note that the time period (1972-2010) and time-frequency (annual) differs from that in Figures 9 and 10 (quarterly, 1960 Q1 to 2022 Q4). The black line plots $\hat{\Delta}_t$, in units of expected percent profit improvement (100 times log points). The blue (dashed), orange (dotted), green (dashed), and red (dash-dotted) lines plot each of the four components of $\hat{\Delta}_t$, corresponding to uncertainty about different variables. The grey shading denotes periods in which $\hat{\Delta}_t < 0$ and thus, according to Proposition 1, quantity-setting is optimal for firms. As described in Section 5, the calculation uses estimates of time-varying volatilities from an annual-frequency CCC GARCH(1,1) model and a calibrated elasticity of demand $\eta = 9$

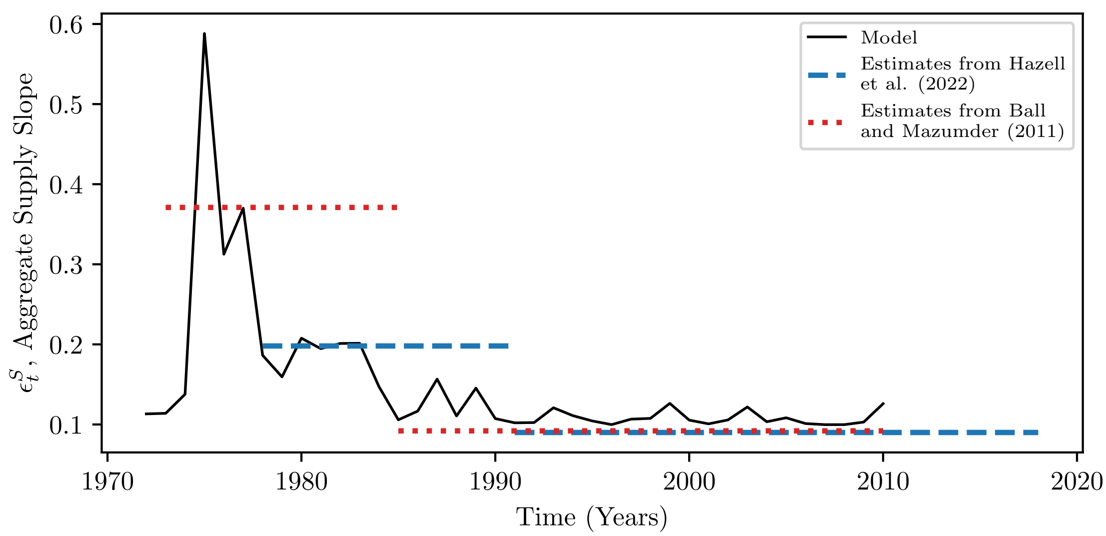
D Additional Tables and Figures

Figure 13: Estimates of Time-Varying Uncertainty



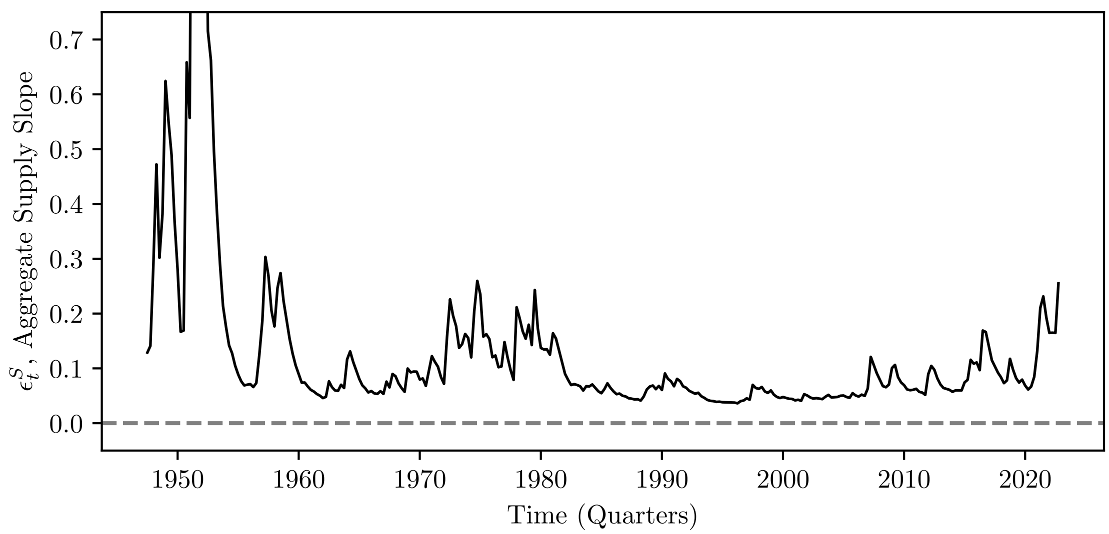
Note: Both panels plot our quarterly time-series estimates of uncertainty, estimated as described in Section 5.1. All lines except for "Demand" (solid blue) are one-quarter-ahead volatility predictions from a constant conditional correlations (CCC) GARCH model. The "Demand" estimates combine the GARCH model's predictions for real GDP uncertainty and TFP uncertainty with an assumption about the relationship between microeconomic (demand) volatility and macroeconomic (productivity) uncertainty, as described in the main text. The left plot shows all series on a common scale, and the right plot zooms in on the series other than demand. Both plots feature spikes that are off the scale of the graph during the Covid-19 lockdown.

