Four decades of research on the open-shop scheduling problem to minimize the makespan

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Abstract

One of the basic scheduling problems, the open-shop scheduling problem has a broad range of applications across different sectors. The problem concerns scheduling a set of jobs, each of which has a set of operations, on a set of different machines. Each machine can process at most one operation at a time and the job processing order on the machines is immaterial, i.e., it has no implication for the scheduling outcome. The aim is to determine a schedule, i.e., the completion times of the operations processed on the machines, such that a performance criterion is optimized. While research on the problem dates back to the 1970s, there have been reviving interests in the computational complexity of variants of the problem and solution methodologies in the past few years. Aiming to provide a complete road map for future research on the open-shop scheduling problem, we present an up-to-date and comprehensive review of studies on the problem that focuses on minimizing the makespan, and discuss potential research opportunities.

Keywords: scheduling; open-shop; review; makespan

1 Introduction

The open-shop scheduling problem is one of the basic scheduling problems, which has a broad range of applications across different sectors. An example of the problem in the health care sector is as follows: Scheduling patients for diagnosing the coronary heart disease. A patient needs to undergo the diagnosis in three stages, namely blood testing, ultrasonic cardiogramming, and coronary computed tomography (CT) scanning, in any order. Each stage requires multiple facilities and/or medical personnel to conduct the diagnosis. In general, the open-shop problem concerns scheduling a set of jobs, each with a set of operations, on a set of different machines, where each machine can process at most one operation at a time and the order of processing the jobs on the machines is immaterial. The scheduler wishes to determine the completion times of the operations on the machines (schedule) such that a performance criterion is optimized.

While research on the open-shop scheduling problem dates back to the 1970s, there have been reviving interests in the computational complexity of variants of the problem and solution methodologies in the past few years. We set out to provide a complete road map for future research on the open-shop scheduling problem, presenting an up-to-date and comprehensive review of studies on the problem that focuses on minimizing the makespan and discussing potential research opportunities.

1.1 Scope and classification

We present a detailed literature review of the classical open-shop scheduling problem. In this problem a set of jobs and a set of machines are given, and the scheduler aims to develop a schedule of jobs' execution on the machines such that a performance criterion is optimized. The jobs are processed on machines, however, the job processing order on the machines is immaterial, i.e., it has no implication for the scheduling outcome. Unless otherwise stated, we assume that all the parameters, e.g., the processing times of job operations on the machines, the due dates, the release times etc, are known and deterministic.

We confine the scope of the paper to the performance criterion (or objective function) of minimizing the makespan, i.e., the maximum completion time of all the jobs. Therefore, we do not review the other performance criteria, such as minimizing the mean flow time, minimizing the maximum lateness, and minimizing the total tardiness, among others. However, a few variants of the problem concern performance criteria that are related to the makespan even though they consider slightly different objective functions. To provide a through review, we discuss those studies as well. We only focus on the open-shop environment, so do not review studies on generalization of the open-shop scheduling environment, e.g., the multi-processor open-shop (Sevast'yanov and Woeginger, 2001), where parallel identical machines process the jobs in every stage. We do not review studies on hybridization of the open-shop scheduling problem with, e.g., the job-shop environment that results in mixed-shop scheduling (Shakhlevich et al., 2000) either; neither do we study the super-shop scheduling setting (Strusevich, 1991) in which some jobs are processed in the flow-shop setting, i.e., all the jobs have the same processing route, some jobs are processed in the job-shop environment, i.e., the jobs have arbitrary processing routes, and some jobs are processed in the open-shop setting, i.e., no specific processing route is imposed on the jobs.

On the open-shop scheduling problem to minimize the makespan, we present a comprehensive review of the pertinent studies that include various constraints and jobs' execution settings. While we have made a meticulous effort to include all the related studies that we are aware of, we might have missed some less relevant papers. In particular, we are only aware of one review paper on the open-shop scheduling problem by Anand and Panneerselvam (2015). Their review study is limited in many aspects. For example, they reviewed papers only up to 2013 and did not consider studies before 1981. Despite reviewing various objective functions that intuitively leads to the expectation that a large number of papers were reviewed, Anand and Panneerselvam (2015) covered only 100 papers, implying that many studies between 1981 and 2013 were missing from their review. We include all the relevant studies conducted between 2014 and 2020. This involves a total number of 47 studies, which is nearly half of the total papers reviewed in Anand and Panneerselvam (2015). In addition,

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we provide an analysis of the publication rate of papers on open-shop scheduling papers to minimize the makespan over the years and the publication outlets.

We organize the rest of the paper as follows: In the next subsection we present an analysis of the reviewed papers. In Section 1.3 we formally define the problem and introduce the main notation that we use throughout the paper, followed by discussion of a number of applications of the open-shop scheduling problem in Section 1.4. In Section 2 we review studies on the open-shop scheduling problem in its basic setting. We review studies on the problem in specific settings of the machines or resources, and jobs in Sections 3 and 4, respectively. We discuss the potential research opportunities in Section 5, followed by concluding the paper in Section 6.

1.2 Analysis of the reviewed papers

We review all the available studies on the open-shop scheduling problem, including the early results published in 1970s. While we have made a meticulous effort to obtain all the relevant studies, we might have missed some less relevant papers.

We collect and review 255 papers in total. Eight papers were published before 1980, followed by a total number of 23 papers that were published in the 1980s. The next decade witnessed a large increase in the interest in the open-shop scheduling problem, with a total of 82 papers published in the 1990s. We also observe that 65 papers appeared in the 2000s and 77 papers in the 2010s. The trend shows that even though open-shop scheduling is a classical scheduling problem and that significant results were presented in the 1980s and 1990s, the problem's properties and the solutions are yet to be fully discovered, so there is still much interest in the problem. Figure 1 shows the number of papers published over time, from 1970 up to the present time. Table 1 reports the outlets that have published more than two papers on open-shop scheduling, where the entries are sorted in non-increasing order of number of published papers. The table shows that conference proceedings, *European Journal of Operational Research*, *Journal of Scheduling*, *Annals of Operations Research*, *Discrete Applied Mathematics*, *Computers & Operations Research*, and *Naval Research Logistics* have published almost 47% of the papers, suggesting that they are the ideal outlets for future research on the problem.

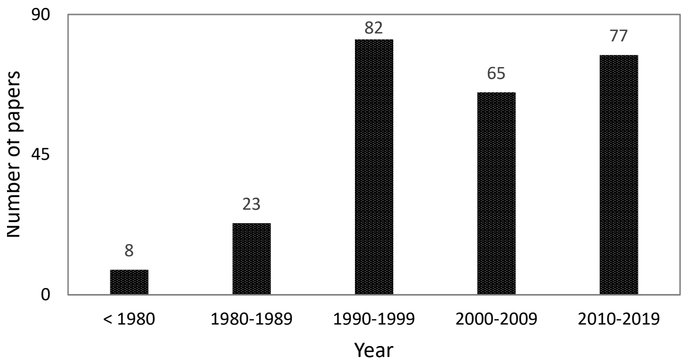

Figure 1: Distribution of the number of papers published in the area of open-shop scheduling in different time periods.

Table 1: Distribution of papers by the major publication outlets that have published more than two papers on the open-shop scheduling problem.

Outlet	Number of papers (%)
Proceedings	30 (11.76)
European Journal of Operational Research	25 (9.80)
Journal of Scheduling	15 (5.88)
Annals of Operations Research	14 (5.49)
Discrete Applied Mathematics	14 (5.49)
Computers & Operations Research	11 (4.31)
Naval Research Logistics	10 (3.92)
Operations Research	8 (3.14)
Operations Research Letters	8 (3.14)
Mathematics of Operations Research	6 (2.35)
The Journal of the Operational Research Society	5 (1.96)
IIE Transactions	4 (1.57)
INFOR: Information Systems and Operational Research	3 (1.18)
The International Journal of Advanced Manufacturing Technology	3 (1.18)
Mathematical Methods of Operations Research	3 (1.18)
Zeitschrift für Operations Research	3 (1.18)
Book chapters, reports, Ph.D. theses, and 65 other journals	93 (36.47)
Total	255 (100)

Next, we formally define the open-shop scheduling problem and introduce the major notation that we use throughout the paper.

1.3 Problem statement and notation

We define the "classical" open-shop scheduling problem as follows: There is a set $N = \{1, \ldots, n\}$ of jobs, each of which has a set of operations, to be processed on a set of different machines $M = \{1, \ldots, m\}$. The operation of job j on machine i is denoted by O_{ij} and its duration, i.e., its processing time, is p_{ij}. If job j is given a release time and a due date, we let r_j and d_j denote them, respectively. The order of processing the jobs on the machines is immaterial, i.e., it has no implication for the scheduling outcome. Each machine can process at most one operation at a time. The aim is to determine a schedule, i.e., the completion times of the job operations on the machines, such that a performance criterion is optimized. In this study we focus on minimizing the makespan, as well as on the performance criteria related to the makespan. It is clear that in the open-shop setting, one can swap the jobs and machines since they play equivalent roles.

It should be noted that there might be additional parameters that are defined in specific settings. We define all the major notation in Table 2.

We use the standard three-field notation $\alpha|\beta|\gamma$ introduced by Graham et al. (1979) for describing scheduling problems throughout the paper, where α, β, and γ represent the scheduling environment, the job characteristics, and the performance criterion, respectively. With respect to the field α, the open-shop scheduling problem is denoted by O. For two, three or a fixed number of machines m, we use $O2$, $O3$, and Om, respectively. However, if the number of machines is a part of the input, i.e., given, we use O. Regarding the field β, various characteristics have been considered in the existing literature that we will review throughout the paper. For example, if preemption of the operations is allowed, i.e., the execution of the operations can be interrupted and will be resumed or re-started later, it is shown by *prmp*. Considering the field γ, we confine to studies that minimize the makespan, i.e., the maximum completion time of the jobs. Let C_{ij} denote the completion time of job j on machine i. Then, the makespan is defined as $C_{max} = \max_{j \in N}\{C_j\}$, where $C_j = \max_{i \in M}\{C_{ij}\}$.

Table 3 lists the major abbreviations that we use throughout the paper. Next, we present several applications of the open-shop scheduling problem.

Table 2: Mathematical notation used throughout the paper.

Notation	Description
Set:	
N	Jobs, $N = \{1, \ldots, n\}$.
M	Machines, $M = \{1, \ldots, m\}$.
Parameter:	
n	Number of jobs.
m	Number of machines.
p_{ij}	Processing time of job j on machine i, $p_{ij} \in \mathbb{Z}^+$.
p_{max}	The longest processing time among all the operations.
p_{min}	The smallest processing time among all the operations.
$p(p_i)$	A common processing time (a common processing time on machine i).
d_j	Due date of job j.
r_j	Release time of job j.
e_j	Rejection cost of job j.
s_{ij}	Setup time of job j on machine i.
s_f	Setup time of batch f.
M_{max}	Maximal machine.
l_i	Load of machine i.
l_{max}	Maximum machine load.
$L(\bar{L})$	Exact (minimum) amount of delay.
α_{ij}	Basic processing time of job j on machine i (in the time-dependent setting).
β_{ij}	Deterioration rate of job j on machine i (in the time-dependent setting).

Table 3: Major abbreviations used throughout the paper sorted in alphabetical order.

Abbreviation	The complete term
ACO	Ant colony optimization
B&B	Branch-and-bound
BS	Beam search
CSP	Constraint satisfaction problem
FPTAS	Fully polynomial-time approximation scheme
GA	Genetic algorithm
PSO	Particle swarm optimization
PTAS	Polynomial-time approximation scheme
SA	Simulated annealing
TS	Tabu search
UET	Unit execution time
VNS	Variable neighbourhood search

1.4 Applications

One of the basic scheduling problems, open-shop scheduling has a broad range of applications across different sectors. In this section we discuss several applications of this scheduling model. For each application, we provide the descriptions of the jobs, operations, and machines in Table 4.

Consider a large automobile workshop with specialized shops and a set of automobiles that require different types of repairs, ranging from general services such as changing oil, rotating tires, and checking electrical parts to specialized services, e.g., painting. No two operations on an automobile can be executed simultaneously due to the different locations of the shops and the characteristics of the operations. It is also clear that the order of performing the operations by the mechanics is immaterial (Adiri and Amit, 1983; Kubzin et al., 2006).

In satellite communication, two or more earth stations may communicate by exchanging data packets. The transmitting earth station sends a packet to the satellite in a specified frequency, i.e., the uplink frequency. The packet is then converted into a different frequency, i.e., the downlink frequency, by the satellite repeater and is sent to the receiving station. The incoming packets from the earth stations are transmitted cyclically and each cycle, called a "frame", takes typically two milliseconds. The order of sending the packets is immaterial. An on-board $n \times n$ switch is used to connect n transmitting stations and n receiving ones. The time needed to transmit data from a transmitting station i to a receiving station j is denoted by p_{ij}, while transmitting (receiving) more than one packet at a time is prohibited. In addition, two stations cannot send a packet at the same time to the same repeater. The problem is then to schedule the transmission of all the data within a short cycle. A schedule is constructed by assigning packets to the repeaters and setting the transmission times. Therefore, p_{ij} is sent through different modes of an on-board switch and each time a fraction of it will be sent over. It is not hard to see that this problem is equivalent to the open-shop scheduling problem (Prins, 1994).

The timetabling problem can also be modelled and solved as the open-shop scheduling problem. de Werra et al. (2000); de Werra et al. (2002) discussed the timetabling issue arising in universities and educational centres, where there are group lectures (given by one teacher to a group of classes) and individual lectures (given by one teacher to one class). Group lecturing takes place in the basic programmes of the universities and schools. In this setting, a class is defined as a group of students following the same programme, and the set of classes is partitioned into a collection of groups such that each class belongs to exactly one group. However, there is a possibility that several groups contain exactly one class. In this context, machines represent the teachers and jobs denote the classes. The jobs are allowed to be preempted and must be processed within k time units. In the context of sport scheduling, Costa (1995) discussed the resemblance between the problem of national hockey league scheduling and the preemptive open-shop scheduling problem. Teams in the league can be split into two sets of M and N. Teams in M only play on the road, whereas teams in N always stay at home. Therefore, M and N may represent machines and jobs, respectively. A game is defined when a team from M matches up with a team from N. Other timetabling applications that can be modelled as the open-shop scheduling problem include scheduling a job fair (Bartholdi and McCroan, 1990), a trade fair (Ernst et al., 2003), school meetings (Rinaldi and Serafini, 2006) and workplace training (Czibula et al., 2016).

The airplane garage task scheduling is another application of the open-shop scheduling problem. Consider a fleet of airplanes. A set of operations need to be performed on each plane in order to prepare it for take-off. Each operation is performed by a technician (a machine) that has the expertise only for that operation. While some operations can be performed simultaneously on a plane, e.g., one technician checks the engine while the other technician inspects the wings, there are operations that cannot be performed concurrently, leading therefore to the classical open-shop setting (Grinshpoun et al., 2014; Grinshpoun et al., 2017). We note that it is called the concurrent open-shop if all the operations of a job can be performed in parallel and it is called the partially concurrent open-shop if only some of the operations of a job can be performed in parallel.

Askin et al. (1994) modelled the problem of assembling k types of printed circuit boards (PCB) as the open-shop scheduling problem. First, the electrical components to be placed on PCBs by machines are assigned to the machines. Then, the PCBs with the same requirements are grouped into families. The objectives include minimizing the makespan of the assembly time and minimizing the flow time. Daganzo (1989) studied the problem of crane scheduling to unload holds for a set of ships at berth. Considering each hold as a single-operation job and each crane as a machine, and assuming that two or more cranes are not allowed to work on a hold simultaneously, the problem can be modelled as classical open-shop scheduling. Another interesting application of the open-shop in this area includes the problem of scheduling chemical tankers' arrivals in ports. An arriving tanker must visit multiple terminals in a port in any order. The aim is to determine the order in which a tanker visits a terminal and the order in which a given terminal services the tankers (Cankaya et al., 2019). This is an important application. For example, Cankaya et al. (2019) pointed out that almost half of the traffic in the Houston ship channel is due to chemical tanker movements that carry liquid cargoes between multiple terminals.

The open-shop scheduling problem can be used to model a range of problems arising in the health care sector. Consider scheduling patients for diagnosing the coronary heart disease. A patient needs to undergo the diagnosis in three stages, namely blood testing, ultrasonic cardiogramming, and coronary computed tomography (CT) scanning, in any order. Each stage requires multiple facilities and/or medical personnel to conduct the diagnosis (Bai et al., 2016). Other problems in health care sector that have been modelled as the open-shop scheduling problem include the endoscopy scheduling problem (Fei et al., 2009), patient scheduling in emergency department laboratories (Azadeh et al., 2014), patient scheduling in medical clinics (Baron et al., 2016), and the rehabilitation scheduling problem (Zhao et al., 2018).

The open-shop scheduling problem is also applicable to the transport sector. For example, Chou and Lin (2007); Vincent et al. (2010) introduced the museum visitor routing problem. By treating the jobs and machines as visitor groups and exhibition rooms, respectively, the problem aims to ensure that each visitor group visits each exhibition room exactly once such that the time by which the last visitor group leaves the museum, i.e., the makespan, is minimized. They modelled the problem as open-shop scheduling in view of the fact that the order of the visits is immaterial. Brandinu and Trautmann (2014) discussed the event-bus scheduling problem that can be modelled as the open-shop problem. Given a number of buses carrying tourists that visit the sites of famous Bollywood movies, the problem's characteristics include parking at most one bus at any location at a time and not having a fixed route for the buses. Wirth and Emde (2018) showed that scheduling trucks on factory premises may be modelled as the open-shop scheduling problem, where a fleet of trucks must visit and deliver parts to a number of premises in a factory in any order. In addition, no two trucks can be handled simultaneously at any destination. In a recent attempt, Ahmadian and Kovalyov (2020) introduced the following application: There are n trucks each delivering a consolidated cargo to m destinations in any order. No two trucks can be handled simultaneously at any destination. The travel time is negligible compared with the service time at any destination.

Next, we introduce the classical open-shop scheduling problem under, e.g., various numbers of machines and preemption. We also review the major solution methods developed to deal with the classical open-shop scheduling problem in Section 2.

2 The classical open-shop scheduling problem

In this section we focus on the classical open-shop scheduling problem. We review variants of the problem in Sections 3 and 4. In the classical variant, we review the basic and fundamental characteristics of the open-shop scheduling problem, such as the number of machines and the preemption/non-preemption conditions. We also review the general solution methods, ranging from exact to heuristic approaches, that have been developed to deal with the open-shop scheduling problem. The solution methods aim to generate a schedule, i.e., determining the start times or completion times of the operations on the machines, that exactly or approximately optimizes the performance criterion.

2.1 The two-machine open-shop

The number of machines vastly impacts the computational complexity of the open-shop scheduling problem. For example, the two-machine problem to minimize the makespan is polynomially solvable (see, e.g., Gonzalez and Sahni, 1976) whereby the same problem under an arbitrary number of machines is strongly NP-hard. Therefore, in this section we review studies that consider the two-machine setting, and in the next section we review studies that consider an arbitrary number of machines.

As discussed, the well-known two-machine open-shop scheduling problem to minimize the makespan is polynomially solvable by the algorithm of Gonzalez and Sahni (1976), which runs in $O(n)$. Cheng and Shakhlevich (2007) stated that the algorithm of Gonzalez and Sahni (1976) constructs an optimal schedule such that each machine has at most one idle time in the interval $[0, C_{max}]$. Interestingly, Dell'Amico and Martello (1996), and Martello (2010) argued that the algorithm is indeed an implementation of the results of Egerváry (1931) published in the early 1930s.

Table 4: Details of the discussed applications of the open-shop scheduling problem.

Application	Job	Machine	Operation	Reference
Timetabling:				
Job fair meeting scheduling	Firms	Students	Meetings	Bartholdi and McCroan (1990)
National hockey league scheduling	Home games	Road games	Games	Costa (1995)
University educational programme scheduling	Teachers	Classes	Lectures	de Werra et al. (2000); de Werra et al. (2002)
Trade fair meeting scheduling	Buyers	Sellers	Meetings	Ernst et al. (2003)
School meeting scheduling	Parents	Teachers	Meetings	Rinaldi and Serafini (2006)
Workplace training scheduling	Apprentices	Practice placements	Trainings	Czibula et al. (2016)
Airplane garage task scheduling	Technicians	Airplanes	Checking or maintenance operations	Grinshpoun et al. (2014); Grinshpoun et al. (2017)
Satellite communication:				
Time slot assignment	A group of packets	Satellite repeaters	Sending packets	Inukai (1979); Prins (1994)
Health care management:				
Endoscopy scheduling	Patients	Endoscopy operating rooms	Gastroscopy and colonoscopy tests	Fei et al. (2009)
Laboratory scheduling	Patients	Place or staff in the laboratory	Medical tests	Azadeh et al. (2014)
Coronary heart disease diagnosis	Patients	Medical equipments	Medical checks	Bai et al. (2016)
Medical clinic scheduling	Patients	Diagnostic stations	Medical tests	Baron et al. (2016)
Rehabilitation scheduling	Patients	Therapists	Therapeutic processes	Zhao et al. (2018)
Transport:				
Crane scheduling	Holds	Cranes	Unloading holds	Daganzo (1989)
Truck scheduling	Trucks	Dock doors	Delivering parts	Cankaya et al. (2019)
Port traffic scheduling	Chemical tankers	Port terminals	Loading and unloading cargoes	Cankaya et al. (2019)
Tourism:				
Museum visitor routing	Visitor groups	Exhibit rooms	Visits	Chou and Lin (2007); Vincent et al. (2010)
Event-bus scheduling	Buses	Locations	Visits	Brandinu and Trautmann (2014)

There are other solution algorithms for the two-machine open-shop scheduling problem to minimize the makespan. One is the longest alternative processing time (LAPT) dispatching rule of Pinedo and Schrage (1982). Moreover, de Werra (1989)'s method partitions the jobs into three batches. An interesting characteristic of the method lies in the fact that the jobs of a batch may have their processing order changed without impacting the optimal makespan. Soper (2015) generalized the method to the case with more than three batches. The generalization solves the two-machine n-job open-shop problem through solving the two-machine flow-shop problem with $n-1$ jobs. The one "omitted" job is then added to the constructed flow-shop schedule, where it is first processed on the second machine before processing all the other jobs, and then on the first machine after processing all the other jobs.

Shakhlevich and Strusevich (1993) presented an $O(n)$-time algorithm for solving the two-machine open-shop problem with arbitrary regular penalty functions associated with "machine usage", i.e., the penalty is a function of the length of the time during which the machines operate. Van Den Akker et al. (2003) assumed that there are given time windows for the machines' availability. By using the algorithm of Shakhlevich and Strusevich (1993), they investigated the existence of a feasible schedule such that all the jobs can be completed before k_1 and k_2, where $[0, k_1]$ and $[0, k_2]$ are the time windows during which machines 1 and 2 are available. It is evident that their study aims to verify whether the given time windows can lead to a feasible schedule. It is also evident that the case of $k_1 = k_2$ leads to makespan minimization. Next, we review studies that focus on an arbitrary number of machines.

2.2 The m-machine open-shop

In reviewing studies on the classical m-machine open-shop scheduling problem, we first consider the studies that focus on the complexity analysis of the problem. Then we report the papers that study the problem with the so-called "dense" schedules, followed by works that aim at reducing the feasible search space.

Complexity

Contrary to the case with two machines, the open-shop scheduling problem with an arbitrary number of machines, i.e., $m \geq 3$, to minimize the makespan is strongly NP-hard (see Gonzalez, 1982; Lawler et al., 1993). However, if the number of machines is fixed, i.e., it is not part of the input, and the problem instance includes at least one job with three operations, it is ordinary NP-hard (Gonzalez and Sahni, 1976). Another ordinary NP-hard case includes the four-machine case with two operations per job, i.e., each job is processed on only two machines (Gonzalez and Sahni, 1976). In general, the complexity of the three-machine problem is still open if each job has exactly two operations. Sevast'yanov and Woeginger (1998) showed that when m is fixed, there exists a $(1+\varepsilon)$-approximation scheme for the problem that is polynomial in the size of the instance, but exponential in m and $1/\varepsilon$. Williamson et al. (1997) showed that no polynomial-time ρ-approximation algorithm, $\rho < \frac{5}{4}$, exists for problem $O||C_{max}$ unless $P = NP$. It has also been conjectured that the optimal solution for $O||C_{max}$ is at most $\frac{3}{2}$ larger than that of $O|prmp|C_{max}$ (Schuurman and Woeginger, 1999). Brucker et al. (2007) showed that problems $O|n=2|C_{max}$ and $O|n=k, prmp|C_{max}$ are polynomially solvable, whereas problem $O|n=3|C_{max}$ is NP-hard. On the other hand, Adiri and Amit (1984) proposed an $O(mn)$-time two-phase algorithm for open-shop scheduling with unit processing (execution) times (UETs) to minimize the makespan and total completion time i.e., $O|UET|(\sum C_j, C_{max})$.

It is known, due to Bárány and Fiala (1982), that the makespan of a solution obtained by any list scheduling heuristic is at most two times greater than the optimal makespan (because Bárány and Fiala (1982) is in Hungarian, we refer the interested reader to Shmoys et al. (1994); Lenstra and Shmoys (1995)). The results also holds when each job has a release time r_j at which it becomes available for processing (Wein, 1991). Certain studies apply the mass polynomial-time reduction from the UET open-shop to the parallel-machine case, and vice versa, in order to determine the complexity of a large number of open-shop scheduling problems. For example, Brucker et al. (1993) showed that the complexity of solving the m-machine open-shop scheduling problem with UET operations is $O(k + n^2 m)$ or $O(k + nm(\log nm)^2)$ if the corresponding parallel-machine problem can be solved in $O(k)$. Their proposed algorithm transforms the problem to an m identical parallel machines under the condition that all the problem's parameters, including the start times, completion times, preemption times, and re-start times of the jobs only take integer values. While the overall complexity of their algorithm is $O(n^2m)$, it can be improved to $O(nm(\log nm)^2)$ by using edge colouring algorithm of Gabow and Kariv (1982) in bipartite graph.

Using a specific matrix of "latin rectangle" (a latin rectangle $LR[n, m, k]$ is a matrix in the format $[n \times m]$ with the entries taken from the set $\{1, \ldots, k\}$, where every entry occurs at most once in every row and column) to present a schedule, Tautenhahn (1994) proposed a polynomial algorithm for the open-shop problem in which every operation of a job has a UET and must be completed by its given due date. They proposed an $O(n^2m)$ algorithm that delivers the optimal schedule for problems $O|UET, C_j \leq d_j|C_{max}$ and $O|UET, C_j \leq d_j|\sum C_j$, and showed that there is a common optimal schedule for both problems. In addition, they used the same algorithm to solve the problem to minimize the number of tardy jobs and the maximum lateness, and improved the earlier result of Liu and Bulfin (1988). Timkovsky (2003) proposed two mass reductions, namely the "open-shopping" and the "open-shop paralleling", which can be applied for any criterion except the total completion time. The former maps the preemptive identical parallel-machine problem with arbitrary jobs' processing times to the UET open-shop problem, and the latter reduces the UET open-shop problem to the non-preemptive identical parallel-machine problem with UET jobs.

Impagliazzo and Paturi (2001) introduced the exponential time hypothesis (ETH) as follows: The 3-SAT (a very strict form of the satisfiability problem) with the parameter n, where n is the number of variables, has no sub-exponential algorithm. That is, 3-SAT cannot be solved in $2^{O(n)}$ time. This conjecture is important because one may use it to derive lower bounds on the running times of algorithms for other combinatorial or computational problems. For example, Jansen et al. (2016) showed that $O3||C_{max}$ cannot be decided in $2^{O(n)}$ time and $O||C_{max}$ does not admit any ρ-approximation algorithm for any $\rho \leq \frac{5}{4}$ in $2^{O(n)}$ time unless ETH is invalid. Sevastianov (2005) presented a 4-parameter complexity analysis of the open-shop scheduling problem. In this approach, instead of establishing the complexity result for a problem based on a single parameter, say, the number of machines or jobs, a set of important parameters are considered and the complexity analysis of sub-problems can be studied. A sub-problem is characterized by each combination of the constraints. That study investigated a complete basis system of the sub-problems of the open-shop with the four parameters of number of jobs, maximum number of operations per job, number of machines, and maximum number of operations on a machine. Using a basis system, one can determine the complexity of any other sub-problems. Kononov et al. (2012) studied the complexity of an infinite class of shop scheduling problems, including the open-shop, with combinations of constraints imposed on the processing times of the operations and the maximum number of operations per job, and an upper bound on the length of the schedule. They presented a finite basis system of sub-problems for these problems that includes ten problems, five of which are polynomially solvable, and the other five are NP-complete. They showed that the problem of deciding the existence of a schedule with a length of at most four for the open-shop with processing times one and two, with at most two operations per job and at most three operations per machine, i.e., $O|p_{ij} \in \{1, 2\}, k_1 \leq 2, k_2 \leq 3|C_{max} \leq 4$, is NP-complete, where k_1 and k_2 represent the maximum number of operations per job and per machine, respectively.

Dense schedules

The open-shop with the "dense" schedule occurs when any machine is idle if there is no job for processing on that machine. In other words, the machine is not kept idle if there is a job that is ready to be processed on the machine (Bárány and Fiala, 1982). Aksjonov (1988) and Shmoys et al. (1994) showed that the worst-case performance ratio for any dense open-shop schedule is 2, since the makespan of a dense schedule cannot be more than the maximum of jobs' durations plus the maximum of machines' loads.

For the m-machine open-shop scheduling problem, Bárány and Fiala (1982) proved that any dense schedule is at most $(m-1)p_{max}$ larger than the optimal solution, where $m \geq 1$ and p_{max} is the longest processing time among all the operations, and that the bound is tight. It should be noted that a general conjecture on the worst-case ratio of the dense schedule is $2-(\frac{1}{m})$ due to Chen and Strusevich (1993). They proved the conjecture for $m = 3$ and proposed two approximation algorithms. The first algorithm is a greedy one that has an approximation bound equal to $\frac{5}{3}$ of the optimal schedule. The second algorithm improves the bound to $\frac{3}{2}$, through reducing the idle time on the third machine and by moving the jobs (other than the last two jobs) on the first two machines to the end of the schedule. Sevast'yanov and Tchernykh (1998) later provided an $O(n)$ algorithm to improve the bound to $\frac{4}{3}$, where they showed that any instance with n jobs can be transformed to an instance with five "aggregated" jobs. The conjecture on the worst-case ratio for the dense schedule has not yet been proved for arbitrary m. Nonetheless, Chen et al. (2012) presented certain types of dense schedules under which the conjecture is always true. Also, they proved the conjecture for $m = 7, 8$. We note that Chen and Yu (2000); Chen and Yu (2001); Chen and Yu (2003) earlier proved the cases where $m = 4, 5, 6$.

Reducing the feasible space

A few studies attempt to decrease the size of the feasible space, i.e., the number of feasible schedules, through identifying subsets of feasible schedules that contain the optimal schedules. Two such subsets include the dense schedules and the so-called "rank-minimal" schedules discussed in Bräsel et al. (1993). If we present the problem by a rank matrix, the rank-minimal schedules are those that the largest value of the rank for all the operations is minimal and is equal to $\max\{n, m\}$. Bräsel and Kleinau (1992) established a one-on-one mapping between the feasible schedules of $O|UET|C_{max}$ and a special latin rectangle called "plan" (here, for every entry $a_{ij} > 1$, there exists element a_{ij} in row i or in column j), and determined the numbers of optimal solutions for $O2|UET|C_{max}$ and $O3|UET|C_{max}$. For small n and m, they proposed an enumeration algorithm that delivers the exact number of feasible (irreducible) schedules for $O||C_{max}$. Bräsel and Kleinau (1996) showed that the set of irreducible sequences contains at least one optimal sequence. They also showed that the active (a schedule is active if no operation can be started earlier without delaying another operation), rank-minimal, and feasible sets are neither disjoint nor equivalent. Later, Bräsel et al. (1999b) and Bräsel et al. (1999a) presented necessary and sufficient conditions for feasibility that can be verified in polynomial time, and developed enumeration algorithms to identify the set of feasible sequences. Andresen and Dhamala (2012) attempted to answer the open question of whether a given open-shop sequence is feasible. They proposed three conjectures that if any of them holds, the given open-shop sequence can be polynomially verified to be feasible. Nevertheless, the problem remains open in general. For a given sequence and under certain assumptions, the conjectures can be verified in $O(n^9m^9)$ and $O(n^5m^52^{n^2m^2})$ time, in order to either find a reducible sequence or show that the sequence is feasible. Next, we review the major results on the preemptive open-shop scheduling problem.

2.3 The preemptive open-shop

In scheduling theory, preemption refers to the situation where the execution of a job or operation can be interrupted so that another job or operation is executed. Depending on the setting, the preempted job may be resumed or re-started. Minimizing the makespan in the two-machine open-shop scheduling problem leads to identical optimal makespan for both the preemptive and non-preemptive variants (Gonzalez and Sahni, 1976). In addition, when the number of machines is more than two, i.e., $m > 2$, the optimal makespan for the preemptive variant can be obtained by either of the two polynomial algorithms by Gonzalez and Sahni (1976), which have the time complexity of $O(k^2)$ and $O(k(\min\{k, m^2\} + m \log n))$, where k is the number of non-zero operations (with non-zero processing time). Gabow and Kariv (1982) later improved the complexity of those algorithms for certain special cases. A similar polynomial algorithm to that of Gonzalez and Sahni (1976) was independently proposed for the preemptive m-machine open-shop by Inukai (1979). Lawler and Labetoulle (1978) proposed a linear program to solve the preemptive open-shop scheduling problem.

Assuming at most $\min\{kn, km, n^3, m^3\}$ preemptions, i.e., an upper bound on the number of preemptions, Gonzalez (1979) showed that the problem can be solved in $O(k + \min\{m^4, n^4, k^2\})$ time. However, if there is a lower bound on the number of preemptions, the problem is NP-hard (Gonzalez and Sahni, 1976). Baptiste et al. (2011) proved the integer preemption property for the two-machine problem. Under that property, if the input data for an instance are integral, then there exists an optimal preemptive schedule where all the interruptions, and start and completion times occur at integral times. Shchepin and Vakhania (2008; 2011; 2015) attempted to minimize the number of preemptions for the problem, where the associate graph is acyclic, such that the problem remains polynomially solvable. They proposed an $O(nm)$-time algorithm if at most $m-2$ preemptions are allowed, and showed that the problem with at most $m-3$ preemptions, i.e., $O|acyclic, (m-3) - prmp|C_{max}$, is NP-hard. They showed that the 2-approximation algorithm for the classical open-shop problem also holds for the non-preemptive acyclic problem, i.e., where the associate graph is acyclic.