Figure 14: The Slope of Aggregate Supply (Annual-Frequency Calculation)



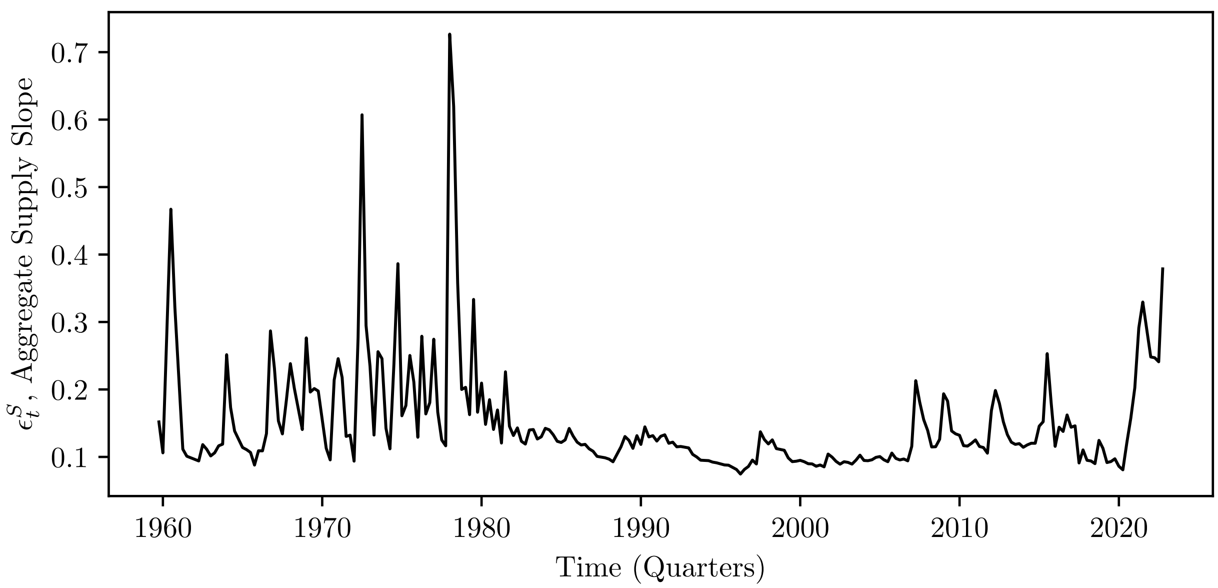
Note: This Figure plots estimates of the inverse elasticity of aggregate supply as measured by Equations 54 and 55. These estimates correspond to our secondary calculation using an annual-frequency GARCH model and a direct measure of microeconomic (demand) uncertainty from Bloom et al. (2018). The blue dashed line indicates the estimates of Hazell et al. (2022), based on state-level estimates of consumer prices and unemployment and an identification strategy that isolates local demand shocks (columns 3 and 4, panel B, of Table II). The red dotted line indicates the estimates of Ball and Mazumder (2011), based on aggregate data (column 4 of Table 3).