Due to the absence of processing routes among the operations in the open-shop setting, it is possible to interrupt a job on a machine and resume it on another one, although such a feature might be undesirable for some real-world applications. Therefore, the case where the interrupted job must wait because it cannot be processed on another machine has also been studied. We may therefore distinguish the machine-preemption-only and the job-preemption-only cases. In the former, interrupted job j on machine i cannot be transferred to other machines, while machine i can process other jobs during the interruption of job j. In the latter, interrupted job j on machine i can be processed on other machines, but machine i cannot process other jobs until job j resumes its processing. We note that due to symmetry of the jobs and machines in the open-shop scheduling environment, both cases, i.e., machine-preemption-only and job-preemption-only, are equivalent. The case of machine-preemption-only is also known as "open-shop with no passing" (Cho and Sahni, 1981) and "open-shop with restricted preemptions" (de Werra and Solot, 1993). Cho and Sahni (1981) showed that finding a feasible schedule for the open-shop with no passing with the common release time and due date of 1, i.e., $O|prmp, r_j = 1, d_j = 1|-$, is NP-hard for $m > 2$. Later, de Werra and Solot (1993) studied $O|prmp|-$ with restricted preemption and showed that obtaining a feasible schedule is NP-hard. de Werra and Erschler (1996) studied restricted preemption and modelled it as a variant of the colouring problem (the graph colouring problem concerns using the smallest number of distinct colours such that no two adjacent edges have the same colour) and proved that the decision problem is NP-complete.

Bräsel and Shakhlevich (1998) proved that the preemptive UET open-shop problem is equivalent to the preemptive identical parallel-machine problem with UET operations. They also discussed that while the number of dense schedules for the preemptive problem is uncountable, i.e., infinite, it is finite for the non-preemptive counterpart. Baptiste et al. (2009) proved a number of properties for the feasible and optimal preemptive schedules for the open-shop problem. For example, they showed that contrary to the results in Gonzalez and Sahni (1976), the maximum number of partial schedules ("slices") in an optimal solution of $O|prmp|C_{max}$, $m \leq n$, can be reduced to $k + m$, where k is the total number of operations (of all the jobs). de Werra et al. (1996) considered the decision problem of scheduling the preemptive open-shop within k time units, in which some operations are pre-assigned to machines at some point. They introduced a few polynomially solvable cases.

In the next section we focus on the open-shop scheduling problem where the jobs or machines follow specific structures.

2.4 The structured open-shop

As discussed earlier, the open-shop scheduling problem is polynomially solvable only for a few variants and under very restricted conditions, see Sections 2.1 and 2.3. It follows that certain structures and properties of those variants are important for solving the problem. Therefore, there is a stream of research that aims to identify special structures for the input data so the problem is polynomially solvable. Five such important special structures have been studied in the literature, namely (1) "machine load", (2) "dominating machine", (3) "bottleneck machine", (4) "proportionate scheduling", and (5) "ordered scheduling". We discuss these structures in the following.

Machine load

By establishing a relationship between the maximum machine load and the maximum operation length, Fiala (1983) showed that an optimal schedule of length l_{max} can be constructed for $O||C_{max}$ in $O(n^2m^3)$ time, if $l_{max} \geq (16m' \log_2 m' + 5m')p_{max}$, where $m' = 2^{\lceil \log_2 m \rceil}$ and p_{max} is the longest processing time among all the operations. Bárány and Fiala (1982) improved l_{max}: If $l_{max} \geq (16m \log_2 m + 26m)p_{max}$, the optimal schedule can be constructed in $O(nm^3)$ time (see Sevast'yanov (1992) for the details). A series of improvements was later proposed. For example, if $l_{max} \geq (\frac{16}{3} m \log_2 m'' + \frac{13}{9} m - \frac{4m}{9m''}(-1)^k)p_{max}$, where $m'' = 2^k$ and $k = \log_2 m$, the optimal schedule is obtained in $O(nm^2 \log_2 m)$ time (Sevast'yanov, 1992), and if $l_{max} \geq (m^2 - 1 + \frac{1}{m-1})p_{max}$, the optimal schedule of length l_{max} can be obtained in $O(n^2m^2)$ time (Sevast'yanov, 1995). If all the processing times are selected from a bounded set of non-negative integers, the $O(nm)$ algorithm by Čepek et al. (1994) solves the restricted case considered by Fiala (1983). If $l_{max} \geq 7p_{max}$, the algorithm by Sevast'yanov (1998) (called the "non-strict vector summation") solves the three-machine open-shop problem in $O(n \log n)$ time.

Dominating machine

Machine i' dominates machine i, if the minimum processing time on machine i' is at least as large as the longest processing time on machine i, i.e., $\min_j\{p_{i'j}\} \geq \max_j\{p_{ij}\}$. Adiri and Aizikowitz (1989) investigated the three-machine problem in the presence of a dominating machine and $n \geq m$. They showed that that case can be solved in $O(n)$ time (so can its flow-shop counterpart). To solve the problem, one may disregard the dominated machine and solve the resulting two-machine problem by using the algorithm by Gonzalez and Sahni (1976). Then, the jobs on the dominated machine can be scheduled without conflicts. Sevast'yanov (1996) introduced two polynomially solvable cases of $O||C_{max}$. He showed that for the open-shop with dominant machine i' of load $l_{i'} = l_{max}$ and $\Delta = l_{max} - l_i \geq (2m-4)p_{max}, i \neq i'$, there exists an optimal schedule of length l_{max} that can be constructed in $O(nm^2)$ time. In addition, if $l_{max} \geq (5.45m - 7)p_{max}$ and $\Delta \geq (m-1)p_{max}$, then a greedy algorithm always exists that delivers an optimal schedule in $O(n^2m^2)$ time. He also presented an approximation algorithm for the problem satisfying $l_{max} \geq (m')p_{max}$, where m' is a function of the number of machines. The algorithm has a running time of $O(nm^2)$ with an absolute performance guarantee of $(\lceil \frac{3m^2-2m}{l_{max}+2mp_{max}} \rceil - 1)p_{max}$. Assuming that $p_{max} = 1$, all the processing times are in the range $[0,1]$ and the machines are numbered in non-decreasing order of their loads, Kononov et al. (1999) showed that the optimal schedule for problem $O3||C_{max}$ can be constructed in $O(n)$ time, if $l_1 = l_2 \geq l_3 + 2$. They also gave the conditions under which the optimal schedule can be obtained polynomially for some cases where $m \geq 4$. For the same problem, Tanaev et al. (2012) also presented an $O(nm)$ algorithm. It should be noted that the problem with a dominating machine, but without the condition of $n \geq m$, is NP-hard.

Bottleneck machine

The bottleneck machine refers to the case where a job is processed on a subset of machines, rather than on all the machines. Here, a machine is a bottleneck if one of the operations of every job needs to be processed on that machine. Gonzalez and Sahni (1976) showed that the four-machine problem is NP-hard even for two-operation jobs, i.e., each job has exactly two operations. This NP-hardness result also holds if one of the machines is a bottleneck. For the three-machine and two-operation open-shop problem with a bottleneck machine, Drobouchevitch and Strusevich (1999) presented a linear-time solution algorithm, although the flow-shop counterpart of the problem is strongly NP-hard (see Herrmann and Lee, 1992). Kyparisis and Koulamas (2000) investigated the m-machine case of the same problem and showed that it can be solved in $O(n + m \log m)$ time if the bottleneck machine i' is also the maximal machine, i.e., $p_{i'j} \geq p_{ij}, \forall j \in N, \forall i \in M \setminus \{i'\}$.

Proportionate scheduling

The proportionate open-shop scheduling problem is characterized by the fact that all the jobs have the same processing time on a given machine, i.e., $p_{ij} = p_i, \forall i \in M$. The first study in this domain is due to Dror (1992). Under the assumption that $p_1 \geq p_2 \geq \cdots \geq p_m$, the results of Dror (1992) include (1) an optimal $O(mn)$ algorithm for $O|prop, n \geq m|C_{max}$, where *prop* denotes the proportionate setting, (2) an optimal $O(m)$ algorithm for $O|prop, n = 2, m > 2|C_{max}$, and (3) an NP-hardness proof for $O|prop, n \geq 3, n < m|C_{max}$. In the three-machine setting, Koulamas and Kyparisis (2015) proved that if $p_1 = p_2$, the problem is polynomially solvable, and remains polynomially solvable even if $p_1 > p_2$ and $l_{max} \leq 2p_1 + p_2$, or if $p_1 > p_2$ and $l_{max} \geq 3p_1 + p_3$. Under the general condition, they proposed an $O(n \log n)$-time $\frac{7}{6}$-approximation algorithm. Naderi et al. (2014) studied problem $O|prop|C_{max}$ and proposed a polynomial algorithm to solve the case where $n \geq m$, and for $m > n$, presented an approximation algorithm with a worst-case performance bound of $2 - \frac{1}{n}$, and provided a heuristic algorithm by converting the problem into a simpler problem called the "machine batch fitting" problem.

Ordered scheduling

The ordered assumption is indeed a generalization of the proportionate case. We call two machines $i, i' \in M, i \neq i'$ ordered and denote it as $M_i > M_{i'}$, if $p_{ij} \geq p_{i'j}, \forall j \in N$. Analogously, jobs $j, j' \in N, j \neq j'$ are ordered $(j > j')$ if the processing time of j is larger than that of j', if $p_{ij} \geq p_{ij'}, \forall i \in M$ (Khatami et al., 2019; Khatami and Salehipour, 2020). An open-shop problem is ordered if it includes both ordered machines and ordered jobs.

Liu and Bulfin (1987) showed that the three-machine ordered open-shop problem, i.e., $O3|ord|C_{max}$, where *ord* denotes the ordered setting, is NP-hard, implying that problems $O3|j > j'|C_{max}$ (with ordered jobs only) and $O3|M_i > M_{i'}|C_{max}$ (with ordered machines only) are NP-hard. They proposed an $O(n)$ algorithm for problem $O3|M_i > M_{i'}|C_{max}$ with the additional constraint that the job with the longest processing time on the first machine is different from the job with the longest processing time on the second machine, i.e., the longest jobs on the machines are different. Kyparisis and Koulamas (1997) proposed an $O(n)$ algorithm for problem $O|M_1 > M_i, m \leq n|C_{max}$ when the ith longest processing time on machine 1 is as large as the processing times of all the operations on machines i through m. They also generalized the results of Liu and Bulfin (1987) for the three-machine open-shop with ordered machines.

2.5 Solution methods

Various solution techniques, including exact, heuristic and meta-heuristic have been proposed for challenging and more general variants of the open-shop scheduling problem. In this section, we first review the characteristics of the well-known benchmark instances for the open-shop, and then we review the available solution techniques.

Benchmark instances

Three well-known sets of benchmark instances are available for the open-shop scheduling problem. These three sets (and sometimes extended variants of them) have been utilized by many researchers for assessing the performance of the algorithms and the solution techniques. The benchmark instances are due to Taillard (1993), Brucker et al. (1997), and Guéret and Prins (1999).

Taillard (1993) introduced the first set of benchmark instances for the open-shop scheduling problem, in which an instance is characterized by the pair (n, m) and consists of six different sizes as follows: $(4,4)$, $(5,5)$, $(7,7)$, $(10,10)$, $(15,15)$, and $(20,20)$, where the processing times are randomly generated from the discrete uniform distribution in the range $U[1,99]$. For each size, they generated ten instances that resulted in a total of 60 instances. This set of instances is considered easy because the trivial lower bound LB_0 (see below) is equal to the optimal makespan for 40 of the larger instances (Malapert et al., 2012). The trivial lower bound on the makespan can be calculated as the maximum of jobs' durations and machines' loads, denoted as LB_0, as follows:

$$LB_0 = \max\{\max_j\{\sum_i p_{ij}\}, \max_i\{\sum_j p_{ij}\}\}. \tag{1}$$

The first term in the lower bound represents the longest job duration and the second term defines the maximum machine load l_{max}, i.e., $l_{max} = \max\{l_i\}$, where $l_i = \sum_j p_{ij}$ represents the load of machine i. It should be noted that the first non-trivial lower bound for the open-shop scheduling problem is due to Guéret and Prins (1999). The lower bound is calculated by determining the optimal makespan for a relaxed version of the problem. We refer the interested reader to Guéret and Prins (1999) for the details.

Brucker et al. (1997) proposed a few measures to capture the hardness of an instance and applied them to Taillard (1993)'s instances. Since they observed that Taillard (1993)'s instances are "easy" to solve, they generated their own instances. They proposed a set of 52 challenging instances. Similar to the instances of Taillard (1993), they considered an equal number of jobs and machines to generate the instances. They proposed eight instances for $n = m = 3, 8$ and nine instances for $n = m = 4, \ldots, 7$. The third benchmark is that of Guéret and Prins (1999), in which there are ten instances for each size of $n = m = 3, \ldots, 10$, leading to 80 instances. It should be noted that the benchmarks proposed by Brucker et al. (1997), and Guéret and Prins (1999) are more challenging than those of Taillard (1993) because the advanced GA by Prins (2000), ACO by Blum (2005), and PSO by Sha and Hsu (2008) struggle to optimally solve all the instances of Brucker et al. (1997) and Guéret and Prins (1999). We refer the interested reader to Malapert et al. (2012) for the details on the performance of exact and heuristic methods in solving the instances of Taillard (1993), Brucker et al. (1997), and Guéret and Prins (1999).

Next, we review the available exact methods, followed by the heuristic and meta-heuristic algorithms that were proposed to solve the general open-shop scheduling problem.

2.5.1 Exact algorithms

The open-shop scheduling problem typically has a larger solution space compared with, e.g., the job-shop and the flow-shop problems, due to its unrestricted job processing order. The free job route also leads to a smaller gap between the optimal makespan and a lower bound (Prins, 2000; Guéret and Prins, 1999). The major exact solution methods to solve the open-shop scheduling problems include the branch-and-bound (B&B) algorithm and the constraint programming technique.

Branch-and-bound based algorithms

Brucker et al. (1997) and Brucker et al. (1999) proposed the first B&B algorithms for the problem. Brucker et al. (1997) used the ideas from Grabowski et al. (1986) for branching and from Carlier and Pinson (1989) for immediate selection of operations and developed a depth-first B&B for the open-shop problem based on a disjunctive graph formulation of the problem. They tested the algorithm on the instances of Taillard (1993). In Brucker et al. (1999), they first reduced an instance of the open-shop to a dedicated parallel-machine scheduling problem and then to a single-machine scheduling problem with positive and negative time-lags. Positive and negative time-lags are used as general timing constraints between the start times of the jobs. They concluded that such a transformation technique is not as efficient as solving the problem directly, i.e., without applying the transformation. Guéret and Prins (1998a) attempted to improve the performance of the B&B method by Brucker et al. (1997). Their proposed technique identifies certain forbidden intervals for the start and completion times of the operations, during which no operation can start or end in an optimal solution. For this reason, $n + m$ subset-sum problems are solved by an efficient dynamic program. Each subset-sum problem corresponds to either a job or a machine. They solved 40 instances of the benchmark of Taillard (1993) and obtained reductions in the number of backtracks for 31 instances. This is equivalent to more than 75% improvement. They also reported improved solutions for certain instances. Guéret et al. (2000) also proposed an advanced backtracking scheme and combined it with the B&B by Brucker et al. (1997). They showed that it reduces the number of backtracks. Dorndorf et al. (2001) designed several consistency tests that reduce the search space of their B&B algorithm. They used the two strategies of "top-down" and "bottom-up" to guide the search. These strategies only differ in the way in which the initial upper bound is chosen. The top-down strategy starts with an upper bound and attempts to improve it by applying the B&B algorithm. In the bottom-up approach, however, a lower bound is selected as a target upper bound and it is incremented until a feasible solution is found. They showed that the proposed algorithm outperforms those of Brucker et al. (1997) and Guéret et al. (2000).

The solution technique by Tamura et al. (2009) includes incorporating the constraint satisfaction problem (CSP) with integer linear constraints into the boolean satisfiability problem (SAT). CSP is defined by a set of variables and a set of constraints where each variable has a domain of possible values and each constraint specifies the allowed combinations of values over a subset of variables. Their method is similar to that of Crawford and Baker (1994) for the job-shop scheduling problem. They showed that the method is able to find the optimal solutions for all 192 instances of the three benchmarks of Taillard (1993), Brucker et al. (1997), and Guéret and Prins (1999), including the three instances of Brucker et al. (1997) that had not been optimally solved earlier.

Constraint programming based techniques

Laborie (2005) proposed a constraint programming method based on detection and resolution of the minimal critical set (MCS). The minimal critical set specifies the minimum requirement for a resource R that would be over-consumed if executed simultaneously. MSCs are chosen in such a way that the size of the search space is minimized. In the disjunctive scheduling context, MCSs are pairs of activities that conflict for the same unary resource. At each node, the branching consists of (1) selecting an MCS according to an estimation of the related reduction of the search space, (2) applying a simplification procedure on each MCS, and (3) branching on its possible precedence in the children nodes until no MCS remains. They managed to solve the 34 and then open instances of Guéret and Prins (1999), and Brucker et al. (1997), respectively.

The constraint programming method by Grimes et al. (2009) sets the makespan equal to $\frac{LB+UB}{2}$ and iteratively solves a feasibility problem until the interval is empty. They optimally solved all the instances of the three benchmarks in reasonable times. The constraint programming algorithm by Malapert et al. (2012) combines randomization with the re-start techniques, and applied efficient propagation and scheduling heuristics. The algorithm not only generates all the known optimal solutions, it also outperforms all the previous methods in terms of the computational effort.

2.5.2 Heuristic and meta-heuristic algorithms

In this section we review some heuristic algorithms developed to tackle the open-shop scheduling problem.

Bräsel et al. (1993) proposed two heuristic algorithms for the m-machine open-shop scheduling problem. Their first algorithm finds the rank-minimal schedules by solving the weighted bipartite maximum cardinality matching problem. Their second algorithm, denoted as "insertion", is indeed a restricted B&B algorithm with $O(n^2m^2)$ time complexity that utilizes the beam search procedure. They showed that their insertion algorithm performs better than the tabu search by Taillard (1993) because it obtains improved solutions for almost all the instances of Taillard (1993). Vestjens et al. (2007) showed that the problem of inserting an additional job into a given schedule of n jobs such that the order of the n jobs in the given schedule does not change and the resulting makespan is as small as possible is strongly NP-hard.

Ramudhin and Marier (1996) generalized the shifting bottleneck procedure (SBP) originally proposed for the job-shop scheduling problem (Adams et al., 1988). Their procedure consists of iteratively selecting a bottleneck job or machine, and performing re-optimization of its execution sequence. The first heuristic of Guéret and Prins (1998b) is a list scheduling algorithm with $O(n^2m^2)$ time complexity. They used two priority rules based on the residual work durations of the operations, similar to those of Prins and Carlier (1986), in order to select the operation to be scheduled at the earliest time t. Indeed, time t is the smallest time at which the operation may be started. Time t is computed during the list scheduling algorithm. Their second heuristic includes two phases. The first phase partitions the operations into subsets by computing each subset as a matching in a weighted bipartite graph. For this reason, a weighted bipartite graph $G(X \cup Y, E, P)$ is constructed, in which each machine i and job j correspond to a vertex in X and a vertex in Y, respectively. Each operation O_{ij} is represented by an edge in E with the associated weight $p_{ij} \in P$. Next successive matchings are extracted from G until G is decomposed, i.e., all the vertices have the zero degree. Each matching specifies a subset of operations that can be performed simultaneously and therefore form a schedule slice. The resulting schedule slices are then concatenated in the order of generation to make a complete schedule (similar to Brucker et al., 1997). The second phase improves the schedule obtained in the first phase.

The iterative improvement procedure of Liaw (1998) generates an initial solution by applying a heuristic that uses the longest total remaining processing on the other machines (LTRPOM) dispatching rule by Pinedo (1995). The improvement method of the procedure separates the sequencing and scheduling problems. The sequencing problem is solved via an iterative procedure that is based on Benders' decomposition, and the scheduling part is shown to be the dual of the longest path problem that can be efficiently solved with a label correcting algorithm. They showed that the procedure is able solve most of their own randomly generated instances within 1% deviation from the trivial lower bound LB_0. The greedy algorithm by Strusevich (1998) for $Om||C_{max}$ generates dense schedules and has an $O(m \log m + nm \min\{n, m\})$ running time. It has been conjectured that the algorithm guarantees the worst-case ratio of $2 - \frac{2}{(m+1)}$ for any m. It has also been shown that the algorithm obtains a schedule at most $\frac{3}{2}$ times of the optimal one and the bound is tight.

Naderi et al. (2010)'s heuristic algorithms remove redundant solutions, which are generated as a result of some encoding methods. For this reason, they proposed four rules. Indeed, these rules lead to four heuristics, all of which apply the insertion operator. They showed that their heuristics have superior performance to the existing algorithms of the longest total processing time (a generalization of the LAPT rule by Pinedo and Schrage (1982) for the two-machine problem), the method by Liaw (1998), and the generation of active and non-delay schedules based on the SPT rule. The so-called rotation schedule by Bai and Tang (2011) is a heuristic that first schedules all the jobs on the first machine in the order 1 to n. Then, on any machine $2 \le i \le m$, the schedule starts with job j to n, and then jobs 1 to $j-1$ are added. The algorithm has several theoretical advantages. First, it is an asymptotically optimal method when the number of jobs n goes to infinity. Second, its worst case ratio is equal to the number of machines m.

The major meta-heuristic methods proposed for the open-shop include the evolutionary algorithms. The first such method is the genetic algorithm (GA) by Fang et al. (1993). Later, Fang et al. (1994) used a simple heuristic rule within GA, chosen from a pool of eight heuristic rules, as well as adaptively choosing the dispatching rules during the course of the algorithm, and showed that the adaptive strategy leads to new best solutions. Several strategies have been investigated to improve the performance of GA. For example, the GA by Louis and Xu (1996) uses the knowledge gained from solving the problem, by storing the past moves, and that by Khuri and Miryala (1999) incorporates the LPT dispatching rule to generate quality solutions. Prins (2000) introduced advanced crossover operators and schedule generators that produce non-delay and active schedules. Other studies to improve the performance of GA include Puente et al. (2003), in which the initial population is seeded by the probabilistic version of the dispatching rules by Liaw (1998), and Senthilkumar and Shahabudeen (2006), who used an operation-based representation and a scheduling algorithm that generates the active schedules. Recently, Rahmani Hosseinabadi et al. (2019) investigated the impact of the crossover and mutation operators, and designed a competitive GA.

GA has also been combined with other algorithms that lead to hybridized methods. Examples include tabu search (TS) (Liaw, 2000) and variable neighbourhood search (VNS) (Zobolas et al., 2009). Ahmadizar and Hosseinabadi Farahani (2012) presented a hybrid GA, in which a local optimization heuristic is applied in order to improve a pre-specified per cent of individuals selected from the population. Therefore, a fraction of the individuals undergo the local optimization heuristic (instead of all of them). They reported quality solutions.

Other proposed evolutionary algorithms include the hybridized constructive algorithms of beam search (BS) and ant colony optimization (ACO), called Beam-ACO (Blum, 2005). In each iteration, each ant uses a probabilistic BS, instead of using the traditional solution construction mechanism of ACO. To this end, adding a new component to a partial solution is performed by using a transition probability, and extending the partial solution is done by a reduced set of feasible solutions. The constructed solution is then improved by a local search. The algorithm outperforms the GAs by Prins (2000) and Liaw (2000). The discrete PSO of Sha and Hsu (2008), in which the algorithm is hybridized with four different decoding operators, has been shown to outperform the earlier meta-heuristics by Prins (2000); Liaw (2000); Blum (2005). Huang and Lin (2011) proposed a bee colony optimization algorithm that uses an idle time-based filtering scheme aiming to terminate the search for solutions with insufficient profitability. As such, their method prefers sequences with shorter idle-times that result in smaller makespan. The parallel GA by Ghosn et al. (2016) solves the non-preemptive open-shop, and the swarm intelligence algorithm by Bouzidi et al. (2019) solves problem $O||C_{max}$. Pongchairerks and Kachitvichyanukul (2016) proposed a two-level PSO. The upper-level of the algorithm tunes the parameters of the lower-level process. They showed that their algorithm, which considers parameterized active schedules, is able to deliver slightly better solutions than, e.g., the beam-ACO by Blum (2005), although they did not discuss the computation effort.

Two other widely applied meta-heuristics include tabu search (TS) and simulated annealing (SA). Alcaide et al. (1997) proposed a TS algorithm for problem $O||C_{max}$. The initial solution is selected from the best solutions generated by using three simple list scheduling algorithms, i.e., the best solution delivered by the three algorithms is used as the initial solution. This solution is then improved by two critical path-based neighbourhood structures that are similar to those by Dell'Amico and Trubian (1993). They conducted numerical studies on instances with up to 25 machines and 250 jobs to assess the performance of the algorithm.

Liaw (1999a) used the disjunctive graph model of Roy and Sussmann (1964) and proposed TS for the problem of finding the critical path on the graph (when the problem is modelled as a disjunctive graph, finding the critical path on the graph results in fining the optimal solution for the associated scheduling problem). The critical path is decomposed into a number of blocks, i.e., a sub-sequence of operations that belong to the same job or that are processed on the same machine. The neighbourhood move includes swapping operations that belong to the same block. Testing the algorithm on random and benchmark instances, they obtained the optimal solutions for almost 97% of the randomly generated instances, and quality including optimal solutions, for the majority of the benchmark instances of Taillard (1993). Jussien and Lhomme (2002) proposed an algorithm based on TS and a filtering technique. Testing the algorithm on the three benchmarks, they obtained superior solutions to Alcaide et al. (1997), Liaw (1999a), and Prins (2000). In addition, the algorithm generates new best solutions for 25 instances of Guéret and Prins (1999), including six optimal solutions. Liaw (1999b) proposed SA that uses the neighbourhoods developed in his earlier work (Liaw, 1999a). Even though the algorithm is capable of finding quality solutions for a wide range of benchmark instances, it has a long computation time for large instances. Another SA is due to Harmanani and Ghosn (2016) that swaps, shifts, and rotates the neighbourhoods.

Colak and Agarwal (2005) proposed a neural network algorithm that uses ten heuristic rules. In this method, the open-shop problem is first converted into a neural network. Then a learning strategy is iteratively applied to improve the solutions. They obtained competitive solutions for the benchmark instances of Taillard (1993).

3 Non-classical resource settings in the open-shop scheduling problem

In Section 2, we introduced the classical open-shop scheduling problem and reviewed studies that consider the problem in the general settings. In this section we review studies that investigate the open-shop scheduling problem in various settings and under different conditions on the machines (processing resources), e.g., machine availability, competing agents, and renewable and non-renewable resources.

3.1 Machine availability

In certain environments the machine may not be always available. The machine availability constraint therefore models the situation in which the machines are not continuously available due to, e.g., maintenance or rest periods. A non-availability period is often called a "hole". For the two-machine problem, the machine availability constraints are represented by $h(k_1, k_2)$, where $k_{1(2)}$ defines the number of holes on machine 1 (2) and $k_1 + k_2 \geq 1$. We refer the interested reader to Breit et al. (2003) for the details. In the following we focus on the case where the machine unavailability durations are constant and known in advance. Vairaktarakis and Sahni (1995) studied the preemptive open-shop scheduling problem with an arbitrary number of holes and showed that when the start times of the holes are undetermined, i.e., the scheduler can decide the start times, the optimal solution can be found in polynomial time. Given pre-determined holes and preemption allowed in non-integer intervals, they also proposed a linear program. The problem is strongly NP-hard, however, if preemption is only possible within integer intervals.

The majority of the research with the machine availability constraint focuses on the non-preemptive open-shop. Under the non-preemptive condition, the three cases of "resumable", "non-resumable", and "semi-resumable" model the different impacts of the holes on the interrupted operations. The resumable setting (denoted as "Re" in Table 5) allows an interrupted operation (caused by a hole) to be resumed after the hole with no penalty. The non-resumable case (denoted as "N-Re" in Table 5) does not allow resumption, so the interrupted operation must be re-started. The semi-resumable refers to a situation in which the interrupted operation is partially re-started after a hole (Ma et al., 2010). We note that, to the best of our knowledge, there is no study on the open-shop scheduling problem in the semi-resumable setting.

Breit et al. (2001) studied the two-machine problem in the resumable setting. They showed that the problem with a single hole, which is not at the beginning of the schedule, i.e., $t_h > 0$ is NP-hard, and proposed a $\frac{4}{3}$-approximation algorithm. It should be noted that if the hole is at the beginning of the schedule, i.e., $t_h = 0$, the problem is solvable in $O(n)$ time due to Lu and Posner (1993). Later, Lorigeon et al. (2002) proposed a pseudo-polynomial time dynamic program for the case where $t_h > 0$ and showed that the problem is ordinary NP-hard. They also proposed a mixed-integer linear program and used CPLEX to solve randomly generated instances with up to 500 jobs within five minutes. They showed that the worst-case performance ratio of any heuristic for the problem is 2. Kubzin et al. (2006) studied two additional cases of the problem: A single hole on each machine and several holes on one machine. They proposed polynomial-time approximation schemes for each case. It is worth mentioning that the two-machine open-shop with two holes on one machine and one hole on the other, i.e., $h(2,1)$ or $h(1,2)$ cannot be approximated unless $P = NP$ (Breit, 2000).

Breit et al. (2003) studied two-machine problem in the non-resumable setting. They showed that the problem with two holes on either machines cannot be approximated in polynomial time unless $P = NP$. They provided a 2- and a $\frac{4}{3}$-approximation algorithms for cases with one hole on each machine, i.e., $h(1,1)$, and one hole on either machines, i.e. $h(1,0)$ or $h(0,1)$. Mosheiov et al. (2018) considered the two-machine problem in the non-resumable setting where a single hole is to be scheduled on either machines and it must be started in a given time window $[k_1, k_2]$. They proposed a $\frac{3}{2}$-approximation algorithm for the problem.

It is also possible that the durations of the holes are not constant, in which case there are "floating" holes, each of which, denoted by $f(k_1, k_2)$, is a time-dependent operation whose duration deteriorates as a function of its start time. Kubzin and Strusevich (2006) studied the two-machine problem with floating holes where there is one hole on each machine, i.e., $f(1,1)$. Using the algorithm by Lu and Posner (1993), they proposed an $O(n)$ time algorithm to optimally solve the problem.

Table 5 summarizes the aforementioned studies with the machine availability constraint.

3.2 Two competing agents

In the two-agent setting, there are two agents each aiming to minimize its own objective, which only depends on its own jobs. Zhao and Wang (2015) considered the two-machine open-shop scheduling problem with two competing agents and deteriorating processing times.

Table 5: Summary of results with machine unavailability.

Problem	Complexity and methods	Reference
$O\|prmp, h_i\|C_{max}$	Polynomial for undetermined holes; if holes are pre-determined, and if preemption is allowed in non-integer intervals, polynomial, and strongly NP-hard otherwise	Vairaktarakis and Sahni (1995)
$O2\|\beta\|C_{max}$, where β is:		
$h(1,0), t_h = 0$	Polynomial in $O(n)$, the result holds for $h(0,1)$ as well	Lu and Posner (1993)
$h(1,0), t_h > 0, Re$	NP-hard, the result holds for $h(0,1)$ as well	Breit et al. (2001)
	Ordinary NP-hard, the result holds for $h(0,1)$ as well	Lorigeon et al. (2002)
$h(1,1), t_h > 0, Re$	PTAS	Kubzin et al. (2006)
$h(k,0), t_h > 0, Re$	PTAS	Kubzin et al. (2006)
$h(2,1), t_h > 0, Re$	Inapproximable, the result holds for $h(1,2)$ as well	Breit (2000)
$h(1,0), t_h > 0, N-Re$	$\frac{4}{3}$-approximation, the result holds for $h(0,1)$ as well	Breit et al. (2003)
$h(1,1), t_h > 0, N-Re$	2-approximation	Breit et al. (2003)
$h(2,0), t_h > 0, N-Re$	Inapproximable, the result holds for $h(0,2)$ as well	Breit et al. (2003)
$h(1,0), t_h \in [k_1,k_2], N-Re$	$\frac{3}{2}$-approximation, the result holds for $h(0,1)$ as well	Mosheiov et al. (2018)
$f(1,1), t_f > 0$	Optimal in $O(n)$	Kubzin and Strusevich (2006)

They proved that minimizing the makespan of one agent, while the makespan of the other agent is bounded by a certain threshold (denoted as k and is a given parameter), is NP-hard. Jiang et al. (2018) considered the two-machine problem with two competing agents to minimize the weighted sum of the makespan of the two agents. They proved that the problem of minimizing $C^A_{max} + kC^B_{max}, k > 0$, where $C^{A(B)}_{max}$ denotes the makespan of agent $A(B)$, is ordinary NP-hard. They also showed that the longest alternative processing time (LAPT) rule provides a 2-approximation algorithm when $k = 1$. Su and Hsiao (2015) studied the more general case where m machines with the machine availability and eligibility constraints exist, preemption is allowed, and the jobs have release times. They considered the same objective function as that of Zhao and Wang (2015). The eligibility constraint models the situation where the number of operations of a job can be fewer than the number of machines. They presented a linear program and proposed a dispatching rule-based heuristic for the problem. Table 6 summarizes the major studies in the context of two competing agents.

Table 6: Summary of the results with two competing agents.

Problem	Complexity and methods	Reference
$O2\|p^{A(B)}_{ij} = \alpha^{A(B)}_{ij}t, C^B_{max} \leq k\|C^A_{max}$	NP-hard	Zhao and Wang (2015)
$O2\|\|C^A_{max} + kC^B_{max}$	Ordinary NP-hard	Jiang et al. (2018)
$O2\|i = 1,2\|C^A_{max} + C^B_{max}$	2-approximation	Jiang et al. (2018)
$O\|C^B_{max} \leq k, r_j, prmp, avail., elig.\|C^A_{max}$	LP model, heuristic	Su and Hsiao (2015)

3.3 Renewable and non-renewable resources

The required resources for processing the jobs and performing the operations can be classified as renewable and non-renewable. Additional non-machine resources, e.g., manpower and tools, can be denoted as $res_{\kappa\chi\delta}$, where κ, χ and δ denote the number of resources, the total amount of available resources per time unit, and the maximum resource requirement of the operations, respectively. In addition, the "." for each of those parameters indicates that the corresponding parameter can take any integer value, whereas a positive integer indicates that the value of the parameter is fixed, i.e., given. Also, res and $tres$ indicate time-dependent and time-independent resources, respectively. The availability or consumption of a time-dependent resource may change over time.