Figure 15: The Slope of Aggregate Supply Since World War II



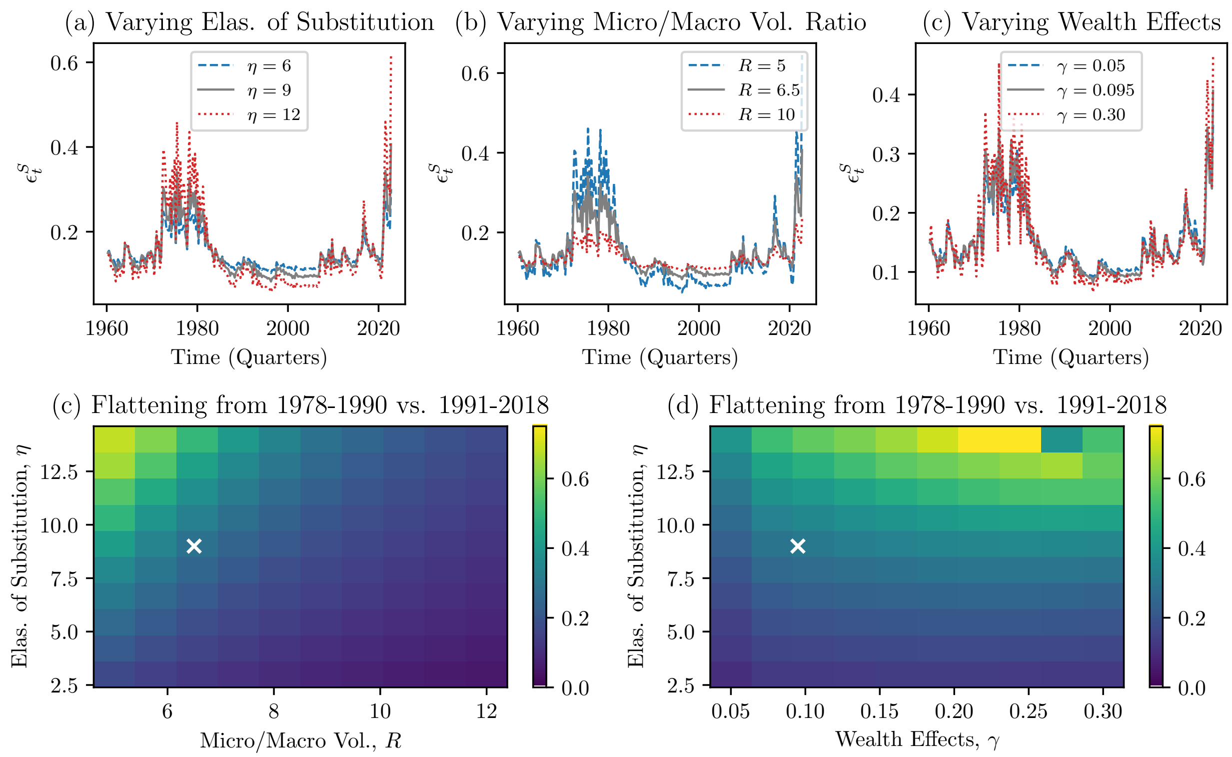
Note: This Figure plots estimates of the inverse elasticity of aggregate supply as measured by Equations 54 and 55. In this calculation, in contrast to the calculation of Figure 4, we extend the calculation back to 1947:Q2. The elasticity of aggregate supply is off the scale of the graph from 1951:Q1 to 1952:Q2. Note that the estimates from 1960 onward numerically differ from the ones in Figure 2 because we re-estimate the GARCH model for macroeconomic uncertainty over the larger sample.

Figure 16: Aggregate Supply with Pseudo-out-of-sample Uncertainty



Note: This Figure plots estimates of the inverse elasticity of aggregate supply as measured by Equations 54 and 55. In this calculation, in contrast to the calculation of Figure 4, we estimate uncertainty about variables in quarter t using a maximum-likelihood estimate of the model with data only up to quarter $t-1$. To cover the beginning of the sample, we use data starting in 1947:Q2.

Figure 17: Sensitivity of Aggregate Supply Estimates to Parameters



Note: This Figure plots the sensitivity of the inverse elasticity of aggregate supply to the elasticity of substitution η (Panel a), to varying ratios of microeconomic to macroeconomic uncertainty R (Panel b), and to varying calibrations of wealth effects (Panel c). Panels (c) and (d) each plot a heat map for the difference in the average slope for aggregate supply from 1970-1980 to 1991-2018, varying different pairs of parameters. In each of these panels, the white "x" denotes our baseline calibration.