Generally speaking, finding a feasible schedule for the resource constrained open-shop scheduling problem, even with one renewable resource over different time periods, is NP-hard, both with and without preemption (de Werra and Solot, 1993). The first studies on the open-shop scheduling problem with renewable resources were conducted by Blazewicz et al. (1983); Blazewicz et al. (1986), who presented the complexity results of some variants of the problem in the UET setting. Cochand et al. (1989); de Werra et al. (1991) considered non-renewable resources. They showed that problem $O|p_{ij} = \{0,1\}, nr\ staircase|C_{max}$ (where "$nr\ staircase$" denotes the non-renewable staircase, where a staircase pattern refers to the situation in which a non-renewable resource such as money may be available only at certain times and in fixed quantities, so its availability follows the staircase pattern) is strongly NP-hard, while problem $O|UET, nr\ staircase|C_{max}$ is solvable in $O(mn)$ time. de Werra (1990) modelled the preemptive and non-preemptive open-shop scheduling problem as a graph, where the nodes represent the resources and the weighted edges denote the jobs. They called it the "chromatic scheduling problem". They proposed an algorithm that solves the preemptive case and showed that the algorithm polynomially solves the non-preemptive variant if the graph has certain properties. de Werra and Blazewicz (1992) and de Werra and Blazewicz (1993) extended the results of de Werra et al. (1991) and studied the preemptive open-shop with a renewable or a non-renewable resource. They showed that the problem is equivalent to the edge colouring problem in the bipartite multi-graph, and presented some cases that are polynomially solvable. Tautenhahn and Woeginger (1997) studied the UET open-shop problem in which the availability of a renewable resource changes over time, i.e., it is time-dependent. They showed that the problem to minimize the regular objective function $\sum f_j(C_j)$, i.e., $O|UET, tres_{\kappa\chi\delta}|\sum f_j(C_j)$, is polynomially solvable if the number of machines, number of resources, and resource demand of each operation are bounded. They also established the complexity status of some cases of the problem to minimize other objective functions. Jurisch and Kubiak (1997) considered $O2|res_{1..}|C_{max}$ and $O2|res_{211}|C_{max}$. For the former, in which each operation needs a number of a single renewable resource, they developed an $O(n^3)$ solution algorithm, which can also solve the preemptive case, i.e., $O2|res_{1..}, prmp|C_{max}$, with no change in the makespan. For the latter, in which each operation requires at most one unit of either resources, they showed that it is NP-hard, and its general version with an arbitrary number of resources, i.e., $O2|res.11|C_{max}$, is strongly NP-hard.

Shabtay and Kaspi (2006) developed an $O(n \log n)$ algorithm for the two-machine open-shop with resource-dependent processing times, in which the availability of additional resources leads to shorter processing times, i.e., $p_{ij} = (w_{ij}/res_{ij})^k$, where w_{ij} is a positive value representing the workload of the operation, res_{ij} is the resource consumed by job j on machine i, and k is a positive constant. The total amount of the resource consumed is upper-bounded by R, i.e., $\sum_{i=1}^{2}\sum_{j=1}^{n} res_{ij} \leq R$, where R is a real (positive) number. They showed that the general case of the problem, i.e., $O|p_{ij} = (w_{ij}/res_{ij})^k, \sum_{i=1}^{2}\sum_{j=1}^{n} res_{ij} \leq R|C_{max}$, is NP-hard when $m \geq 3$, and presented a fully polynomial-time approximation scheme (FPTAS) when preemption is allowed. Oulamara et al. (2013) considered the two-machine open-shop in which every job, before its execution, undergoes a preparation phase. This phase requires a number of renewable resources that have capacity limits. Hence, the processing time of a job on each machine consists of two parts, preparation times, denoted by p'_{1j}, p'_{2j}, and execution times, denoted by p''_{1j}, p''_{2j}. It is evident that preparation always precedes execution. The problem is denoted as $O2|res^{prp}_{...}|C_{max}$, where prp indicates that the resource consumption is related to the preparation phase. They showed that the problem is strongly NP-hard, and NP-hard for the non-preemptive and preemptive cases, respectively. They also proved that the problem remains NP-hard even either the preparation times or the processing times are constant, i.e., $p'_{1j}, p'_{2j} = p'$ or $p''_{1j}, p''_{2j} = p''$. However, if both the preparation and processing times are constant, the problem is polynomially solvable by a reduction to perfect matching in a specific bipartite graph.

Certain jobs cannot be processed simultaneously on different machines. This is referred to as the open-shop with conflict graphs (OSC) and the jobs that cannot be processed simultaneously are presented by adjacent vertices in the conflict graph. Tellache and Boudhar (2017) showed that OSC is equivalent to problem $O|res^{idnt}_{.11}|C_{max}$, where $idnt$ denotes that all the operations of a job have identical resource requirements. They proposed a heuristic algorithm for the general OSC problem. Also, they showed that the two-machine OSC problem is strongly NP-hard if $p_{1j} = p_{2j} = p_j, p_j \in \{1, 2, 3\}$, and the conflict graph is the complement of the bipartite graph. This implies that $O2|res^{idnt}_{.11}, p_{ij} \in \{1, 2, 3\}|C_{max}$ is also strongly NP-hard. They then proved that the three-machine OSC problem is strongly NP-hard for any arbitrary conflict graph even with UET operations, which leads to the strong NP-hardness of $O3|res^{idnt}_{.11}, UET|C_{max}$. They also identified some polynomially solvable cases: (1) the two-machine OSC problem with any arbitrary conflict graph is solvable in $O(n^{2.5})$ time if $p_{ij} \in \{0, 1, 2\}$, which results in the same complexity status for problem $O2|res^{idnt}_{.11}, p_{ij} \in \{0, k, 2k\}|C_{max}$ for any fixed given integer k, (2) the preemptive two-machine OSC problem is solvable in $O(n^3)$ time for any arbitrary conflict graph, and (3) the three-machine OSC problem with UET operations is solvable in $O(n^{2.5})$ time if the conflict graph is a complement of a triangle-free graph. We summarize those reviewed studies in Table 7.

Table 7: Summary of the major studies with resource constraints.

Problem	Complexity and methods	Reference
$O2\|res_{...}, UET\|C_{max}$	$O(n^{2.5})$	Blazewicz et al. (1983)
$O2\|res_{111}, chain, UET\|C_{max}$	Strongly NP-hard	Blazewicz et al. (1986)
$O3\|res_{1..}, UET\|C_{max}$	Strongly NP-hard	Blazewicz et al. (1986)
$O3\|res_{.11}, UET\|C_{max}$	Strongly NP-hard	Blazewicz et al. (1986)
$O\|p_{ij} = \{0, 1\}, nr\ staircase\|C_{max}$	Strongly NP-hard	de Werra et al. (1991); Cochand et al. (1989)
$O\|UET, nr\ staircase\|C_{max}$	$O(mn)$	de Werra et al. (1991); Cochand et al. (1989)
$O2\|res_{1..}; prmp\|C_{max}$	$O(n^3)$	Jurisch and Kubiak (1997)
$O2\|res_{1..}\|C_{max}$	$O(n^3)$	Jurisch and Kubiak (1997)
$O2\|res_{122}\|C_{max}$	NP-hard	Jurisch and Kubiak (1997)
$O2\|res_{.11}\|C_{max}$	Strongly NP-hard	Jurisch and Kubiak (1997)
$O2\|UET, tres_{.11}\|C_{max}$	Strongly NP-hard	Tautenhahn and Woeginger (1997)
$O2\|UET, tres_{1..}\|C_{max}$	Strongly NP-hard	Tautenhahn and Woeginger (1997)
$O\|UET, tres_{1.1}\|C_{max}$	Strongly NP-hard	Tautenhahn and Woeginger (1997)
$O2\|p_{ij} = (w_{ij}/res_{ij})^k, \sum_{i=1}^{2}\sum_{j=1}^{n} res_{ij} \leq R\|C_{max}$	$O(n \log n)$	Shabtay and Kaspi (2006)
$O3\|p_{ij} = (w_{ij}/res_{ij})^k, \sum_{i=1}^{2}\sum_{j=1}^{n} res_{ij} \leq R\|C_{max}$	NP-hard	Shabtay and Kaspi (2006)
$O\|p_{ij} = (w_{ij}/res_{ij})^k, \sum_{i=1}^{2}\sum_{j=1}^{n} res_{ij} \leq R, prmp\|C_{max}$	FPTAS	Shabtay and Kaspi (2006)
$O2\|res^{prp}_{...}\|C_{max}$	Strongly NP-hard	Oulamara et al. (2013)
$O2\|res^{prp}_{...}; prmp\|C_{max}$	NP-hard	Oulamara et al. (2013)
$O2\|res^{idnt}_{.11}, p_{ij} \in \{1, 2, 3\}\|C_{max}$	Strongly NP-hard	Tellache and Boudhar (2017)
$O3\|res^{idnt}_{.11}, UET\|C_{max}$	Strongly NP-hard	Tellache and Boudhar (2017)

3.4 Transfer resource

The basic assumption for many scheduling problems is that as soon as a job is completed on a machine, it can be instantly processed by the next machine. In real-world practice, however, there might be a "time lag" or "delay" between the completion time of a job on the preceding machine and the start time of the job on the succeeding machine. The delay is typically incurred due to the transport time between the two machines that may need a set of transporters. The delay may also be caused by processing operations that are not in the form of machines, such as cooling procedures. Different approaches have been proposed to study such environments. We review the related studies in this section by grouping them into interstage delay and transport delay. The former studies concern delay due to technological reasons, while the latter studies deal with delay due to transporting jobs between machines.

3.4.1 Interstage delay

Delay can be due to technological reasons. For example, a cooling process may cause the delay. In the open-shop setting, such a delay has been characterized as either minimal or exact. The minimal delay is where the time lag is at least equal to the amount of the delay. We denote a minimum amount of delay by $\bar{L}$. The exact delay describes the situation where a succeeding operation should be started after its preceding operation, plus an exact amount of time. This case is commonly referred to as the coupled task scheduling problem (Khatami et al., 2020). We let L present an exact amount of delay. Also, the delay is "symmetric" if it has the same value from machine i to machine i' and from machine i' to machine i. For simplicity, for a two-machine problem with job-independent delay, we let $\tau = \bar{L}_{12}$ and $\sigma = \bar{L}_{21}$. If all the delays are identical, it is called the uniform delay.

Rayward-Smith and Rebaine (1992) investigated the non-preemptive open-shop scheduling problem with minimal delay. They proved that the two-machine problem is NP-hard even for uniform delay. The problem with an arbitrary number of machines is NP-hard with symmetric delay even for UET operations. Given an arbitrary number of machines, UET operations, and uniform delay, however, they presented a polynomial-time algorithm. Dell'Amico and Vaessens (1996) showed that the non-preemptive two-machine problem with symmetric delay is strongly NP-hard even if the processing time of each job is identical on both machines. Yu et al. (2004) showed that the non-preemptive two-machine problem with minimal symmetric delay is strongly NP-hard even with UET jobs.

Rebaine and Strusevich (1999) studied the two-machine problem with machine-dependent but job-independent delay, denoted by $O2|\tau, \sigma|C_{max}$. They developed an $\frac{8}{5}$-approximation algorithm, which achieves the ratio of $\frac{3}{2}$ if $\tau = \sigma$. They showed that both bounds are tight. Brucker et al. (2004) showed that the problem is solvable in constant time if all the operations have the same processing time, i.e., $p_{ij} = p$. By extending the algorithm by Pinedo and Schrage (1982), they also proposed an $O(n)$-time algorithm for $O2|\tau_j = \sigma_j, \max\{\tau_j\} \leq p_{min}, n \geq 6|C_{max}$, where p_{min} is the smallest processing time (> 0) of all the operation. Later, Strusevich (1999) investigated the general job-dependent case of the problem, i.e., $O2|\tau_j = \sigma_j|C_{max}$, and presented an $O(n \log n)$-time $\frac{3}{2}$-approximation algorithm and showed that the bound is tight. The on-line version of the problem, in which there is no *a priori* information on the release, processing, and transport times, was studied by Zhang and van de Velde (2010). They proposed a greedy algorithm that produces non-delay schedules. They proved that the competitive ratio of the greedy algorithm is 2 and the bound is tight, and no on-line delay and non-delay algorithm performs any better. In other words, the greedy algorithm is the best possible choice for the problem. Also, they studied the semi on-line version of $O2|\max\{\bar{L}_j\} \leq p_{min}|C_{max}$, again with no *a priori* information on the release, processing, and transport times, and showed that the competitive ratio of the greedy algorithm is $\frac{5}{3}$ and the bound is tight, and no on-line non-delay algorithm has superior performance. In addition, they showed that no on-line delay algorithm has a competitive ratio better than $\sqrt{2}$. Zhang and Velde (2010) proposed the first polynomial-time approximation scheme (PTAS) for this class of problems if $\max\{\bar{L}_j\} \leq kp_{min}$ for any constant $k > 0$. Munier-Kordon and Rebaine (2010) studied two solvable cases (either polynomially or pseudo-polynomially) of the problem with minimal integral delay and UET operations. For the first case that includes distinct delays the study proposed an $O(n \log n)$-time algorithm. For the second case, in which the delay can take only two distinct values, they presented a pseudo-polynomial algorithm. They also proposed two heuristics with the ratio of $\frac{3}{2} - \frac{1}{2n}$ and asymptotic worst-case ratio of $\frac{5}{4}$, respectively.

Recently, Ageev (2018) considered the problem with exact delay. They showed that the two-machine problem is NP-hard because the problem with exact delay is indeed a generalization of the no-wait environment. It should be noted that the two-machine no-wait open-shop scheduling problem is NP-hard (Giaro, 2001). Ageev (2018) also proved that the existence of a $(1.5 - \varepsilon)$-approximation algorithm for the special case of $p_{1j} = p_{2j}$ implies $P = NP$ for any $\varepsilon > 0$. When the delay takes only two values, they showed that there is no approximation algorithm with a ratio better than $(1.25 - \varepsilon)$ for any $\varepsilon > 0$. Table 8 summarizes the major studies considering interstage delay.

Table 8: Summary of results concerning interstage delay.

Problem	Complexity and method	Reference
$O2\|UET, \tau_j = \sigma_j\|C_{max}$	Strongly NP-hard	Yu et al. (2004)
$O2\|p_{1j} = p_{2j}, \tau_j = \sigma_j\|C_{max}$	Strongly NP-hard	Dell'Amico and Vaessens (1996)
$O2\|\tau_j = \sigma_j\|C_{max}$	$\frac{3}{2}$-approximation algorithm	Strusevich (1999)
$O2\|UET, \tau_j = \sigma_j\|C_{max}$	$\frac{3}{2} - \frac{1}{2n}$ and $\frac{5}{4}$-approximation algorithms	Munier-Kordon and Rebaine (2010)
	$O(n \log n)$ if all delays are distinct, and pseudo-polynomial if $\bar{L}_j \in \{\bar{L}_1, \bar{L}_2\}$	Munier-Kordon and Rebaine (2010)
$O2\|\tau = \sigma\|C_{max}$	NP-hard, remains NP-hard even if the processing times on one of the machines are all zero (Strusevich, 1999)	Rayward-Smith and Rebaine (1992)
	$\frac{3}{2}$-approximation algorithm	Rebaine and Strusevich (1999)
$O2\|\tau, \sigma\|C_{max}$	$\frac{8}{5}$-approximation algorithm	Rebaine and Strusevich (1999)
$O2\|p_{ij} = p, \tau, \sigma\|C_{max}$	$O(1)$	Brucker et al. (2004)
$O2\|\tau_j = \sigma_j, \max\{\tau_j\} \leq p_{min}, n \geq 6\|C_{max}$	$O(n)$	Rebaine and Strusevich (1999)
$O\|UET, \bar{L}_{ii'} = \bar{L}_{i'i}\|C_{max}$	NP-hard	Rayward-Smith and Rebaine (1992)
$O\|UET, \bar{L}_{ii'} = \bar{L}_{i'i} = \bar{L}\|C_{max}$	Polynomial	Rayward-Smith and Rebaine (1992)
$O2\|\max\{\bar{L}_j\} \leq p_{min}, on-line\|C_{max}$	2-competitive algorithm, $\frac{5}{3}$-competitive for the semi on-line case	Zhang and van de Velde (2010)
$O2\|\max\{\bar{L}_j\} \leq kp_{min}, on-line\|C_{max}$	PTAS	Zhang and Velde (2010)
$O2\|L_j\|C_{max}$	$(1.5 - \varepsilon)$-approximation algorithm means $P = NP$	Ageev (2018)

3.4.2 Transport delay

Delays may also be caused by transporting jobs between machines. The routing open-shop scheduling problem includes the transport time (Averbakh et al., 2005; Averbakh et al., 2006), where the jobs are located at the nodes of an undirected transport network represented by graph $G = (V, E)$, and the machines travel, typically at a unit speed, between the jobs. So, not only the processing times of the operations but also the travel times between the jobs should be considered. Starting at the depot the machines will return to the depot after finishing all the jobs. The problem is denoted as $RO||C_{max}$. The makespan is calculated as the time span between the start time of processing or moving of machines and the returning time of the last machine to the depot after performing all of its operations.

In a two-node network of two machines and n jobs, where more than one job can be located at a vertex, Averbakh et al. (2005) proposed a linear-time approximation algorithm with the worst-case performance ratio of $\frac{6}{5}$. Chernykh and Pyatkin (2017) presented certain specific results regarding the approximation algorithm of Averbakh et al. (2005). For example, they determined how the maximal ratio of the optimal makespan to a standard lower bound depends on the jobs' load distribution between the two nodes. They also presented an approximation algorithm with the same worst case as that by Averbakh et al. (2005); however, the worst case is completely specific to the load distribution. Later, Averbakh et al. (2006) showed that problem $RO2||V| = 2|C_{max}$, i.e., in a two-node network, is NP-hard both for two machines, and for two jobs and m machines. By excluding the assumption concerning the number of nodes, they proposed a $(1 + \frac{\rho}{2})$-approximation and an $(\frac{(m+1)}{2} + \rho)$-approximation algorithm for the two-machine and m machine variants, respectively, where a ρ-approximate solution for TSP, $\rho \leq 2$, is given. Those approximation ratios were later improved by Yu et al. (2011) to $\max\{\lceil \frac{m}{2} \rceil, \frac{4}{3}\rho\} + \frac{1}{3}$ and $\max\{\frac{4\rho+3}{2\rho+3}, \frac{2\rho+2}{3}\}$. Kononov (2012) proposed an FPTAS for $RO2||V| = 2|C_{max}$, and also introduced conditions under which the problem is solvable in linear time. Pyatkin and Chernykh (2012) showed that $RO2|prmp, |V| = 2|C_{max}$ is solvable in linear time, where $RO|prmp, |V| = 2|C_{max}$ is strongly NP-hard, i.e., with an arbitrary number of machines. It is conjectured that the two-node routing open-shop with UET and unit travel times is polynomially solvable if $m = n$ (Golovachev and Pyatkin, 2019).

A number of studies addressed the triangular network, i.e., $|V| = 3$. For example, Chernykh and Kuzevanov (2013) presented an $\frac{11}{10}$-approximation algorithm for $RO2||V| = 3, prmp|C_{max}$, and Chernykh and Lgotina (2016), and Chernykh and Lgotina (2019) studied the non-preemptive variant with identical and unrelated travel speeds, and proposed $\frac{6}{5}$ and $\frac{5}{4}$-approximation algorithms, respectively, which run in linear time.

Several studies investigated the variant with an arbitrary number of machines. Chernykh et al. (2013) use a $\frac{3}{2}$-approximation algorithm, originally proposed for TSP, and developed a ρ-approximation algorithm for $RO||C_{max}$, where $\rho = O(\sqrt{m})$, and a $\frac{13}{8}$-approximation algorithm for $RO2||C_{max}$. By reducing the original problem to the classical flow-shop scheduling problem, Yu and Zhang (2011) improved the results of Chernykh et al. (2013) and presented an $O(\log m (\log\log m)^{1+\varepsilon})$-approximation algorithm for $RO||C_{max}$. Later, Kononov (2015) showed that there exists an $O(\log m)$-approximation algorithm for $RO||C_{max}$. Bevern and Pyatkin (2016) proposed a fixed-parameter algorithm for the case with UET jobs and showed that it can be solved in $2^{|V||M|^2 \log |V||M|}.poly(|N|)$ time, where M and N are the machine and job sets, respectively.

In the classical parallel-machine scheduling models, machines can have identical, uniform, or unrelated speeds (Pinedo, 2016). Inspired by the classical parallel-machine scheduling models, Chernykh (2016) relaxed the assumption of unit speed. Each machine therefore has its own unrelated speed, leading to different travel times for the machines. The problem is denoted as $RO2|Rtt|C_{max}$. He presented an approximation algorithm. Also, he proposed a linear-time algorithm for the case where the transport network has a tree and the depot is not pre-defined, i.e., it has to be chosen. In Yu et al. (2011)'s study, the machines do not return to the depot. The makespan is then the maximum completion time of all the jobs. They proposed an $(\frac{8-3\rho}{5-2\rho})$-approximation algorithm for the two-machine case, where $\rho \leq 2$ is the approximation factor for the shortest Hamiltonian path problem. They also presented a ρ'-approximation algorithm for the m-machine case, where

$$\rho' = \begin{cases} \max\{\lceil \frac{m}{2} \rceil, 2\} + \frac{1}{2}, & m \leq 6, \\ \lceil \frac{m}{2} \rceil + \frac{1}{3}, & m > 6. \end{cases} \tag{2}$$

It should be noted that the routing open-shop scheduling problem can also be seen as a variant of the open-shop with sequence-dependent families or batch setup times. That is, the travel times of the machines between the nodes can be modelled as the sequence-dependent setup time and, because a node may include several jobs, a family or a batch of jobs is visible. Whether the routing open-shop problem parameterized with number of batches is fixed-parameter tractable is an open question (Mnich and Bevern, 2018). Table 9 summarizes the major studies considering transport delay.

Table 9: Summary of the results on transport delay.

Problem	Complexity and methods	Reference
$RO2\|\|V\| = 2\|C_{max}$	NP-hard and $\frac{6}{5}$-approximation algorithm	Averbakh et al. (2006); Averbakh et al. (2006)
$RO\|\|C_{max}$	Strongly NP-hard (when neither m nor n is bounded)	Averbakh et al. (2006)
$RO2\|\|V\| = 2\|C_{max}$	FPTAS	Kononov (2012)
$RO2\|prmp, \|V\| = 2\|C_{max}$	$O(n)$	Pyatkin and Chernykh (2012)
$RO\|prmp, \|V\| = 2\|C_{max}$	Strongly NP-hard	Pyatkin and Chernykh (2012)
$RO2\|\|V\| = 3, prmp\|C_{max}$	$\frac{11}{10}$-approximation algorithm	Chernykh and Kuzevanov (2013)
$RO2\|\|V\| = 3\|C_{max}$	$\frac{6}{5}$-approximation algorithm	Chernykh and Lgotina (2016)
$RO2\|\|C_{max}$	$\frac{13}{8}$-approximation algorithm	Chernykh et al. (2013)
$OR2\|Rtt, tree, variable - depot\|C_{max}$	$O(n)$	Chernykh (2016)
$RO\|\|V\| = 3, n = 1, Qtt\|C_{max}$(single job routing open shop with uniform travel times)	NP-hard	Chernykh (2016)
$RO2\|\|V\| = 3, Rtt\|C_{max}$	$\frac{5}{4}$-approximation algorithm	Chernykh and Lgotina (2019)
$RO\|\|C_{max}$	$O(\log m)$-approximation algorithm	Kononov (2015)

3.4.3 Interstage resource

The interstage resource is different from the transport resource because in the former the transporter moves the jobs between the processing machines, so the machines are fixed. In the latter, however, the machines move to the jobs to process them, meaning that the jobs are fixed. It is typical that a set of transporters deliver products (jobs) between the machines. Lee and Strusevich (2005) studied this problem with an uncapacitated interstage transporter for the two-machine problem. The transporter can carry the jobs between machines 1 and 2, i.e., both from machine 1 to 2 and from machine 2 to 1, where the transport times are denoted by τ and σ, respectively. The problem is then denoted by $TO2|k = 1, c \geq n|C_{max}$, where k is the number of transporters and $c, c \geq n$ is the capacity of the transporters. The transporter has indeed infinite capacity because $c \geq n$. They proposed an approximation algorithm with a ratio of 2, which can be improved to $\frac{5}{3}$ if the transport times are symmetric, i.e., $\tau = \sigma$. Lushchakova et al. (2009) generalized the problem: The transporter first carries the jobs to one of the machines at the beginning of the schedule and then collects all of them at the end of the schedule. Therefore, the objective function is to minimize the time when all the completed jobs are collected by the transporter, denoted by K, and the problem is denoted by $TO2|\nu = 1, c \geq n|K$. They proposed a $\frac{7}{5}$-approximation algorithm and showed that the bound is tight. Table 10 summarizes the major studies considering interstage transporters.

Table 10: Summary of the studies considering interstage transporters.

Problem	Complexity and methods	Reference
$TO2\|\tau, \sigma, k = 1, c \geq n\|C_{max}$	2-approximation algorithm	Lee and Strusevich (2005)
$TO2\|\tau = \sigma, k = 1, c \geq n\|C_{max}$	$\frac{5}{3}$-approximation algorithm	Lee and Strusevich (2005)
$TO2\|k = 1, c \geq n\|K$	$\frac{7}{5}$-approximation algorithm	Lushchakova et al. (2009)

3.5 No-wait, no-idle, and blocking open-shops

In this section we review studies that consider no-wait, no-idle, and blocking in the open-shop scheduling problem. No-wait means that the processing of a job/operation must immediately start after completion of the job on the preceding machine. In addition, if each machine processes the operations with no idle time, then the problem is denoted as no-wait no-idle. Blocking occurs when there is no (or limited) intermediate storage between the machines.

The two-machine preemptive no-wait open-shop scheduling problem to minimize the makespan, denoted as $O2|prmp, no-wait|C_{max}$, has been shown to be strongly NP-hard (Strusevich, 1991; Hall and Sriskandarajah, 1996). Also, the two-machine non-preemptive no-wait open-shop, i.e., $O2|no-wait|C_{max}$, in which the jobs must visit both machines, is known to be NP-hard in the strong sense (Sahni and Cho, 1979). Gonzalez (1982) later showed that the problem with an arbitrary number of machines, i.e., $O|no-wait|C_{max}$, is strongly NP-hard even if all the non-zero operations have identical positive processing times. They proved that for the two-machine case if all the jobs go through both machines and have identical processing times, the problem is, however, trivial. Brucker et al. (1993) showed that the no-wait UET open-shop can be transformed to the m identical parallel-machine problem. They showed that the following problems are polynomially solvable: $O|UET, r_j, no-wait|C_{max}$, $O|UET, tree, no-wait|C_{max}$, and $O2|UET, prec, no-wait|C_{max}$. They also showed the NP-hardness of $O|UET, prec, no-wait|C_{max}$ by using the same transformation. Sidney and Sriskandarajah (1999) showed that the algorithm by Gilmore and Gomory (1964), which optimally solves the two-machine no-wait flow-shop problem in $O(n \log n)$ time, has a tight bound of $\frac{3}{2}$ for the two-machine open-shop problem. Panwalkar and Koulamas (2014) showed that two variants of the two-machine problem with proportionate processing times, i.e., $O2|p_{ij} = p_j, no-wait|C_{max}$ and $O2|p_{ij} = p_j + s_i, no-wait|C_{max}$, can be solved in $O(n \log n)$ time, where s_i is a machine-specific setup time. In addition, they significantly improved the exhaustive enumeration efforts required for the problems $O2|no-wait|C_{max}$ and $O2|p_{ij} = \frac{p_j}{v_i}, no-wait|C_{max}$, where $v_i \geq 1$ is the speed of machine i.

In regard to producing schedules, the B&B algorithm by Liaw et al. (2005) optimally solves the two-machine no-wait problem with up to 100 jobs in a reasonable amount of time. The heuristic algorithm by Liaw et al. (2005) has an $O(n^3)$ running time and is able to solve larger instances with up to 1,000 jobs within a few seconds. Naderi and Zandieh (2014) proposed three mixed-integer linear programs for the no-wait problem. To overcome the limitation of the well-known permutation and rank matrix encoding for the open-shop scheduling problem, which likely generates infeasible solutions for the no-wait problem, they proposed a new encoding scheme. They also proposed the meta-heuristic algorithms of VNS and GA.

Recall that in the no-wait no-idle environment, each job is processed with no delay between its operations and each machine processes the operations with no idle time. The no-wait no-idle is also known as "compact" scheduling. Giaro et al. (1999) studied the compact open-shop with zero-one time operations. They showed that the problem is equivalent to the strongly NP-hard colouring problem in the bipartite graph. They also proved that there is no approximation algorithm for the problem with a ratio better than $\frac{6}{5}$. Giaro (2001) showed that the decision problem as to whether a compact schedule exists for the two-machine open-shop is strongly NP-complete. They proved that the compact open-shop problem with an arbitrary number of machines is NP-hard when the scheduling graph is a path or cycle. Also, the compact open-shop on a tree, i.e., the associated graph is a tree, with two machines is NP-hard, and that with an arbitrary number of machines is strongly NP-hard. A linear-time 2-approximation algorithm has been presented for the latter problem. Billaut et al. (2019) showed that the two-machine compact open-shop scheduling problem is strongly NP-hard.

In the blocking open-shop scheduling problem, there is no intermediate storage between the machines. Yao et al. (2000) argued that the two-machine blocking open-shop is essentially the two-machine no-wait open-shop, because a schedule for the former can be converted to a schedule for the latter with the same makespan. They evaluated the performance of several heuristics for the two-machine blocking open-shop problem: The algorithm by Sidney and Sriskandarajah (1999), two heuristics for the matching problem, and a random search algorithm. They concluded that the random search heuristic outperforms the others. Mejía et al. (2018) studied the open-shop setting in which there are no buffers between the machines, so blocking may occur. They modelled the problem as a Petri Net and proposed a graph search algorithm. Table 11 summarizes the major studies considering the no-wait, no-idle, and blocking settings.

Table 11: Summary of the studies considering the no-wait, no-idle, and blocking settings.

Problem	Complexity and methods	Reference
$O2\|no-wait\|C_{max}$	Strongly NP-hard	Strusevich (1991); Hall and Sriskandarajah (1996)
$O2\|no-wait\|C_{max}$	Strongly NP-hard when jobs must visit both machines	Sahni and Cho (1979)
	The algorithm if Gilmore and Gomory (1964) has a tight bound of $\frac{3}{2}$	Sidney and Sriskandarajah (1999)
	Heuristics	Yao et al. (2000)
	B&B	Liaw et al. (2005)
$O2\|no-wait\|C_{max}$	Polynomial when all non-zero operations have equal length and jobs must visit both machines	Gonzalez (1982)
$O2\|p_{ij} = p_j, no-wait\|C_{max}$	$O(n \log n)$, result holds for the case of $p_{ij} = p_j + s_i$ as well	Panwalkar and Koulamas (2014)
$O\|no-wait\|C_{max}$	Strongly NP-hard even if all non-zero operations have equal length	Gonzalez (1982)
	MILP, VNS and GA	Naderi and Zandieh (2014)
$O\|UET, prec, no-wait\|C_{max}$	NP-hard	Brucker et al. (1993)
$O\|UET, r_j, no-wait\|C_{max}$	Polynomial	Brucker et al. (1993)
$O\|UET, tree, no-wait\|C_{max}$	Polynomial	Brucker et al. (1993)
$O2\|UET, prec, no-wait\|C_{max}$	Polynomial	Brucker et al. (1993)
$O2\|no-idle, no-wait\|C_{max}$	Ordinary NP-hard if the scheduling graph is in the shape of tree, 2-approximation algorithm	Giaro (2001)
	Strongly NP-hard	Billaut et al. (2019)
$O\|p_{ij} \in \{0,1\}, no-idle, no-wait\|C_{max}$	Strongly NP-hard, inapproximable within factor of $\frac{6}{5}$ unless $P = NP$	Giaro et al. (1999)
$O\|no-idle, no-wait\|C_{max}$	Strongly NP-hard if the scheduling graph is in the shape of tree, 2-approximation algorithm, NP-hard when the scheduling graph is a path or cycle	Giaro (2001)
$O2, S\|no-wait\|C_{max}$	NP-hard	Glass et al. (2000)

4 Non-classical job settings in the open-shop scheduling problem

After reviewing the non-classical resource settings in Section 3, in this section we review studies the open-shop scheduling problem in the non-classical settings and with constraints on the jobs, e.g., the precedence constraint, job batching and rejection, and assumptions on the jobs' release and processing times. We start with the precedence constraint.

4.1 Precedence constraint

The precedence constraint stipulates that a job or an operation can be executed only if all of its predecessors have already completed their execution. The open-shop scheduling problem with UET operations and the precedence constraint has been studied in the literature. For example, Bräsel et al. (1994) presented an $O(mn)$ algorithm for the two-machine open-shop with UET jobs where the precedence relation between the jobs follows a tree, i.e., if job j precedes job j', the last operation of job j must be processed before the first operation of job j', i.e., $O2|UET, tree|C_{max}$. They also presented a linear-time algorithm to solve problem $O2|UET, prec|C_{max}$, i.e., where the precedence constraint follows a general (arbitrary) form (Bräsel et al., 1996). Coffman Jr and Timkovsky (2002) studied the "ideal" schedules in which the objective includes simultaneously minimizing the maximum and total completion time (or flow time) of the jobs. For the two-machine case, they showed that the ideal schedule for $O2|prec, r_j, UET|C_{max}, \sum C_j$ and $O2|no-wait, prec, r_j, UET|C_{max}, \sum C_j$ can be obtained polynomially by an extension of Coffman Jr and Graham (1972)'s algorithm. Zhang et al. (2018) presented an $O(n^2)$-time $(2-\frac{2}{m})$-approximation algorithm for the m-machine open-shop with UET jobs and an arbitrary precedence constraint, i.e., $Om|prec, UET|C_{max}, m \geq 3$. Nonetheless, the complexity of the problem is still open even for $m = 3$ (see, e.g., Prot and Bellenguez-Morineau (2018)).

Shafransky and Strusevich (1998) considered the precedence constraint in the form of a given sequence of jobs' operations on only one of the machines (denoted as $GS(1)$ in Table 12). They showed that even though the preemptive case of the problem is solvable in $O(n)$ time for an arbitrary number of machines, the general case is strongly NP-hard for an arbitrary number of machines. They proved, however, that it is ordinary NP-hard if there are two machines, and proposed a pseudo-polynomial time dynamic program and used it to construct an FPTAS. They also designed a heuristic algorithm with a worst ratio of $\frac{5}{4}$. Table 12 summarizes the major studies on the open-shop scheduling problem with the precedence constraint.

Table 12: Summary of results for problems with the precedence constraint.

Problem	Complexity and methods	Reference
$O2\|UET, tree\|C_{max}$	$O(nm)$	Bräsel et al. (1994)
$O2\|UET, prec\|C_{max}$	$O(n)$	Bräsel et al. (1996)
$O2\|prec, r_j, UET\|C_{max} \sum C_j$	Polynomial	Coffman Jr and Timkovsky (2002)
$O2\|nowait, prec, r_j, UET\|C_{max} \sum C_j$	Polynomial	Coffman Jr and Timkovsky (2002)
$Om\|prec, UET\|C_{max}, m \geq 3$	$O(n^2)$-time $(2-\dfrac{2}{m})$-approximation algorithm	Zhang et al. (2018)
$O2\|GS(1)\|C_{max}$	Ordinary NP-hard, $\frac{5}{4}$-approximation algorithm, FPTAS	Shafransky and Strusevich (1998)
$O\|GS(1)\|C_{max}$	Strongly NP-hard	Shafransky and Strusevich (1998)
$O\|GS(1), prmp\|C_{max}$	Polynomially solvable in $O(n)$	Shafransky and Strusevich (1998)

4.2 Batch processing and setup time

There are a number of studies that consider processing the jobs in batches, setup time, and their combination, which we review in this section.

Strusevich (1993) studied the two-machine open-shop scheduling problem where each operation has three stages, namely setup, processing, and removal. The setup stage of an operation can only be started if the removal stage of the preceding operation is completed. The processing stages of a job cannot be performed simultaneously; however, the other stages can overlap. They proposed an algorithm based on the one by Gonzalez and Sahni (1976) that solves the problem in $O(n)$ time. Glass et al. (2000) considered the two-machine open-shop problem where the setup stages of the operations are performed by a single server that is different from the processing machines. The problem is denoted as $O2, S||C_{max}$ and they proved that it is strongly NP-hard. Also, they showed that even the no-wait version of the problem is NP-hard. Mosheiov and Oron (2008) studied the m-machine open-shop with identical processing times and identical setup times, i.e., $p_{ij} = p, s_{ij} = s, \forall j \in N, \forall i \in M$, to minimize the makespan and the flow time. They proposed an $O(n)$-time heuristic for the former problem and a constant time algorithm based on the number of batches for the latter problem. Roshanaei et al. (2010) considered the general sequence-dependent setup time in the m-machine setting. They adapted two dispatching rules-based heuristics provided by Pinedo (2016) and Liaw (1998). They also proposed several heuristics, including a multi-neighbourhood search SA algorithm, and hybridization of SA and local search. Their multi-neighborhood search SA performs better than pure SA, VNS, and the GA of Senthilkumar and Shahabudeen (2006).

Glass et al. (2001) investigated batching and sequencing in the two-machine open-shop. Batching refers to forming batches of jobs to be processed together, while sequencing refers to determining the order in which the batches are processed. They proved that there exists an optimal schedule with at most three consistent batches, where the batches are consistent if they are identical on the two machines. They also showed that if the optimal schedule consists of a single batch, then the schedule can be easily obtained. In addition, the case with two batches has been shown to be ordinary NP-hard because it is pseudo-polynomially solvable, and the case with three batches has been conjectured not to be easier than the case with two batches (Glass et al., 2001). It is known that the algorithm by Gribkovskaia et al. (2006), which is based on the work by de Werra (1989) for the generic two-machine open-shop problem, delivers an optimal schedule in $O(n)$ time if the optimal schedule consists of three consistent batches. To conclude, the problem can be solved in linear time if one or three consistent batches are in the optimal schedule, while it is NP-hard if there are two consistent batches. We note that the schedules with more than three batches need not be considered because Glass et al. (2001) has showed that an optimal schedule with at most three consistent batches always exits.

Potts et al. (2001) extended the work of Glass et al. (2001) and studied the two-machine open-shop where the processing time of a batch is the maximum of the processing times of the operations in the batch, denoted as "max-batch", and the number of jobs in a batch on the machines is bounded above by k_1 and k_2, respectively. The problem is denoted as $O2|max-batch, b_1 = k_1, b_2 = k_2|C_{max}$, where b_i denotes the maximum number of jobs in a batch on machine i. They showed that if $k_1 = k_2 = k$, the problem is ordinary NP-hard, as it can be pseudo-polynomially solved by a dynamic program. Khormali et al. (2012) studied the open-shop with the parallel-batching constraint and non-identical jobs, i.e., the jobs in a batch are processed simultaneously and they have different processing times. The processing time of a batch is determined by the longest job in the batch. They proposed heuristics, including an SA and a GA. Mor et al. (2012) investigated open-shop batch scheduling with UET operations and machine-dependent setup times. They showed that the equal allocation policy, i.e., assigning an identical number of jobs to the batches is optimal for the two- and three-machine problems. The equal allocation policy may not necessarily be optimal for problems with more than three machines. Nonetheless, the policy still delivers good quality schedules.

In manufacturing systems jobs might be grouped according to their processing similarities/characteristics, and are processed in groups. This modelling is conceptually very similar to batch processing and is called "group technology" (GT). Ben-Arieh and Dror are the first to expand GT to open-shop scheduling (Ben-Arieh and Dror, 1989; Ben-Arieh and Dror, 1991). They proposed a two-phase algorithm in which the jobs are first grouped and then scheduled. Kleinau (1993) investigated the two-machine open-shop with family setup times, where the jobs are partitioned into groups and setup times do not occur between jobs of the same group. However, the setup time arises when a machine switches from processing a job of one group to a job of another group. The problem is denoted as $O2|batch\ setup|C_{max}$. They showed that it is NP-hard and presented several polynomially solvable cases. Later, Strusevich (2000) proposed a linear-time $\frac{5}{4}$-approximation algorithm for the same problem. Billaut et al. (2008) showed that to obtain the optimal solution for the same problem, a group should not be split more than once. They applied this observation and designed a heuristic with a worst-case performance ratio of $\frac{6}{5}$ and showed that the bound is tight. Blazewicz and Kovalyov (2002) studied the two-machine open-shop with at least two groups such that processing of a group incurs a setup time s_f, which is neither sequence nor machine dependent. They showed that the problem to minimize the makespan is NP-hard even if there is no setup time, and $p_{1j} = p_{2j}, \forall j \in N$, i.e., $O2|s_f = 0, p_{1j} = p_{2j}|C_{max}$. The two-machine open-shop with two conflicting criteria of flexibility and makespan was investigated by Esswein et al. (2005). The aim is to find a schedule with the smallest makespan among the schedules with k consistent sequential groups on each machine. The problem is denoted as $O2|G(k_1, k_2)|C_{max}$, where $G(k_1, k_2)$ implies that there are at most k_1 (k_2) groups on machine 1 (2). Problem $O2|G(2, 2)|C_{max}$ is NP-hard due to Glass et al. (2001); however, if there exists job j such that $p_{1j} + p_{2j} \geq \max\{l_1, l_2\}$, i.e., the total processing time of job j on both machines is greater than the maximum load of the machines, the linear-time algorithm by de Werra (1989) was shown to be optimal (Esswein et al., 2005). Otherwise, the algorithm produces an optimal schedule for problem $O2|G(3, 3)|C_{max}$. Table 13 summarizes the major studies considering batch processing and setup times.

Table 13: Summary of studies with batch processing and setup times.

Problem	Complexity and methods	Reference
$O2\|batch\ (family)\ setup\|C_{max}$	NP-hard	Kleinau (1993)
$O2\|batch\ (family)\ setup\|C_{max}$	$\frac{6}{5}$-approximation algorithm	Billaut et al. (2008)
$O2\|sum-batch\|C_{max}$	NP-hard, FPTAS	Glass et al. (2001)
$O2\|max-batch, b_1 = 1, b_2 = 2\|C_{max}$	$O(n \log n)$	Potts et al. (2001)
$O2\|max-batch, b_1 = 1, b_2 = k\|C_{max}$	$O(n^{k(k-1)})$	Potts et al. (2001)
$O2\|max-batch, b_1 = k, b_2 = n\|C_{max}$	NP-hard (For fixed k and $k \geq 1$)	Potts et al. (2001)
$O2\|max-batch, b_1 = 1, b_2 = n\|C_{max}$	$O(n^3 \sum_{j=1}^{n} p_{1j})$	Potts et al. (2001)
$O2\|max-batch, b_1 = k, b_2 = n\|C_{max}$	$O(n^4 k^4 (\sum_{j=1}^{n} p_{1j})^2))$	Potts et al. (2001)
$O2\|max-batch, b_1 = n, b_2 = n\|C_{max}$	$O(n)$	Potts et al. (2001)
$O2\|s_f = 0, p_{1j} = p_{2j}\|C_{max}$	NP-hard	Blazewicz and Kovalyov (2002)
$O2, S\|\|C_{max}$	Strongly NP-hard	Glass et al. (2000)
$O2\|G(2, 2)\|C_{max}$	NP-hard	Glass et al. (2001)
$O2\|G(3, 3)\|C_{max}$	$O(n)$	Esswein et al. (2005)

4.3 Rejection

Rejection occurs when the scheduler is allowed to reject some jobs for execution, typically by incurring a penalty. Hoogeveen et al. (2003) investigated the preemptive open-shop scheduling problem with job rejection. The objective function is to minimize the makespan of the accepted jobs plus the total rejection cost of the rejected jobs, dented as $Rjct$. Even the two-machine problem, i.e., $O2|prmp|C_{max} + Rjct$, is ordinary NP-hard, but for problem $Om|prmp|C_{max} + Rjct$, a pseudo-polynomial dynamic program and an FPTAS when the number of machines is fixed exist (Hoogeveen et al., 2003). When the number of machines is part of the input, i.e., problem $O|prmp|C_{max} + Rjct$, Hoogeveen et al. (2003) showed that it is strongly NP-hard and gave a 1.58-approximation algorithm.

Shabtay et al. (2013) posed the question as to whether the non-preemptive variant of the problem, i.e., $O2||C_{max} + Rjct$, is polynomially solvable. To answer the question, Zhang et al. (2016) showed that it is indeed ordinary NP-hard, even for certain special cases. These special cases include identical processing times on the first (second) machine, i.e., $p_{1j} = p$ ($p_{2j} = p$), and identical rejection cost, i.e., $e_j = e$, where e_j is the rejection cost of job j, and $p > 0$ and $e > 0$ are constants. They proposed a pseudo-polynomial time dynamic program for the problem and an FPTAS algorithm with $O(\frac{n^6}{\varepsilon^2})$ time complexity. In addition, they showed that the 2-approximation algorithm by Shabtay and Gasper (2012) for the flow-shop setting is also valid for its open-shop counterpart.

Koulamas and Panwalkar (2015) studied the two-machine problem of selecting a subset of jobs with a given cardinality to minimize the makespan. The problem is denoted as $O2|k\ jobs|C_{max}$. For a given $k = 1, \ldots, n$, the problem consists of selecting the best subset of k jobs from the set of n jobs. The problem is ordinary NP-hard because the similar problem to minimize the number of tardy jobs is ordinary NP-hard (Józefowska et al., 1994). Koulamas and Panwalkar (2015) proposed an $O(n^2)$ algorithm to solve $O2|k\ jobs, M_1 = M_{max}|C_{max}$, i.e., machine 1 is the maximal.

Table 14: Summary of studies with job rejection.

Problem	Complexity and methods	Reference
$O2\|prmp\|C_{max} + Rjct$	Ordinary NP-hard, FPTAS	Hoogeveen et al. (2003)
$O\|prmp\|C_{max} + Rjct$	Strongly NP-hard, 1.58-approximation algorithm	Hoogeveen et al. (2003)
$O2\|\|C_{max} + Rjct$	Ordinary NP-hard, FPTAS with $O(\frac{n^6}{\varepsilon^2})$	Zhang et al. (2016)
$O2\|k\ jobs\|C_{max}$	Ordinary NP-hard	Józefowska et al. (1994)
$O2\|k\ jobs, M_1 = M_{max}\|C_{max}$	$O(n^2)$	Koulamas and Panwalkar (2015)

4.4 Release time and on-line scheduling

A realistic assumption for scheduling problems is that all the jobs may not be available at time 0, but at different times known as the release times. The release time of job j is denoted by r_j. Typically, the problem with release times is more challenging than its counterpart without release times. For example, (Graham et al., 1979; Lawler et al., 1981) have shown that the non-preemptive two-machine open-shop with release times is strongly NP-hard.

Cho and Sahni (1981) obtained feasible schedules for the preemptive open-shop scheduling problem with two distinct release times and identical due-dates for all the jobs, denoted as $O|r_j \in \{r_1, r_2\}, d_j = d, prmp|-$. Here, the common due date d, also known as "deadline", is the time by which all the operations of the jobs must be completed. They transformed the problem to an instance of the network flow problem with upper and lower bounds on the edges, and proposed an $O(n^3 + m^4)$ solution algorithm, where $n \geq m$ and $m > 2$. Zhan et al. (2011) provided another network flow formulation of the preemptive open-shop scheduling problem with release times and applied the maximum flow algorithm to solve it. Lu and Posner (1993) investigated the two-machine preemptive problem with two distinct release times: Some jobs have zero release times while the other jobs have a positive common release time. They developed an approach with an average-case complexity being polynomial in the number of jobs. Considering the feasibility problem of the preemptive open-shop with general release times and due dates, Sedeño-Noda et al. (2006) proposed an $O(\min\{n, m\}n^2m)$-time network flow algorithm. Pinedo (1995) (see also Pinedo (2016)) showed that the optimization version of the problem can be solved with a linear program utilizing the processing times matrix. Sedeño-Noda et al. (2006) presented the first strongly polynomial combinatorial algorithm to solve the original feasibility problem. Kubale (1997) studied the zero-one m-machine open-shop problem with integer release times and due-dates, i.e., $O|p_{ij} \in \{0, 1\}, r_j, d_j \in \mathbb{Z}^+|C_{max}$. They showed that the problem is NP-hard and proposed polynomial-time algorithms to solve two special cases: (1) all the operations have UET and (2) at most $m + n$ operations have UET.

As stated in Section 2, the dense schedules are conjectured to have a $2 - \frac{1}{m}$ bound for the classical open-shop. Chen et al. (2008) extended the conjecture for the three-machine open-shop with release times, i.e., $O3|r_j|C_{max}$. They showed that any dense schedule provides a $\frac{7}{4}$-approximation for the problem. They tightened the bound to $\frac{5}{3}$ for the two special cases where the processing times are machine-independent and where each job consists of at most two operations.

On-line scheduling differs from scheduling with release times in the sense that the data of the jobs (e.g., release times, processing times etc) are available only when the jobs arrive in the system. The quality of an on-line algorithm is assessed by comparing its solution to the optimal solution of the off-line counterpart, and is often referred to as the "competitive" ratio. Bai and Tang (2013) proved that the dense schedules are asymptotically optimal for both the off-line and on-line open-shops with release times if the problem size is large enough. For both the off-line and on-line settings, they developed a heuristic that constructs the schedule based on combining the SPT dispatching rule and the dense schedules. Their experiments showed that such a dispatching rule-based heuristic obtains the optimal schedule when the number of jobs goes to infinity.

Chen and Woeginger (1995) investigated on-line scheduling of the two-machine open-shop in both preemptive and non-preemptive settings. For the preemptive case, they proposed a $\frac{4}{3}$-competitive algorithm and showed that no better ratio exists. For the non-preemptive setting, their algorithm has a worst-case ratio of 1.875. In addition, they showed that no on-line algorithm has a better ratio than $\frac{1}{2}(1 + \sqrt{5}) \approx 1.618$. The two- and three-machine variants of the problem were investigated by Chen et al. (2001). In the preemptive three-machine setting, they provided an algorithm with a best possible competitive ratio of $\frac{27}{19}$. In the non-preemptive two-machine setting, they only considered permutation schedules, in which the sequences of the jobs on both machines are identical. They improved the results of Chen and Woeginger (1995) and proposed a permutation algorithm with a performance ratio of 1.848 and showed that the ratio of any such algorithm is never less than $\frac{(23-2\sqrt{13})}{9} \approx 1.754$. The study of Liu et al. (2010) on on-line scheduling of the preemptive two-machine open-shop assumes that the processing times are bounded (although still unknown before the jobs arrive), i.e., $1 \leq p_{ij} \leq k$ and $k \geq 1$ is a constant. They proposed an on-line algorithm with a competitive ratio of $\frac{5k-1}{4k}$ and showed that it is optimal, implying that no on-line algorithm with a smaller competitive ratio exists.

Chen et al. (1998) categorized the on-line open-shop problem into the clairvoyant and non-clairvoyant settings. In the clairvoyant setting, the processing time of a job is known upon its arrival, whereas in the non-clairvoyant setting, this is unknown until the job is fully processed. They presented a $\frac{5}{4}$-approximation algorithm for the preemptive clairvoyant variant, a greedy algorithm with a worst-case performance ratio of $\frac{3}{2}$ for the non-preemptive clairvoyant variant, and for both preemptive and non-preemptive non-clairvoyant variants. They claimed that generalization of their greedy algorithm to $m \geq 3$ yields a worst-case performance ratio of $2 - \frac{1}{m}$ for all of the four aforementioned variants. Table 15 shows the results for the cases with release times and on-line scheduling.

4.5 Start time

In this section we review studies that consider additional constraints on the start times of the operations. de Werra et al. (1993) studied the preemptive open-shop scheduling problem with the additional constraint that some operations must be processed simultaneously through requiring that the start times of such operations are the same. They showed that the decision problem is NP-complete. de Werra et al. (1993) and later de Werra and Erschler (1996) proposed polynomial solution algorithms for special variants of the problem. Middendorf (1998) studied the two-machine open-shop scheduling problem with "coordinated" start times, i.e., when one machine starts processing an operation, the other machine either has to be idle at that time or start processing another operation. He showed that if the constraint is imposed on both machines (instead of either of the machines), the problem is polynomially solvable; however, it is NP-hard if the constraint is imposed on either of the machines. In addition, he showed that the problem in the no-wait setting is NP-hard regardless

Table 15: Summary of studies with release times and on-line scheduling.

Problem	Complexity and method	Reference
$O2\|r_j\|C_{max}$	Strongly NP-hard	Graham et al. (1979) and Lawler et al. (1981)
$O3\|r_j\|C_{max}$	$\frac{7}{4}$-approximation	Chen et al. (2008)
$O3\|p_{ij} = p_j, r_j\|C_{max}$	$\frac{5}{3}$-approximation, result holds for $O3\|r_j\|C_{max}$ when each job consists of at most 2 operations	Chen et al. (2008)
$O\|p_{ij} \in \{0,1\}, r_j, d_j\|C_{max}$	NP-hard	Kubale (1997)
$O\|UET, r_j, d_j\|C_{max}$	NP-hard, result holds when $p_{ij} \in \{0,1\}$ and the number of UET operations is at most $m+n$	Kubale (1997)
$O2\|r_j \in \{0, r\}, d_j = d, prmp\|C_{max}$	Polynomial algorithm	Lu and Posner (1993)
$O\|r_j \in \{r_1, r_2\}, d_j = d, prmp\|-$	$O(n^3+m^4)$-algorithm, where $n \geq m$ and $m > 2$	Cho and Sahni (1981)
$O\|r_j, d_j, prmp\|-$	$O(\min\{n,m\}n^2m)$-time network flow algorithm	Sedeño-Noda et al. (2006)
$O\|r_j, d_j, prmp\|C_{max}$	Linear program	Pinedo (1995)
	Strongly polynomial algorithm	Sedeño-Noda et al. (2006)
$O\|r_j, prmp\|C_{max}$	MILP	Zhan et al. (2011)
$O\|on-line\|C_{max}$	1.875-competitive algorithm	Chen and Woeginger (1995)
$O\|on-line, prmp\|C_{max}$	$\frac{4}{3}$-competitive algorithm	Chen and Woeginger (1995)
$O2\|on-line\|C_{max}$	1.848-competitive permutation algorithm	Chen et al. (2001)
$O3\|on-line, prmp\|C_{max}$	1.754-competitive algorithm	Chen et al. (2001)
$O\|on-line, clairvoyant\|C_{max}$	$\frac{5}{4}$-approximation algorithm	Chen et al. (1998)
$O\|on-line, non-clairvoyant\|C_{max}$	$\frac{3}{2}$-approximation algorithm, result holds for preemptive case as well, in both clairvoyant and non-clairvoyant settings	Chen et al. (1998)
$O2\|on-line, 1 \leq p_{ij} \leq k, prmp\|C_{max}$	$\frac{5k-1}{4k}$-competitive algorithm	Liu et al. (2010)
$O\|on-line, r_j\|C_{max}$	Heuristic	Bai and Tang (2013)

of whether the constraint is imposed on either of the machines or on both machines. There are situations where a pair of jobs cannot be performed at the same time, for which the disjunctive constraint (DC) can be imposed to model the corresponding scheduling problem. We refer the interested reader to Hassan et al. (2018) for a branch-and-cut algorithm for the m-machine open-shop scheduling problem with disjunctive constraints.

The execution of the jobs can also be overlapping. Here, the operations of a job can be processed simultaneously by more than one machine. That is called "concurrent scheduling". Wagneur and Sriskandarajah (1993) showed that, for any regular objective function, the permutation schedules are dominant in the concurrent setting. They further showed that the schedule is immaterial for the objective function of minimizing the makespan. Grinshpoun et al. (2014); Grinshpoun et al. (2017) introduced the partial concurrent open-shop scheduling problem, which models the setting in which only subsets of jobs' operations can be processed simultaneously. They showed that the problem is NP-hard even with one UET job. They proposed a heuristic algorithm for the general problem. Ilani et al. (2017) showed that the partial concurrent open-shop problem with integral processing times, UET operations, and preemption at integral time points is equivalent to the NP-hard set-colouring problem (also known as the graph multi-colouring problem, see Caramia and Dell'Olmo (2001)). They investigated the polynomially solvable cases and proposed a constructive heuristic algorithm for the case with uniform jobs, i.e., all the jobs have the same processing time and conflict graph. Ilani et al. (2018) studied the preemptive variant with limited resources and proposed an algorithm that is optimal for some special cases that consist of two resources.
Table 16 shows the results for cases with start time constraints.

Table 16: Summary of studies with start time constraints.

Problem	Complexity and method	Reference
$O2\|coordinated\ machines\|C_{max}$	Polynomial, becomes NP-hard if only one machine is coordinated	Middendorf (1998)
$O2\|no-wait, coordinated\ machines\|C_{max}$	NP-hard, remains NP-hard even if only one machine is coordinated	Middendorf (1998)
$O\|DC\|C_{max}$	ILP, Branch and cut	Hassan et al. (2018)
$O\|conc\|C_{max}$	Polynomial	Wagneur and Sriskandarajah (1993)
$O\|UET, n = 1, p-conc\|C_{max}$	NP-hard	Grinshpoun et al. (2014)
	Heuristic	Grinshpoun et al. (2017)
$O\|UET, p-conc, prmp\|C_{max}$	NP-hard, polynomial if conflict graph is a perfect graph	Ilani et al. (2017)
$O\|UET, p-conc, prmp, res\|C_{max}$	Optimal with two resources	Ilani et al. (2018)

4.6 Processing time

Various forms of the processing times for the jobs' operations have been considered in the literature, e.g., known and unknown processing times, as well as variable processing times. In the route-dependent open-shop scheduling problem, the processing times of a job's operations depend on the route on which the job passes through the machines. Adiri and Amit (1983) investigated the two-machine route-dependent open-shop scheduling problem, denoted as $O2|RD|C_{max}$, where RD stands for route dependency. They showed that the problem is ordinary NP-hard and, for the case where one machine dominates the other, they proposed an $O(n)$ solution algorithm. Strusevich et al. (1999) also proposed a $\frac{3}{2}$-approximation algorithm for the problem that runs in $O(n^2)$ and showed that the bound is tight.

The majority of scheduling studies assume that the job processing or operation times are constant and known *a priori*. However, there are studies that consider scheduling with variable processing or operation times. For example, Kononov and Gawiejnowicz (2001) studied the two- and three-machine open-shop with both simple and general linear deteriorating jobs. Here, the processing time of a job depends on its start time. They showed that the two-machine problem with general linear deterioration, i.e., $O2|p_{ij} = \alpha_{ij} + \beta_{ij}t|C_{max}$, is NP-hard, where t is the job's start time, and α_{ij} and β_{ij} are the basic processing time and the deterioration rate of job j on machine i. They also showed that the three-machine problem with simple linear deterioration, i.e., $p_{ij} = \beta_{ij}t$ is also ordinary NP-hard even if the jobs have equal deterioration rates on one of the machines. Mosheiov (2002) and Li (2011) also studied simple linear deterioration. Mosheiov (2002) proposed an $O(n)$ algorithm for $O2|p_{ij} = \beta_{ij}t|C_{max}$ and showed that the problem with three or more machines is NP-hard. Li (2011) studied a different variant of the two-machine open-shop with simple linear deterioration, where one of the machines is a non-bottleneck and has infinite capacity. The problem is denoted as $O2|NB, p_{ij} = \beta_{ij}t|C_{max}$, where NB indicates one of the machines is a non-bottleneck. They proposed an $O(n^2 \sum_{j=1}^{n} \log(1 + \beta_j)/\varepsilon)$-time FPTAS, where β_j is the deteriorating rate of job j on the machine with finite capacity.

The uncertainty of the processing times can be addressed by stochastic models, in which random variables are used to model the random attributes such as processing times, due-dates, machine breakdowns etc. The scheduler aims to determine the order to process the jobs on the machines so that the expected value of a performance criterion, e.g., makespan, is optimized. In this context, the priority rule used by the scheduler to generate a sequence (order) of the jobs for processing on the machines is called a "policy". A policy is classified into "static" and "dynamic", based on the amount of information available to the scheduler. If the list of the jobs is provided at the beginning of the planning horizon, the policy is referred to as static, whereas it is called dynamic if new information is available at any moment and the scheduler may then update the remaining schedule accordingly. Several studies have attempted to find the optimal dynamic policy for minimizing the expected makespan. Here, almost all studies consider the two-machine open-shop. Emmons (1973) was the first to study the two-machine open-shop with identical jobs, where the processing time of each job's operation is an independent and identically distributed (iid) random variable that follows the exponential distribution with parameters λ and μ, due to the different speeds of the machines. They considered the policy under which the scheduler is not allowed to preempt and always gives priority to the jobs that have not yet received processing on either machine, which is known as the "longest expected remaining processing time (LERPT)" first policy. By using LERPT, they obtained a closed form expression for the expected makespan. An important and popular policy, LERPT has attracted much research attention. For example, Pinedo and Ross (1982) showed that (1) LERPT minimizes the expected makespan for both the preemptive and non-preemptive two-machine open-shop where the processing times of the jobs on the machines are statistically independent and follow the exponential distribution with the rate μ_j, i.e., $O2|p_{ij} \sim \exp(\mu_j)|E(C_{max})$, and different machine speeds, (2) LERPT minimizes the expected makespan when preemption is not allowed and the distribution of the processing times is "new better than used" (NBU), i.e., $O2|p_{ij} \sim \Phi_k|E(C_{max})$, where $\Phi_k(t' + t)/\Phi_k(t) \geq \Phi_k(t), \forall t, t' \geq 0, k = 1, 2$, and (3) LERPT is the optimal policy when the job processing times on both machines are identical following the distribution Φ and preemption is not allowed. Chung and Mohanty (1988) showed that LERPT minimizes the expected makespan for the two-machine open-shop with identical jobs where the job arrivals and processing times follow the Poisson and exponential distributions, respectively. Pinedo and Weber (1984) provided several bounds on the expected makespan for the two-machine open-shop under various distributions for the job processing times.

A number of studies have extended the aforementioned models. For example, Frostig (1991) proved that for the two-machine open-shop where the processing times on a machine are iid random variables with the exponential distribution, the optimal static policy for minimizing the makespan includes assigning an equal number of jobs to be first processed by either machine. Righter (1997) showed that for the two-machine open-shop with the restriction that some jobs are processed on both machines and the remaining ones are processed only on one machine, LERPT minimizes the expected makespan in certain preemptive and non-preemptive settings. For the m-machine open-shop with random processing times, Koryakin (2003) showed that for a fixed m and an increasing number of jobs, if distribution Φ_{ij} satisfies several conditions, then the Sevast'yanov (1995)'s algorithm almost always constructs an optimal schedule. Alcaide et al. (2005) studied a stochastic open-shop subject to random machine breakdowns, where the remaining processing times of the preempted jobs are also random variables. They proposed a heuristic to minimize the expected makespan by solving a sequence of stochastic open-shop problems without breakdowns. At every point in time, the heuristic only takes into account all the available information up to that point.

Recently, Knust et al. (2019) introduced the concept of "pliability" for jobs, meaning that the total processing time of a job's operations is given, while the actual processing time of an operations is a decision variable to be decided by the scheduler. They presented three types of pliability: (1) unrestricted, denoted by $plbl$, under which the processing time of a job's operation must be less than or equal to the total processing time of the job, (2) restricted with a common lower bound, denoted by $plbl(\underline{p})$, under which the processing time of a job's operation must be at least greater than or equal to the given minimum length $\underline{p}$, e.g., the minimum length of an operation must be at least two time units, and (3) restricted, i.e., $plbl(\underline{p}_{ij}, \bar{p}_{ij})$, under which each operation has its own lower and upper bounds $\underline{p}_{ij}$ and $\bar{p}_{ij}$. They analyzed the complexity status of open-shop scheduling with unrestricted and restricted pliable jobs and obtained the results reported in Table 17.

Table 17: Summary of studies with processing time constraints.

Problem	Complexity and method	Reference
$O2\|RD\|C_{max}$	NP-hard	Adiri and Amit (1983)
$O2\|RD, p_{1,min} \geq p_{2,max}\|C_{max}$	$O(n)$	Adiri and Amit (1983)
$O2\|RD\|C_{max}$	$\frac{3}{2}$-approximation algorithm	Strusevich et al. (1999)
$O2\|p_{ij} = \alpha_{ij} + \beta_{ij}t\|C_{max}$ (linear deterioration)	NP-hard	Kononov and Gawiejnowicz (2001)
$O2\|p_{ij} = \beta_{ij}t\|C_{max}$ (simple linear deterioration)	$O(n)$	Mosheiov (2002)
$O3\|p_{ij} = \beta_{ij}t, \beta_{3j} = \beta\|C_{max}$	NP-hard	Kononov and Gawiejnowicz (2001)
$O\|p_{ij} = \beta_{ij}t\|C_{max}$	NP-hard	Mosheiov (2002)
$O2\|NB, p_{ij} = \beta_{ij}t\|C_{max}$	FPTAS	Li (2011)
$O2\|plbl\|C_{max}$ $(n \leq m)$	$O(n)$	Knust et al. (2019)
$O2\|plbl(\underline{p})\|C_{max}$ $(n \leq m)$	$O(n)$	Knust et al. (2019)
$O2\|plbl(\underline{p}_{ij}, \bar{p}_{ij})\|C_{max}$ $(n \leq m)$	$O(n)$	Knust et al. (2019)
$O\|plbl\|C_{max}$ $(n \leq m)$	$O(n)$	Knust et al. (2019)
$O\|plbl\|C_{max}$ $(n > m)$	$O(1)$	Knust et al. (2019)
$O\|plbl(\underline{p})\|C_{max}$ $(n > m)$	$O(1)$	Knust et al. (2019)
$O\|plbl(\underline{p}_{ij}, \bar{p}_{ij})\|C_{max}$ $(n \leq m)$	Strongly NP-hard	Knust et al. (2019)

5 Future research directions

In this section we highlight the main open problems, as well as potential areas for future research on the open-shop scheduling problem, which we discussed in detail in the previous sections.

Related to the general structure of the open-shop scheduling problem, we highlight two themes. The first theme refers to the basic assumption that the jobs visit each machine only once. This assumption may not hold in certain real-world applications. For example, the manufacturing setting called the re-entrant shop has jobs visiting certain machines more than once. Even though this setting was first observed in the electronic industry (Graves et al., 1983) and has been extensively studied in the flow-shop and job-shop settings ever since (see, e.g., Wang et al. (1997); Pan and Chen (2005)), we are not aware of any related study in the open-shop environment. The second theme focuses on the coupled task scheduling problem within the open-shop. Recently, the coupled task scheduling problem was introduced in the open-shop setting (Ageev, 2018) and, as such, it is still in the early stage of development. Given the recently emerging applications of the coupled task scheduling problem in the health care sector (see, e.g., Condotta and Shakhlevich (2014), Lamé et al. (2016), and Khatami and Salehipour (2019), and the references therein), we believe further research in this area is worthy.

We detailed the available solution methods for the classical open-shop scheduling problem, and discussed the state-of-the-art methods that can optimally solve all the instances in the three well-known benchmarks in a reasonable amount of time in Section 2.5. It is noted that two benchmarks of Brucker et al. (1997) and Guéret and Prins (1999) include a maximum number of ten jobs and machines only. We therefore believe designing more challenging instances is important. The new challenging instances may also necessitate the development of advanced solution methods. Regarding the approximation algorithms for the classical open-shop with an arbitrary number of machines, i.e., $O||C_{max}$, albeit the greedy algorithm by Bárány and Fiala (1982) provides a worst-case performance ratio of 2, its gap with the inapproximability result of Williamson et al. (1997) (where $\rho < \frac{5}{4}$) is still open. Furthermore, the vexing conjecture that the optimal solution for $O||C_{max}$ is at most $\frac{3}{2}$ larger than that of its preemption variant, i.e., $O|pmtn|C_{max}$, remains unsettled, and in the words of Woeginger (2005) "settling this conjecture would mean a major breakthrough in the area".

Regarding the non-classical resource models, we suggest a number of potential research themes. To the best of our knowledge, there is no published research on the open-shop scheduling problem with machine availability in the semi-resumable setting (see Section 3.1). We do not find any study considering the open-shop scheduling problem with more than two agents (see Section 3.2) either. So there is a research gap concerning multi-agent open-shop scheduling with resource considerations. Conducting research to fill this gap is of both theoretical and practical interests because, in the multi-agent setting, each agent is interested in processing a subset of the jobs, while all the jobs share the same resources and the agents need to compete to optimize their own performance criteria (Agnetis et al., 2014). Finally, The available studies on interstage transporters (Section 3.4.1) are limited to only two machines. Future work could extend the model to more than two machines.

We identify a number of research avenues in the non-classical job settings. For example, the studies reported in Section 4.2 consider sequence-dependent setup times. We are not aware of any published research considering sequence-independent setup times for makespan minimization. Also, almost all the studies on batch processing deal with only two machines, showing a research gap for problems with more than two machines. The second avenue lies in the area of time-dependent processing times (see Section 4.6). While a few studies consider time-dependent processing times due to job deterioration, there is no study considering time-dependent processing times due to "job shortening" functions, in which the processing time of a job is a decreasing function of its' start time.

6 Concluding remarks

It has been more than 40 years since the open-shop scheduling problem was first introduced to the scheduling community. While the early years witnessed only a few studies on this topic, the majority of research has been developed in the last 30 years. In particular, there has been a considerable increase in research publications, both theoretical and practical, in the past few years, suggesting that open-shop scheduling is a thriving area of scheduling research that provides many opportunities for fruitful research. We believe that our work provides a comprehensive and timely survey of research on open-shop scheduling, which can be a valuable resource for understanding the development trajectory and guiding future research on the topic.

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